Cosmological Constant Problem: Deflation During Inflation

B. Koch

with

F. Canales, C. Laporte, & A. Rincon,

based on:

JCAP no 1, 21, 2020, ArXiv:1812.10526

Frankfurt, Nuclear Physics Colloquium

16.01. 2020

acknowledge:

VRI, Fondecyt

Collaboration

- Angel Rincon, Cristobal Laporte, & Felipe Canales

René Araneda

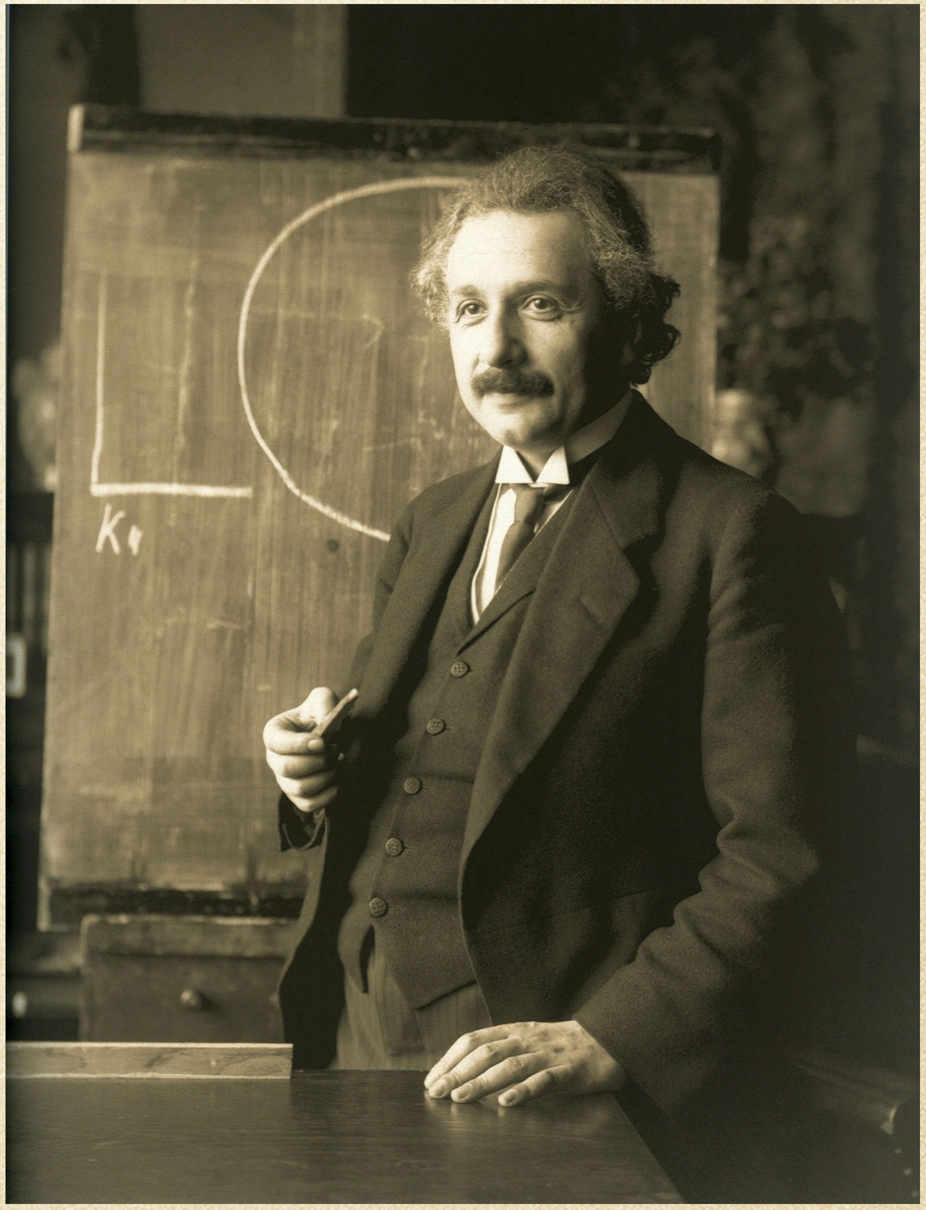
Content

- Cosmological constant problem, status
- Conceptual problem, in evolving Universe
- Scale dependent framework & evolving Universe
- Possible solution: Deflation during inflation
- Link to Asymptotic Safety
- Conclusion

The CCP 1.0

The CCP 1.0

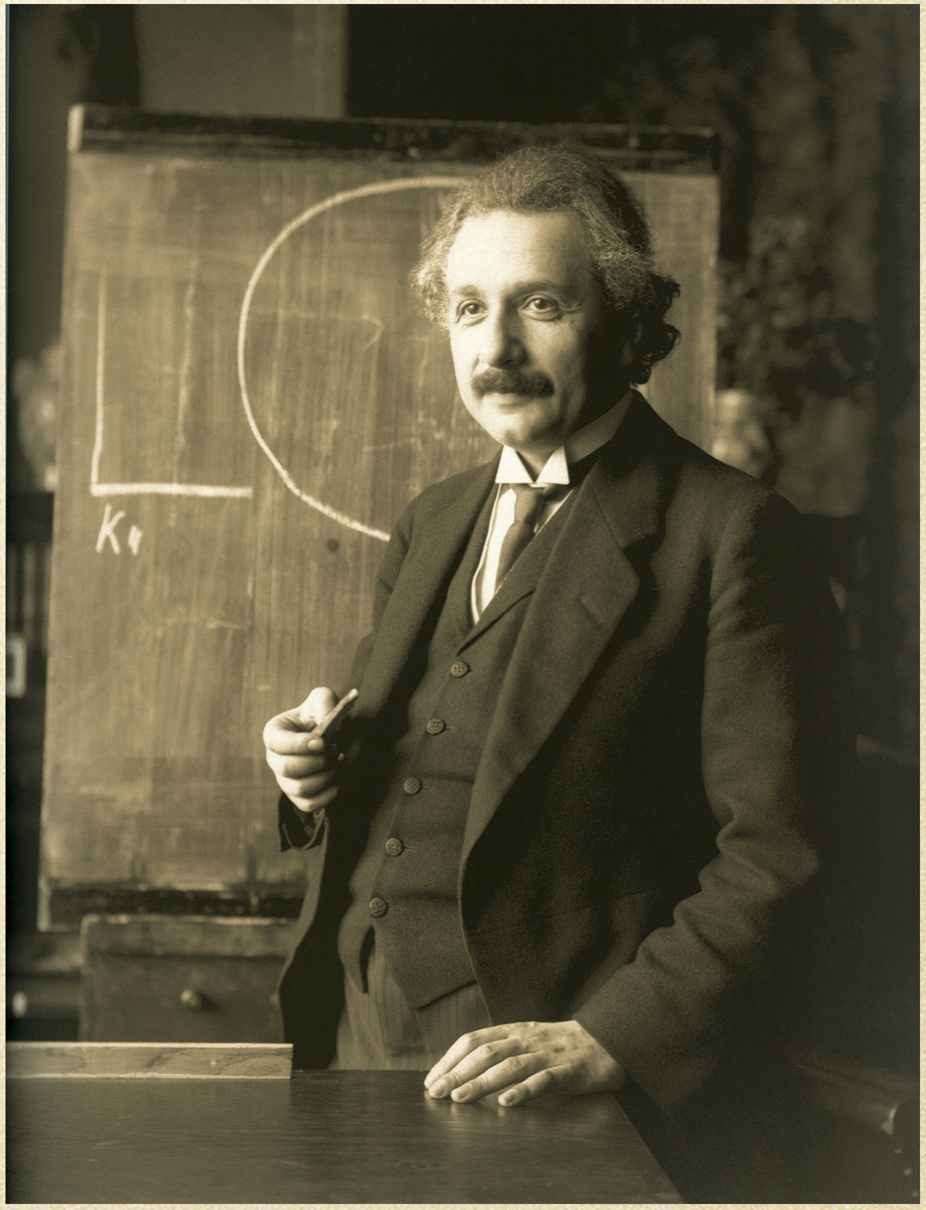
Albert Einstein



The CCP 1.0

Albert Einstein

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$



The CCP 1.0

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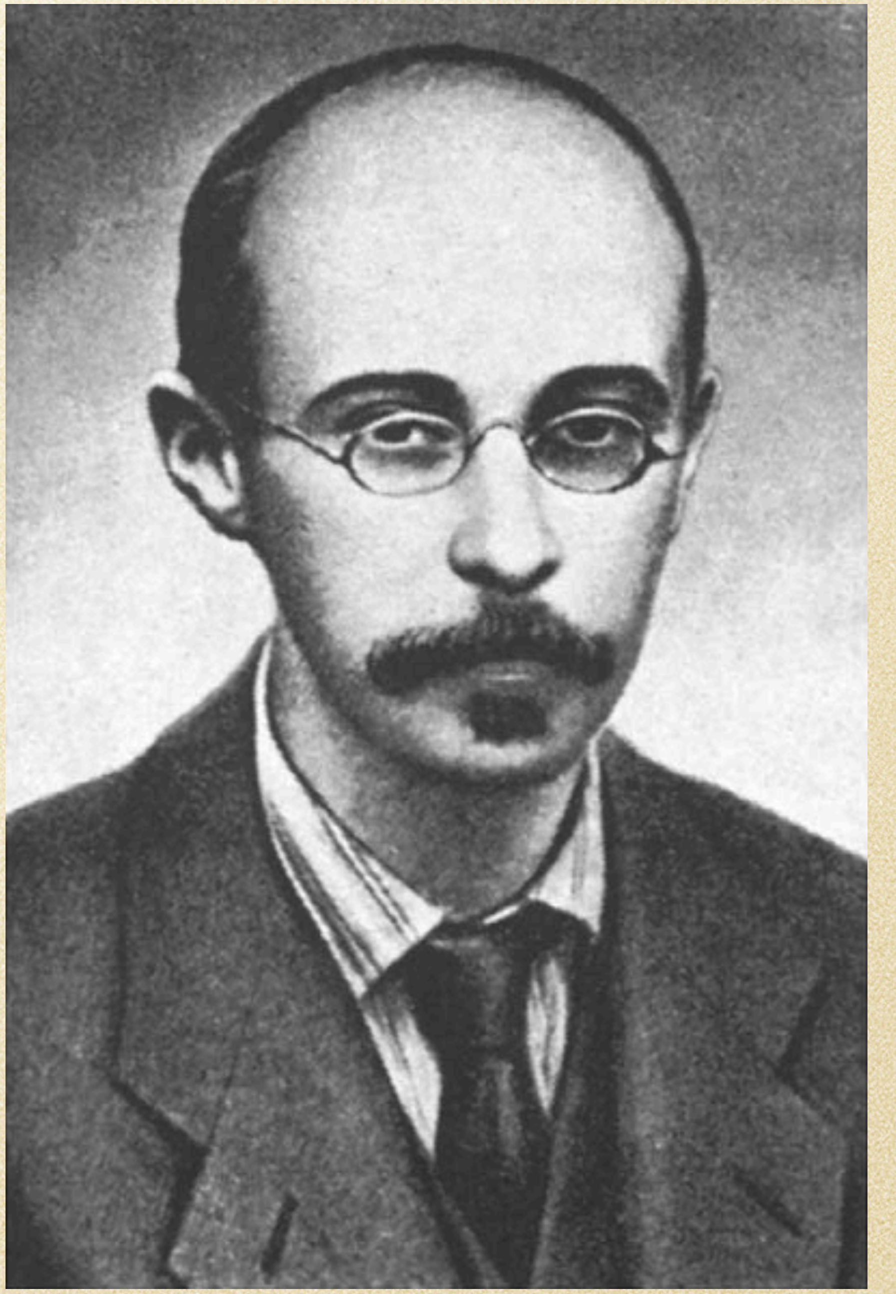
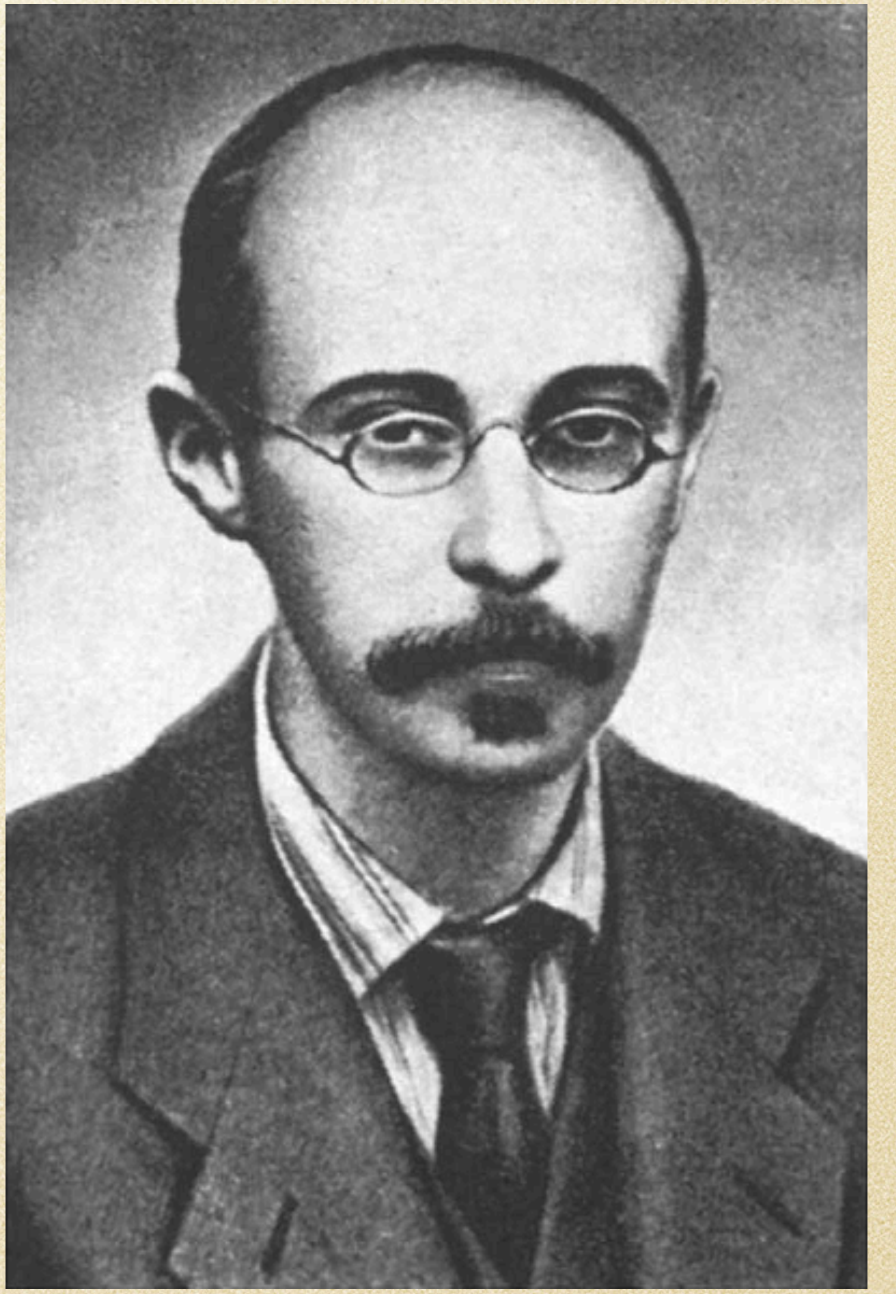
Alexander Friedmann



The CCP 1.0

Alexander Friedmann

$$ds^2 = a(t)ds_3^2 - dt^2$$



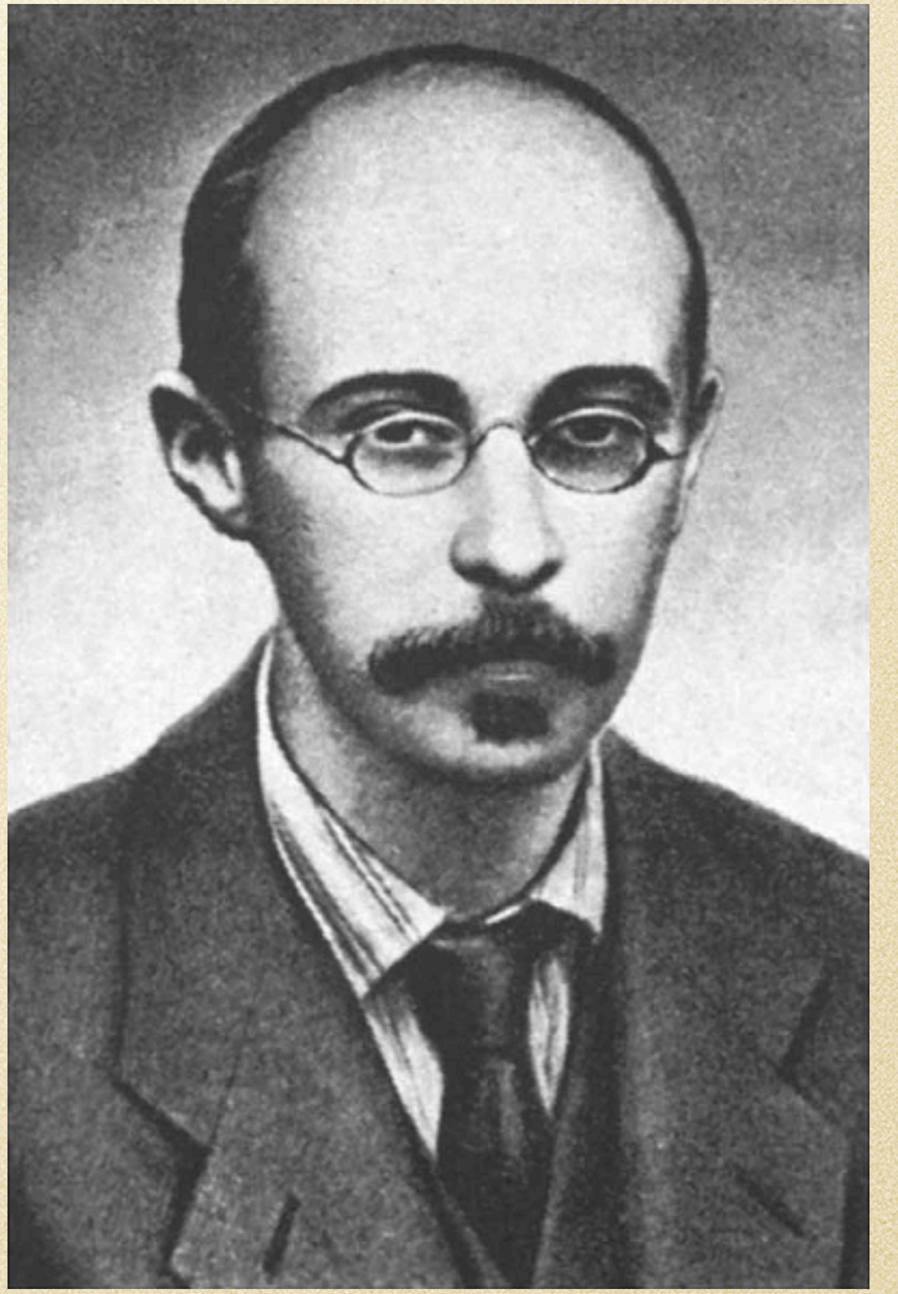
The CCP 1.0

Alexander Friedmann

$$ds^2 = a(t)ds_3^2 - dt^2$$

$$\frac{\dot{a}^2 + k}{a^3} = \frac{1}{3}8\pi G\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$



