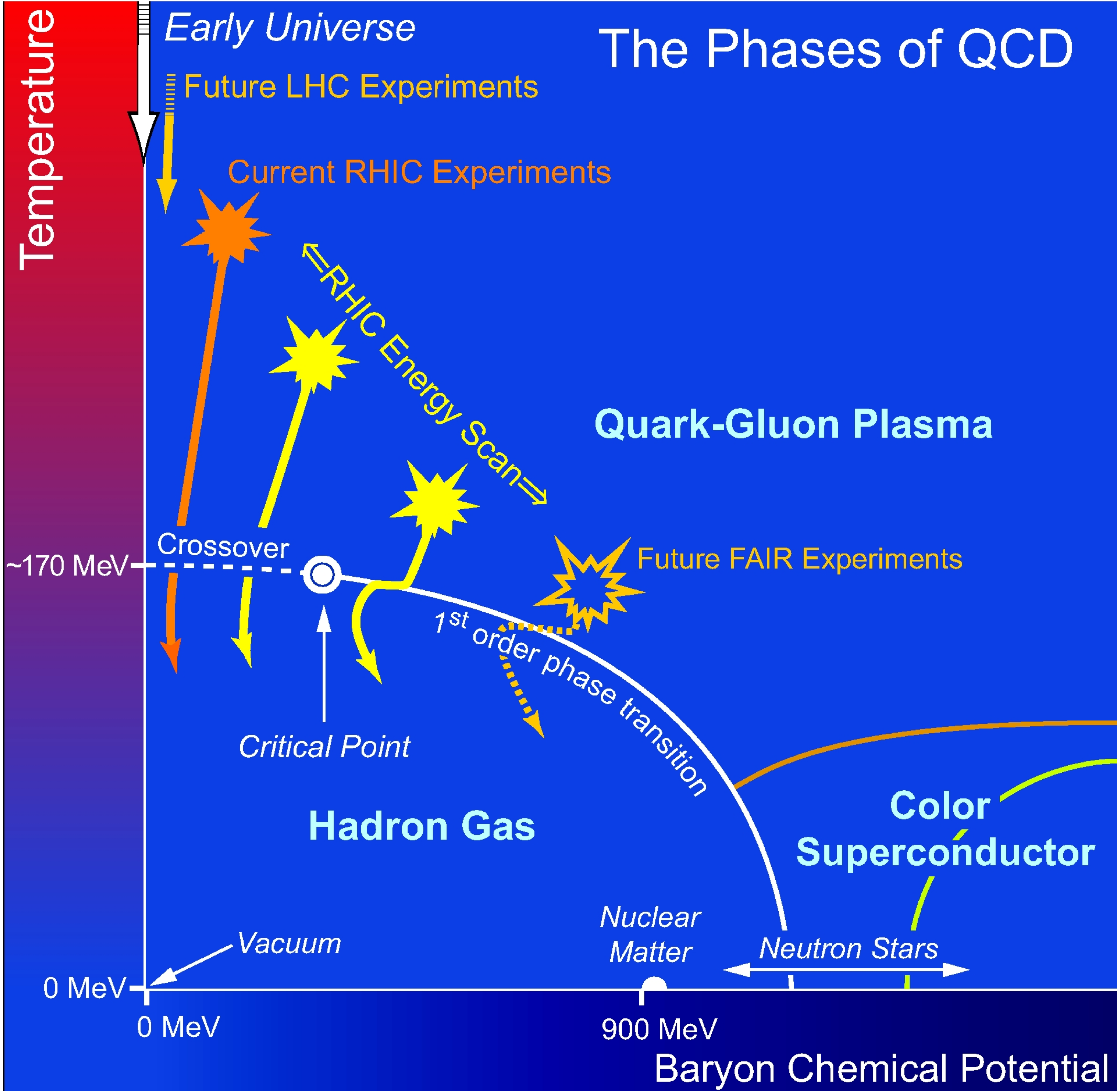
Gertrude Stein about Oakland, California, ~ 1890:

"There's no there, there."

Beam Energy Scan at RHIC:

There *is* a there, there

But what is it?


The Phases of QCD
Temperature
Early Universe
Future LHC Experiments
Current RHIC Experiments
⇐RHIC Energy Scan⇒
Quark-Gluon Plasma
Future FAIR Experiments
~170 MeV
Crossover
Critical Point
1st order phase transition
Hadron Gas
Color Superconductor
Nuclear Matter
Neutron Stars
Vacuum
0 MeV
0 MeV
900 MeV
Baryon Chemical Potential

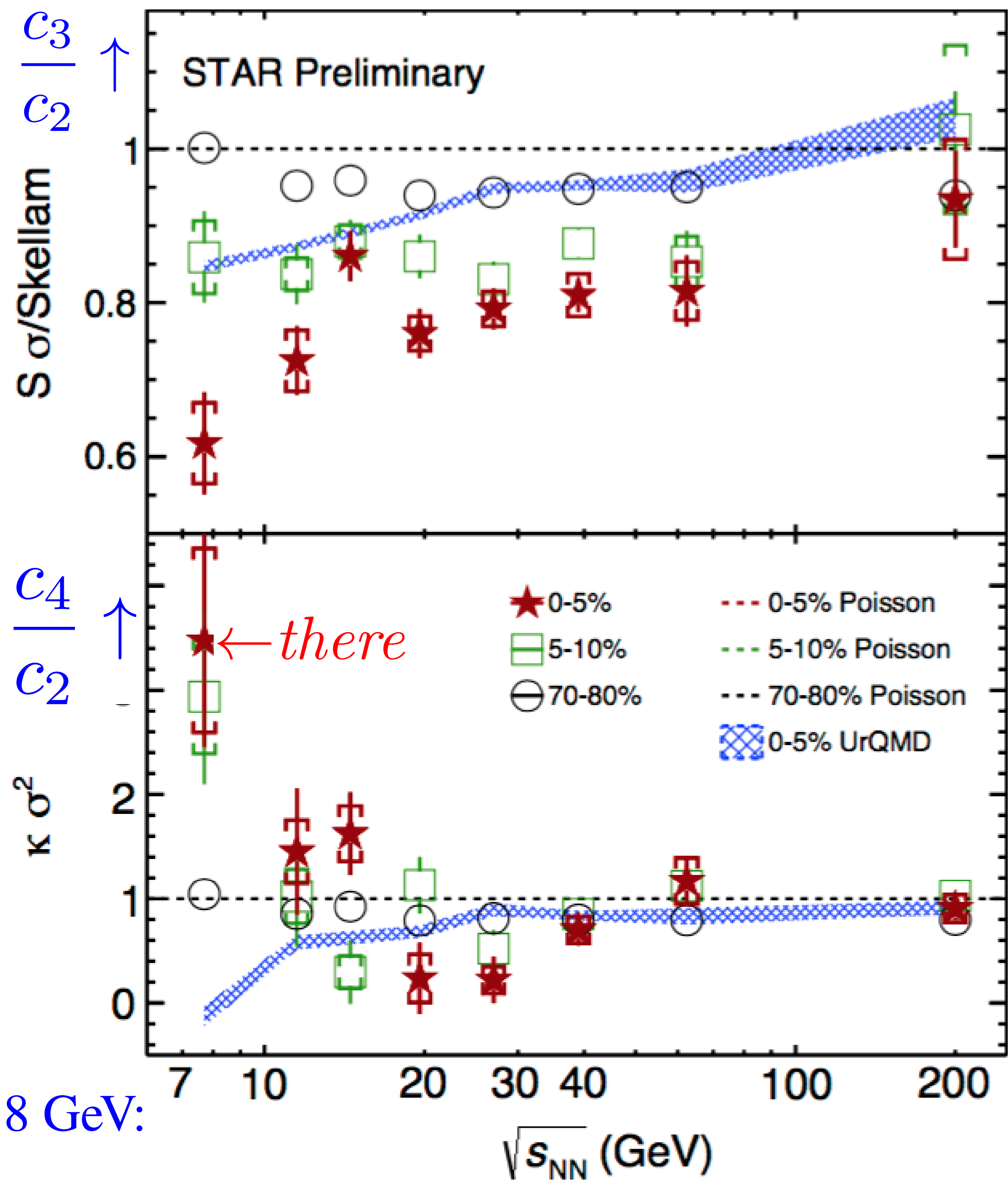
Beam Energy Scan @ RHIC, down to √s/A = 7 GeV

Exp.'y, measure moments of pressure w.r.t. μ = quark chemical potential:

$$c_n = \frac{\partial^n}{\partial \mu^n} p(T,\mu)$$

Ratio of $4^{th}/2^{nd}$ moments:
~ 1 above 40 Gev, dips below 1,
BIG increase from 19 to 7 GeV

The first "*there*"



N.B.: increase is due to p_{tr} *above* .8 GeV: *weird* if critical endpoint

Slice & Dice the moments with convolution correlators

Bill Llope, CPOD '17, STAR:

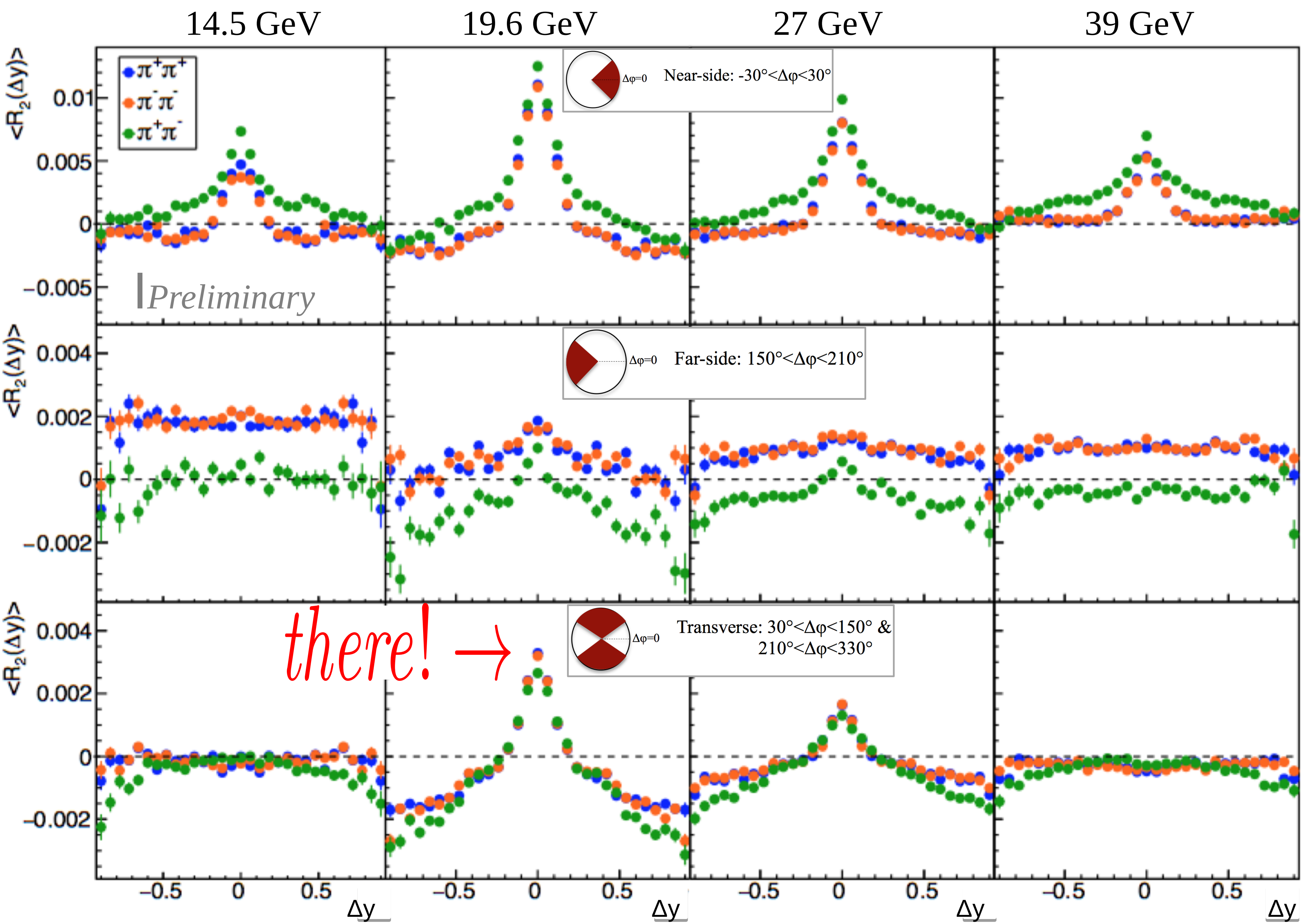
Consider two-particle correlations,
along the beam axis (rapidity y) and w.r.t. angle transverse to the beam (θ)

$$R_2 = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_2(y_2)} - 1$$

Integral of R_2 w.r.t. rapidities y_1 and y_2 is related to c_2 moment

Berger, NPB 85, '75; Carruthers & Sarcevic PRL 63, '89; M Jacob, Phys Rep 315, '99
Bzdak, 1108.0882; Bzdak & Teaney 1210.1965; Jia, Radhakrishnan & Zhou, 1506.03496
Ling & Stephanov, 1512.09125; Bzdak, Koch, & Strodthoff 1607.07375

The there, *there*



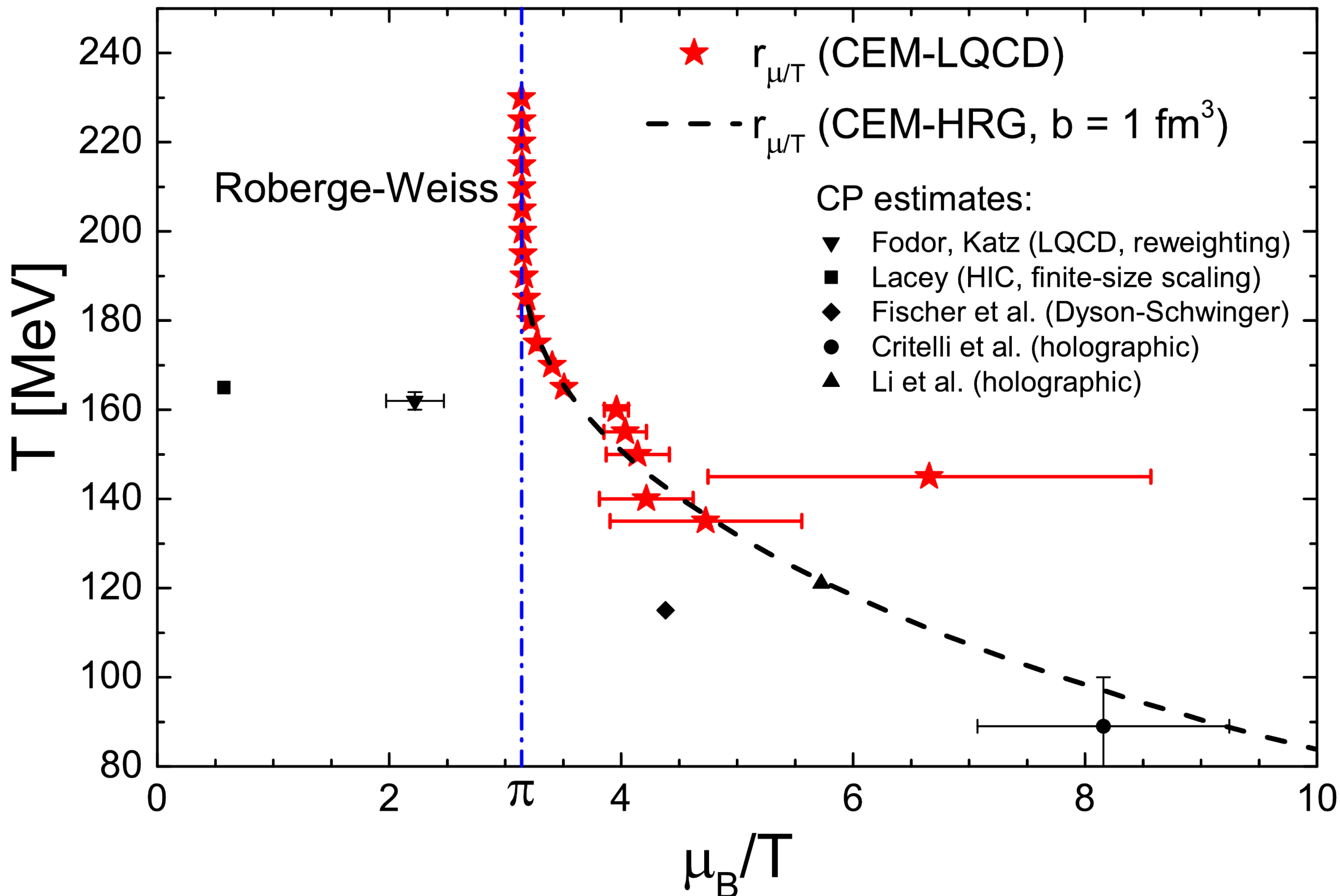
Lattice: *no* critical point nearby

Vovchenko, Steinheimer, Philipsen, Stoecker 1711.0126:

Cluster Expansion Method (CEM) for baryon fluctuations on the lattice:

(not Taylor expansion in powers of μ, powers asymptotic behavior in μ.)

No critical endpoint accessible by experiment: *so what is it?*



Matrix models & a (pseudo-) Lifshitz point in QCD

Chiral matrix model: marrying

a linear sigma model, for the chiral transition

plus a “matrix model”, to characterize deconfinement

RDP & VV Skokov, 1604.00002

Quarkyonic chiral spirals and a (pseudo-) Lifshitz point in QCD:

RDP, VV Skokov & A Tsvelik, 1712.x

Fluctuations from a pseudo-Lifshitz point at low energies?

Finite size effects for baryon # cumulants:

G Almasi, VV Skokov, & RDP, 1612.04416

Tetraquarks in QCD: two chiral order parameters, two chiral transitions?

RDP & VV Skokov 1606.04111

Solution for SU(∞): RDP & VV Skokov; 1205.0545

S Lin, RDP & VV Skokov, 1301.7432;

H Nishimura, RDP & VV Skokov, 1712.04465

Matrix model for deconfinement

Polyakov Loop: $$\ell = \frac{1}{3} \mathrm{tr}\, \mathcal{P} \exp \left(ig \int_0^{1/T} A_0 \, d\tau \right)$$

Simplest approximation to give a non-trivial loop: constant, diagonal A_0:

$$A_0^{cl} = \frac{2\pi T}{3\, g} \lambda_3 \, q(T) \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Depends upon single function, $q(T)$, fixed from pressure(T).

Only need two parameters to fit pressure, then compute

Matrix model for pure glue

To one loop order, Stefan-Boltzmann + potential for q

$$\mathcal{V}_{pert}(q) = \frac{2\pi^2}{3} T^4 \left(-\frac{4}{15} + \sum_{a,b} q_{ab}^2 (1 - q_{ab})^2\right) \; , \; q_{ab} = |q_a - q_b|_{mod\, 1}$$

From lattice data for pure glue, assume non-pert. potential ~ T^2:

$$\mathcal{V}_{non}(q) = \frac{2\pi^2}{3} T^2 \, T_d^2 \sum_{a,b} \left(-\, c_1 \, q_{ab}(1 - q_{ab}) - \, c_2 \, q_{ab}^2 (1 - q_{ab})^2 + \, c_3\right)$$

From lattice for pure glue: T_d = 270 MeV.

Constant term ~ c_3 most important for T > 1.2 T_d.

q's only matter for T < 1.2 T_d : *narrow* transition region

Dumitru, Guo, Hidaka, Korthals-Altes & RDP, 1011.3820 & 1205.0137 + ….

Chiral symmetry

For 3 flavors of massless quarks,

$$\mathcal{L}^{qk} = \overline{q}\, \not{D} q = \overline{q}_L\, \not{D} q_L + \overline{q}_L\, \not{D} q_L \;,\; q_{L,R} = \frac{1 \pm \gamma_5}{2}\, q$$

Classically, global flavor symmetry of $SU(3)_L$ x $SU(3)_R$ x $U(1)_A$,

$$q_L \to \mathrm{e}^{-i\alpha/2} U_L q_L \;,\; q_R \to \mathrm{e}^{+i\alpha/2} U_R q_R$$

Simplest order parameter for χ sym. breaking: a,b… = flavor. A,B… = color

$$\Phi^{ab} = \overline{q}_L^{bA} q_R^{aA} \qquad\qquad \Phi \to \mathrm{e}^{+i\alpha} U_R\, \Phi\, U_L^\dagger$$

Quantum mechanically, axial $U(1)_A$ is broken by instantons +…. to $Z(3)_A$ at T=0
’t Hooft instanton vertex is invariant under $Z(3)_A$:

$$\det \Phi \to \mathrm{e}^{3i\alpha} \det \Phi$$

As T → ∞, $U(1)_A$ approximately restored as $1/T^{7 \to 9}$.

Usual linear sigma model

Linear sigma model for Φ:

$$\mathcal{V}_\Phi = m^2 \mathrm{tr}\left(\Phi^\dagger \Phi\right) - \, c_A \left(\det\, \Phi + \mathrm{c.c.}\right) + \lambda\, \mathrm{tr}\left(\Phi^\dagger \Phi\right)^2$$

Drop $(\mathrm{tr}\ \Phi^+\Phi)^2$: fits show coefficient is *really* small

Mass, quartic terms $U(1)_A$ invariant; det Φ *only* under $Z(3)_A$.

For light but massive quarks, need to add

$$\mathcal{V}_H^0 = -\,\mathrm{tr}\left(H\left(\Phi^\dagger + \Phi\right)\right)$$

So $m_\pi^2 \sim H$, etc. Standard linear sigma model.

Chiral matrix model

Quarks generate potential in "q", so *must* couple Φ to quarks: $P_{L,R} = (1 \pm \gamma_5)/2$

$$\mathcal{L}_{\Phi}^{qk} = \overline{q} \left(\not{D} + \mu\gamma^0 + y \left(\Phi \mathcal{P}_L + \Phi^\dagger \mathcal{P}_{\mathcal{R}} \right) \right) q$$

Use matrix model from pure glue, with *same* $T_d = 270$ MeV.

With quarks, T_d is *just* a parameter in a potential, *not* deconfining T_c.

From quark loop, need logarithmic term in Φ:

$$\mathcal{V}_{\Phi}^{log} = \kappa \, \mathrm{tr} \left((\Phi^\dagger \Phi)^2 \, \log \left(\frac{M^2}{\Phi^\dagger \Phi} \right) \right)$$

To 1 loop order, $\kappa = 3y^4/(16\,\pi^2)$; y is a free parameter, fit to T_χ.

Log term complicates things, results similar to that for $\kappa = 0$.

New symmetry breaking term

With usual symmetry breaking, at high T,

$$\mathcal{V}^{eff} \approx -h\,\phi + \frac{y^2T^2}{12}\,\phi^2 + \ldots, \; T \to \infty$$

1st term SB'g, 2nd quark fluctuations.
But then at high T, no symmetry breaking!

$$\phi \sim \frac{12h}{y^2T^2}\;,\; m_{qk} \sim y\,\phi \sim \frac{1}{T^2}$$

Solve by adding a new term *by hand*

$$\mathcal{V}^{eff} \sim \; h\,\phi - \frac{y}{6}\,m_0\,T^2\,\phi + \frac{y^2T^2}{12}\,\phi^2 + \ldots$$

So $\varphi \sim m_0/y$ at high T, $m_{qk} \sim m_0$. In QCD,

$$\mathcal{V}_h^T = -\frac{m_{qk}}{V}\left(\mathrm{tr}\,\frac{1}{\not{D} + \mu\gamma^0 + y\Phi_{ii}}\bigg|_{T\neq 0} - (T=0)\right)$$

Solution at T = 0

Consider first the SU(3) symmetric case, $h_u = h_d = h_s$.
Spectrum. 0^-: singlet η' & octet π. 0^+: singlet σ and octet a_0.
Satisfy a 't Hooft relation:

$$m_{\eta'}^2 - m_\pi^2 = m_{a_0}^2 - m_\sigma^2$$

The anomaly moves η' *up* from the π, but also moves σ *down* from the a_0.

QCD: ⟨Φ⟩ = (Σ_u, Σ_u, Σ_s). From:

$$f_\pi = 93\ ,\ m_\pi = 140\ ,\ m_K = 495\ ,\ m_\eta = 540\ ,\ m_{\eta'} = 960$$

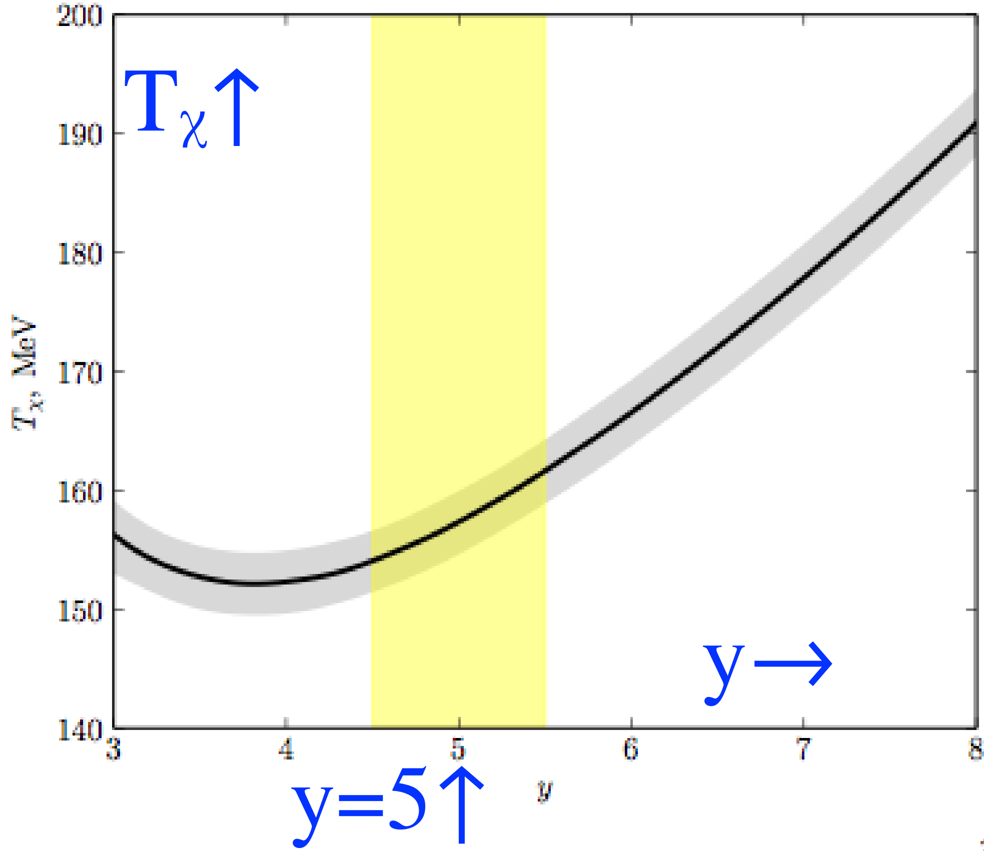
Determine:

$$\Sigma_u = 46\ ,\ \Sigma_s = 76\ ,\ h_u = (97)^3\ ,\ h_s = (305)^3\ ,\ c_A = 4560$$

$$m^2 = (538)^2 - 121 y^4\ ,\ \lambda = 18 + 0.04\, y^4$$

One free parameter, Yukawa coupling "y", fix from T_χ.

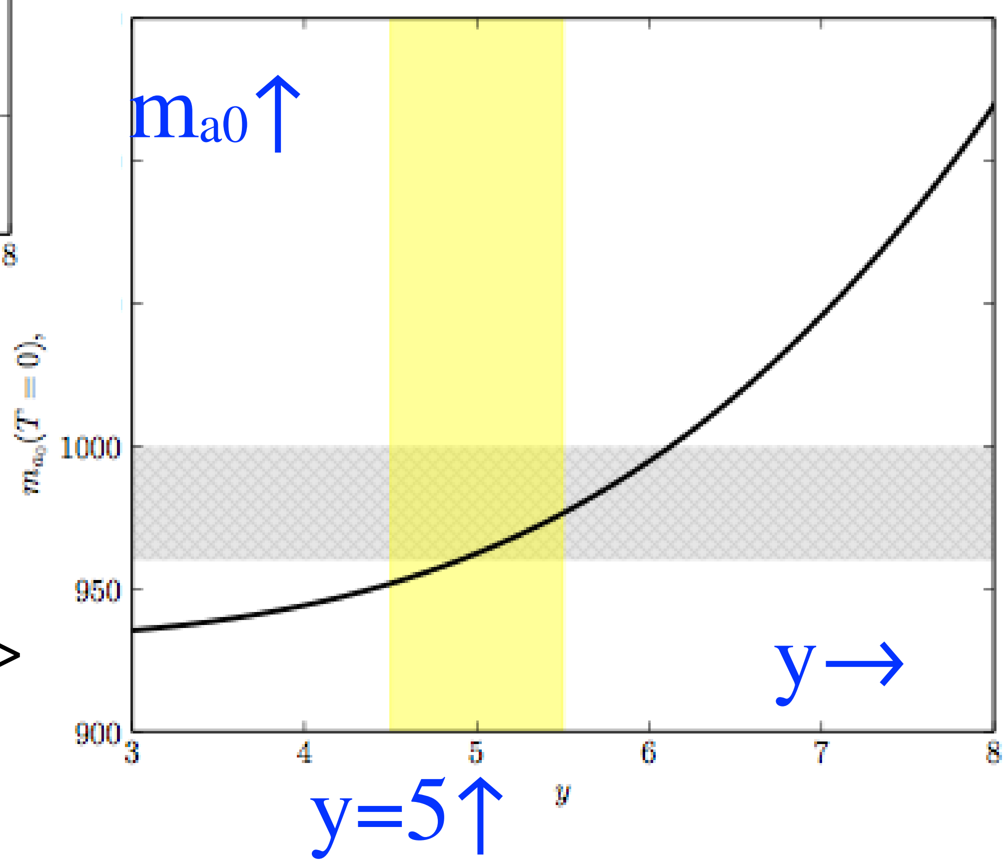
Varying the Yukawa coupling



T_χ defined from maximum in light quark suscep., $d\Sigma_u/dT$

<= Grey band: vary T_d from 260 -> 280

<= Yellow band = y: 4.5 -> 5.5



Grey band: experimental uncertainty in the mass of the a_0 =>

Solution at T ≠ 0

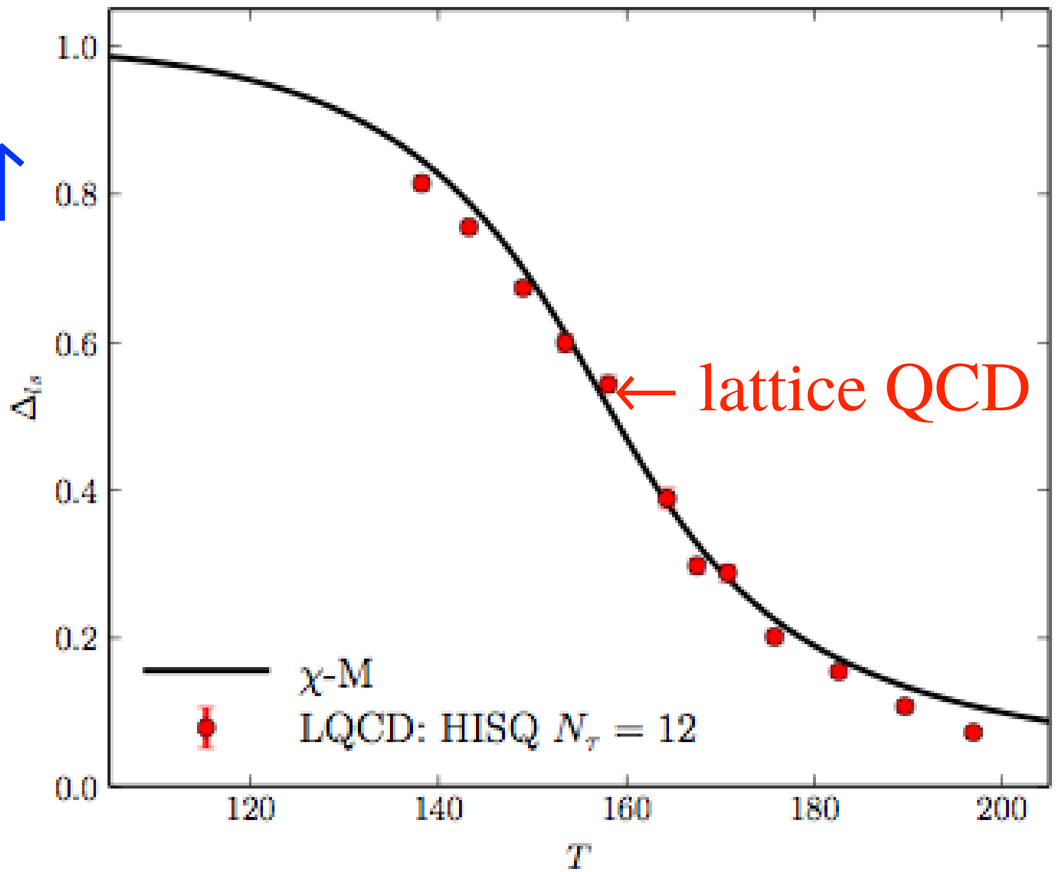
To eliminate u.v. divergences,
lattice uses substracted condensates

$$\Delta_{u,s}^{lattice}(T) = \frac{\langle \overline{q}q \rangle_{u,T} - (m_u/m_s)\langle \overline{q}q \rangle_{s,T}}{\langle \overline{q}q \rangle_{u,0} - (m_u/m_s)\langle \overline{q}q \rangle_{s,0}}$$

In the chiral-matrix (χ-M) model
use this to fix y = 5.

$$\Delta_{u,s}^{\chi-M}(T) = \frac{\Sigma_u(T) - (h_u/h_s)\Sigma_s(T)}{\Sigma_u(0) - (h_u/h_s)\Sigma_s(0)}$$

$\Delta_{u,s}{}^{\chi\text{-}M}$ ↑



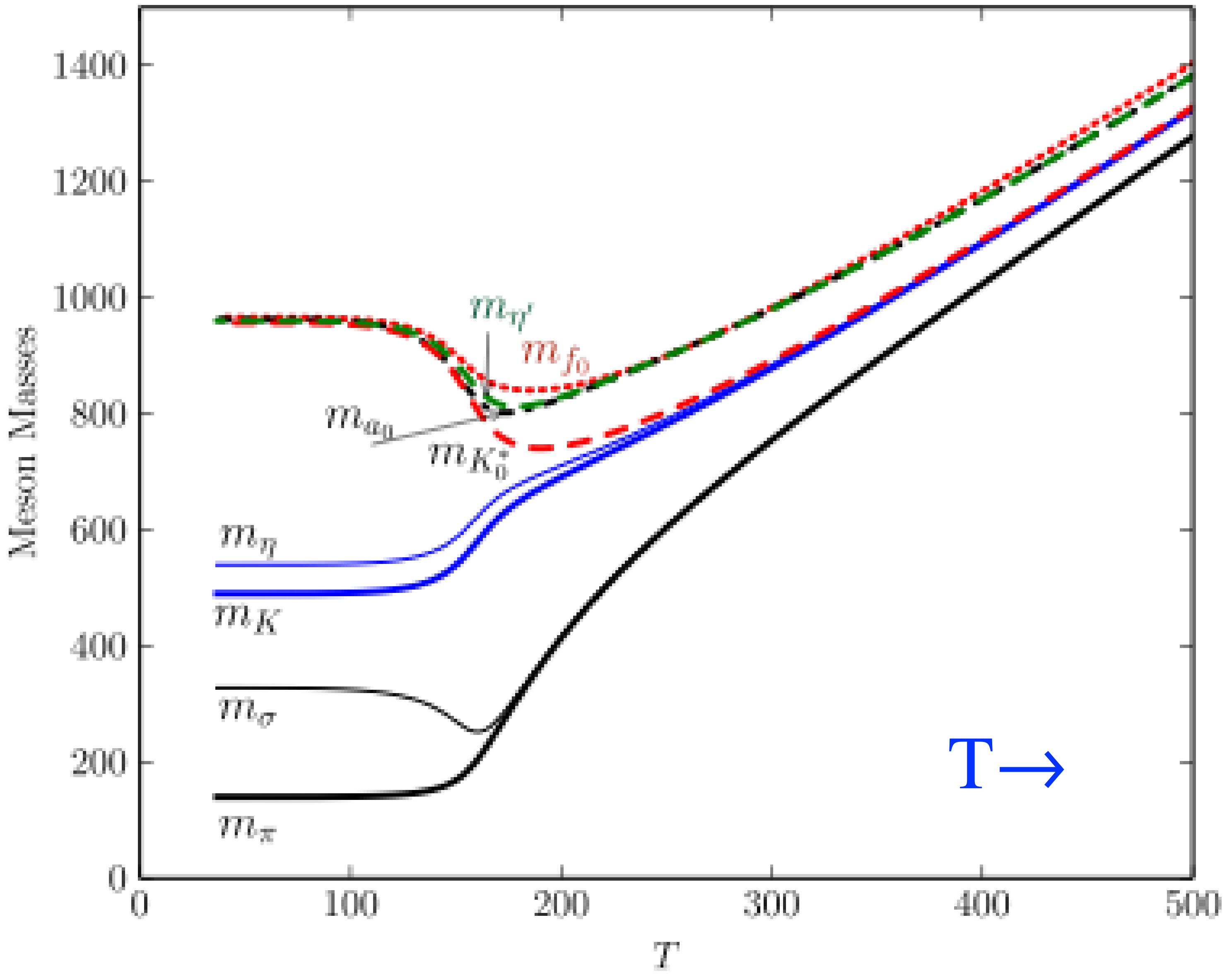
T→

Bazavov et al,
1407.6387
1701.03548

Meson masses vs T

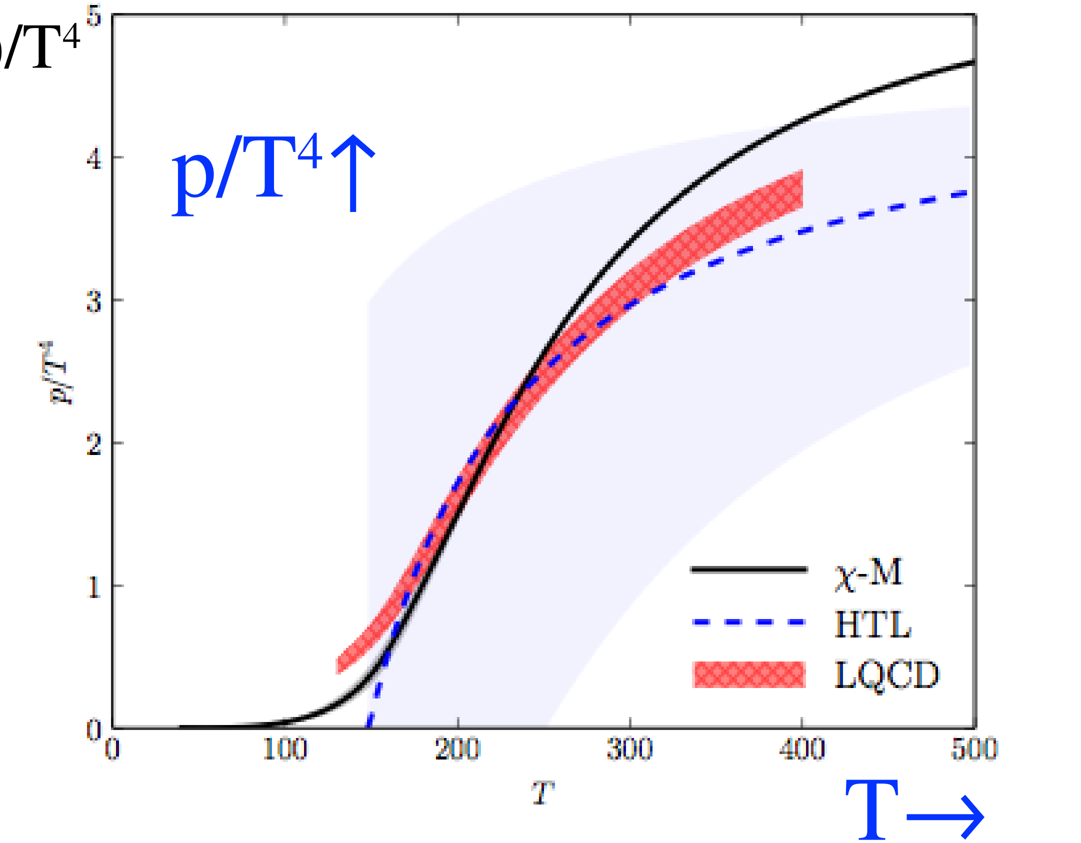
Usual pattern for $m_u = m_d \neq m_s$. y = 5.