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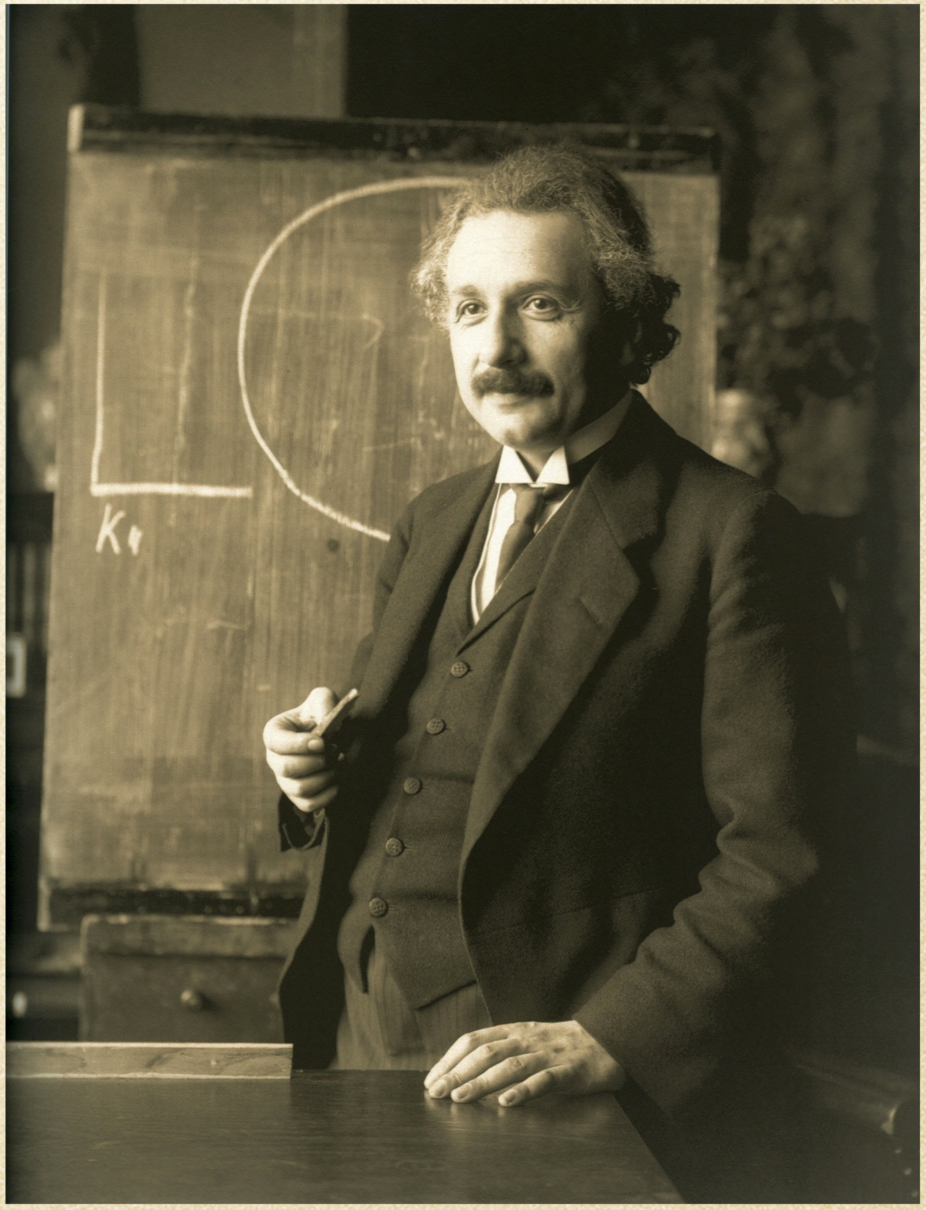

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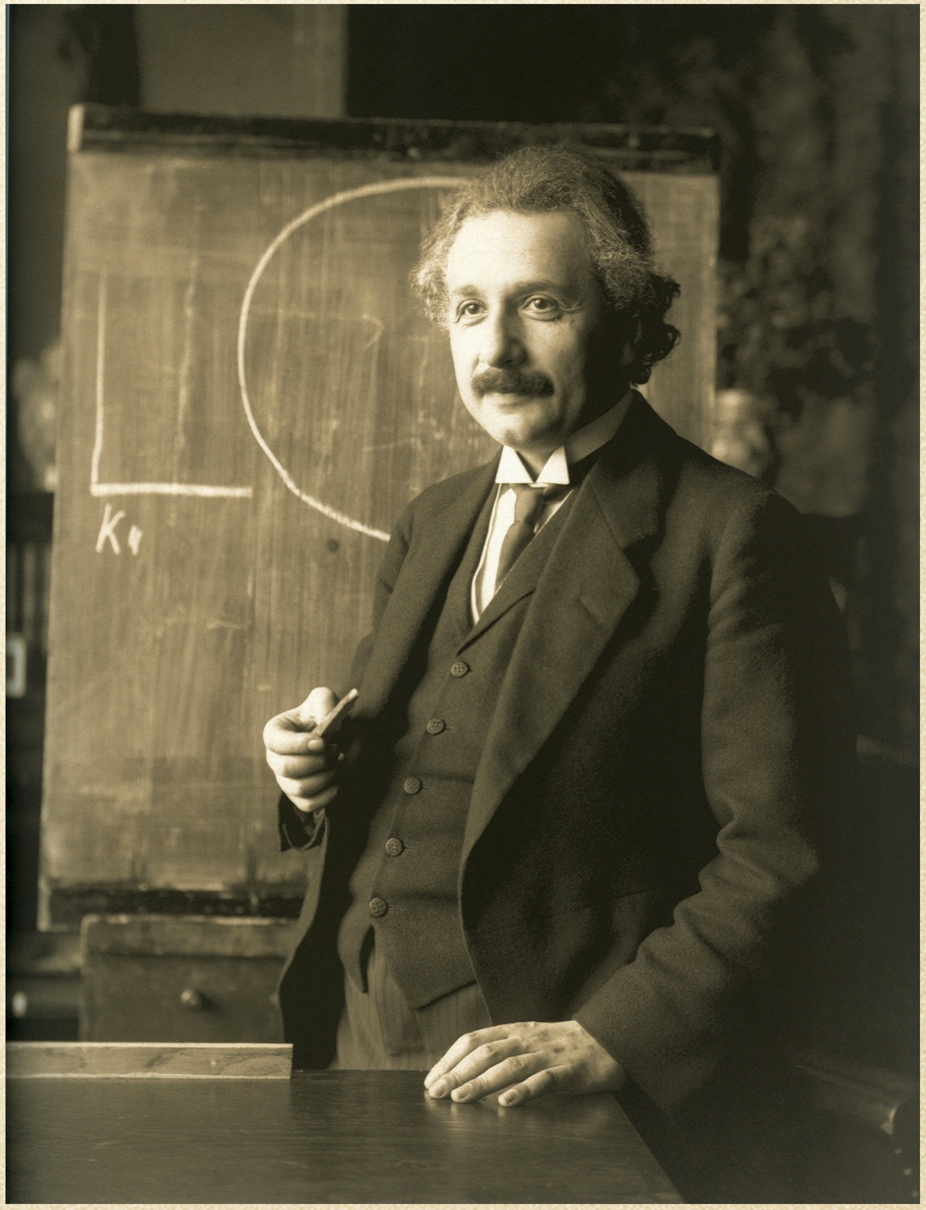

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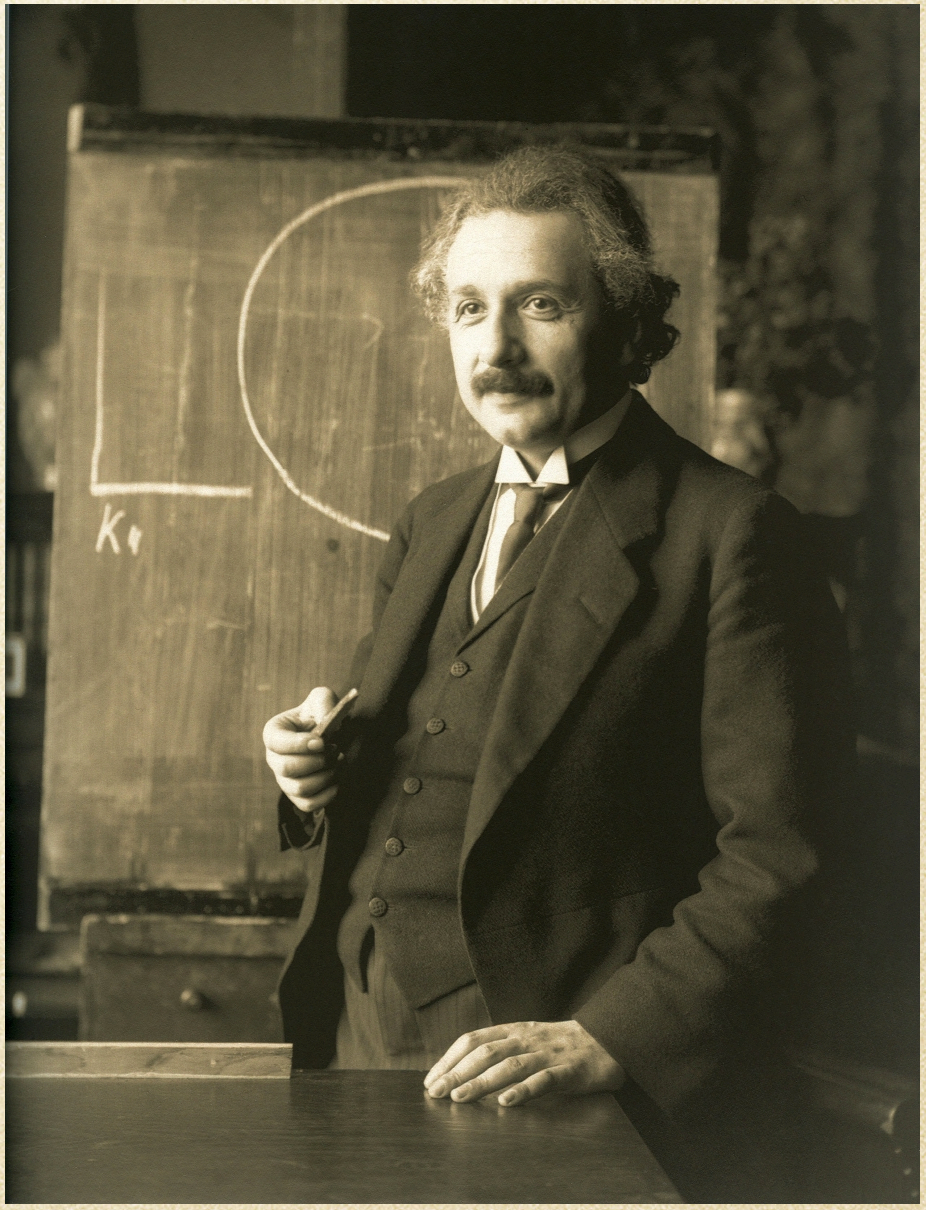

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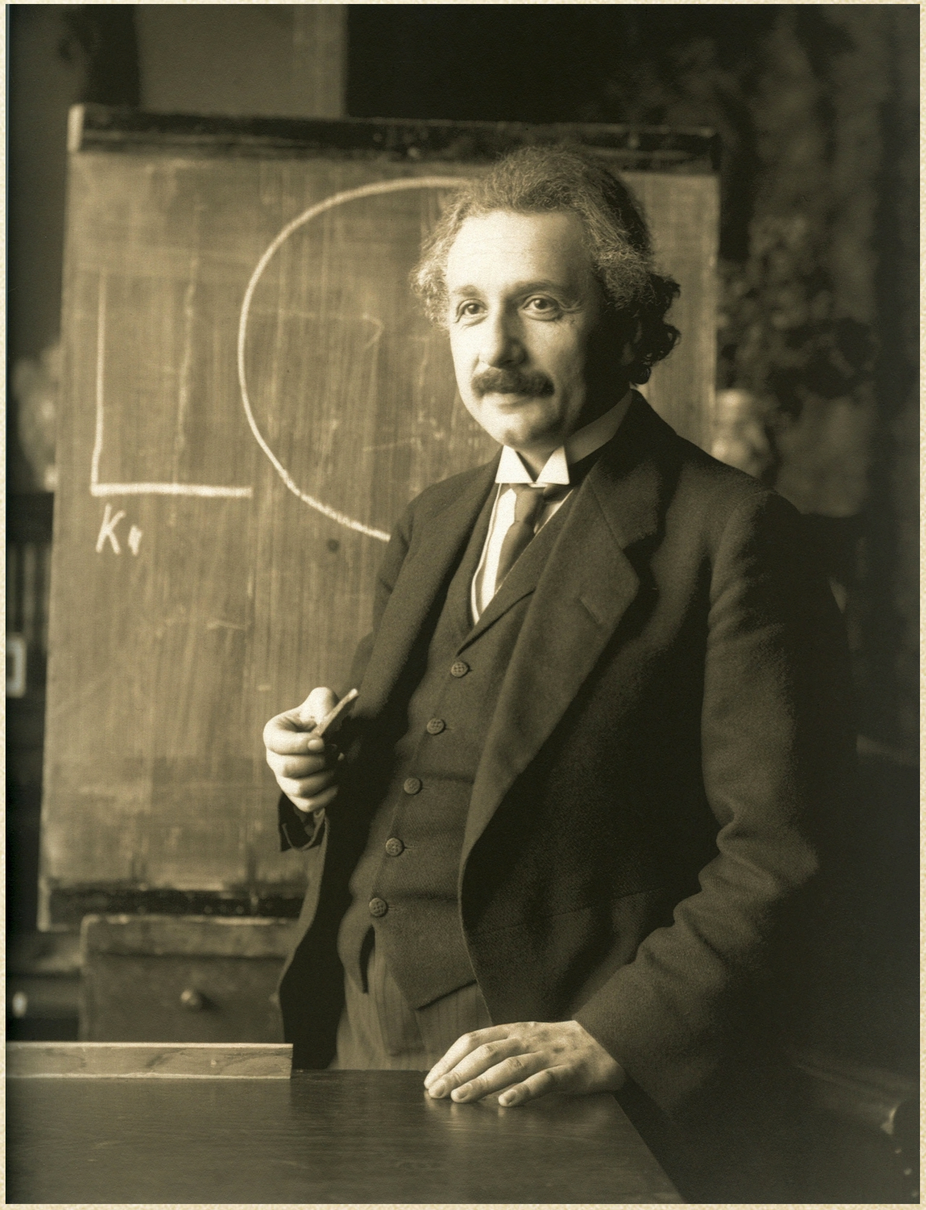