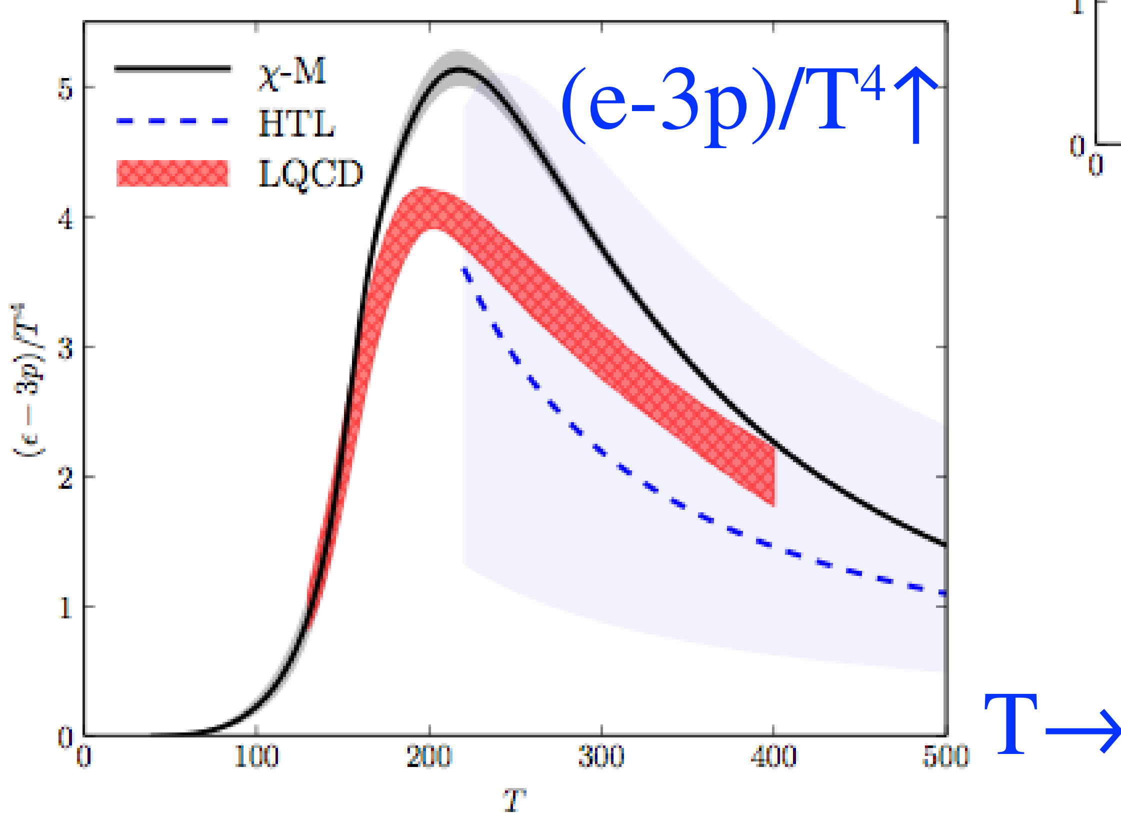
$U(1)_A$ breaking persists to high T, unphysical.



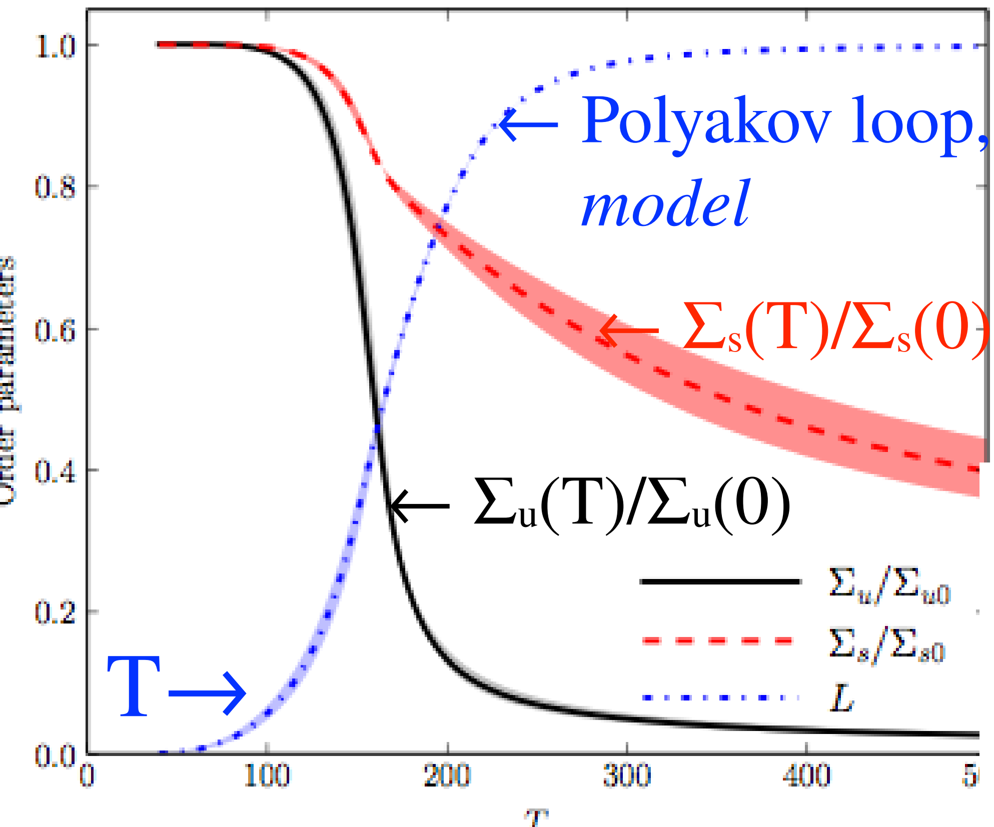
Pressure, interaction measure vs T

Pressure and interaction measure, $(e-3p)/T^4$
χ-M model,
Lattice, Bazavov et al, 1407.6387
Hard Thermal Loop (HTL)
(blue region = change ren. scale)
Andersen et al, 1511.04660





Order parameters, chiral and deconfining

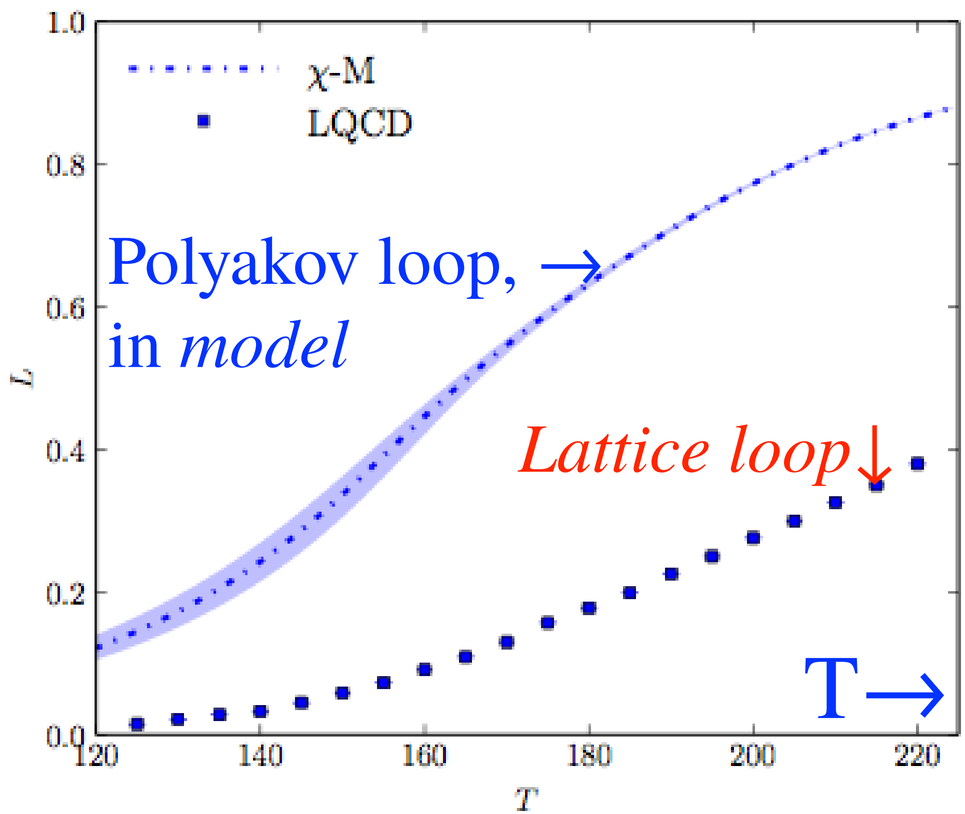


Chiral matrix model:

Chiral and deconfining order parameters
are *strongly* correlated

But Polyakov loop from lattice
Petreczky & Schadler, 1509.07874
is *much* smaller than in model.

Persistent discrepancy,
also in pure gauge.
What's up with lattice loop?



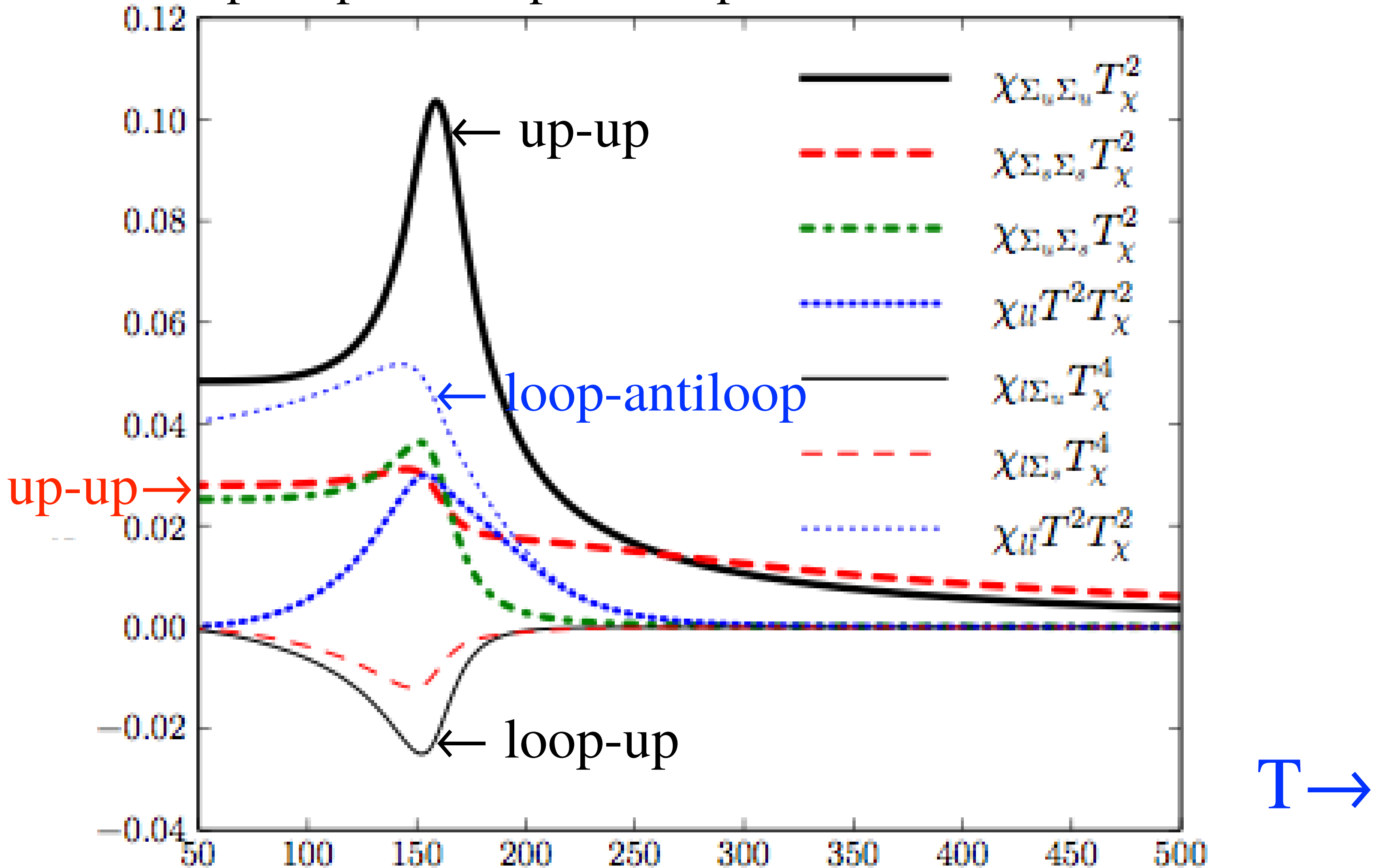
Susceptibilities, chiral and deconfining

Largest peak for up-up; strange-strange small.

In QCD, notable peaks for loop-up & loop-loop, *strongly* correlated with up-up

In chiral limit: loop-up suscep. *diverges*. Sasaki, Friman, Redlich ph/0611147

loop-loop and loop-antiloop finite

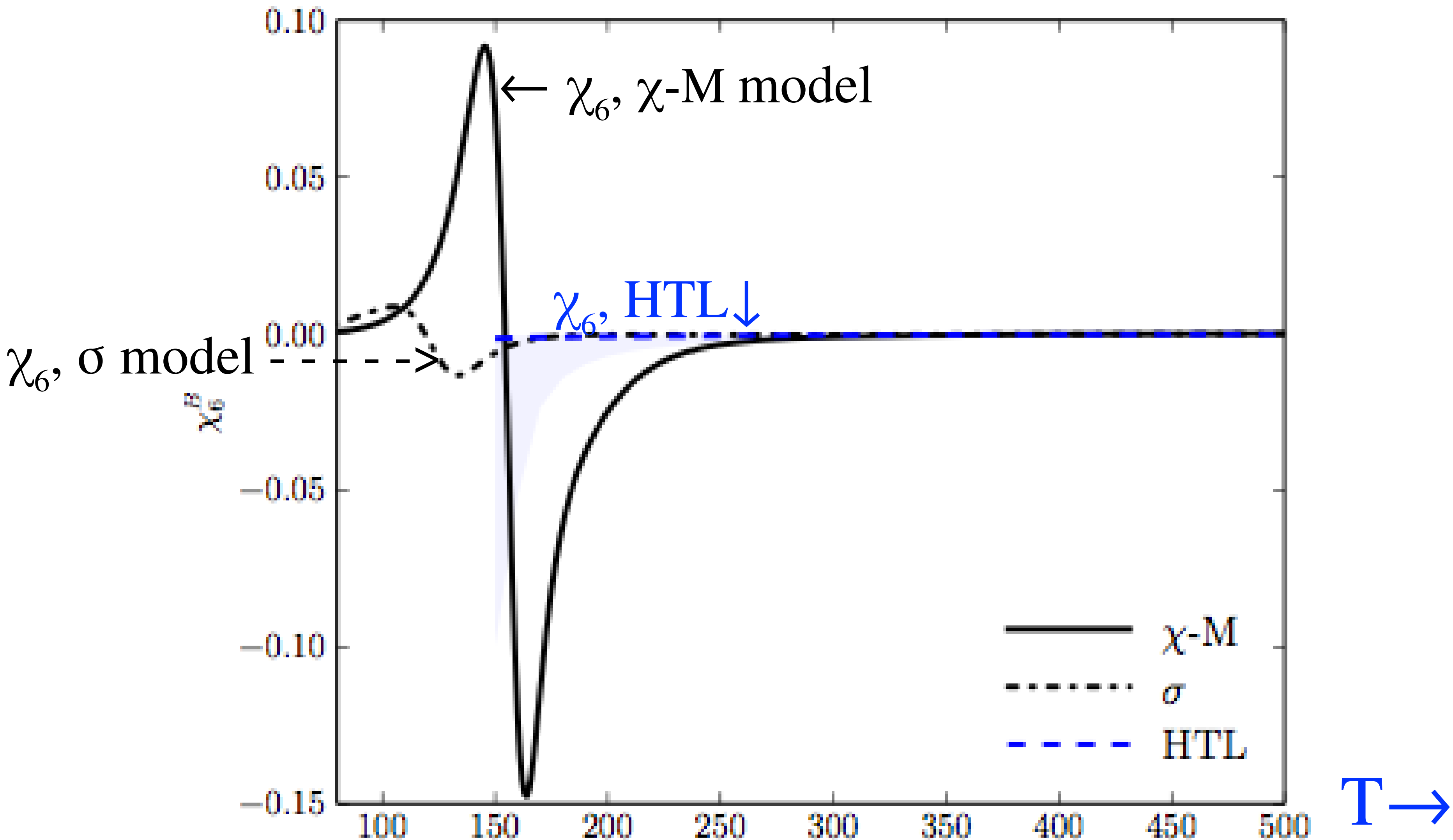


6th order baryon susceptibility

In χ-M model, χ_6 shows *non*-monotonic behavior near T_χ.

In HTL, χ_6 is very small (because m=0)

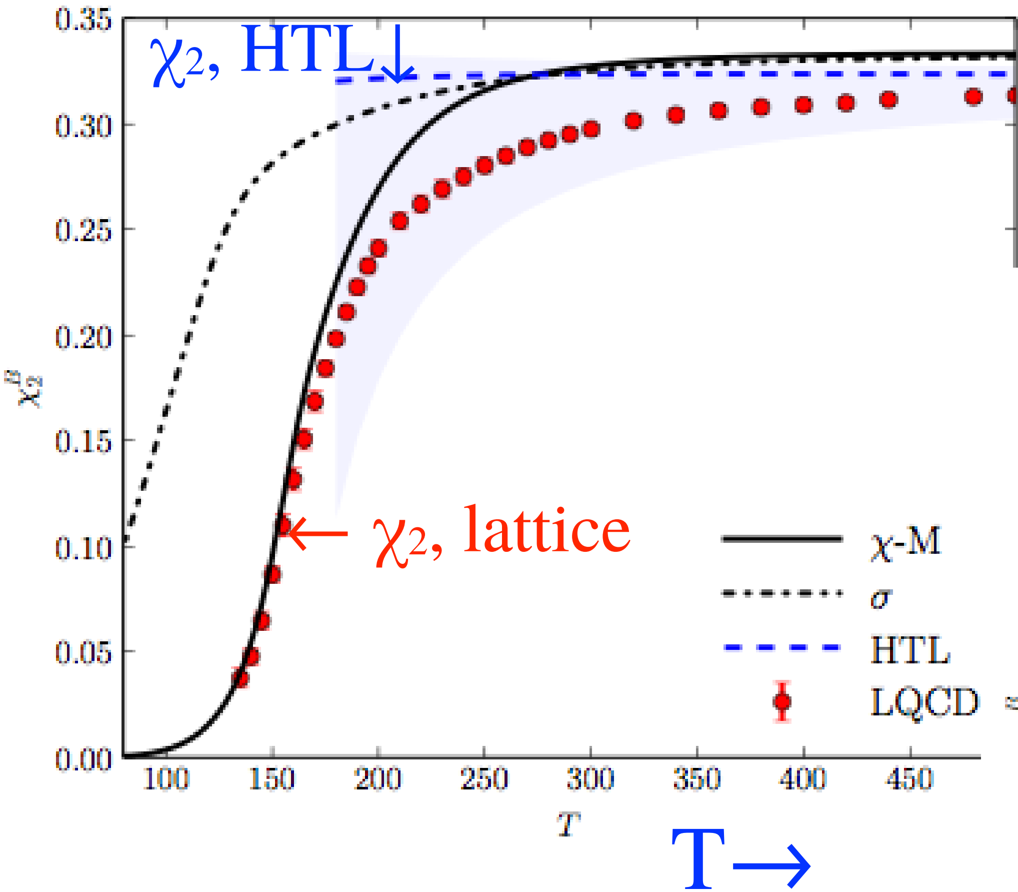
σ model: including change in Σ_u, but *not* in loop. Change in χ_6 *much* smaller.