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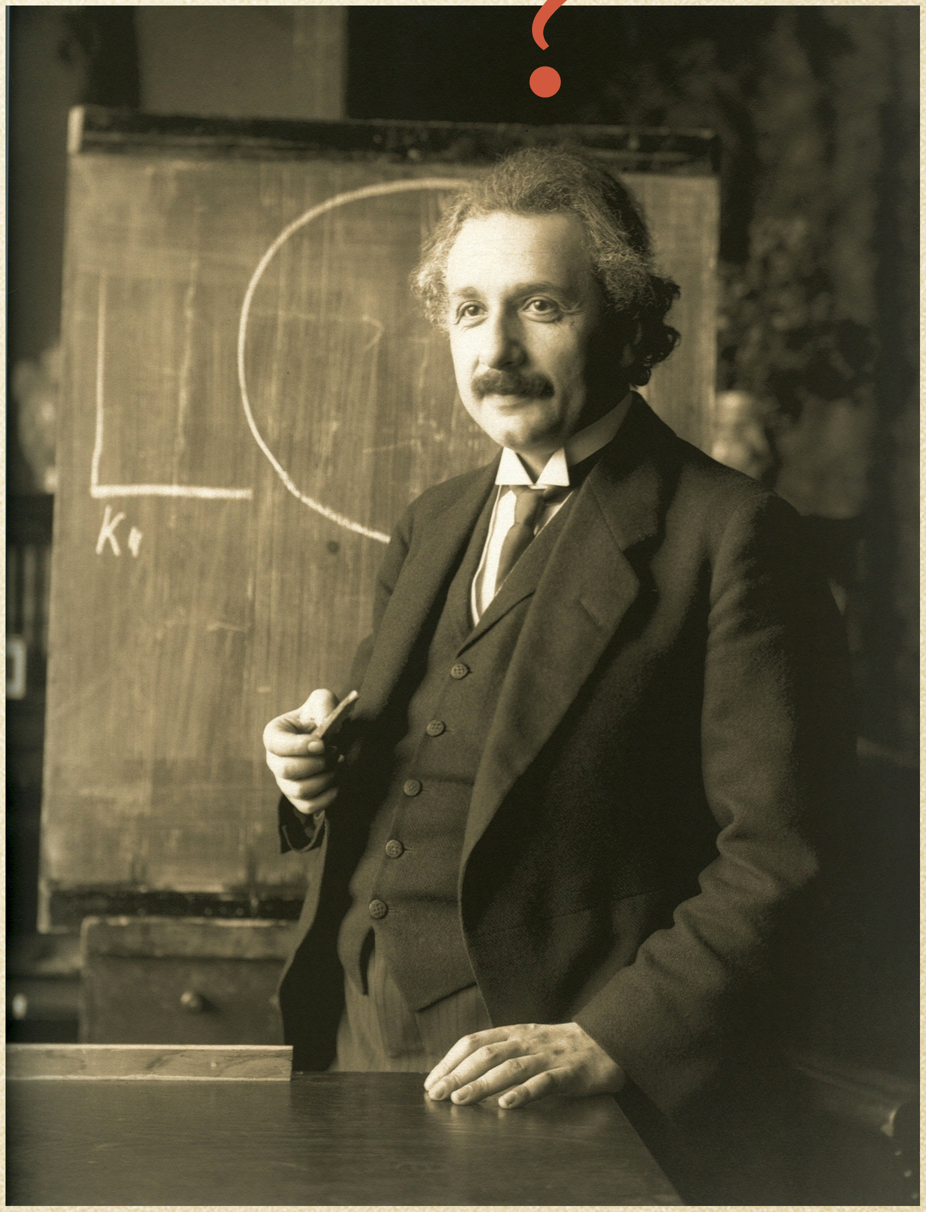

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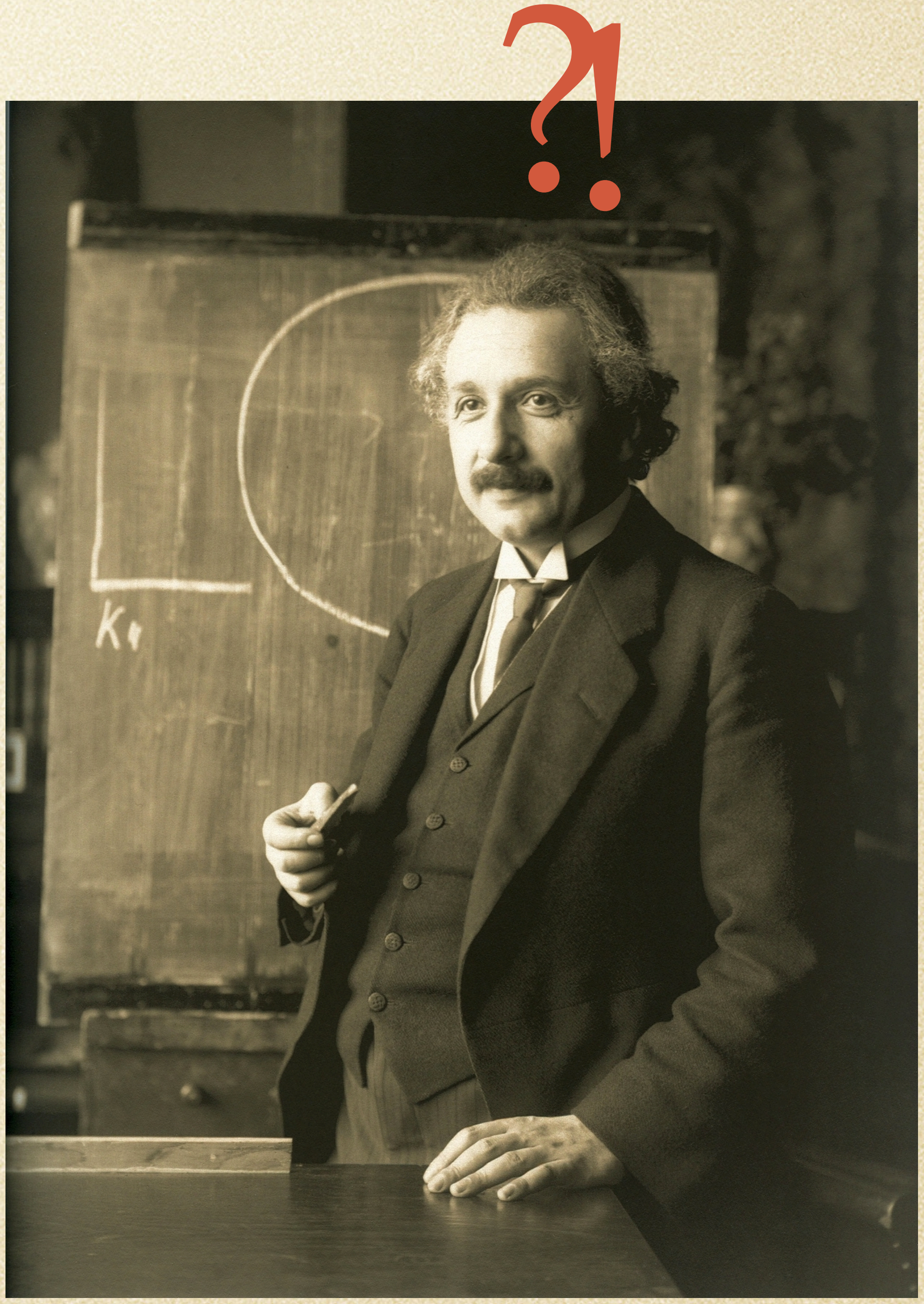

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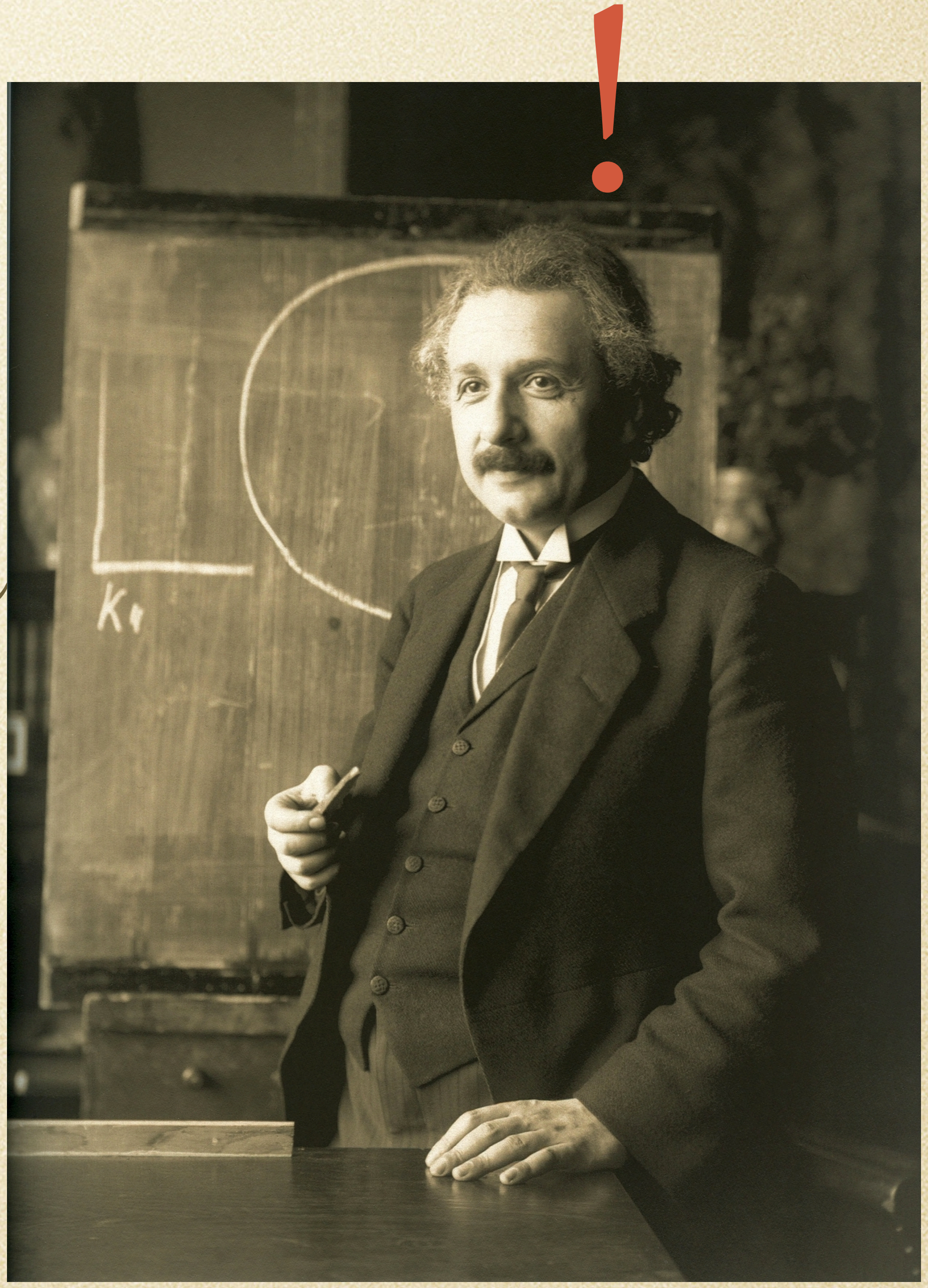

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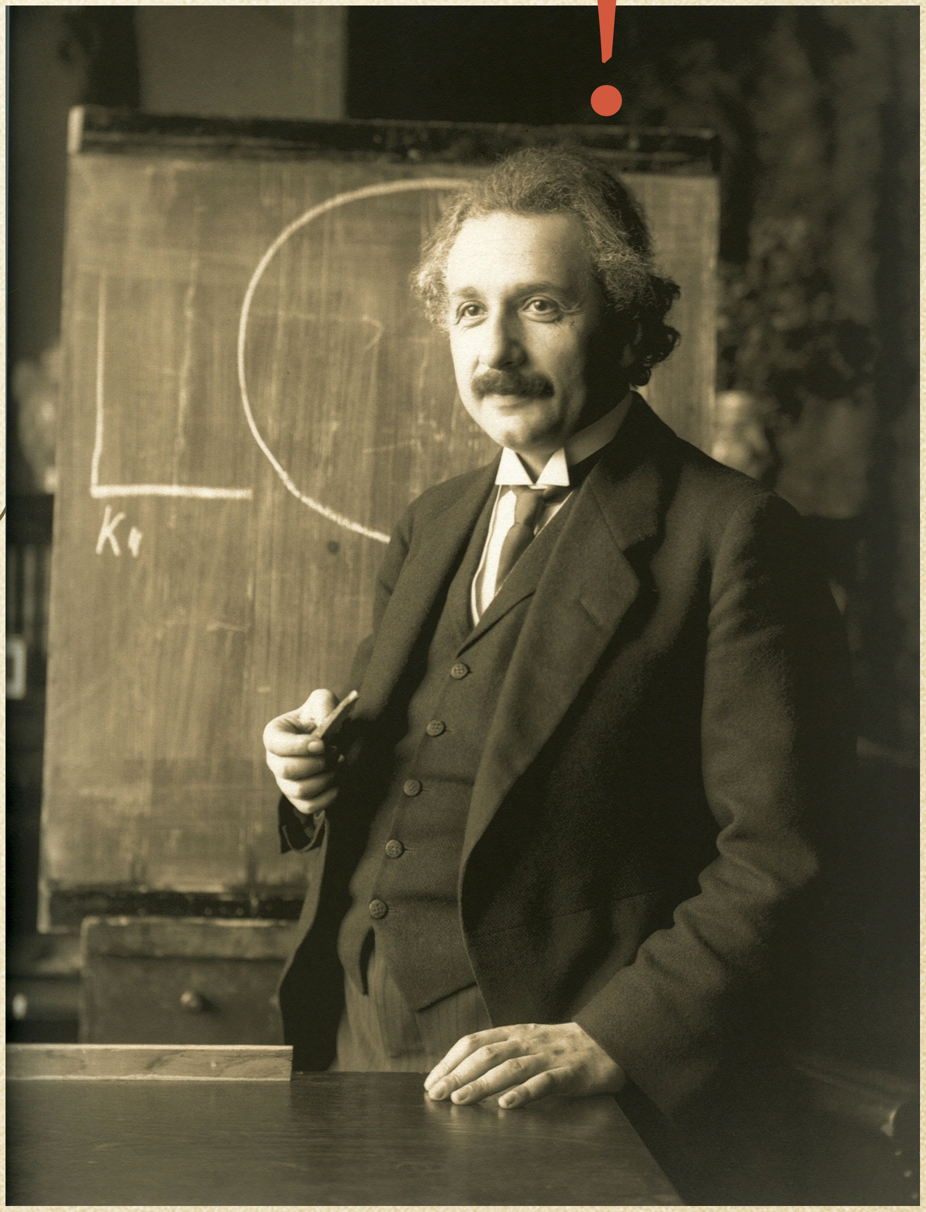

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

!

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3} \equiv 0$$

static possible $\dot{a} \equiv 0$

!

The CCP 2.0

The CCP 2.0

Edwin Hubble, Georges Lemaítre

The CCP 2.0

Edwin Hubble, Georges Lemaítre measurement:

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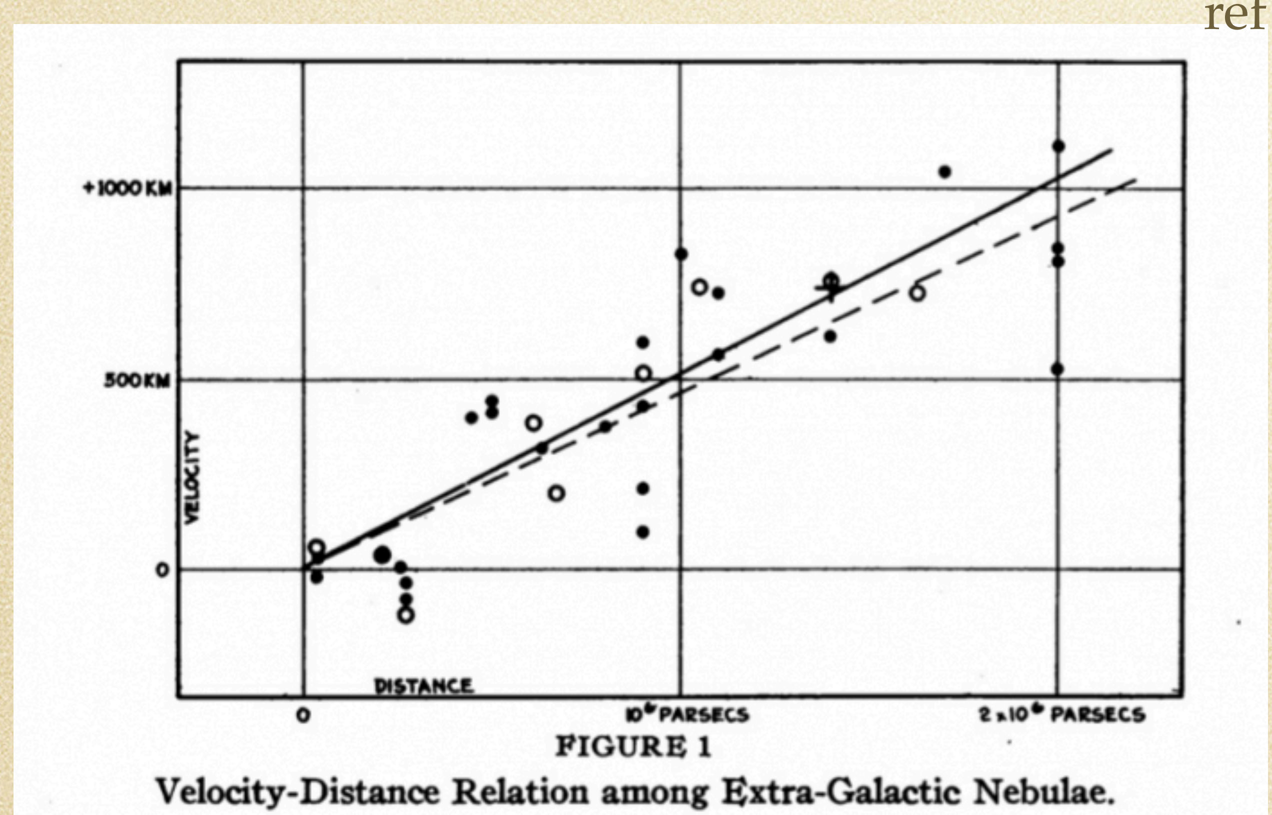
Edwin Hubble, Georges Lemaítre

measurement:

ref [1]



1927,1929

The CCP 2.0

Edwin Hubble

measurement:

$$\dot{a} > 0$$

The CCP 2.0

Edwin Hubble

measurement:

$$\dot{a} > 0$$

later:

$$\dot{a} = 67.66 \pm 0.42 \frac{km/s}{Mpc}$$

(Planck collaboration 2018)

The CCP 2.0

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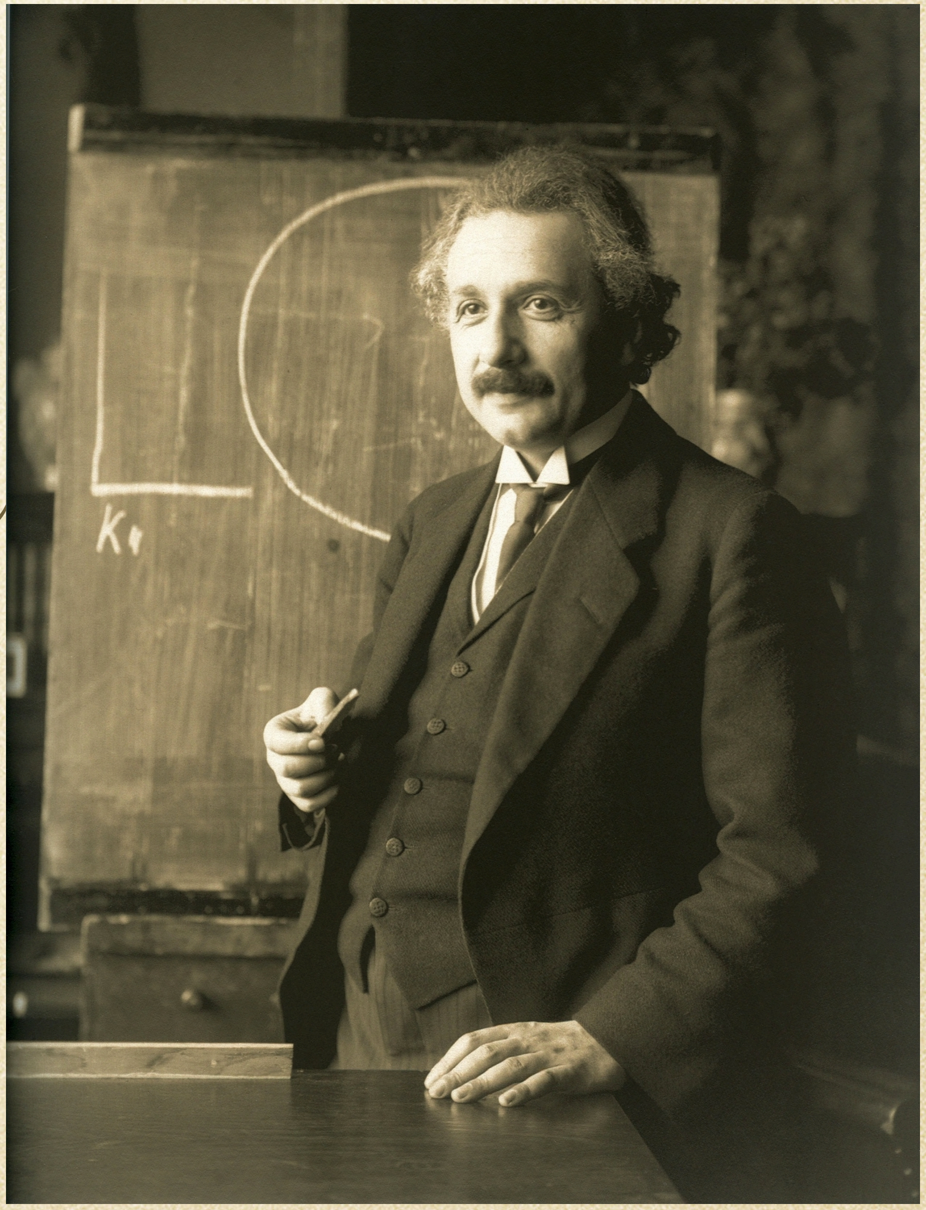

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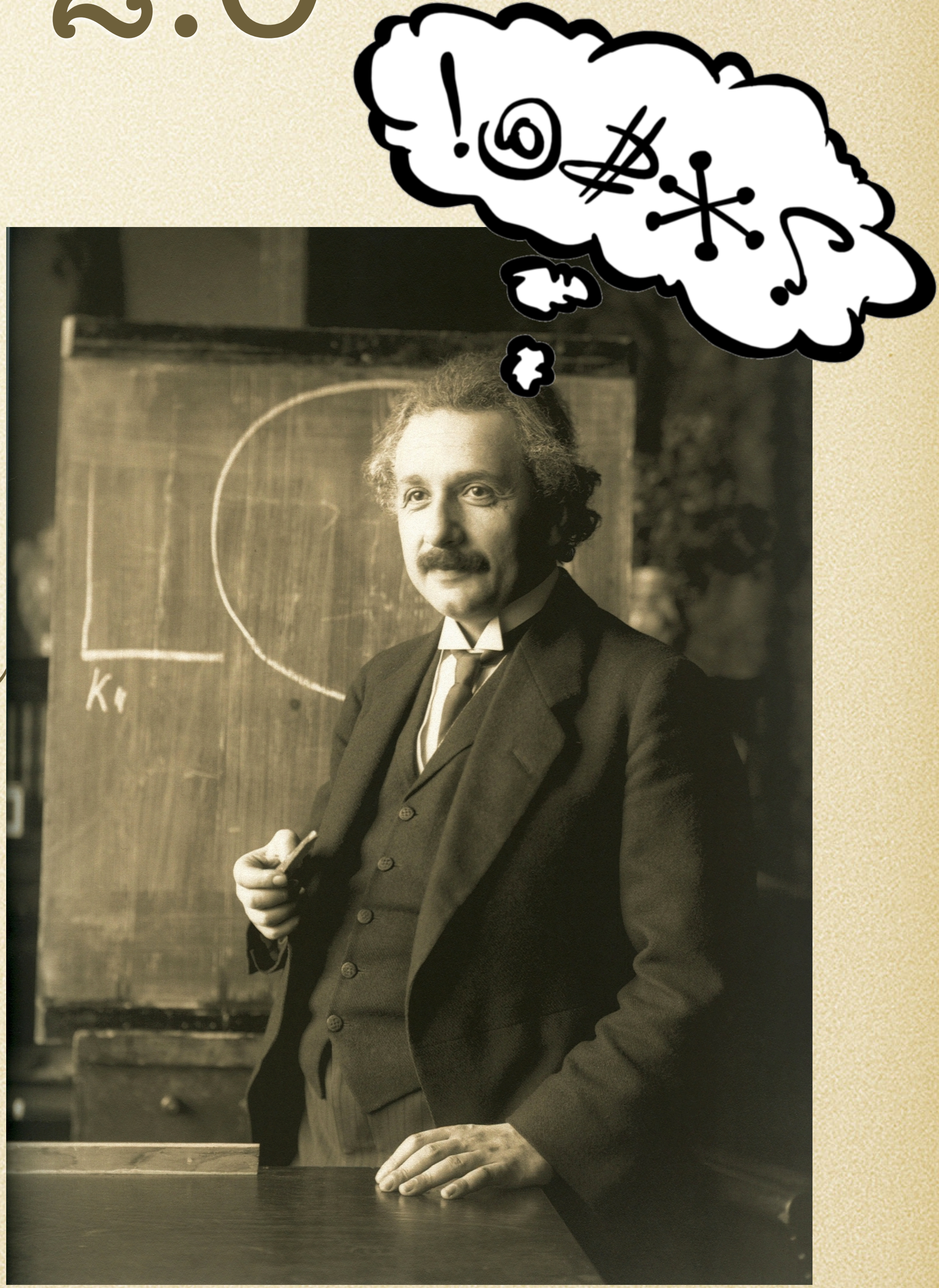

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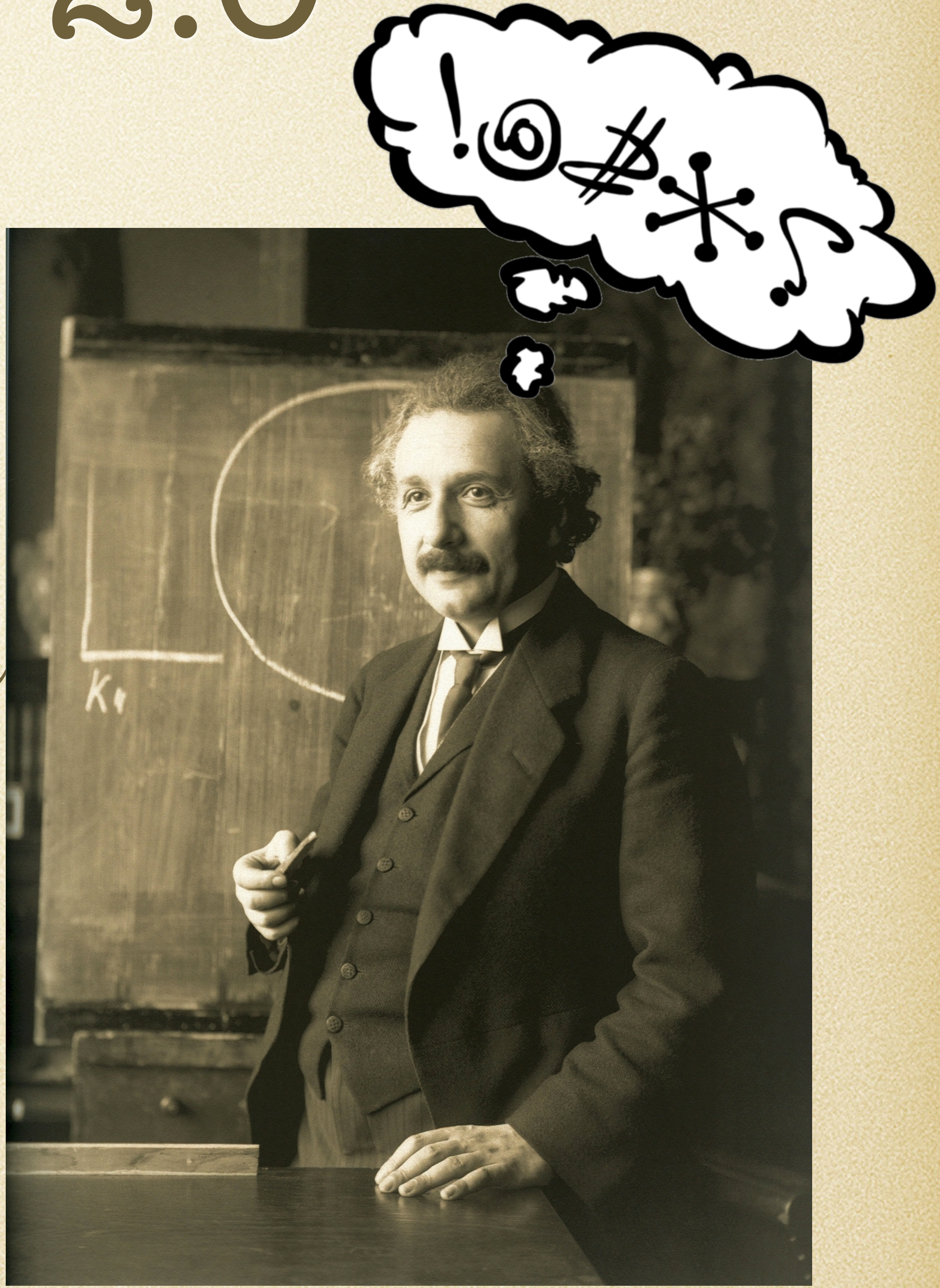

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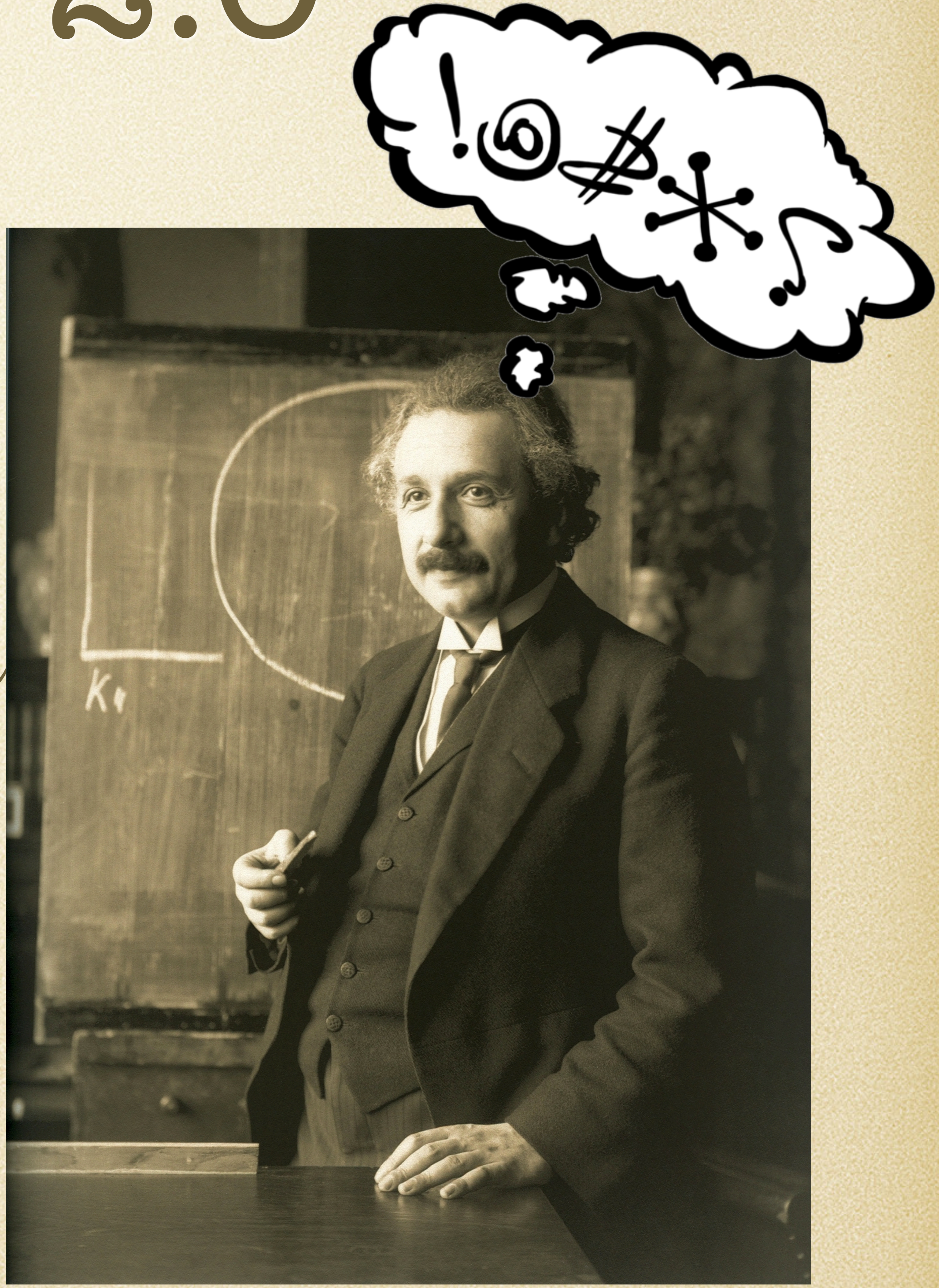
not static $\dot{a} \neq 0$