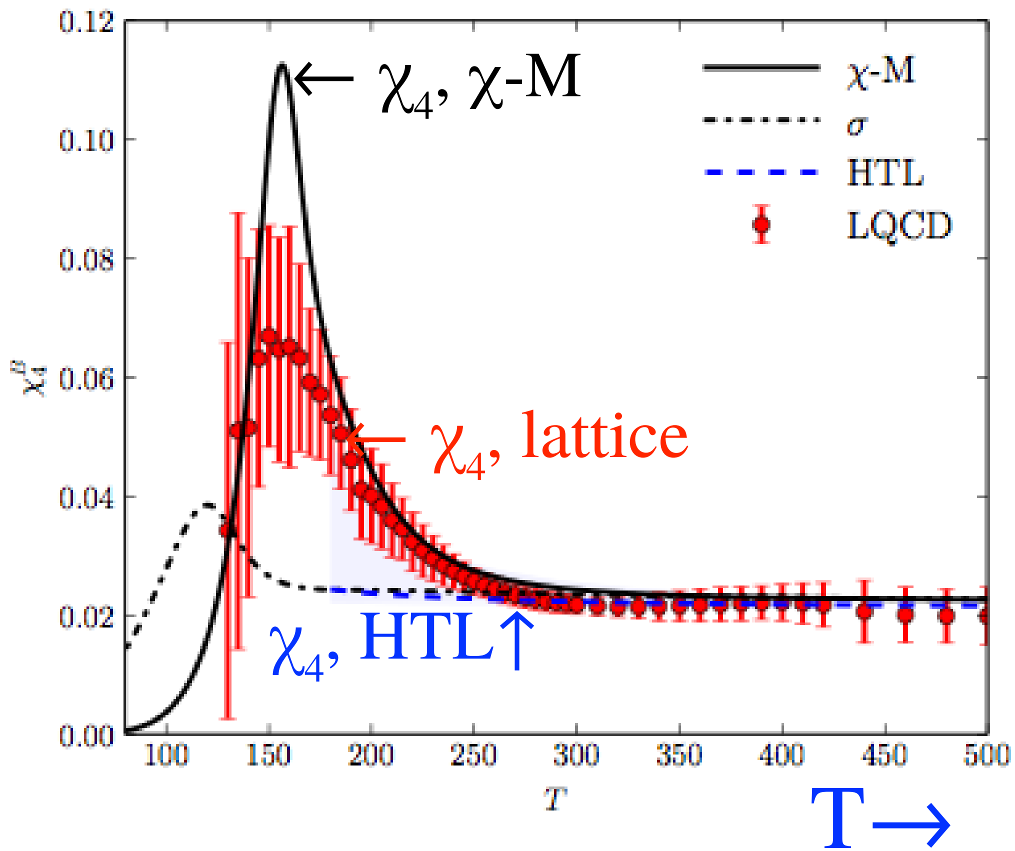


Baryon susceptibilities: 2nd & 4th

As evaluated at μ = 0, lattice ok.
Baryon $\mu_B = 3\ \mu_q$.

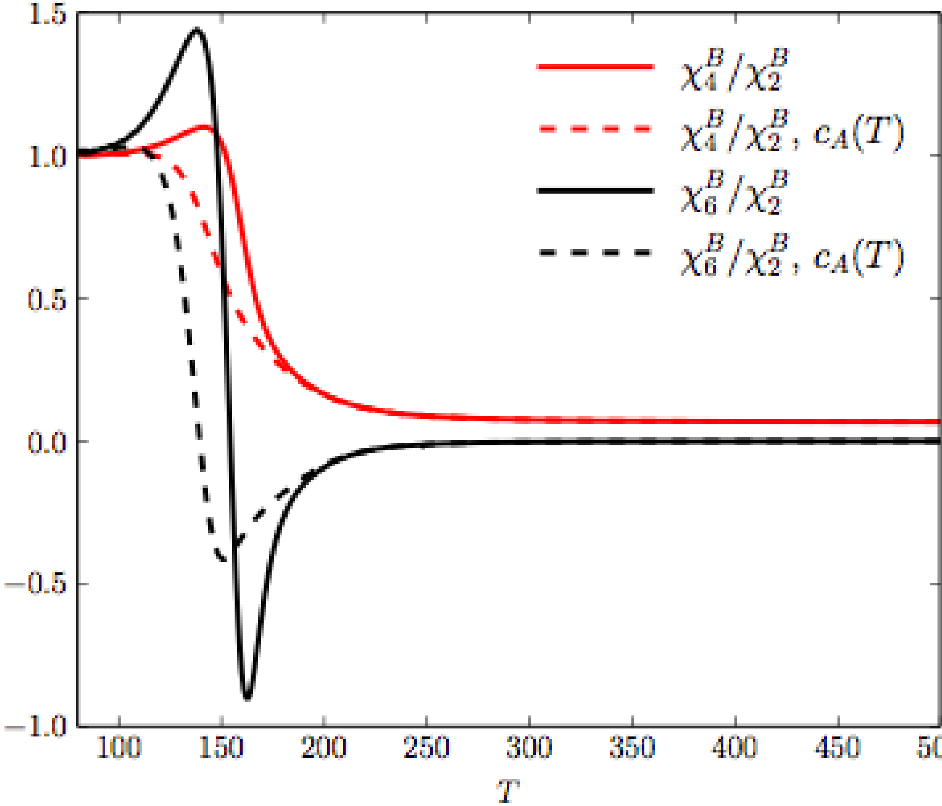
$$\chi_n^B(T) = T^{n-4}\ \frac{\partial^n}{\partial \mu_B^n}\, p(T, \mu_B)\Big|_{\mu_B=0}$$





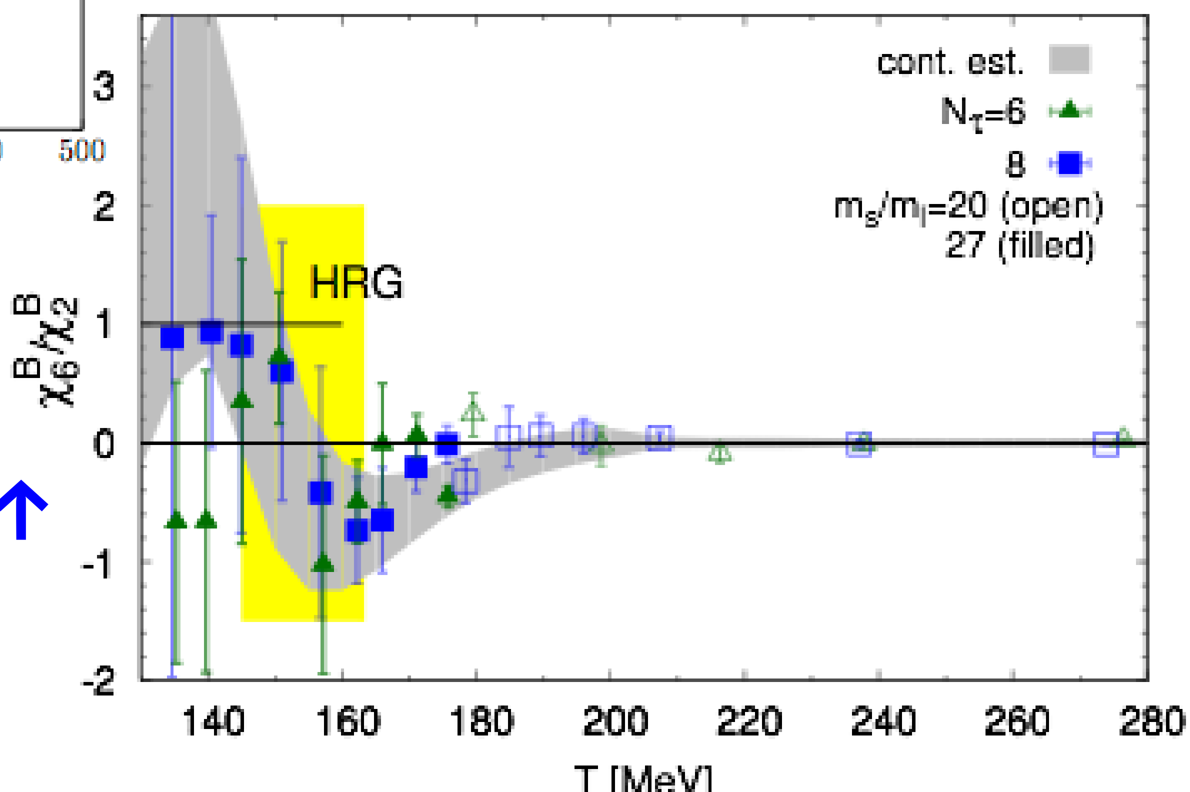
Lattice: Bazavov et al, 1701.04325

Ratios of moments, vs Columbia lattice



Left: ratio of χ_4/χ_2 and χ_6/χ_2 in χ-M model

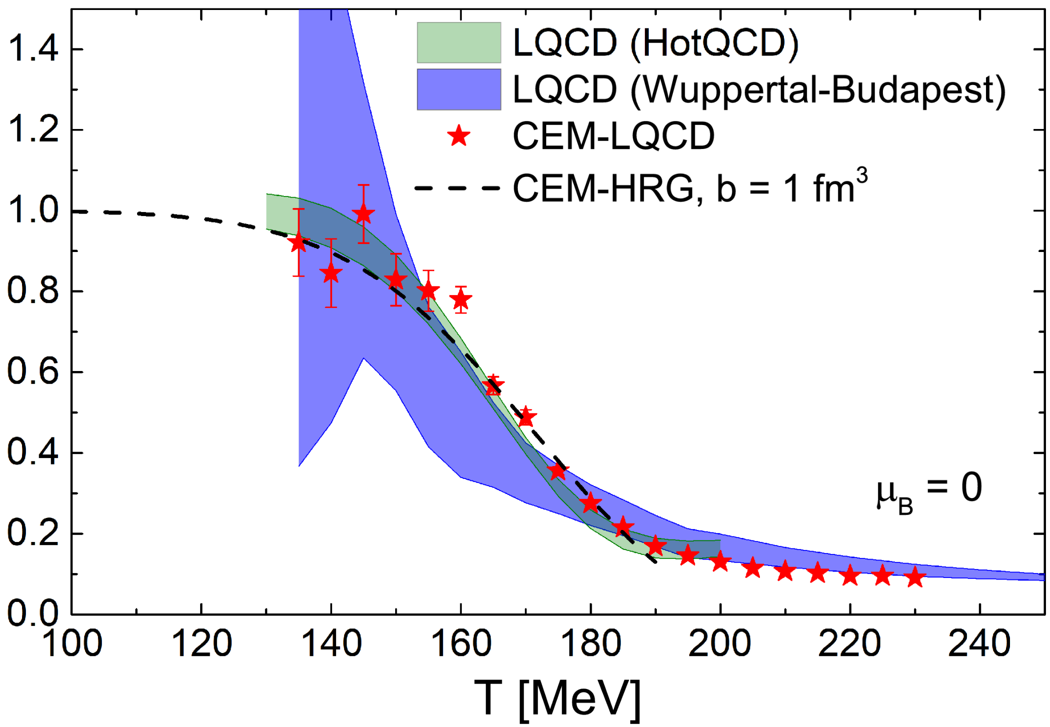
Bazavov et al, 1701.04325



χ_6/χ_2 ↑

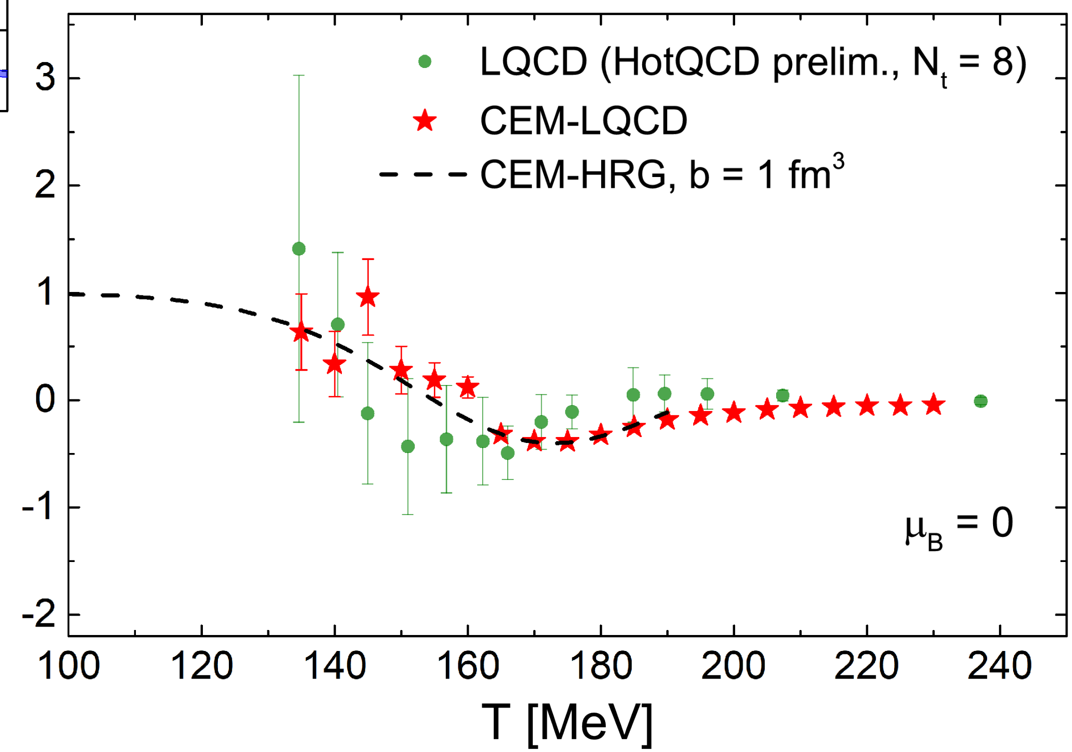
Lattice moments, Frankfurt

Vovchenko, Steinheimer, Philipsen, Stoecker 1711.0126:



↑χ_4/χ_2

χ_6/χ_2↑



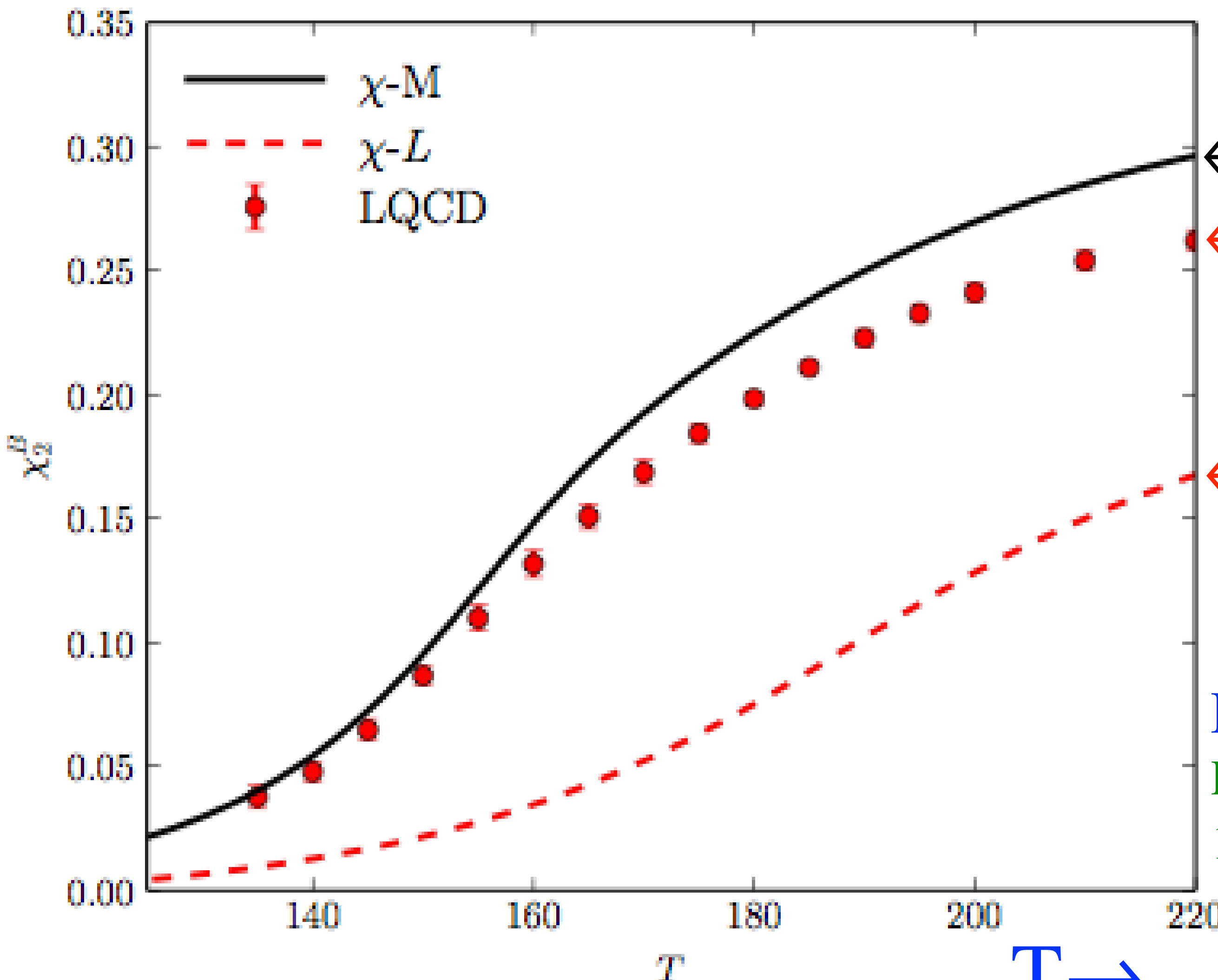
What's up with the lattice loop?

Looked at *wide* variety of variations on χ-M models.

Below: χ_2 from chiral matrix model, lattice,

and fitting the loop to the lattice value, then computing χ_2.

If the lattice loop is right, then χ_2 is too small.