$\ddot{a} < 0$



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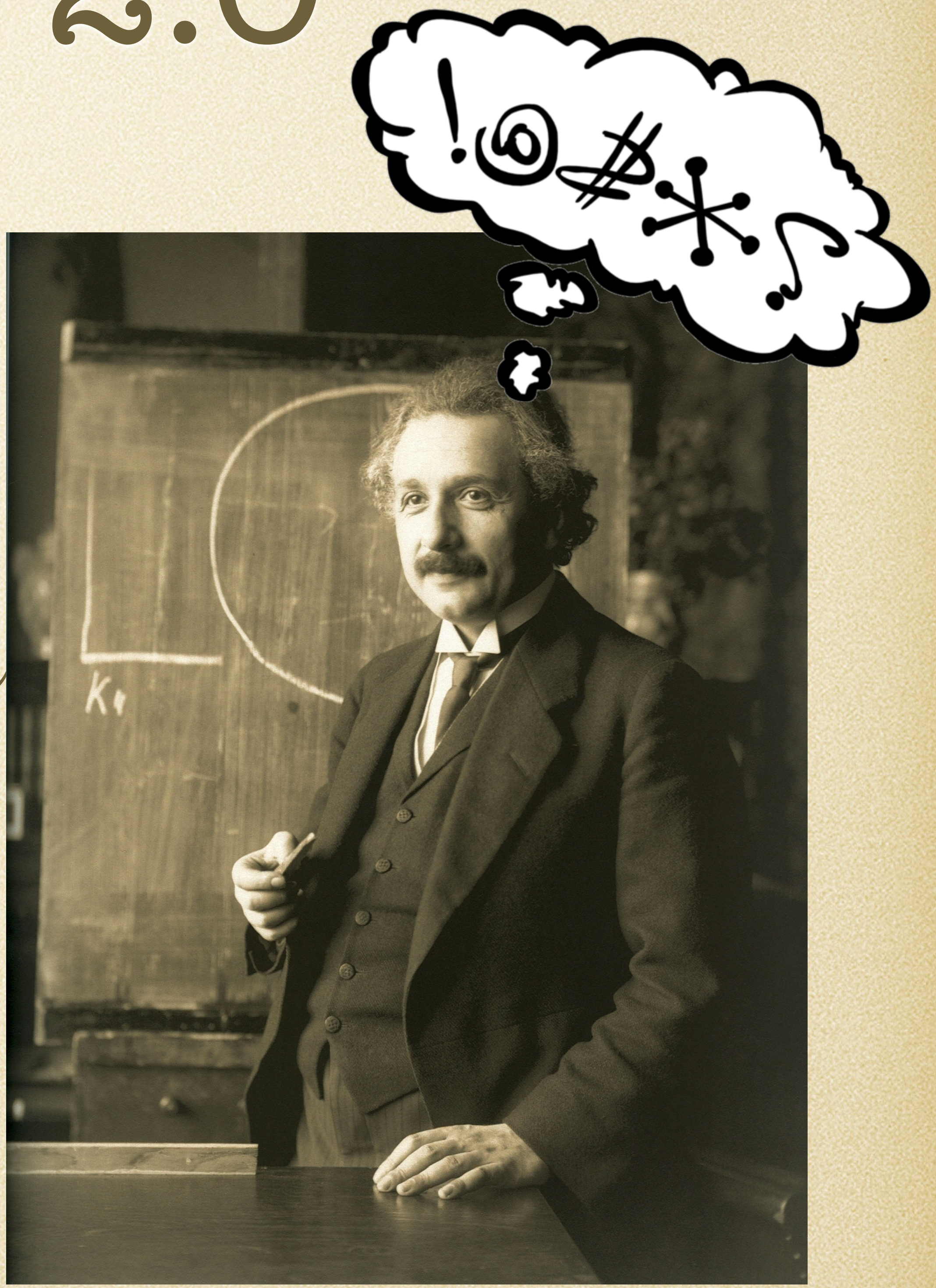

"biggest blunder"

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The CCP 3.0

S. Perlmutter, A. Riess, B. Schmidt, & others

ref [2]

The CCP 3.0

S. Perlmutter, A. Riess, B. Schmidt, & others

measurements: ref [2]

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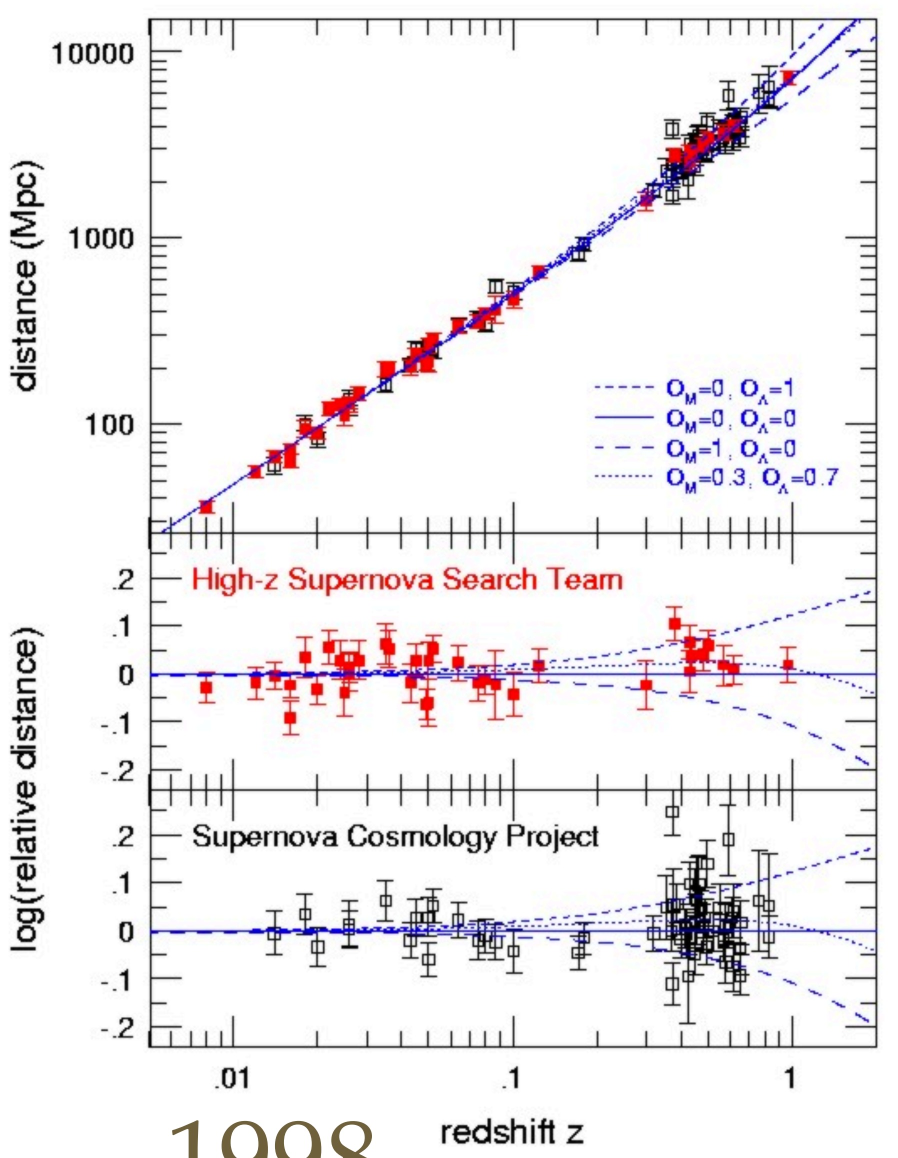
S. Perlmutter, A. Riess, B. Schmidt, & others

mea

ref [2]



1998

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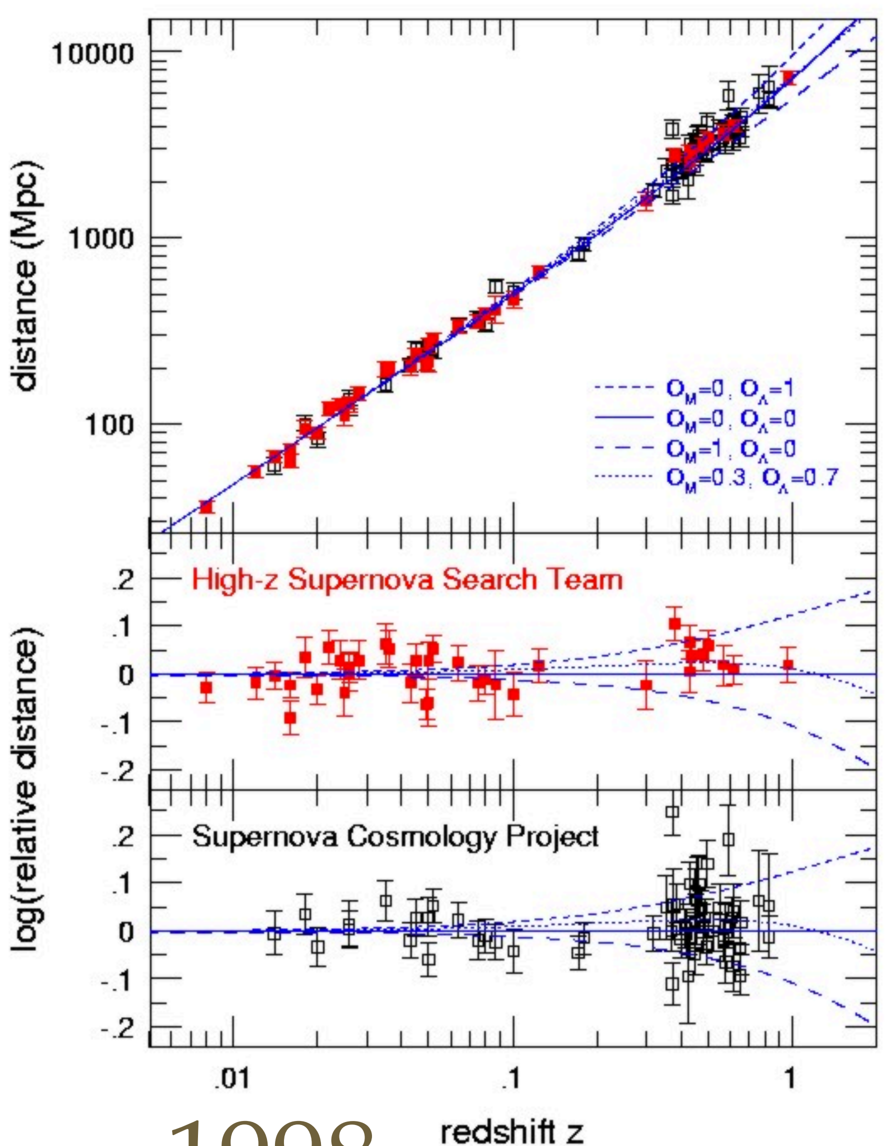
S. Perlmutter, A. Riess, B. Schmidt, & others

mea

ref [2]



1998

$\dot{a} \neq 0$ $\ddot{a} < 0$

The CCP 3.0

S. Perlmutter, A. Riess, B. Schmidt, & others

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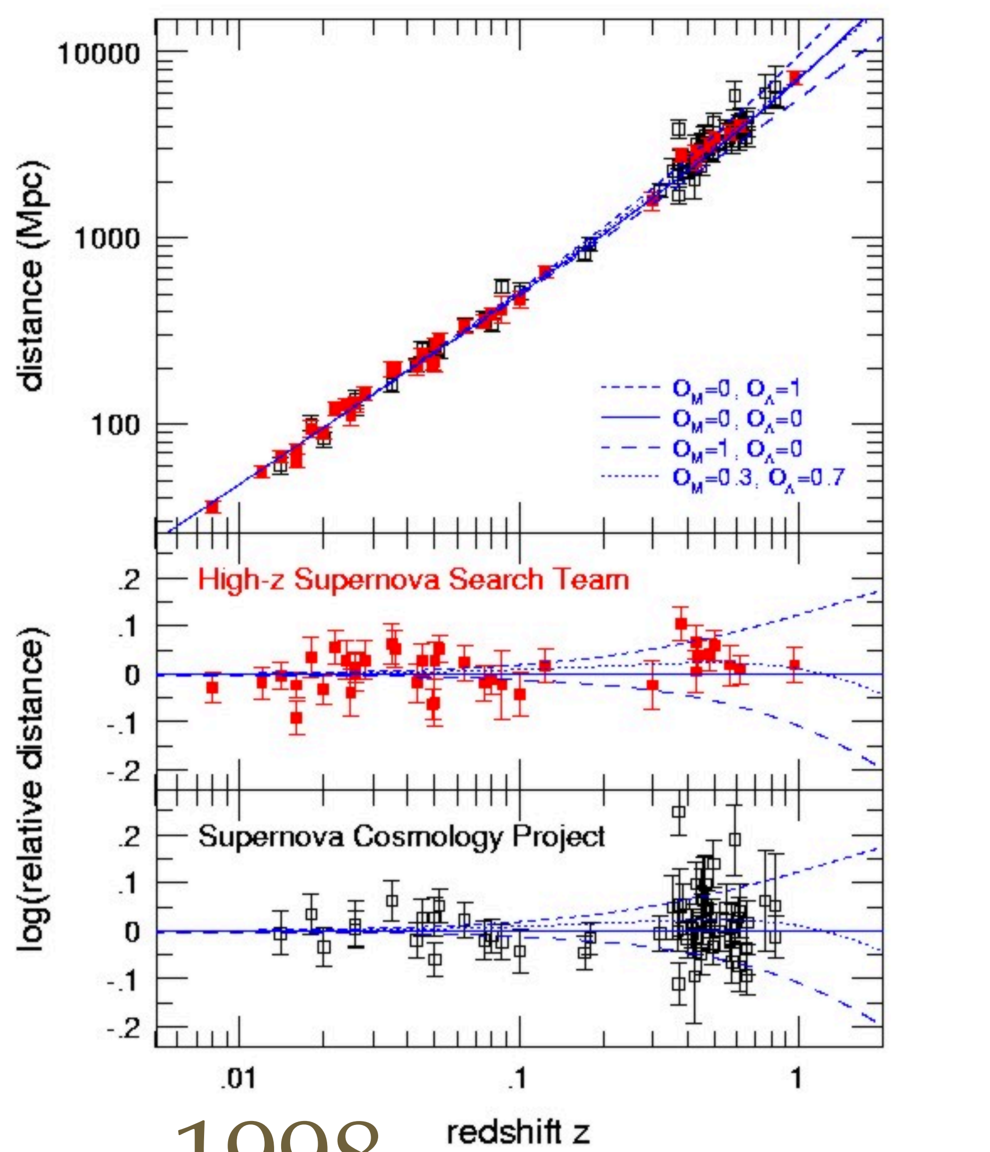

1998

$\dot{a} \neq 0$

~~$\dot{a}$ 0~~

$\ddot{a} > 0$!

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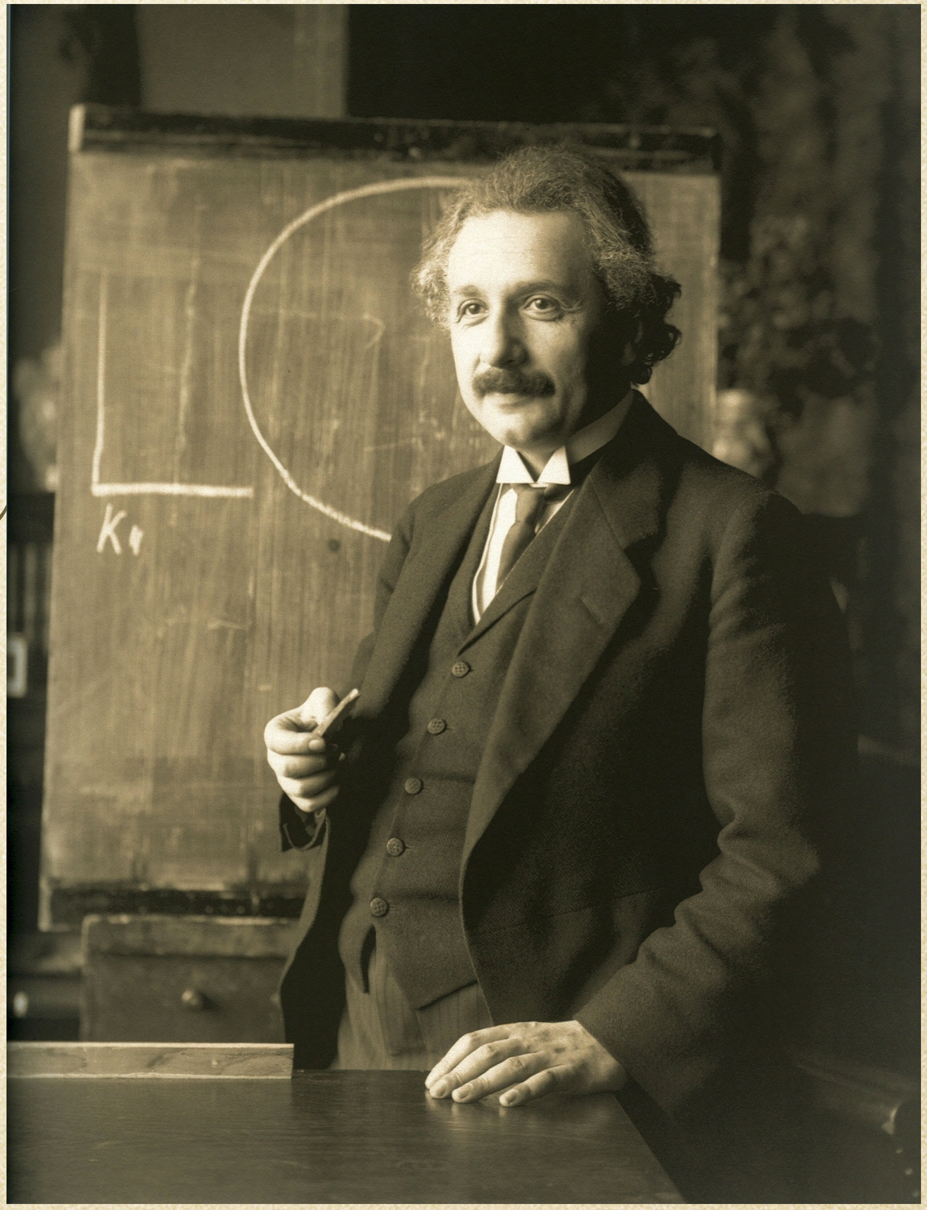

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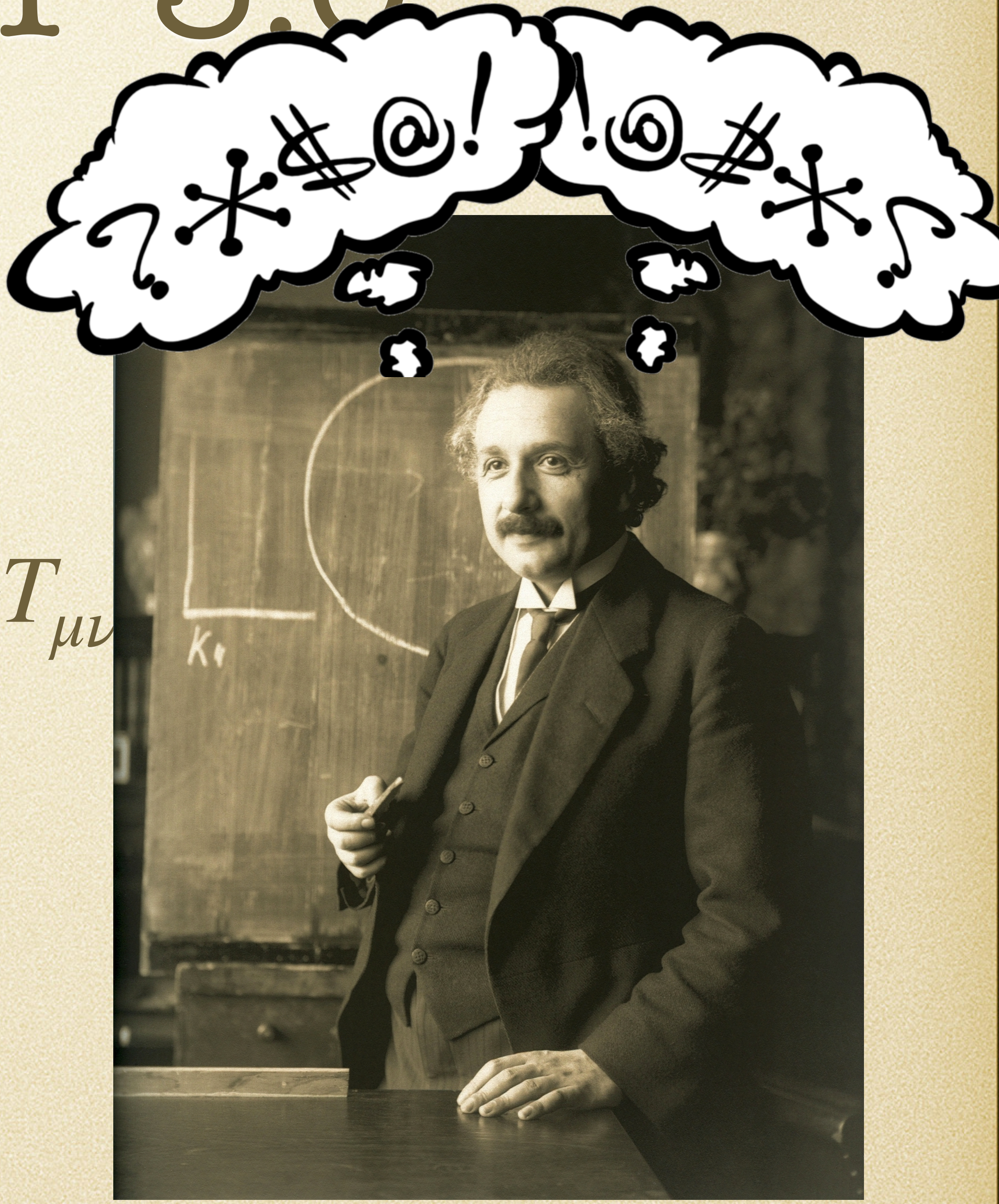

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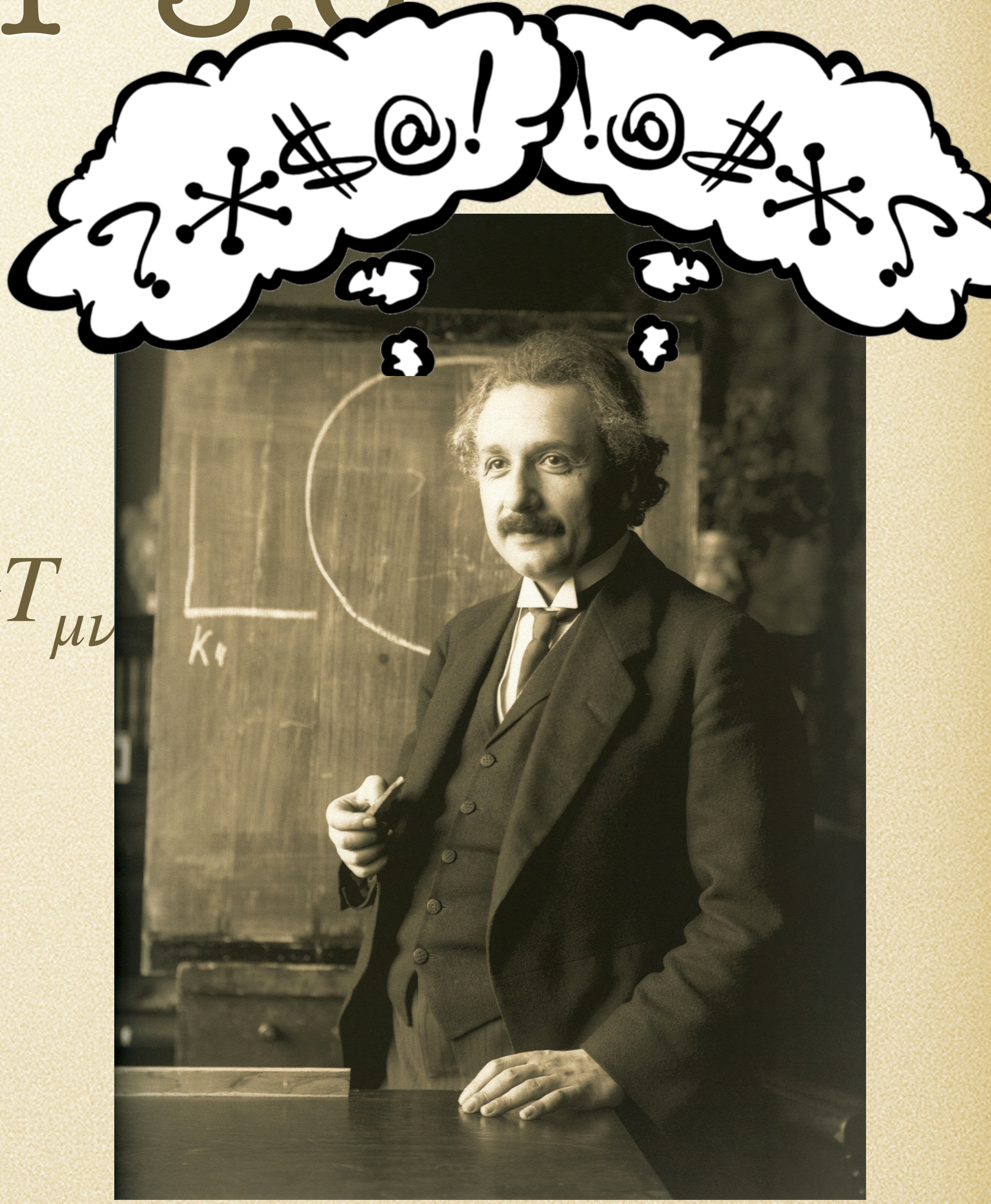

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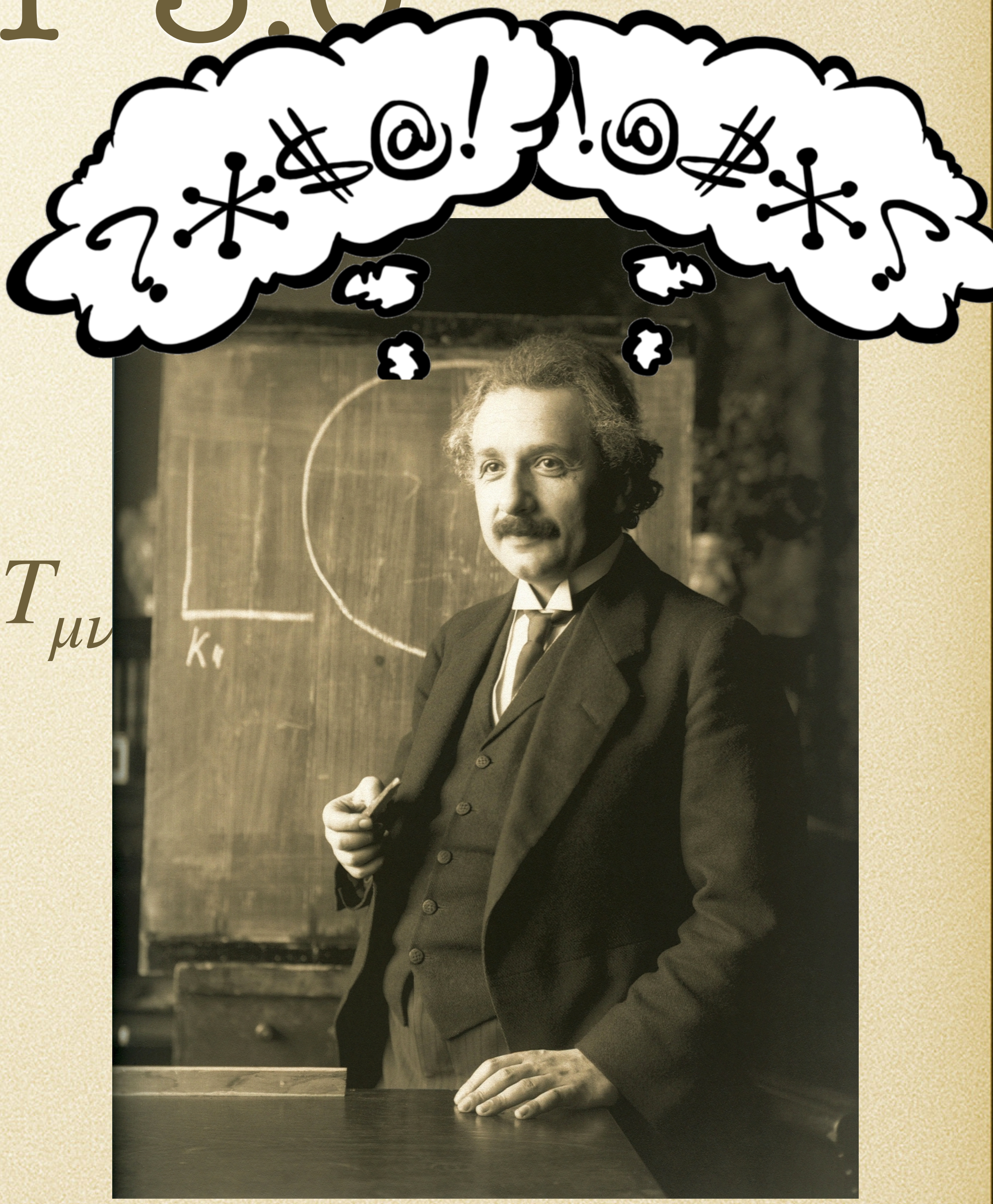

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$$

$$\Lambda > 0 \Rightarrow \ddot{a} > 0$$

The CCP 3.0

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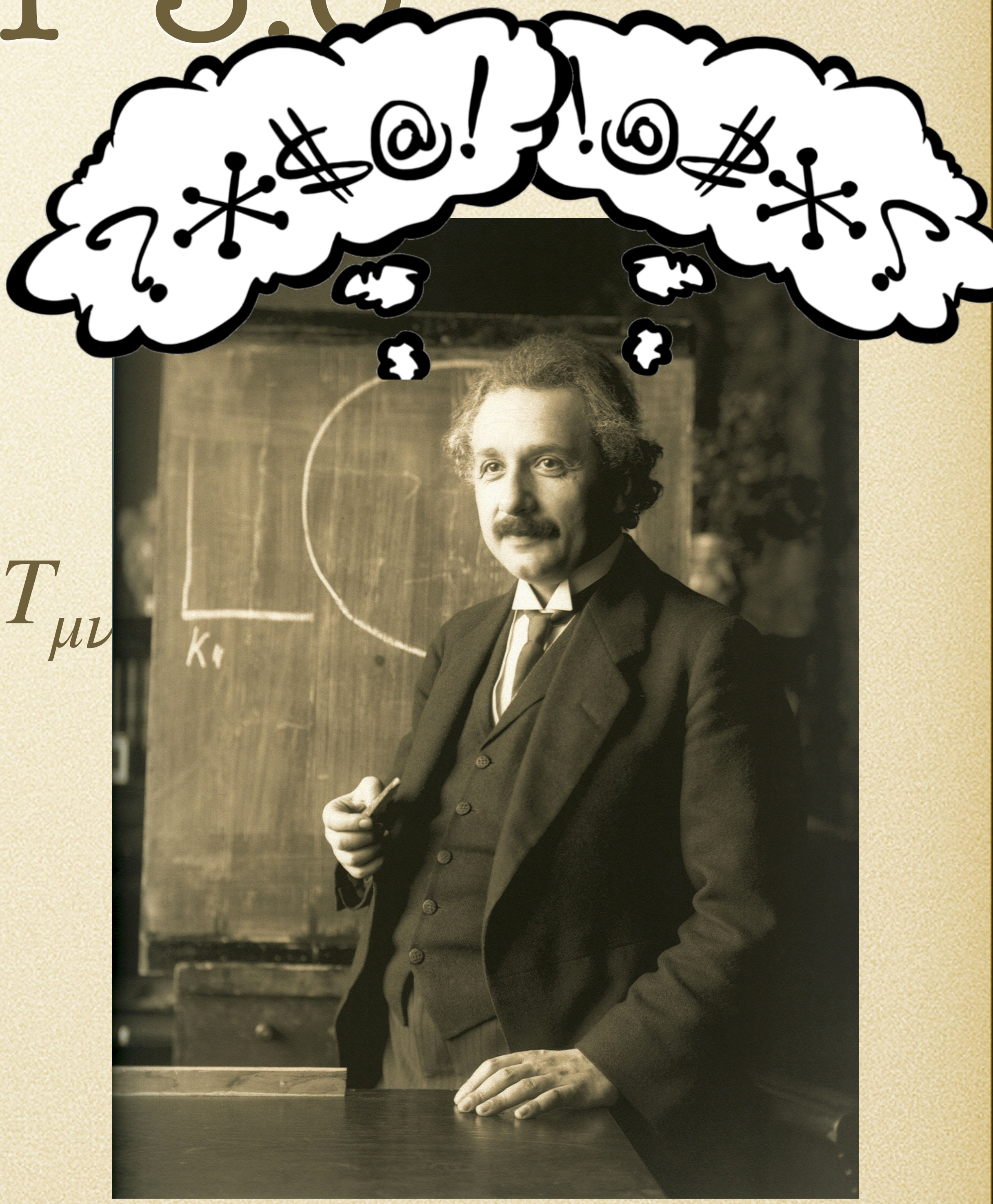

"???"

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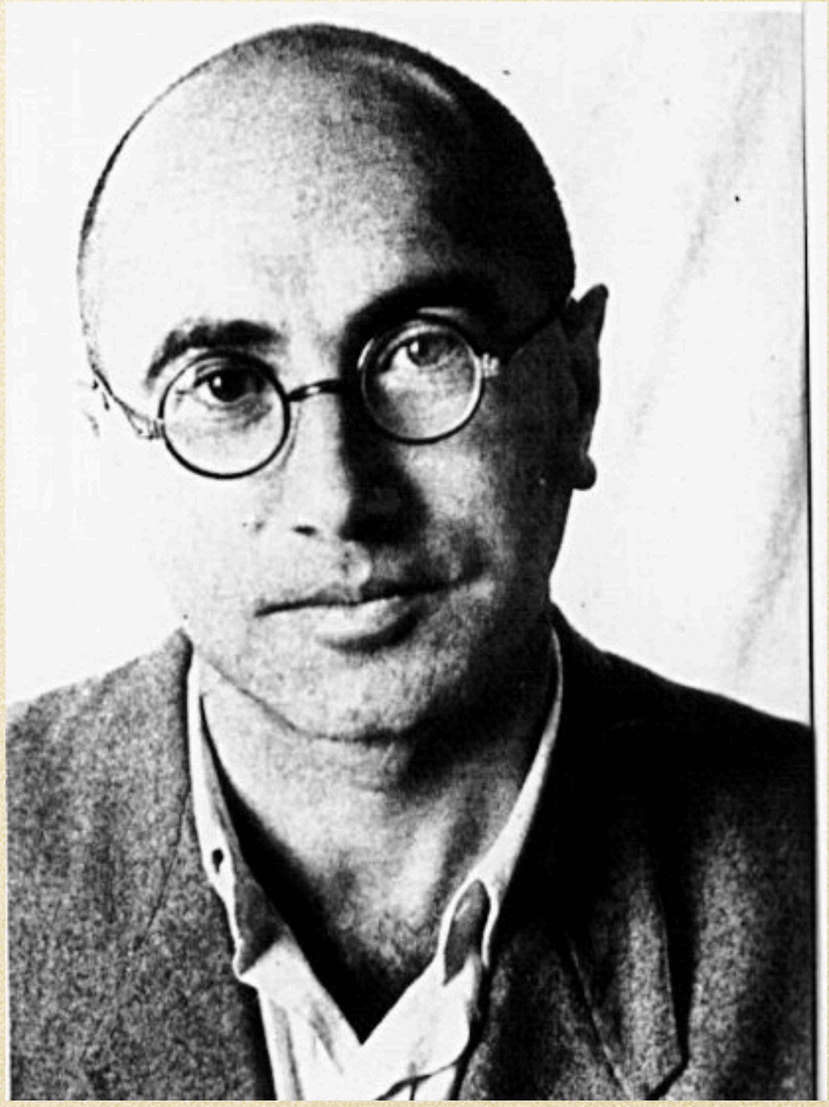
Yakov Zeldovich