← χ_2, our model

← χ_2, lattice

← χ_2, *fitting* to lattice loop

Lattice loop:
Petreczky & Schadler, 1509.07874

T→

Quarkyonic & 1-D patches

Cold, quark matter as "Quarkyonic" matter: McLerran & RDP 0706.2191

Fermi surface ~ *confined*, deep in Fermi sea ~ perturbative

Valid at large N_c: $N_c = 3$? At $T \neq 0$, $\mu = 0$: $\Lambda_{ren} \sim 2\,\pi\,T$

We suggest: $T = 0$, $\mu \neq 0$: quarkyonic for $\mu_{quark} < 1$ GeV, for *any* N_c, N_f

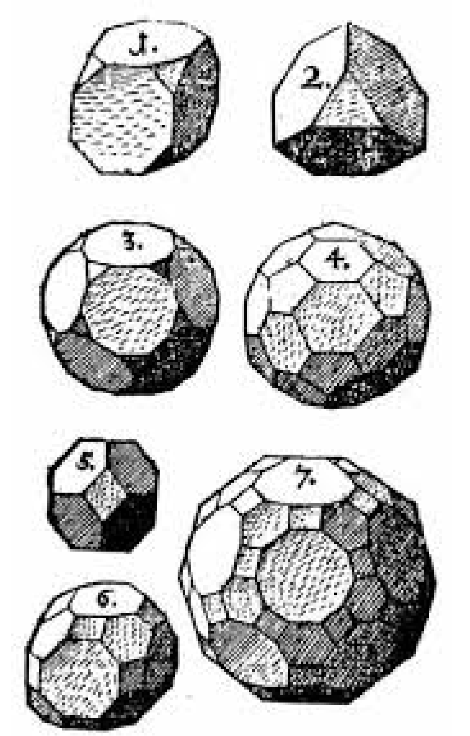
At $\mu \neq 0$, $T << \mu$ confining potential ~ $1/(p^2)^2$ tends to form 1-dim *patches* of chiral spirals in effective 1-dim theory

Kojo, Hidaka, McLerran & RDP 0912.3800;

Kojo, RDP & Tsvelik 1007.0248;

Kojo, Hidaka, Fukushima, McLerran, RDP 1107.2124;

RDP, Skokov & Tsvelik 1712.x



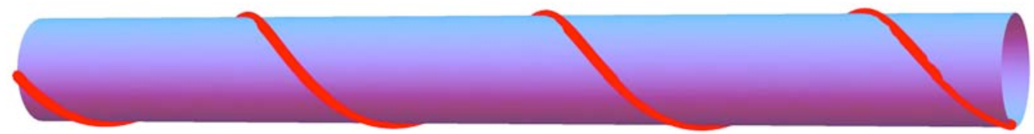
Width of patch ~ Λ_{QCD}, so for large μ,

Fermi surface is covered with patches

Chiral Spirals in 1+1 dimensions

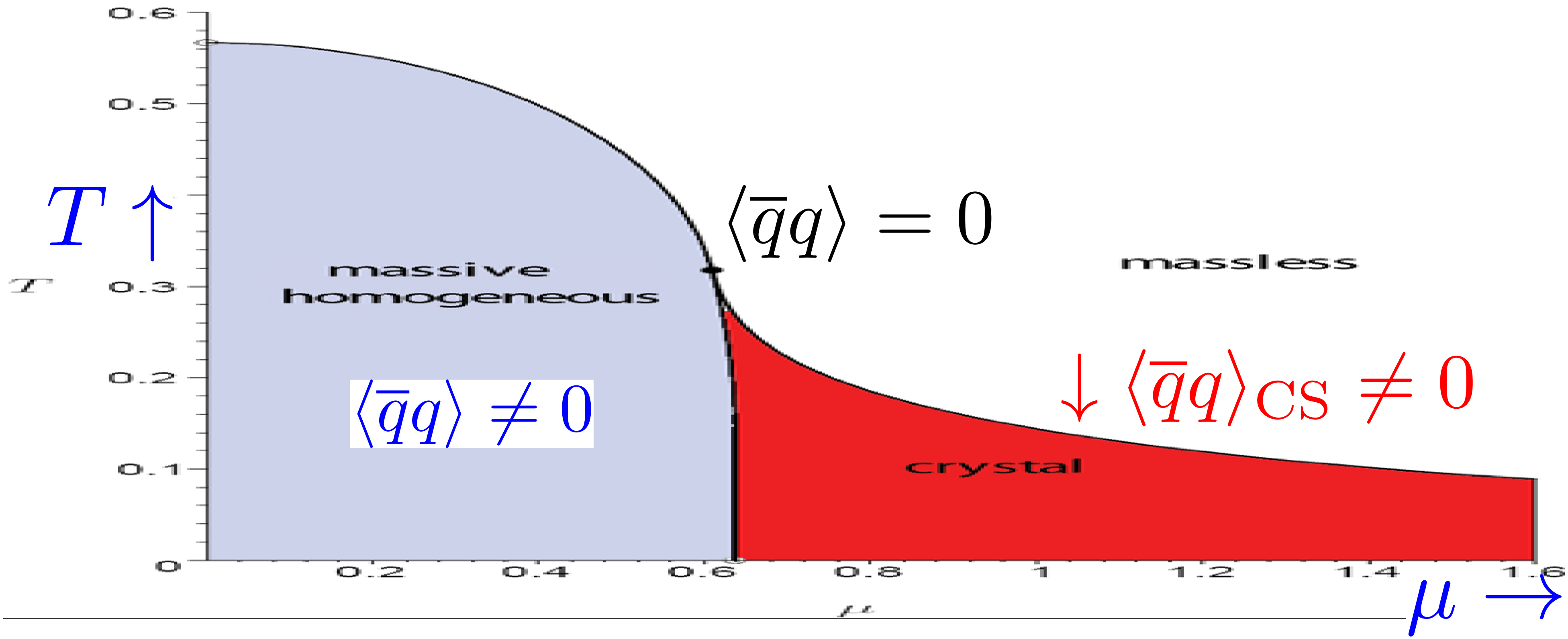
Chiral Spiral (CS) ~ Migdal's pion condensate:

$$(\sigma, \pi^0) = f_\pi(\cos(k_0 z), \sin(k_0 z))$$



Ubiquitous in 1+1 dimensions: Basar, Dunne & Thies, 0903.1868; Dunne & Thies 1309.2443+ ...

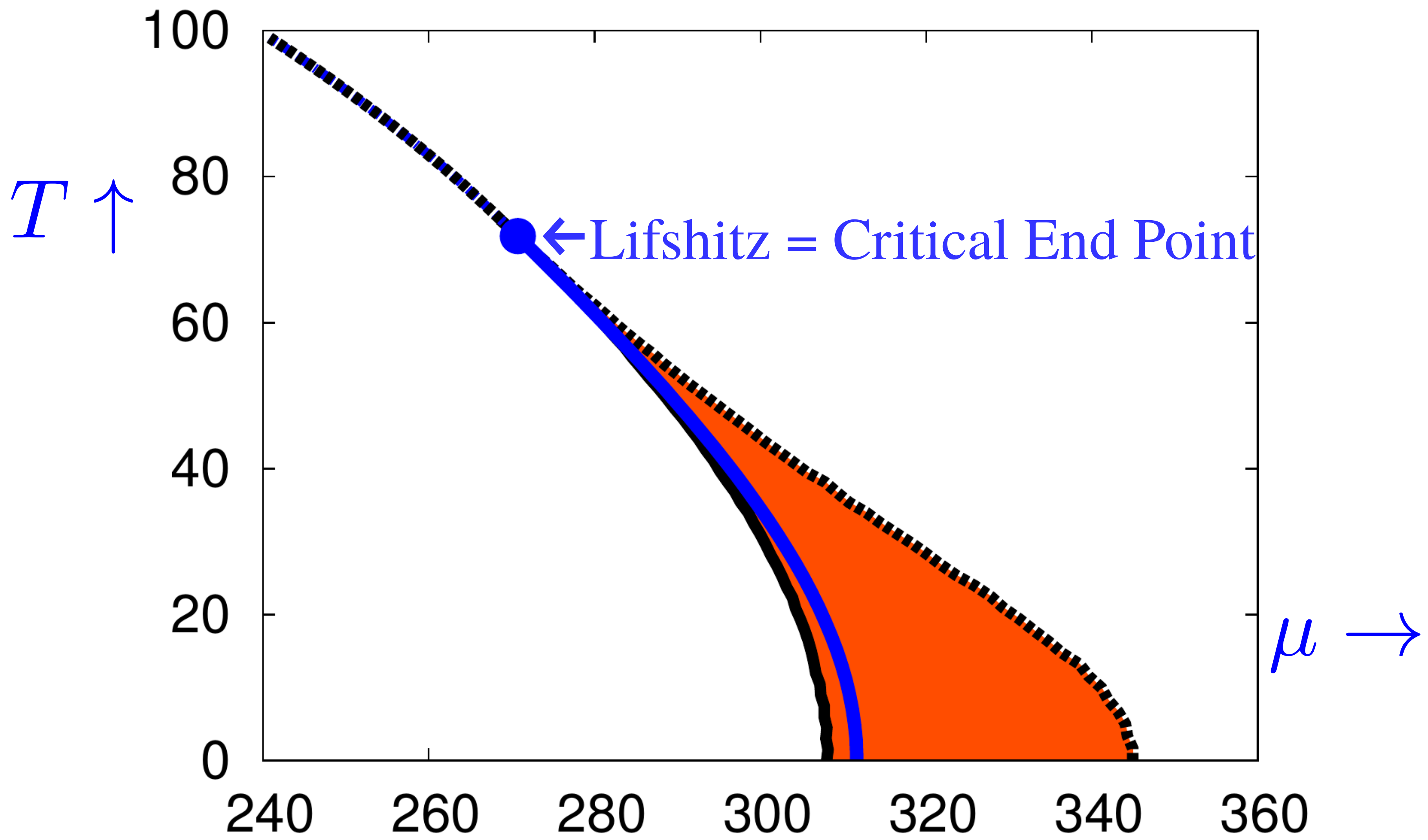
Wealth of exact solutions, phase diagrams...



Chiral Spirals in 3+1 dimensions

In 3+1, *common* in NJL models: Nickel, 0902.1778 + ...Buballa & Carignano 1406.1367 + ...

In reduction to 1-dim, $\Gamma_5^{\text{1-dim}} = \gamma_0 \gamma_z$, so chiral spiral between $\overline{q}q$ & $\overline{q}\gamma_0\gamma_z\gamma_5 q$



Fluctuations in Chiral Spirals

In Chiral Spiral, <φ>≠0 *locally* but <φ>=0 *globally*.

Spon. breaking of global symmetry => interactions of Goldstone Bosons ~ ∂^2

In CS, spon. bkg's of global *plus* rotational sym. implies interactions in transverse momenta ~ $\partial_\perp{}^2$ *cancel*. Interactions ~ $(\partial_\perp{}^2)^2$ ~ $\partial_\perp{}^4$. U = GB:

$$\mathcal{L}_{\rm CS} = f_\pi^2 |(\partial_z - i k_0) U|^2 + \kappa |\partial_\perp^2 U|^2 + \ldots$$

Hidaka, Kamikado, Kanazawa & Noumi 1505.00848; Nitta, Sasaki & Yokokura 1706.02938

Transverse fluctuations *dis*order: *large* fluctuations about k_z ~ k_0 :

$$\int d^2 k_\perp \, dk_z \, \frac{1}{(k_z - k_0)^2 + (k_\perp^2)^2} \sim \int d^2 k_\perp \, \frac{1}{k_\perp^2} \sim \log \Lambda_{\rm IR}$$

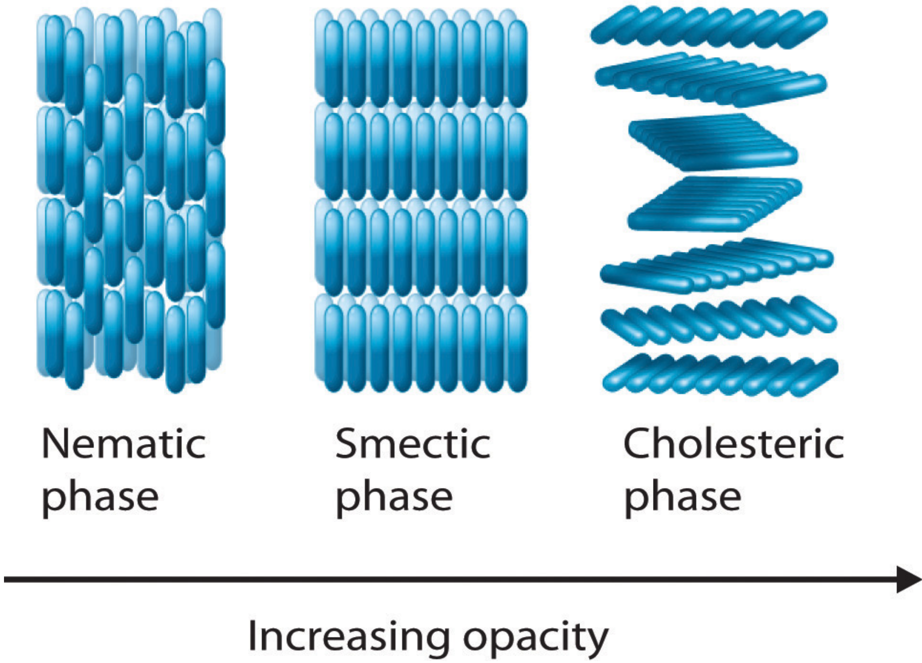
No true long range order (Landau-Peierls) ~ *smectic liquid crystal*

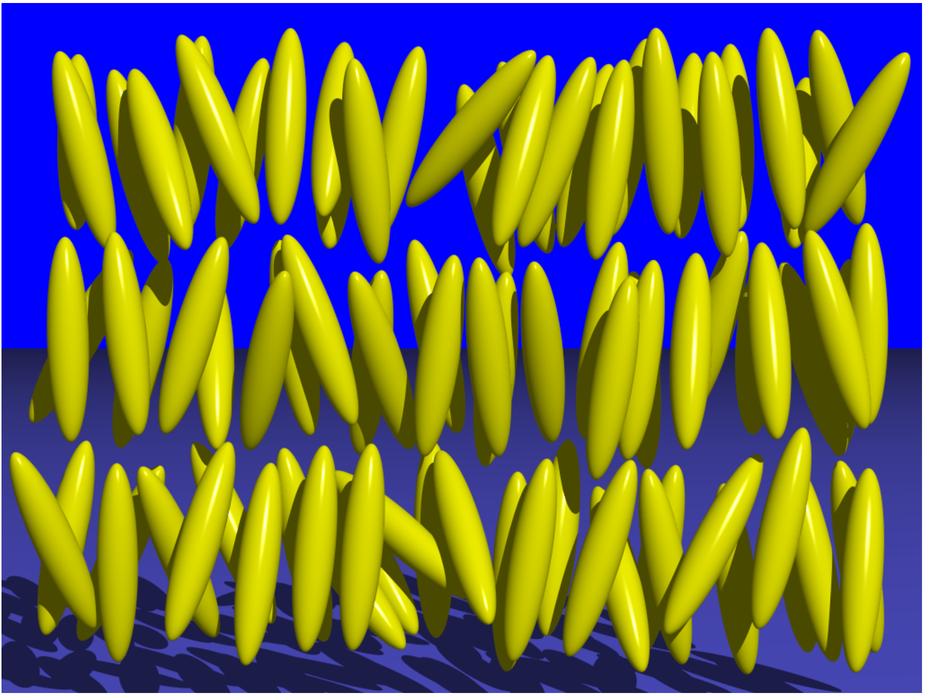
Varieties of liquid crystals

Nematics: rotational ordering (vector with no direction)

Smectic: rotational ordering and in planes disordered in the planes ("liquid")

Cholesteric: chiral ordering (with twist)





Smectic something like patches in QCD

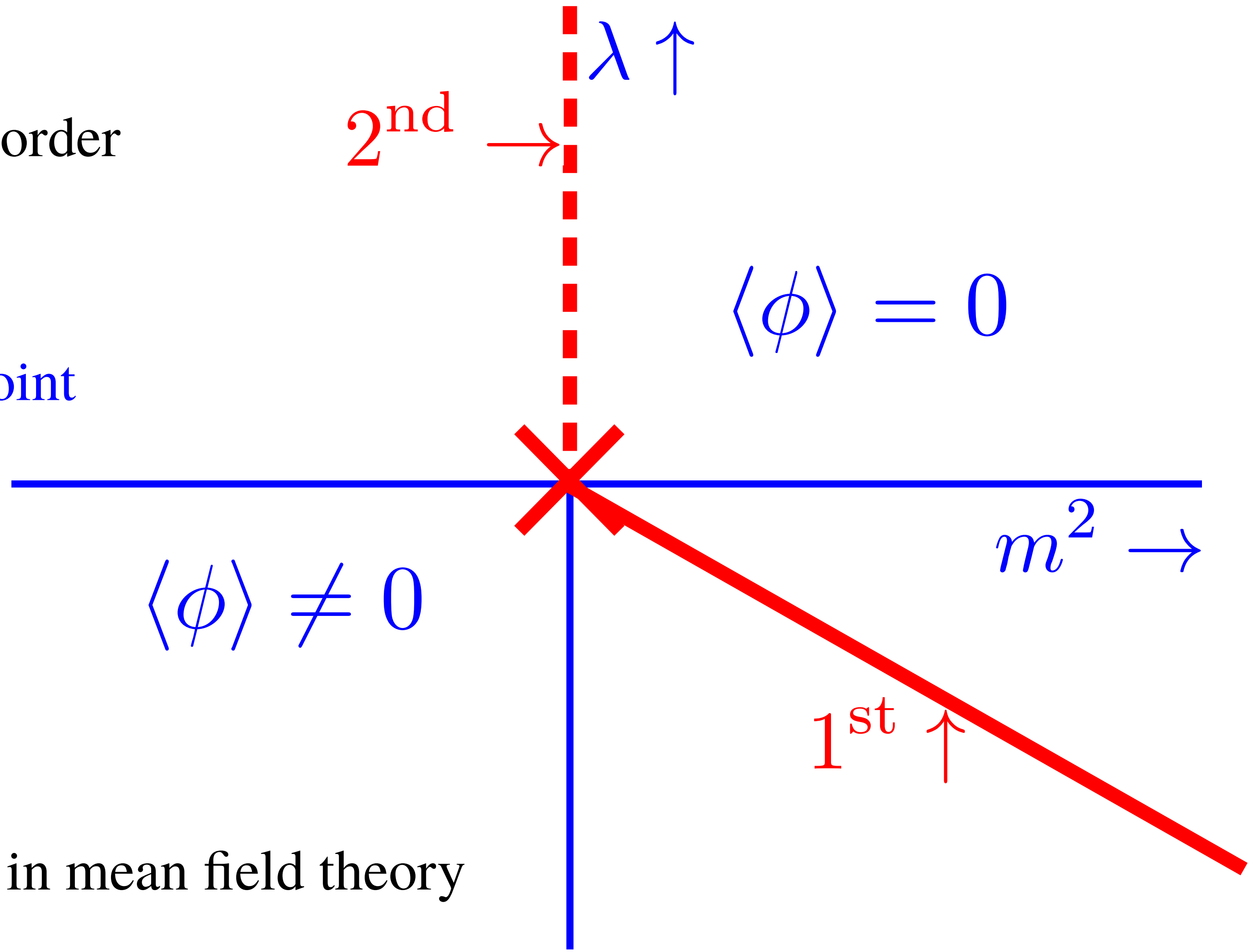
Smectic – nematic transition has analogy, to follow (1^{st} order from reduction to 1-dim)

Standard phase diagram

Trade T & μ for m^2 & λ.