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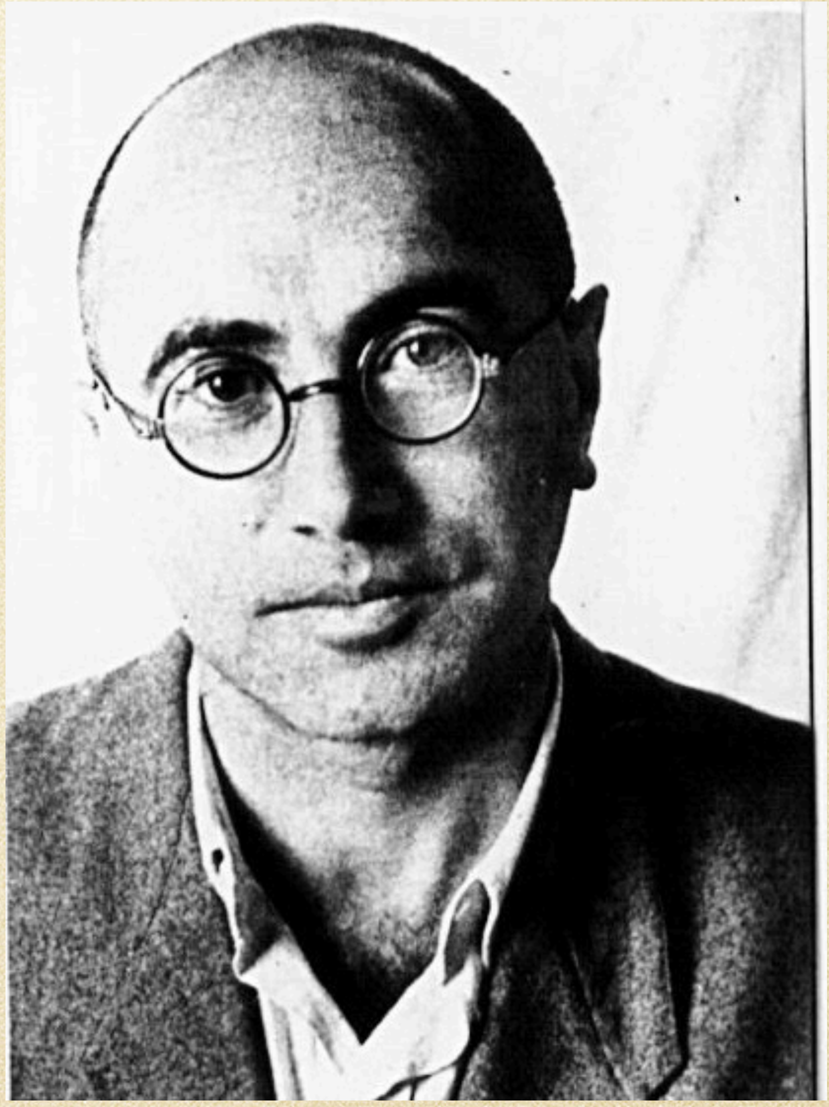
Yakov Zeldovich

Quantum fluctuations predict value of Λ

1967



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Steven Weinberg

The CCP 3.0

Steven Weinberg

Quantum fluctuations predict value of Λ

Problem since 1998 ref [3]

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Quantum fluctuations predict value of Λ

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Quantum fluctuations
predict value of Λ

$$\left(\frac{\Lambda}{G}\right)_{eff} \sim M^4(1+\dots)$$

ref [3]

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ref [3]

effective
quantum value

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Quantum fluctuations
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ref [3]

effective
quantum value

cut-off
mass scale of QFT

The CCP 3.0

Quantum fluctuations
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ref [3]

effective
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different
methods

cut-off
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The CCP 3.0

Quantum fluctuations
predict value of Λ

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ref [3]

effective quantum value

different methods

cut-off mass scale of QFT

use: $G \sim \dfrac{1}{M_P^2}$

The CCP 3.0

Quantum fluctuations
predict value of Λ

$$(\Lambda)_{eff} \sim \frac{M^4}{M_P^2}(1 + \dots)$$

effective
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Planck mass

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Quantum fluctuations
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Quantum fluctuations predict value of Λ

Highest physical mass scale

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Quantum fluctuations predict value of Λ

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Observed value

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Quantum fluctuations predict value of Λ

Highest physical mass scale

$$(\Lambda)_{eff} \sim \frac{M^4}{M_P^2}(1 + \ldots)$$

Observed value

$$\Lambda_o = \frac{\rho_c}{M_P^2} \approx \frac{10^{-47} GeV^4}{M_P^2}$$

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Quantum fluctuations predict value of Λ

Highest physical mass scale

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Observed value

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observed critical energy density

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Problem as a ratio:

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Problem as a ratio: $\ln\left(\frac{(\Lambda)_{eff}}{\Lambda_0}\right) \sim \ln\left(\frac{M^4}{\rho_c}\right)$

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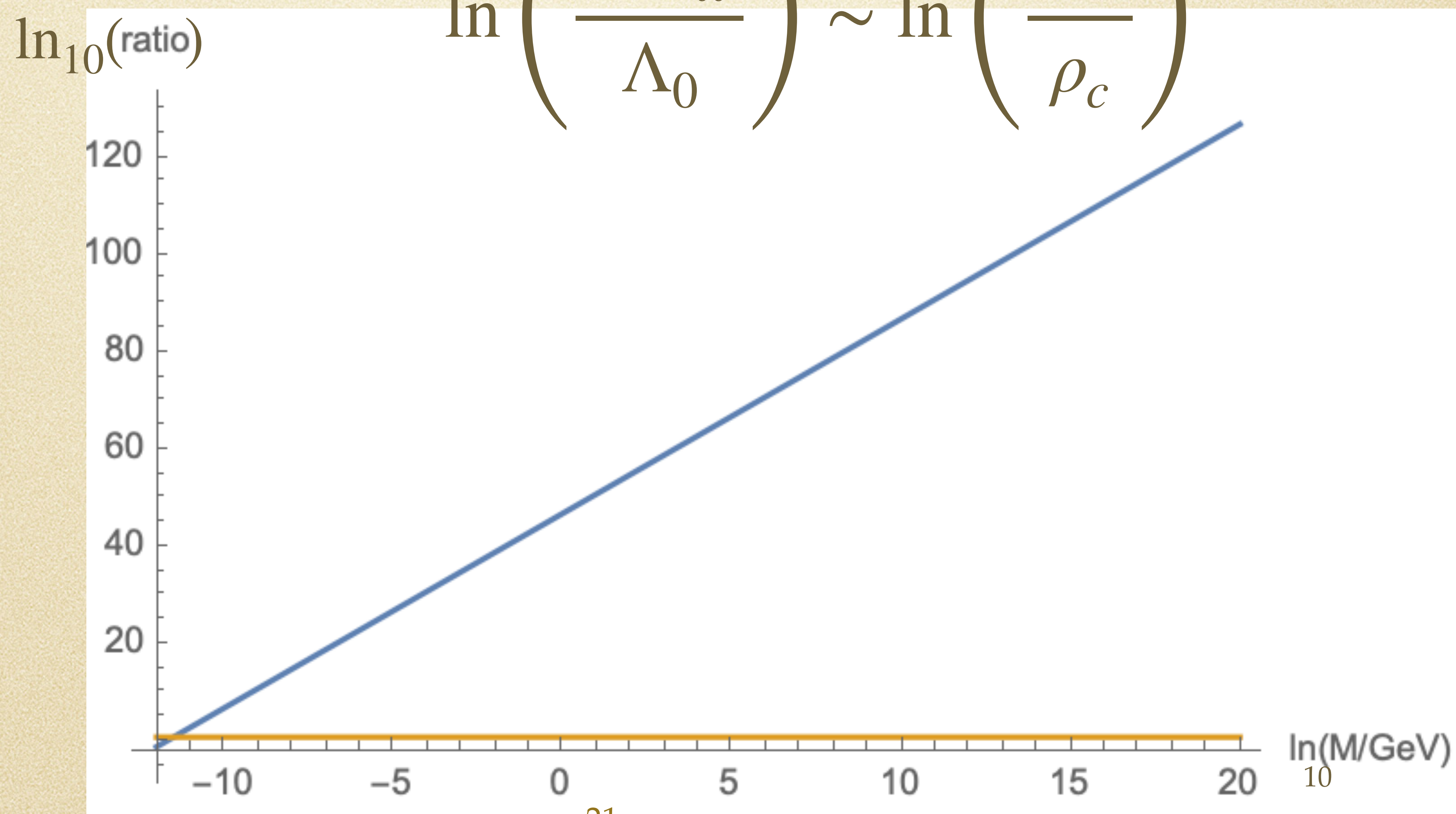

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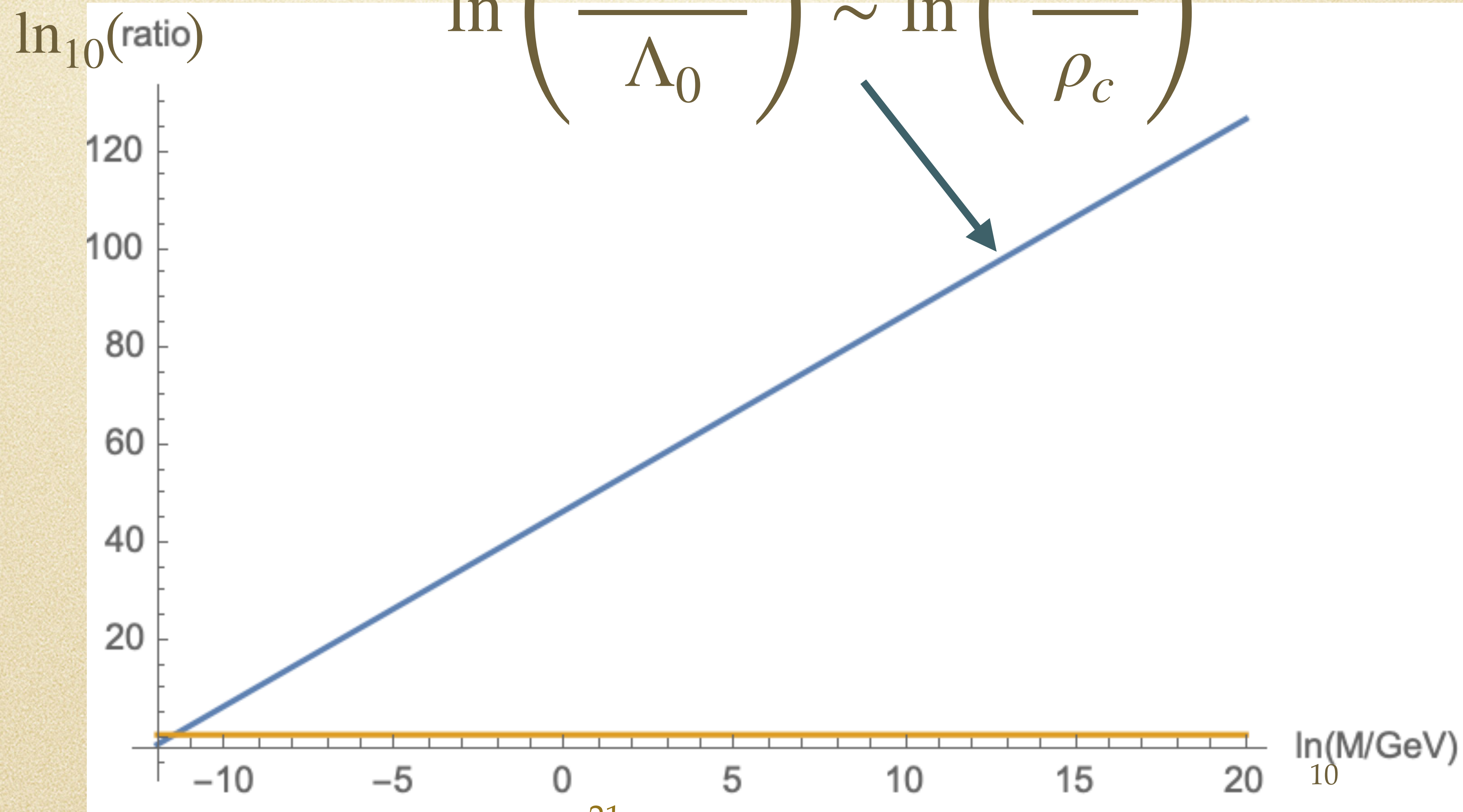

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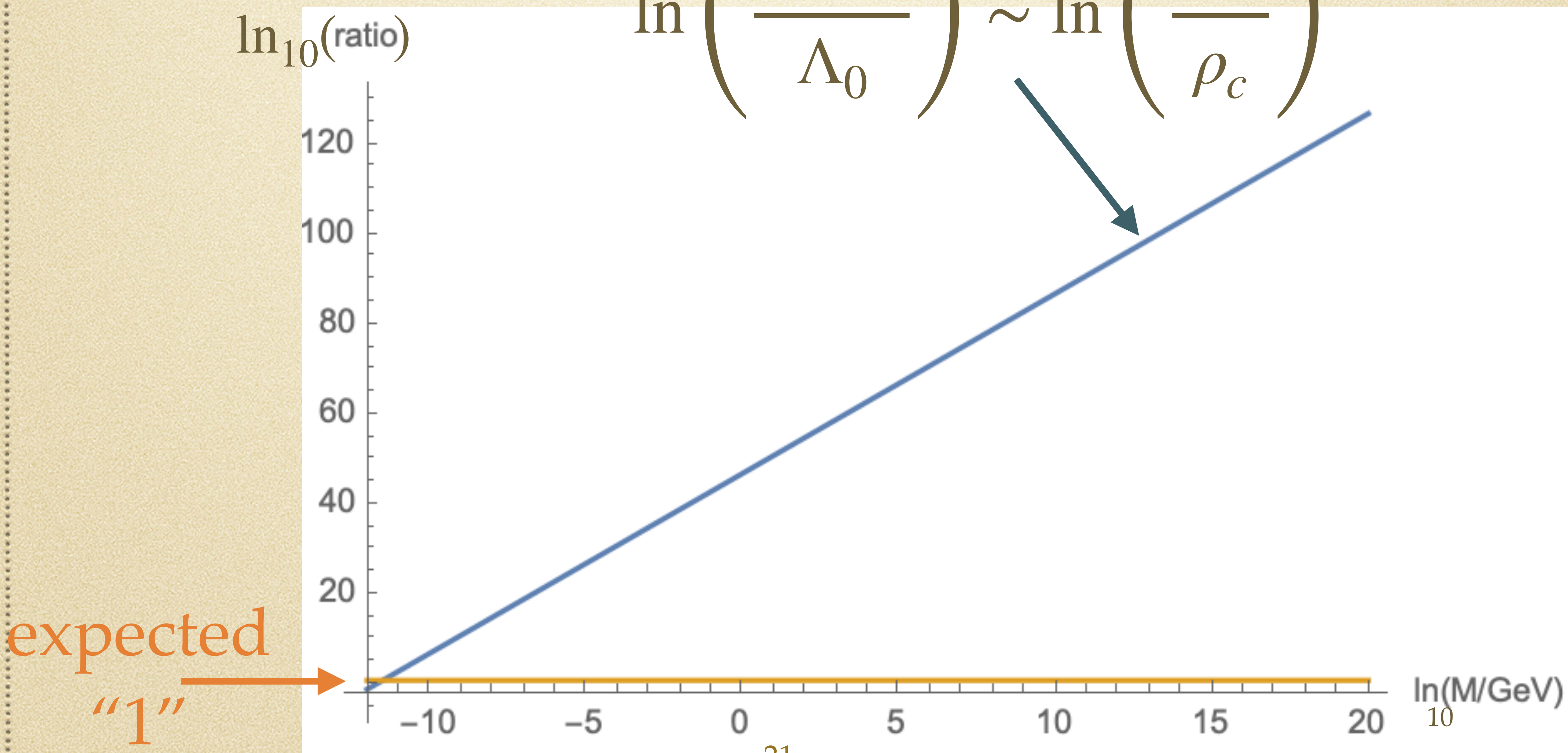