Two phases, symmetric & broken

$$\mathcal{L} = (\partial_\mu \phi)^2 + m^2\phi^2 + \lambda\phi^4 + \kappa\phi^6$$



$m^2 = 0$, $\lambda > 0$: usual 2^{nd} order

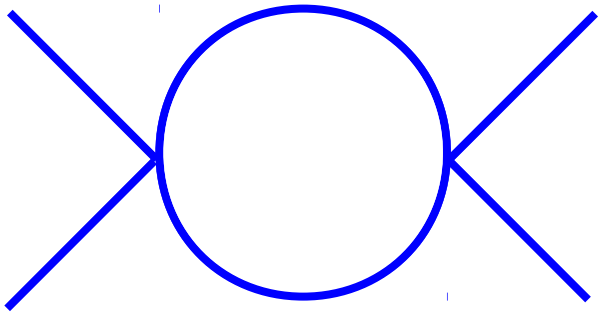
$m^2 = \lambda = 0$: tricritical point

$m^2 > 0$, $\lambda < 0$: 1^{st} order in mean field theory

Usual critical dimensions

φ^4: $d_{upper} = 4$: expand in $d = 4 - \varepsilon$ dimensions

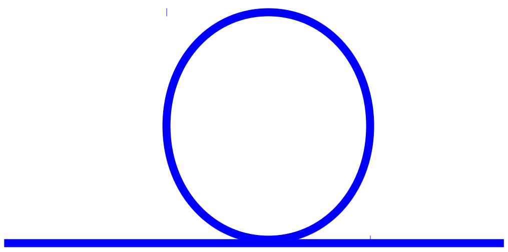
$$\int d^4k \, \frac{1}{(k^2)^2} \sim \log \Lambda_{\mathrm{UV}}$$



φ^4: $d_{lower} = 2$: expand in $2 + \varepsilon$ dimensions

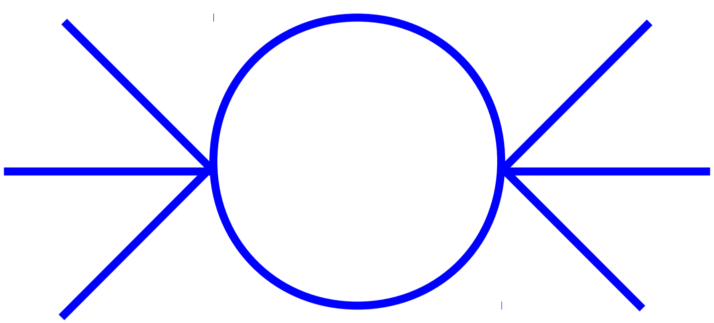
always disordered when $d < 2$

$$\int d^2k \, \frac{1}{k^2} \sim \log \Lambda_{\mathrm{IR}}$$



φ^6: $d_{critical} = 3$: at tricritical point, log corrections

$$\int d^3k_1 \int d^3k_2 \, \frac{1}{(k_1)^2(k_2)^2(k_1+k_2)^2} \sim \log \Lambda_{\mathrm{UV}}$$



Lifshitz points

To get a Chiral Spiral (CS):

$$\mathcal{L}_{CS} = (\partial_0 \phi)^2 + Z(\partial_i \phi)^2 + \frac{1}{M^2}(\partial_i^2 \phi)^2 + m^2 \phi^2 + \lambda \phi^4$$

Need higher (spatial) derivatives for stability. Then CS occurs when Z < 0.

Can*not* have higher derivatives in time or theory is acausal.

In gravity, models with higher derivatives are renormalizable:

$$\mathcal{L}_{\rm ren.gravity} = \frac{1}{16\pi G} R + \alpha_1 R^2 + \alpha_2 R^2_{\mu\nu}$$

but acausal. Hořava-Lifshitz gravity: add higher derivatives only in space

Hořava 0901.3775 + ...

$$\mathcal{L}_{\rm Horava-Lifshitz} = \frac{1}{16\pi G} R + \beta_1 R^2_{ij} + \ldots$$

Only two time derivatives, so causal. Flows into Einstein gravity in the infrared.

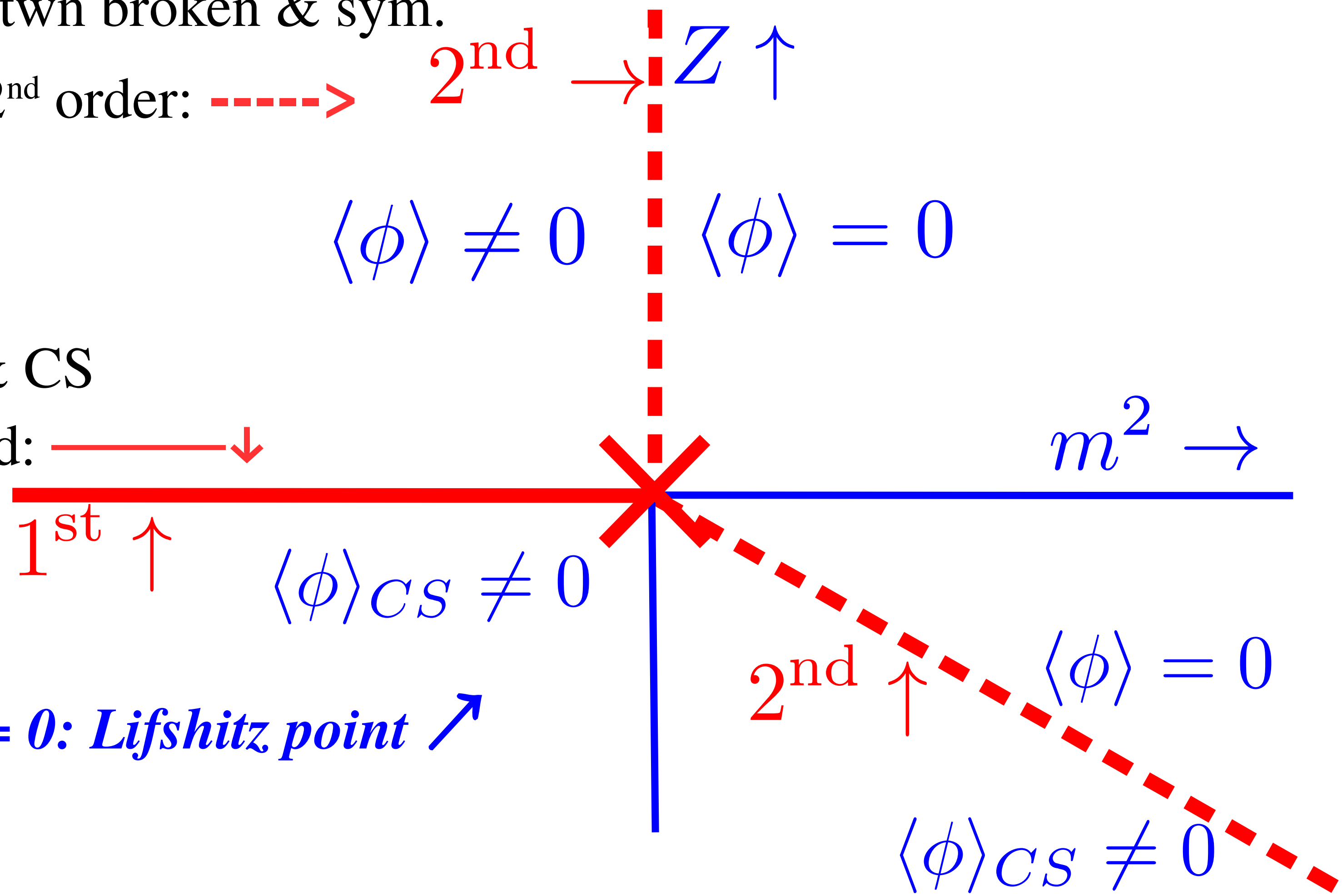
Lifshitz phase diagram in mean field theory

Phase diagram in Z & m^2: *three* phases, symmetric, broken, *and* Chiral Spiral

Hornreich, Luban, Shtrikman, PRL '75, Hornreich J. Magn. Matter '80...Diehl, cond_mat/0205284 + ...

$m^2 = 0$, $Z > 0$: btwn broken & sym.

usual 2nd order: ----->



$m^2 < 0$, $Z = 0$: broken & CS

1st order in mean field: ⟶↓

X at origin, $m^2 = Z = 0$: Lifshitz point ↗

$m^2 > 0$, $Z < 0$: btwn CS & broken ----- ↗

2nd order in mean field, but *fluctuations?*

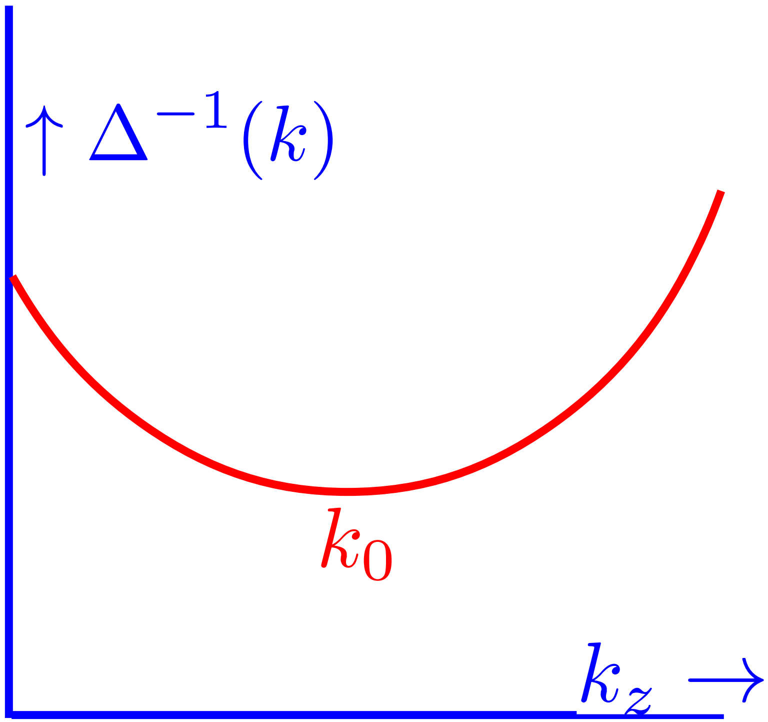
Symmetric to CS: 1D (Brazovski) fluctuations

Consider $m^2 > 0$, $Z < 0$: minimum in propagator at *non*zero momentum

Brazovski '75; Hohenberg & Swift '95 + ... ;

Lee, Nakano, Tsue, Tatsumi & Friman, 1504.03185; Yoshiike, Lee & Tatsumi 1702.01511

$$\Delta^{-1} = m^2 + Z\,k^2 + k^4/M^2$$
$$= m^2_{\rm eff} - 2\,Z\,k_z^2 + (k_\perp^2)^2/M^2$$



$k=(k_\perp, k_z - k_0)$: *no* terms in $k_\perp^2$, *only* $(k_\perp^2)^2$.

Due to spon. breaking of rotational sym.

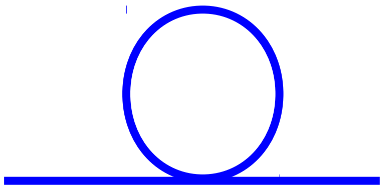
$$\int d^3k\,\frac{1}{k_z^2 + m^2_{\rm eff} + \ldots} \sim M^2 \int \frac{dk_z}{k_z^2 + m^2_{\rm eff}} \sim \frac{M^2}{m_{\rm eff}}$$

Effective reduction to 1-dim for any spatial dimension d, any global symmetry

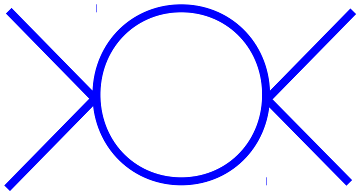
1^{st} order transition in 1-dim.

Strong infrared fluctuations in 1-dim., both in the mass:

$$\Delta m^2 \sim \lambda \int d^3k \, \frac{1}{k_z^2 + m_{\rm eff}^2 + \ldots} \sim \lambda \, \frac{M}{m_{\rm eff}}$$



and for the coupling constant:

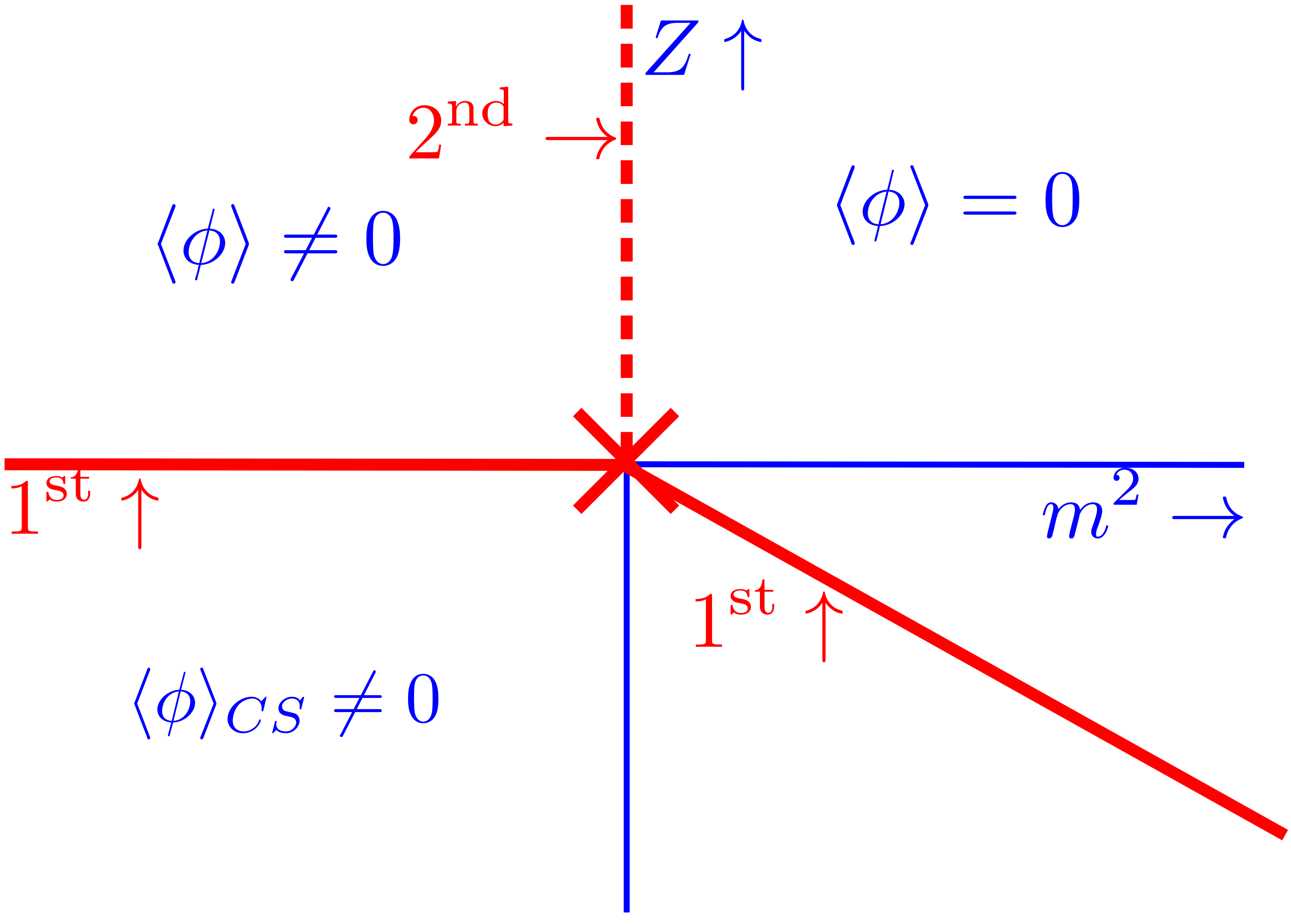


$$\Delta\lambda \sim -\lambda^2 \int \frac{d^3k}{(k_z^2 + m_{\rm eff}^2 + \ldots)^2} \sim -\lambda^2 M^3 \int_{m_{\rm eff}} \frac{dk_z}{k_z^4} \sim -\lambda \, \frac{M^3}{m_{\rm eff}^3}$$

Can*not* tune $m_{\rm eff}^2$ to 0: $\lambda_{\rm eff}$ goes negative, 1^{st} order trans. induced by fluctuations

Not like other 1st order fluc-ind'd trans's: just that in 1-d, $m_{\rm eff}^2 \neq 0$ always

Lifshitz phase diagram, with eff. 1-D fluc.'s



What about fluctuations at the Lifshitz point?

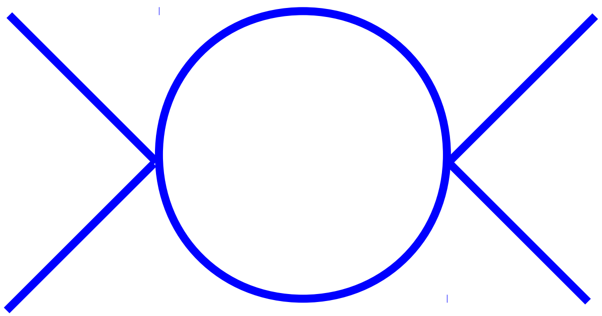
Critical dimensions at the Lifshitz point

At the Lifshitz point, Z=m=0,
massless propagator ~ $1/k^4$

$$\mathcal{L}_{\rm Lifshitz} = (\partial^2\phi)^2 + \lambda\phi^4$$

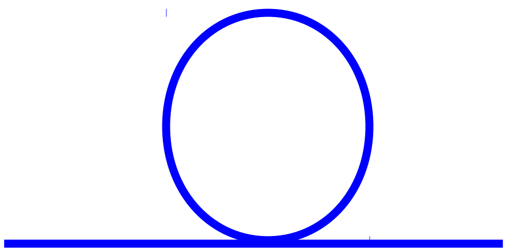
$d_{upper} = 8$: expand in d = 8 – ε dimensions

$$\int d^8k\, \frac{1}{(k^4)^2} \sim \log \Lambda_{\rm UV}$$



$d_{lower} = 4$: expand in d = 4 + ε dimensions

$$\int d^4k\, \frac{1}{k^4} \sim \log \Lambda_{\rm IR}$$



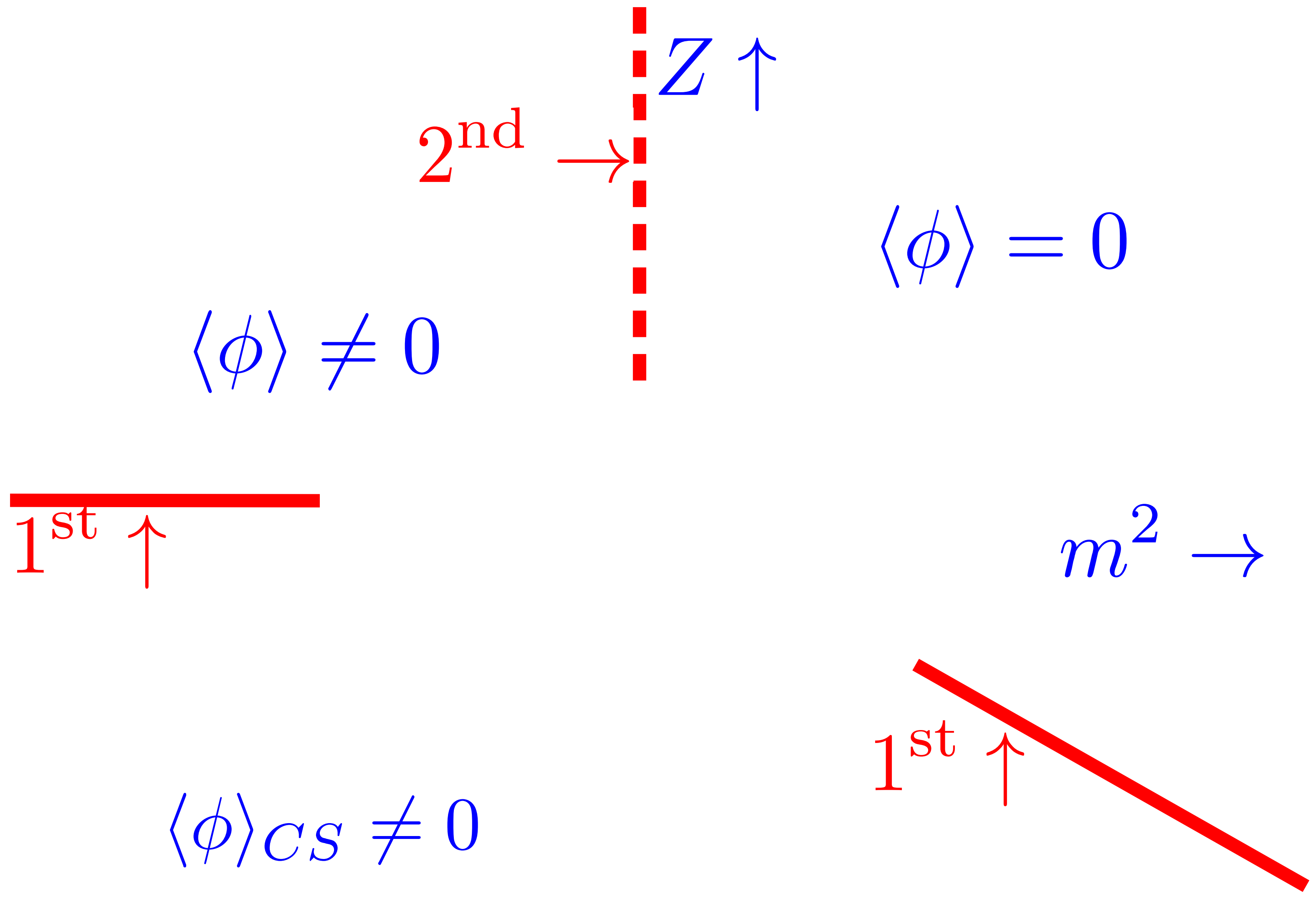
$d = 3 < d_{lower}$: there is *NO* (isotropic) Lifshitz point in *three* dimensions

...+ Bonanno & Zappala, 1412.7046; Zappala, 1703.00791

Infrared fluctuations *always* generate a mass gap *dynamically*.

Phase diagram with*out* a Lifshitz point?

Have three phases, three lines of phase transition far from the would be Lifshitz point. *How can they connect?*

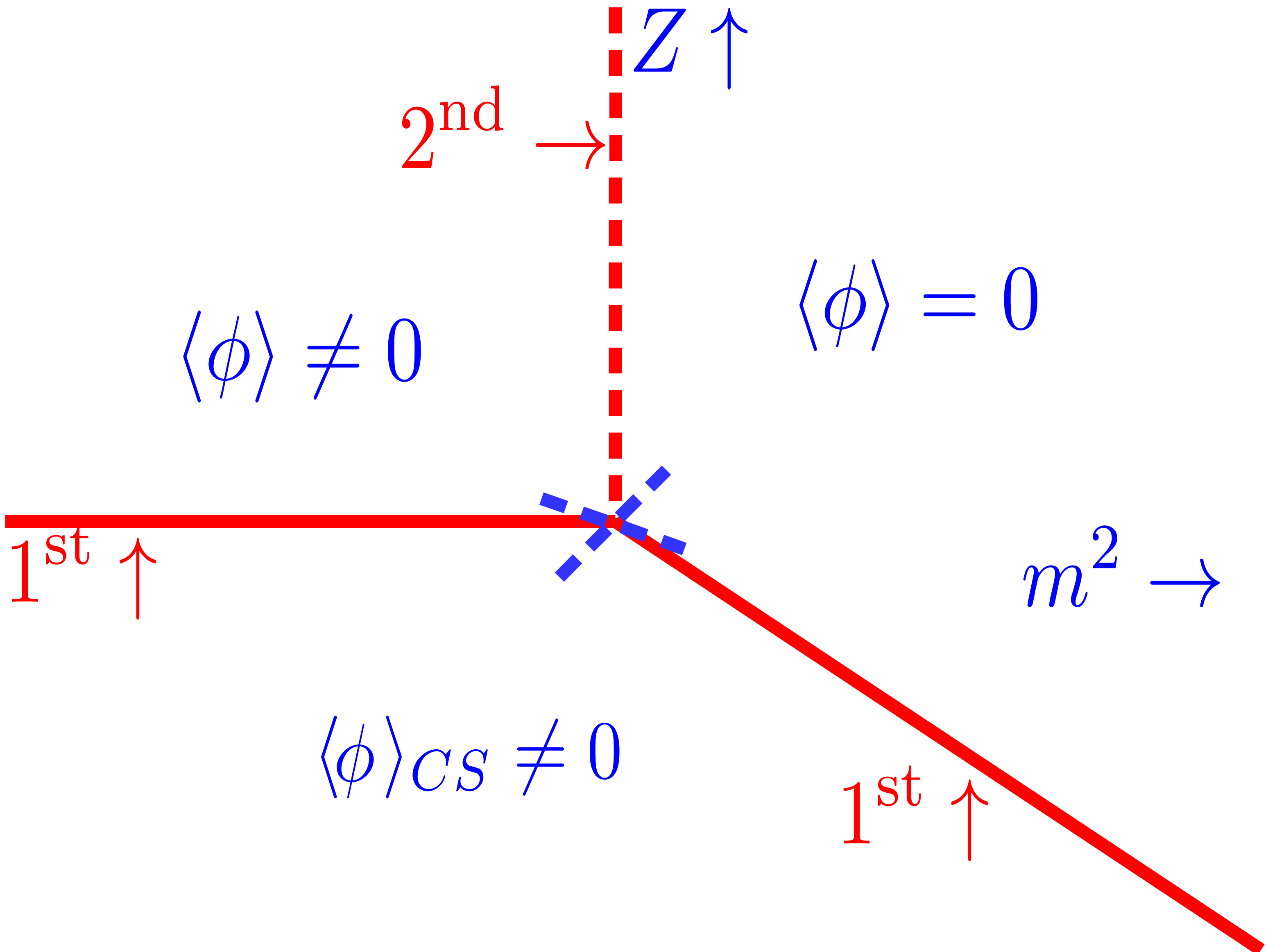


A: looks like Lifshitz point, but isn't

All three lines connect at a "pseudo"-Lifshitz point.

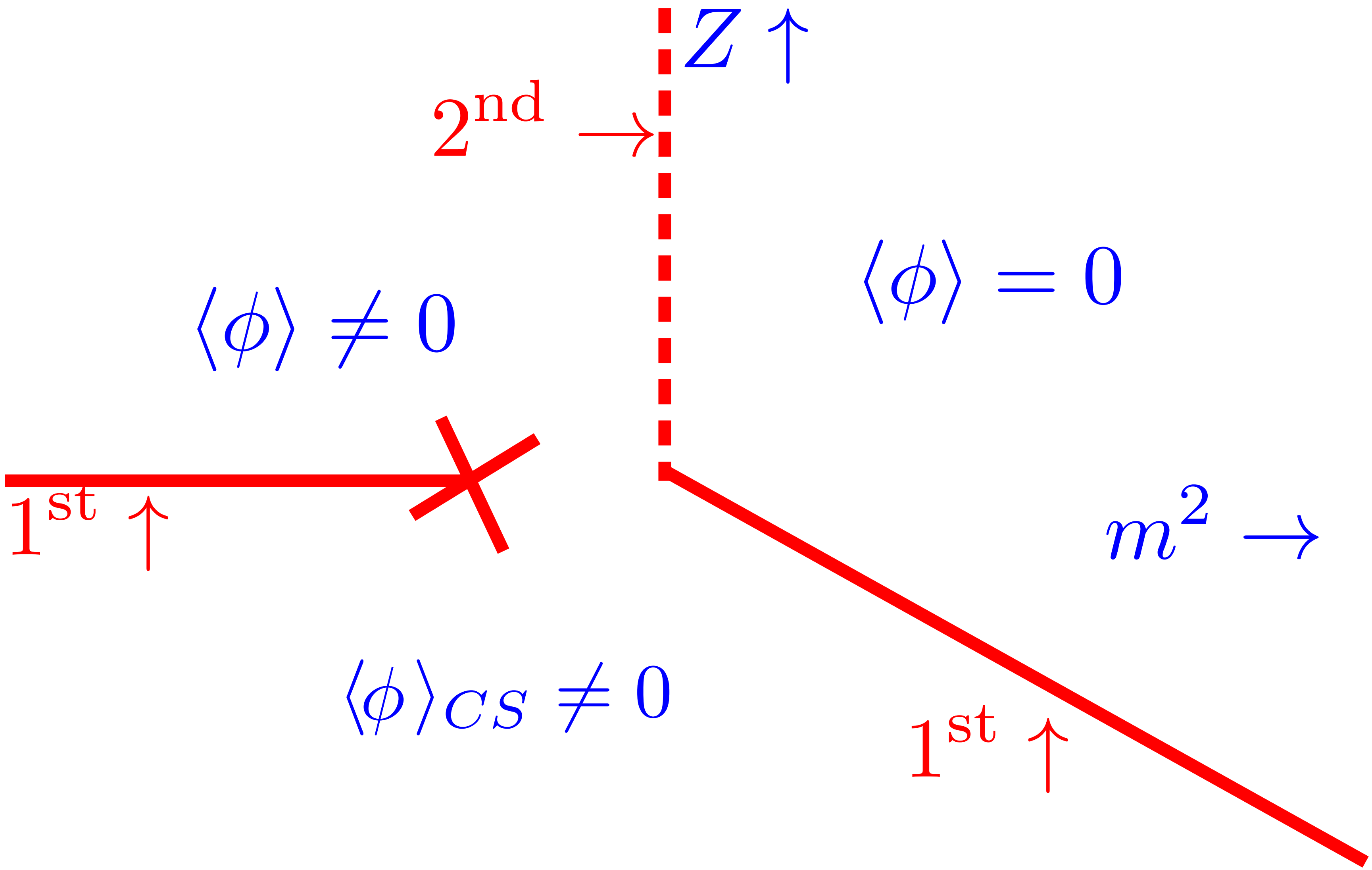
As terminus of 2nd order line, $m^2 = 0$. So at pseudo-Lifshitz point, $Z \neq 0$

Why do fluctuations drive symmetric-CS transition 1st order if $Z \neq 0$?



B: 1st order line between broken/CS phases ends

Crossover between broken and CS phases? But <φ>≠0 in the broken phase, and <φ>=0 for a Chiral Spiral. Crossover seems unlikely, unless fluctuations are *small* (so long range order in CS phase)

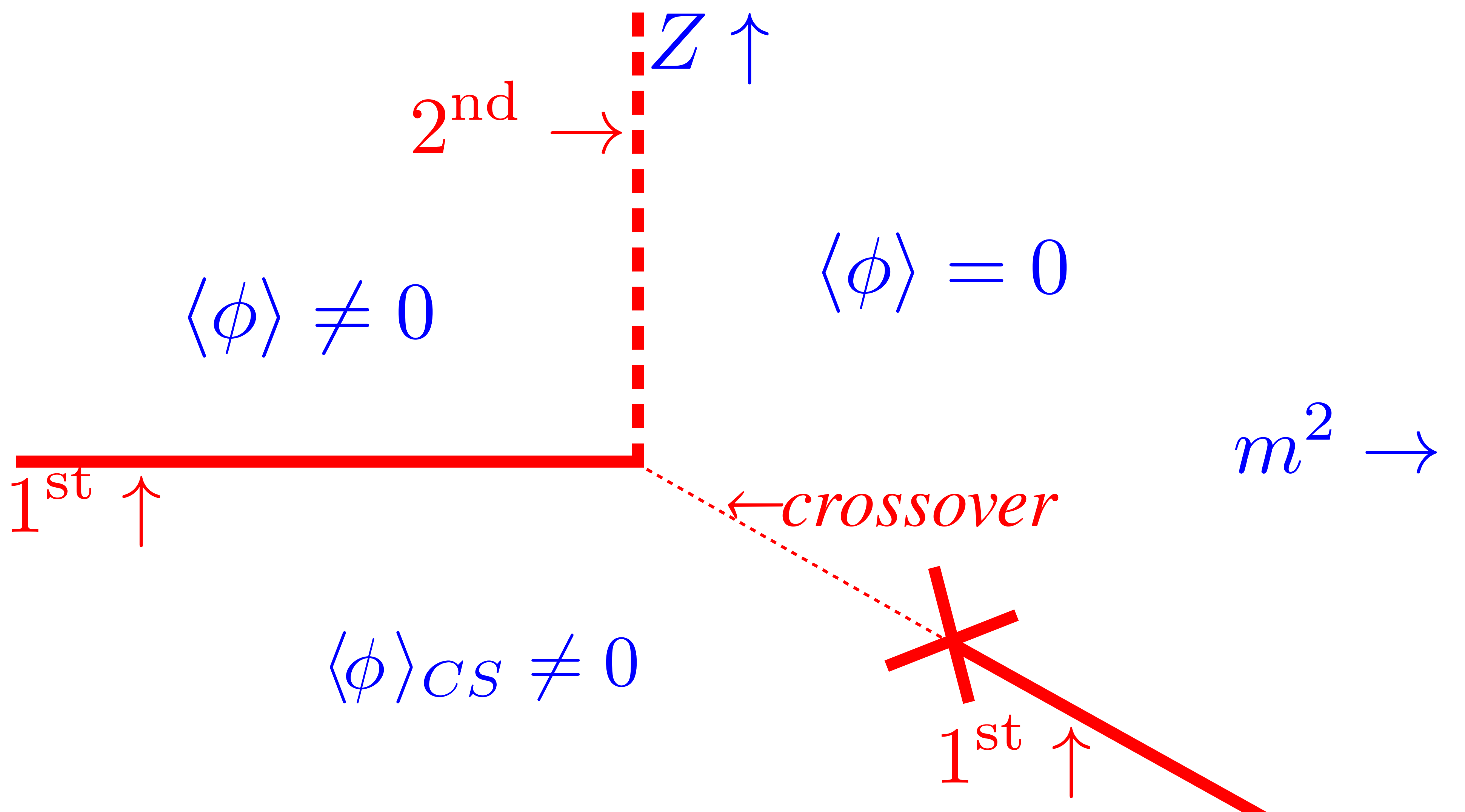


C: Brazovski 1st order CS/sym. line ends

Chiral spiral has *no* long range order, so *when* fluctuations are large, possible to have just *crossover* between CS & symmetric phases.

Brazikovski 1st order line ends in critical endpoint.

Novel tricritical point where 2nd order line joins to 1st order, at small Z.



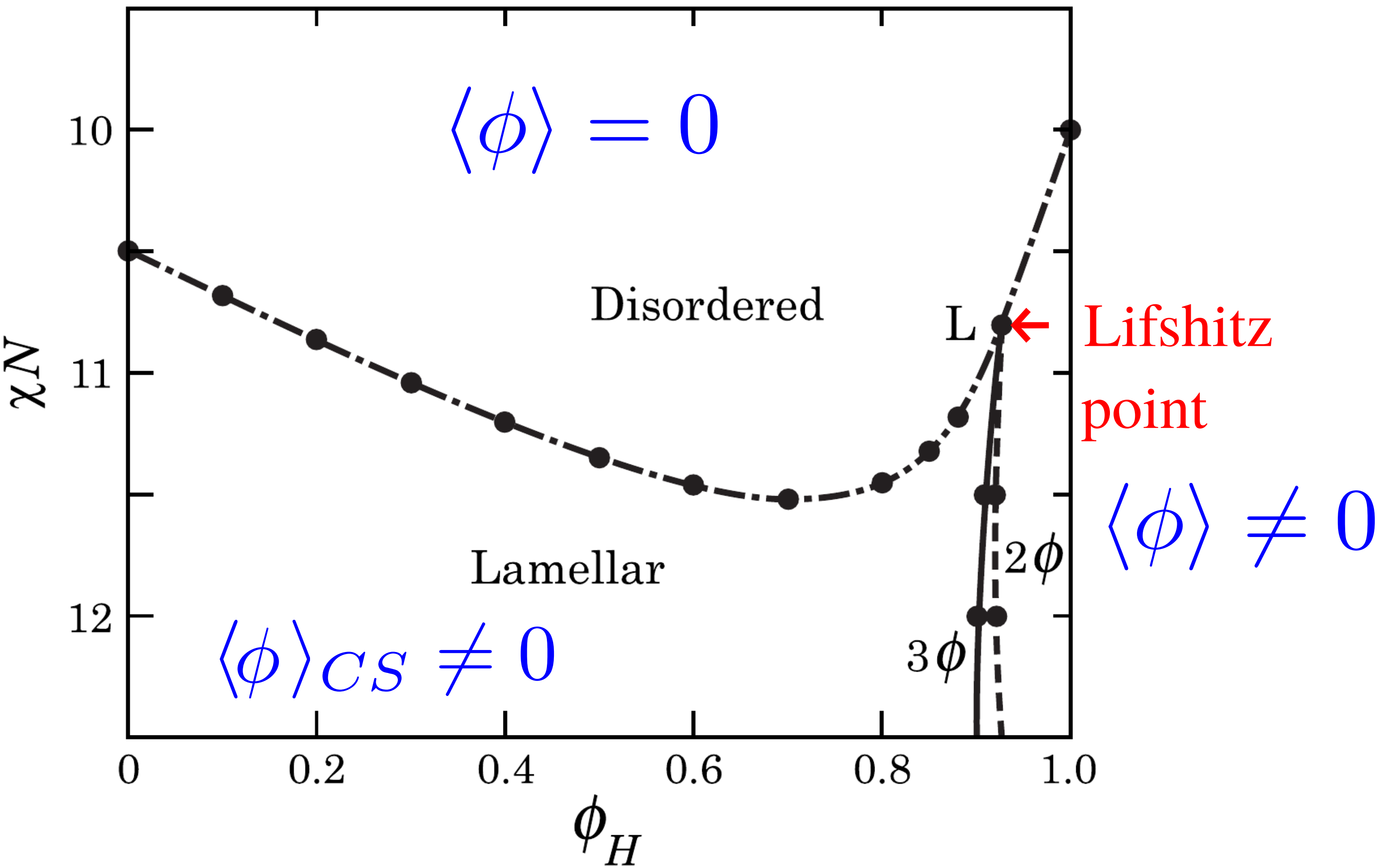
Lifshitz points in inhomogenous polymers: mean field

Fredrickson & Bates, Jour. Polymer Sci. 35, 2775 (1997);

Fredrickson, "The equilibrium theory of inhomogenous polymers", pg. 390.

Polymers A & B, for blend with A, B, & A+B

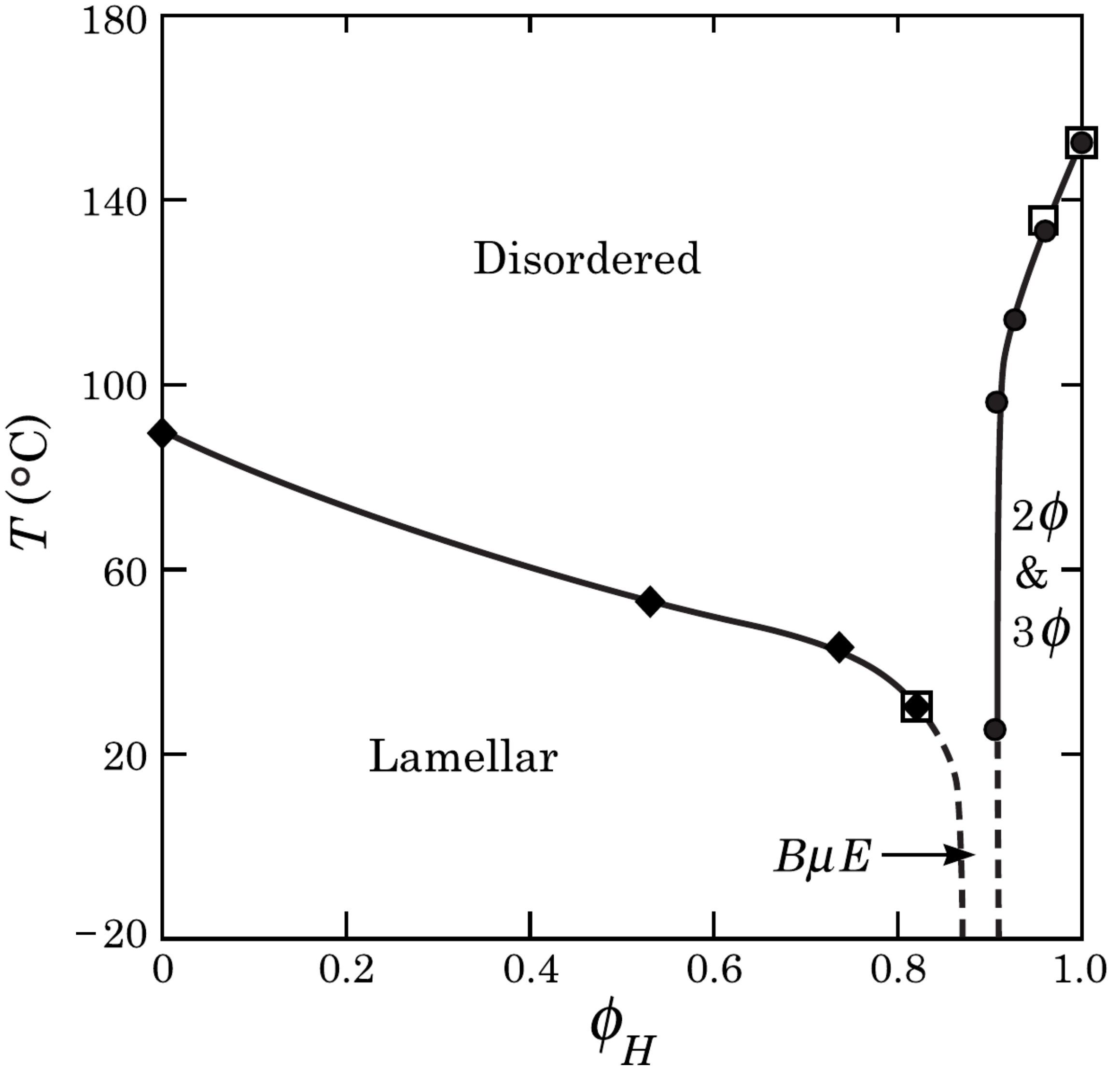
Have disordered, separated, and "lamellar" phases



Inhomogenous polymers: *no* Lifshitz point

From both experiment & numerical simulations, Lifshitz point wiped out by fluctuations: instead a “bicontinuous microemulsion”, BμE, appears

“structured, fluctuating disordered phase”

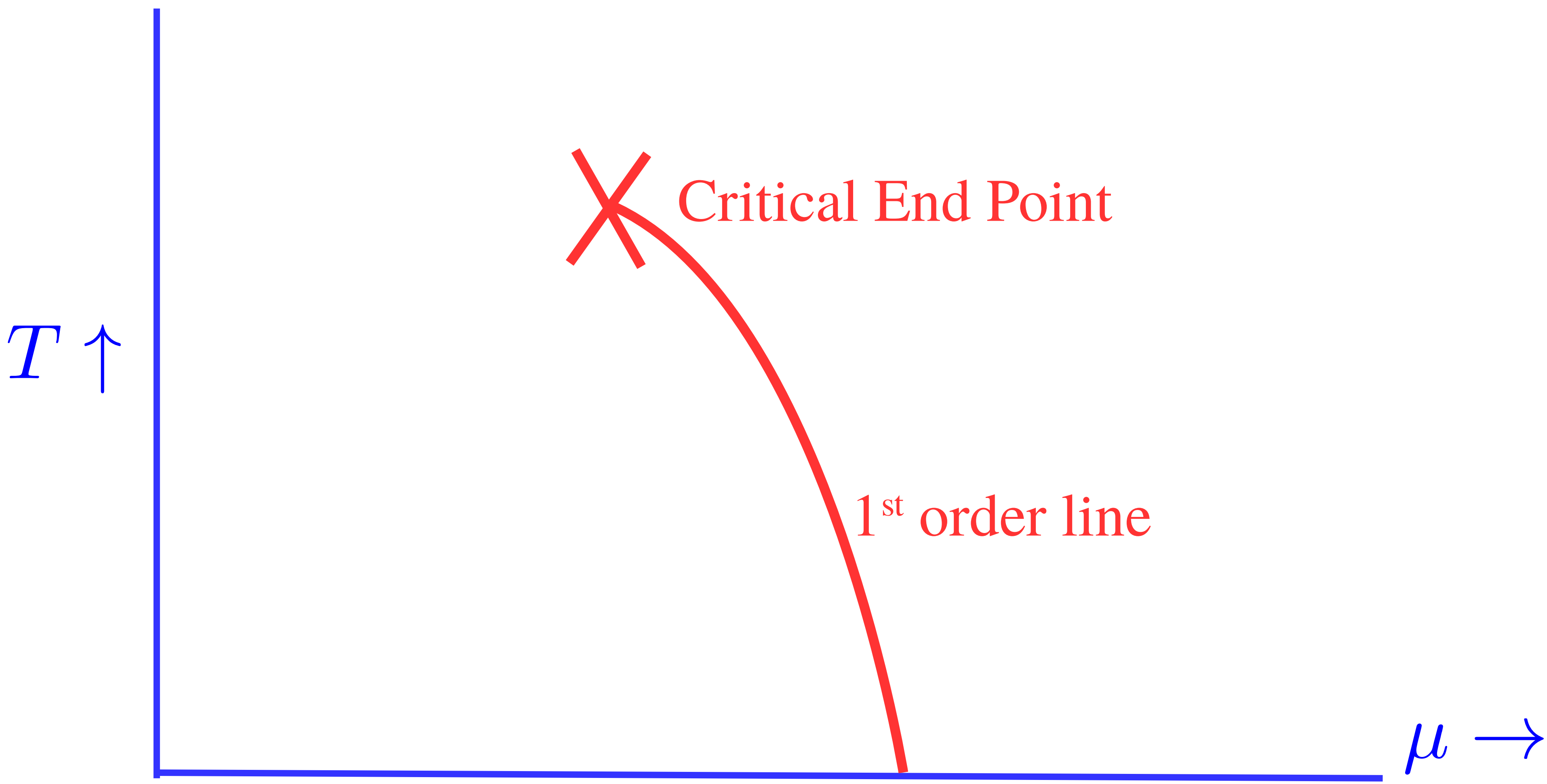


Phase diagram for QCD in T & μ: usual picture

Two phases, one Critical End Point (CEP)

between crossover and line of 1st order transitions

Ising fixed point, dominated by *massless* fluctuations at CEP

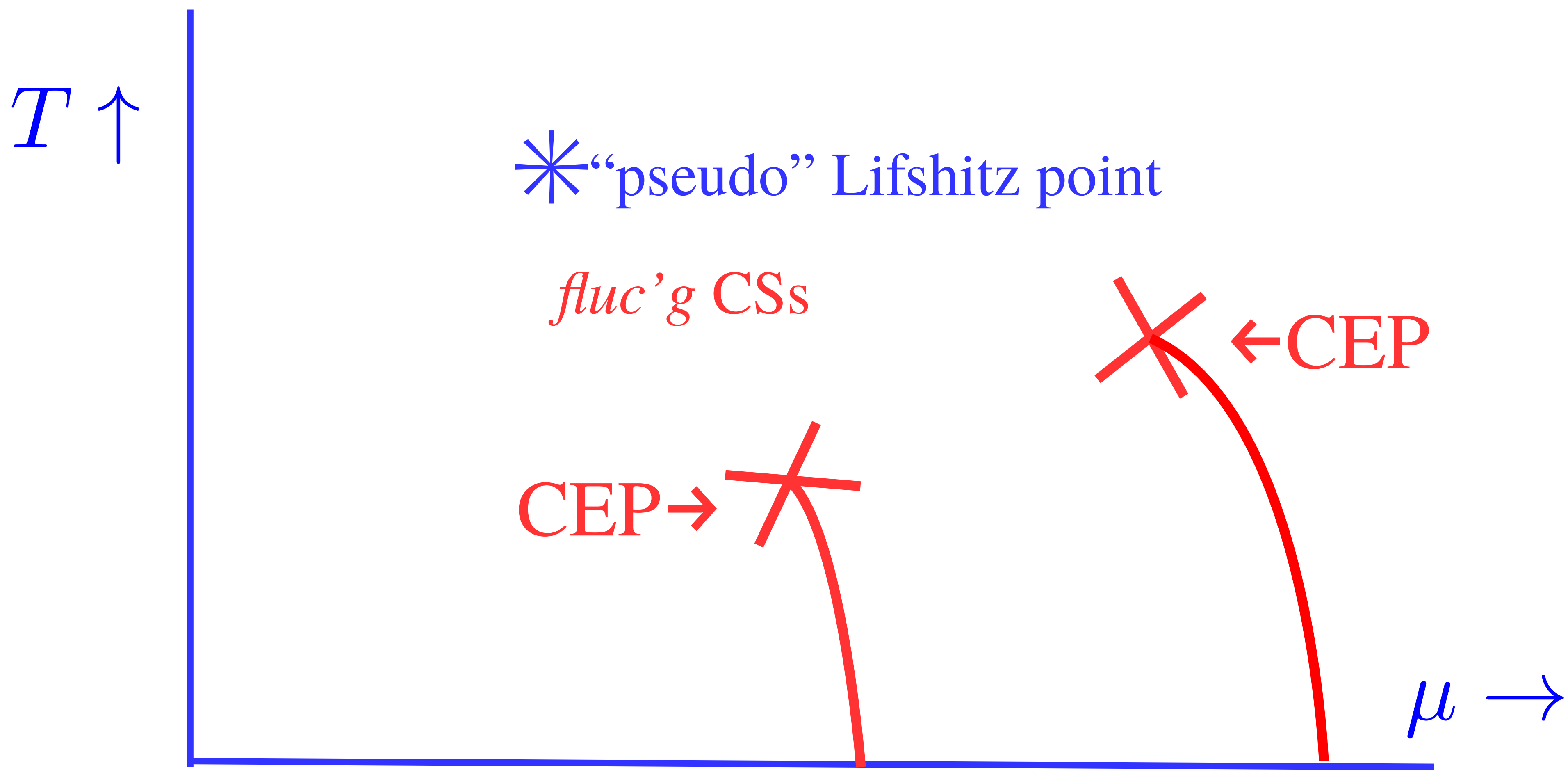


Phase Diagram with Chiral Spirals

Now *three* phases. If model "C", *two* 1st order lines and *two* CEP's

"Pseudo" Lifshitz point with *large* fluctuations.

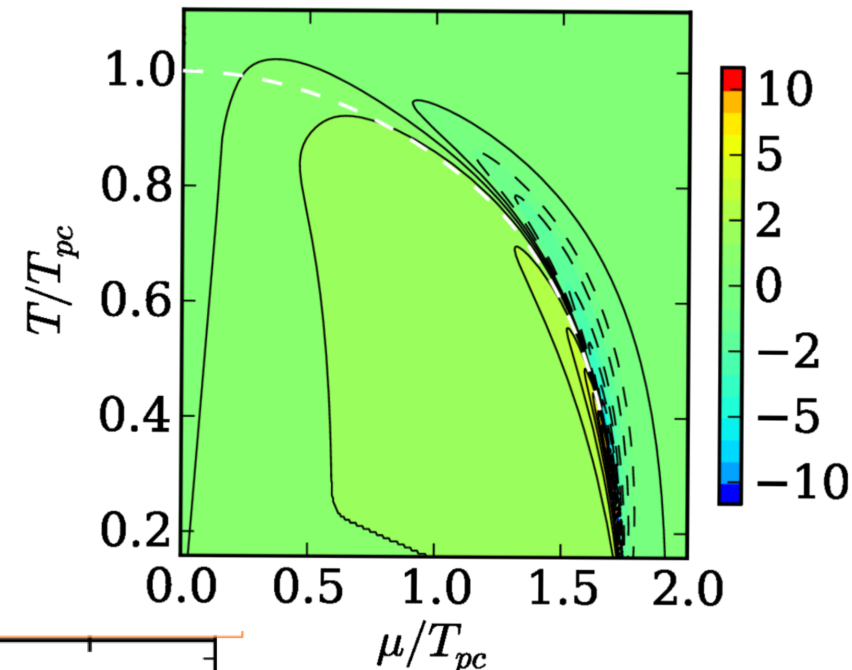
In CS, large fluc.'s at *non*zero momenta, ~ k_0 .



Beam Energy Scan and cumulants

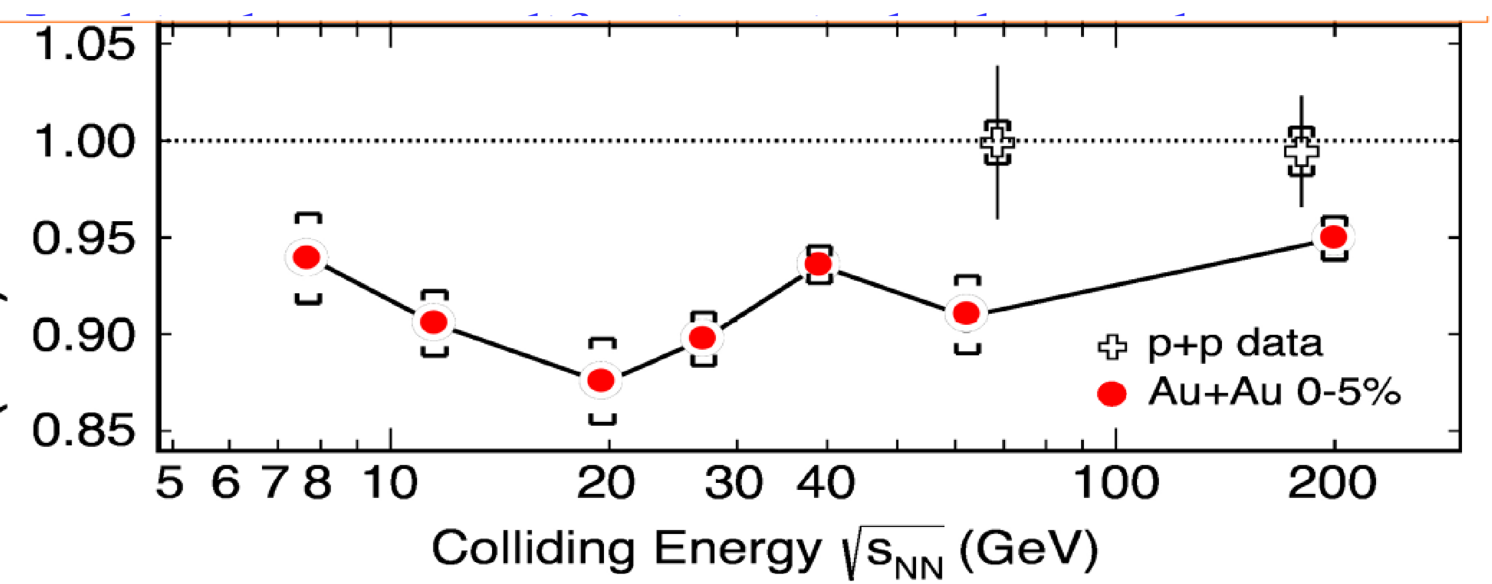
To look for Critical End Point, typically compute cumulants

$$c_n = \frac{\partial^n}{\partial \mu^n} p(T,\mu)$$



Expectation from theory, to right: corrections to c_3 are *positive*

But STAR finds that the corrections to c_3, below, are *negative*



$\frac{c_3}{c_2} \uparrow$

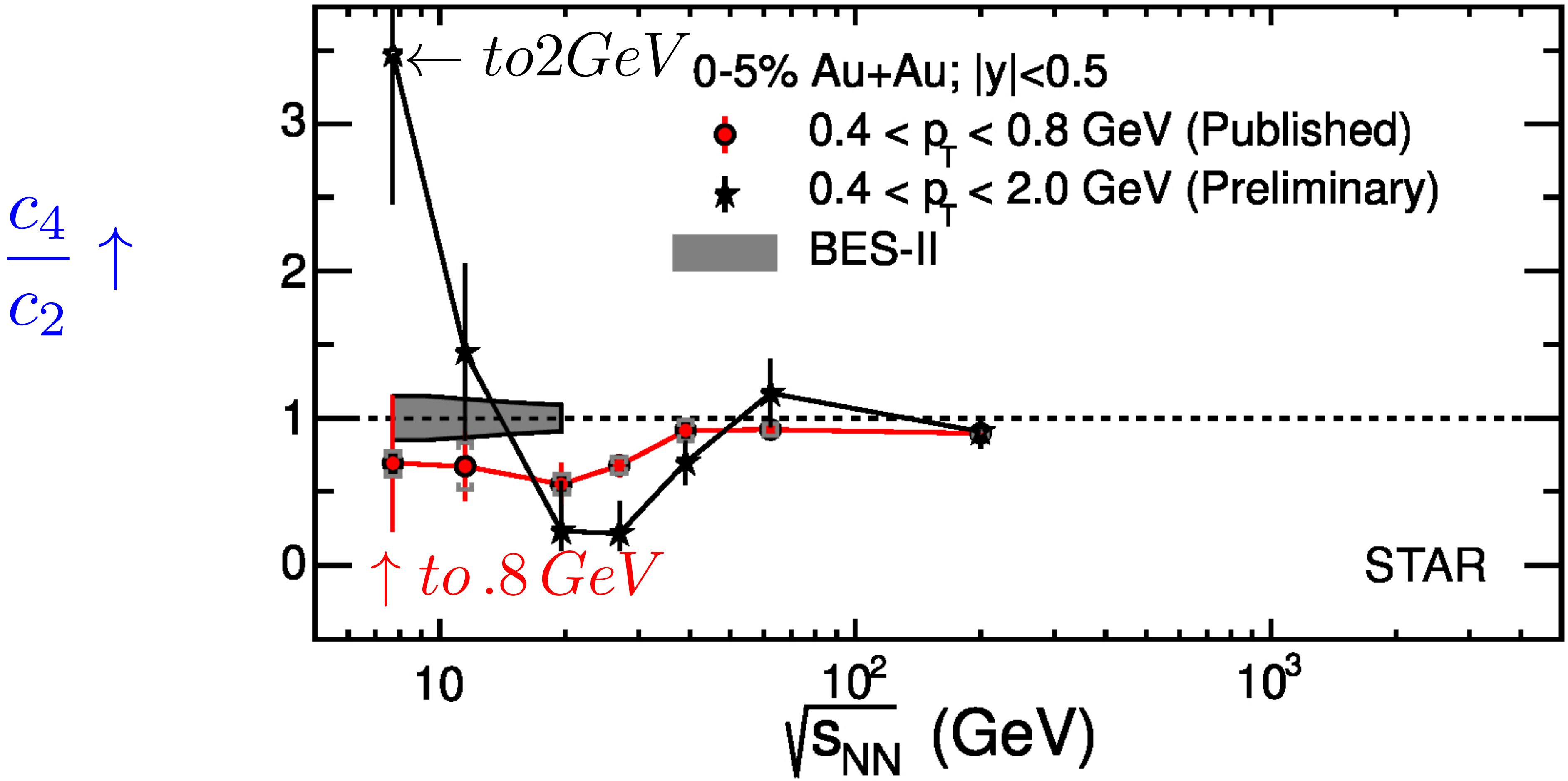
Divided by Skellam

Fluctuations at 7 GeV

Beam Energy Scan, down to 7 GeV.

Fluctuations *MUCH* larger when up to 2 GeV than to 0.8 GeV

Trivial multiplicity scaling? ... or first evidence for a Chiral Spiral?!



STAR: fig. 14,https://drupal.star.bnl.gov/STAR/files/STAR_iTPC_proposal_06_09_2015.pdf

Suggestion for experiment

For *any* sort of periodic structure (1D, 2D, 3D...),

fluctuations concentrated about some characteristic momentum k_0

So "slice and dice": bin in intervals, 0 to .5 GeV, .5 to 1., etc.

If peak in fluctuations in a bin not including zero, *may* be evidence for $k_0 \neq 0$.

If periodic structure, fluctuations must go *up* as $\sqrt{s}$ goes *down,* since μ increases

NJL models and Lifshitz points

Consider Nambu-Jona-Lasino models.

Nickel, 0902,1778 & 0906.5295 + ... + Buballa & Carignano 1406.1367

$$\mathcal{L}_{\mathrm{NJL}} = \overline{\psi}(\partial\!\!\!/ + g\sigma)\psi + \sigma^2$$

Integrating over ψ,

$$\log(\partial\!\!\!/ + g\sigma) \sim \ldots + \kappa_1((\partial\sigma)^2 + \sigma^4) + \kappa_2((\partial^2\sigma)^2 + \sigma^2(\partial\sigma)^2 + \sigma^6) + \ldots$$

Consequently, in NJL @ 1-loop, *tricritical* = *Lifshitz point.*

Above due to scaling $\partial \to \xi\partial\ ,\ \sigma \to \xi\sigma$.

Special to including only σ at one loop.

Not generic: violated by the inclusion of more fields, to two loop order, etc.