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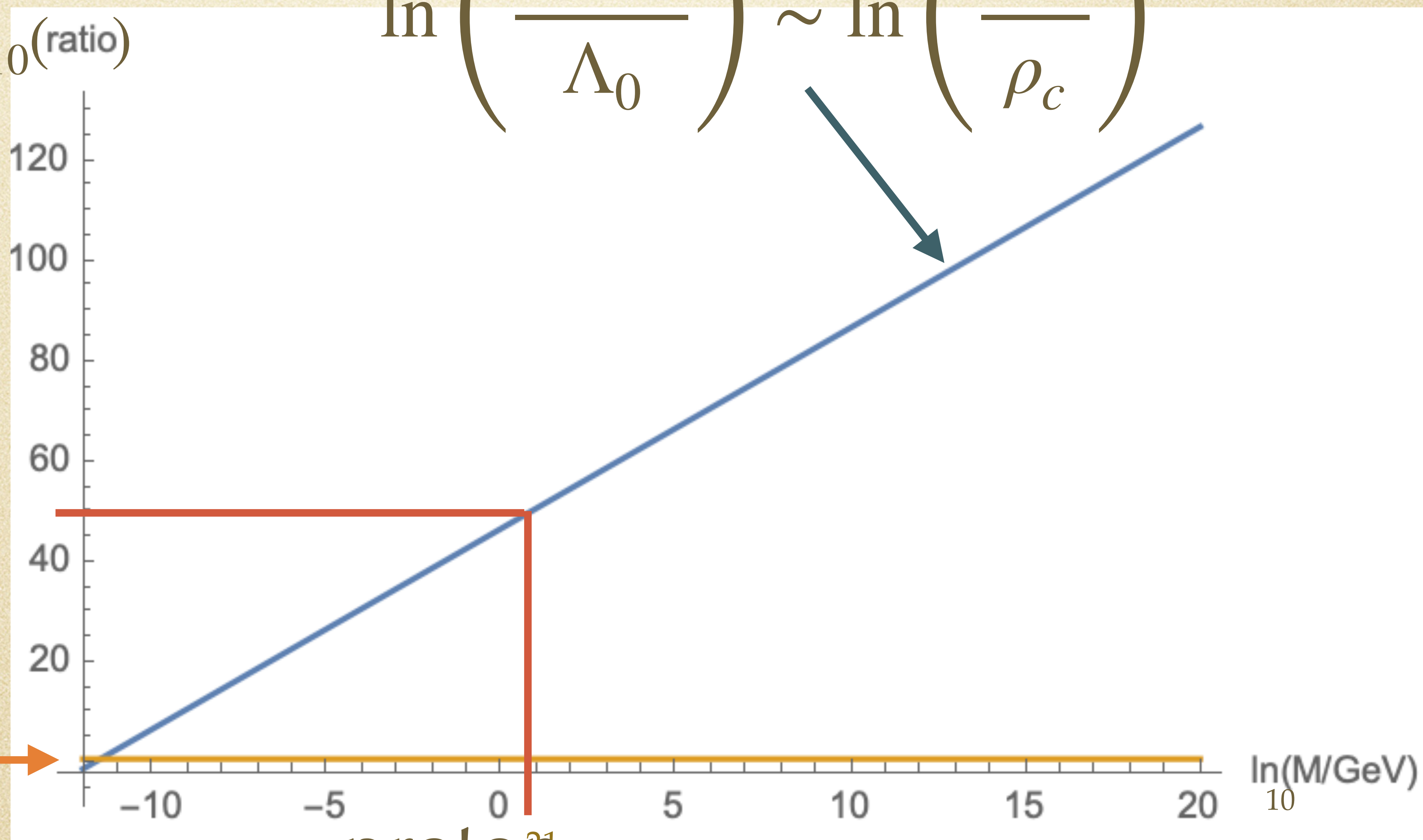

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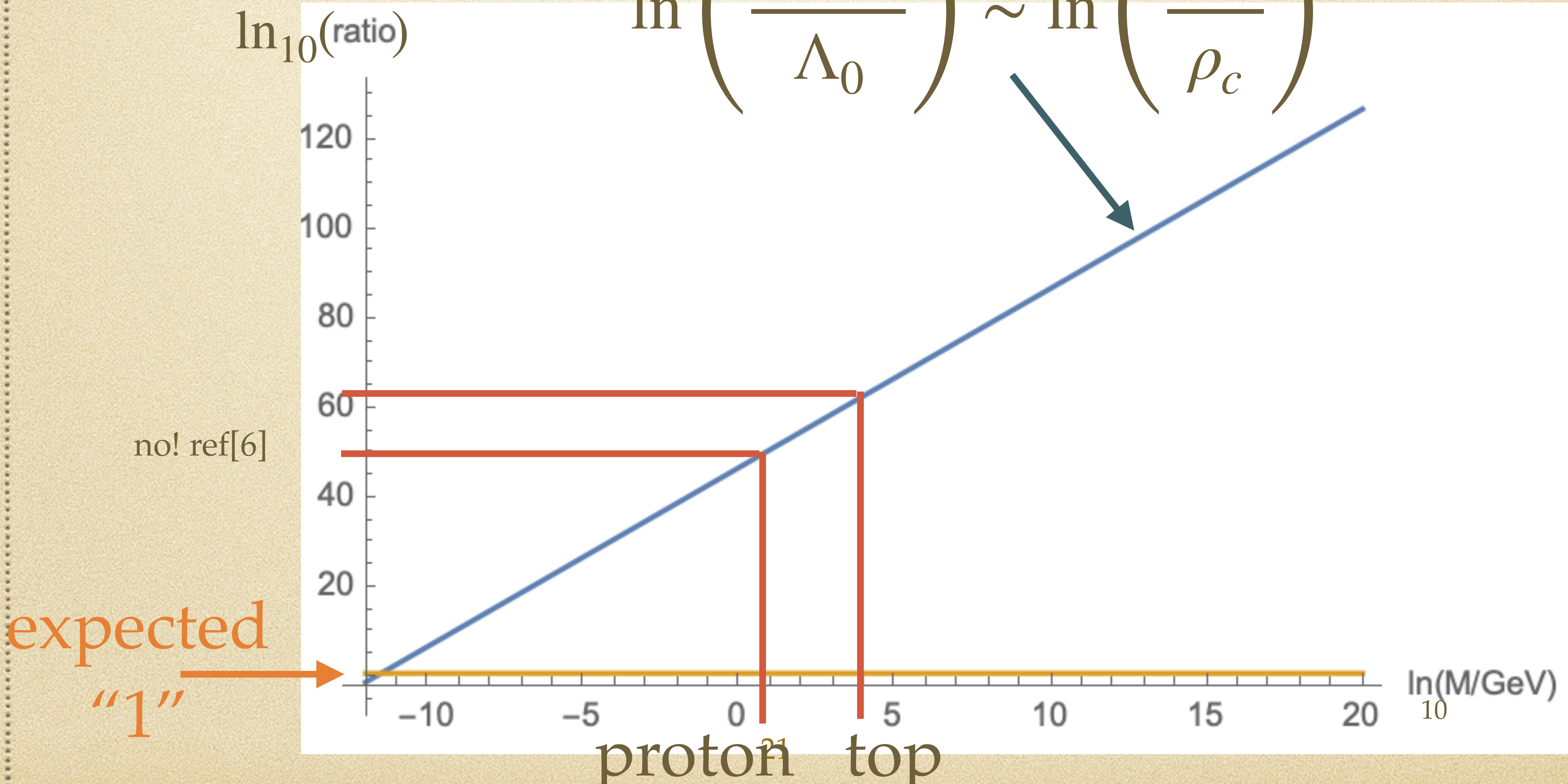

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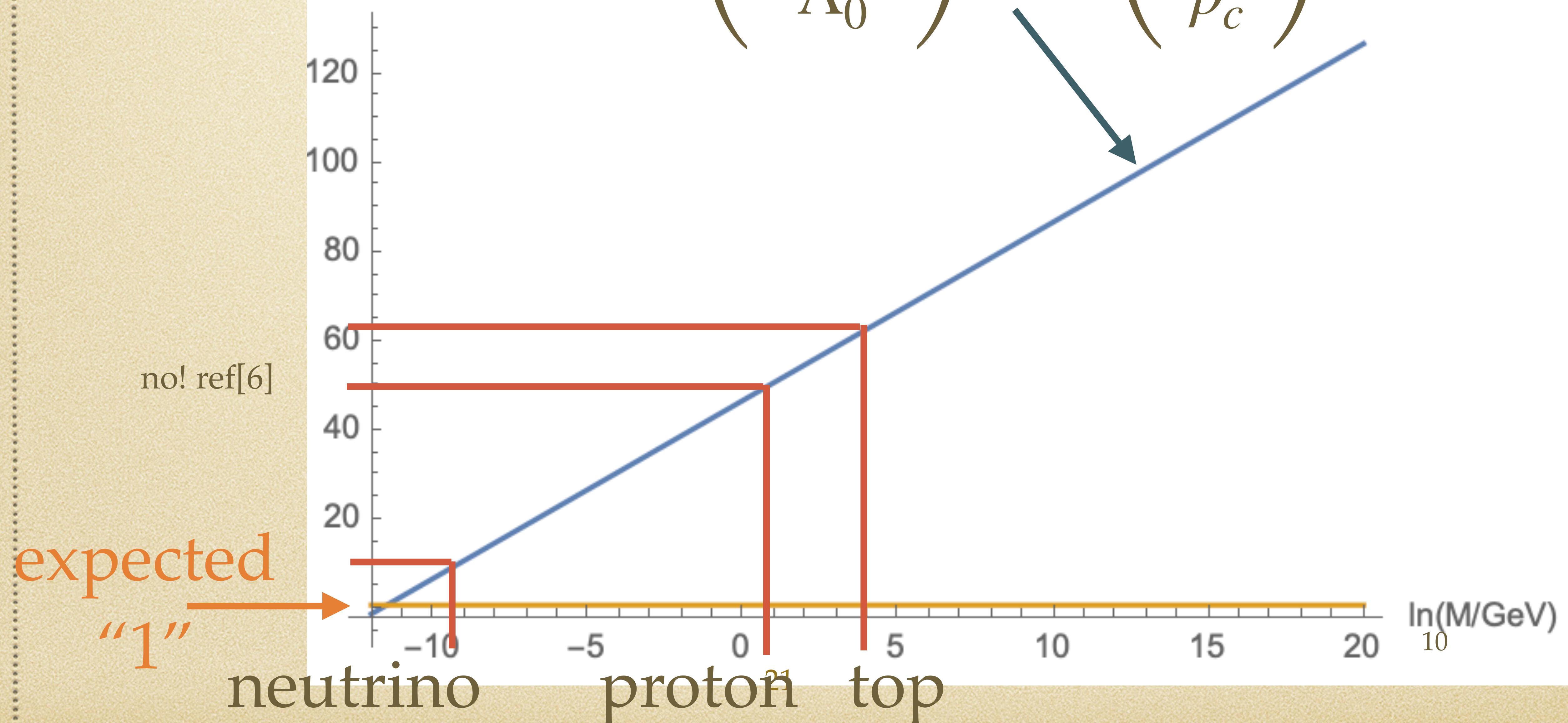

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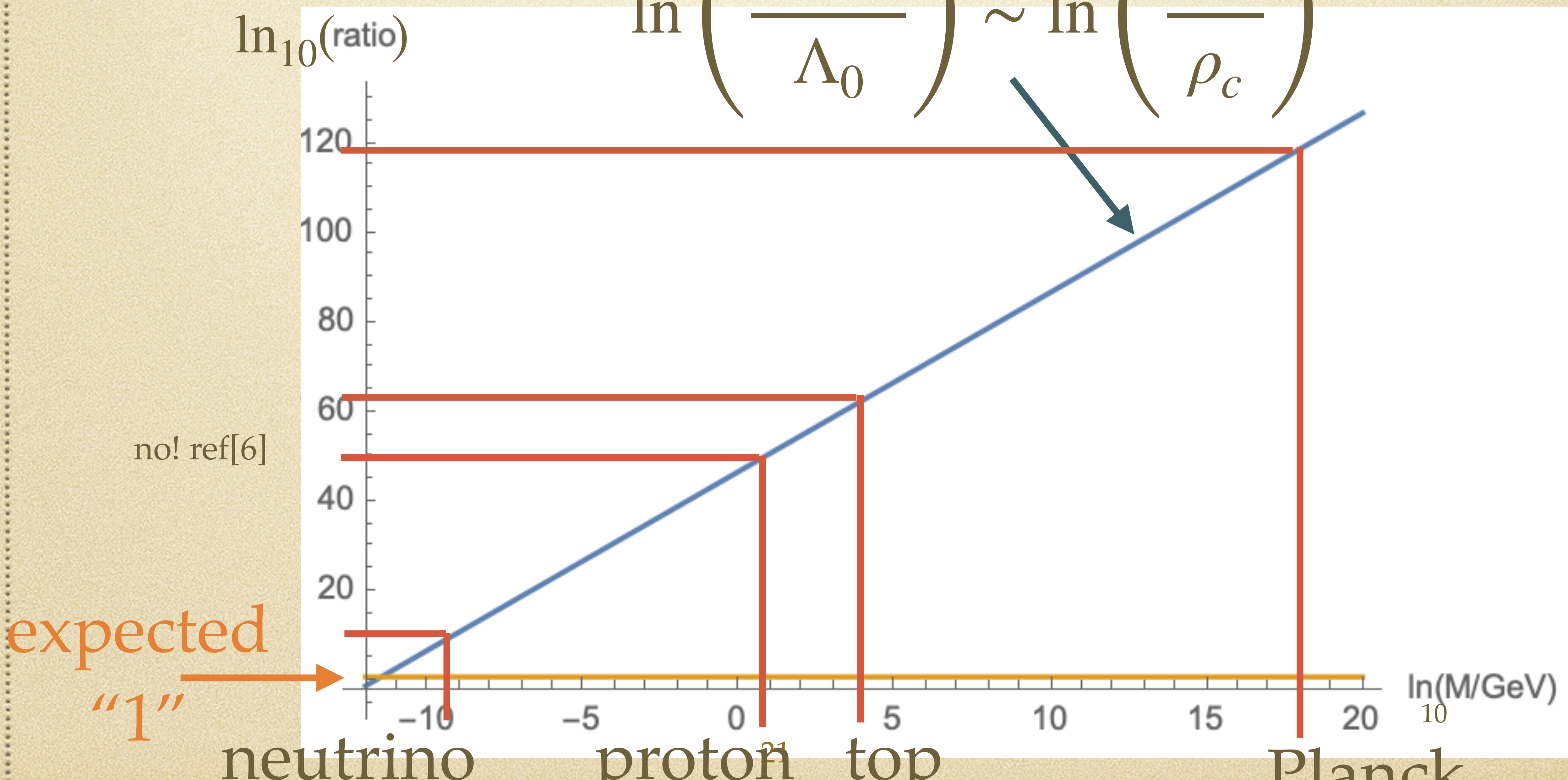

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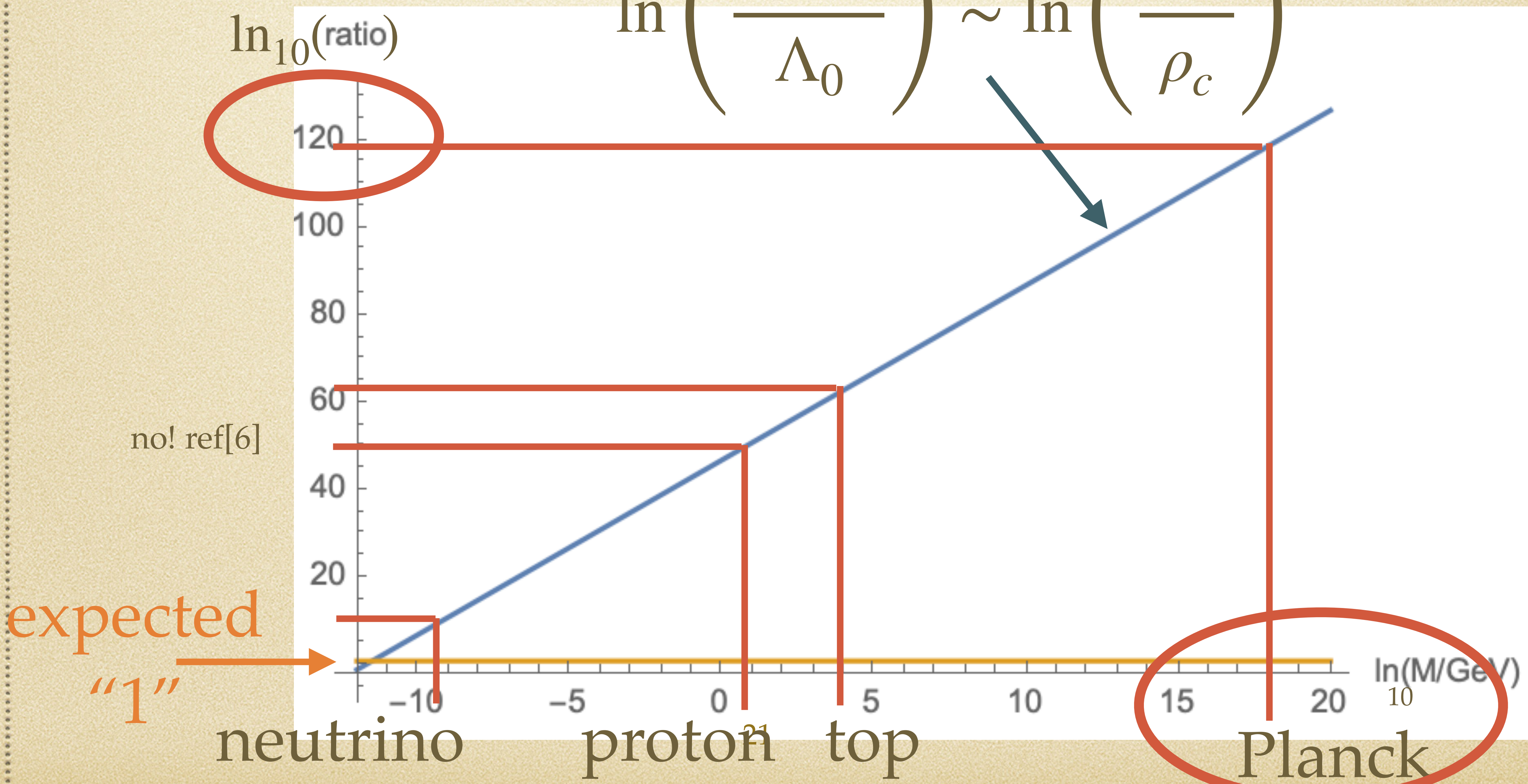

The CCP 3.0

Problem as a ratio:

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Problem as a ratio:

$$\frac{(\Lambda)_{eff}}{\Lambda_0} \sim \frac{1}{G_N \cdot \Lambda_0} \sim \frac{M_P^4}{\rho_c} \approx 10^{120}$$

The CCP 3.0

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we try to address
this problem

The CCP 3.0

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assuming there are quantum fluctuations
of gravity associated to the Planck scale

Evolving Universe Issue

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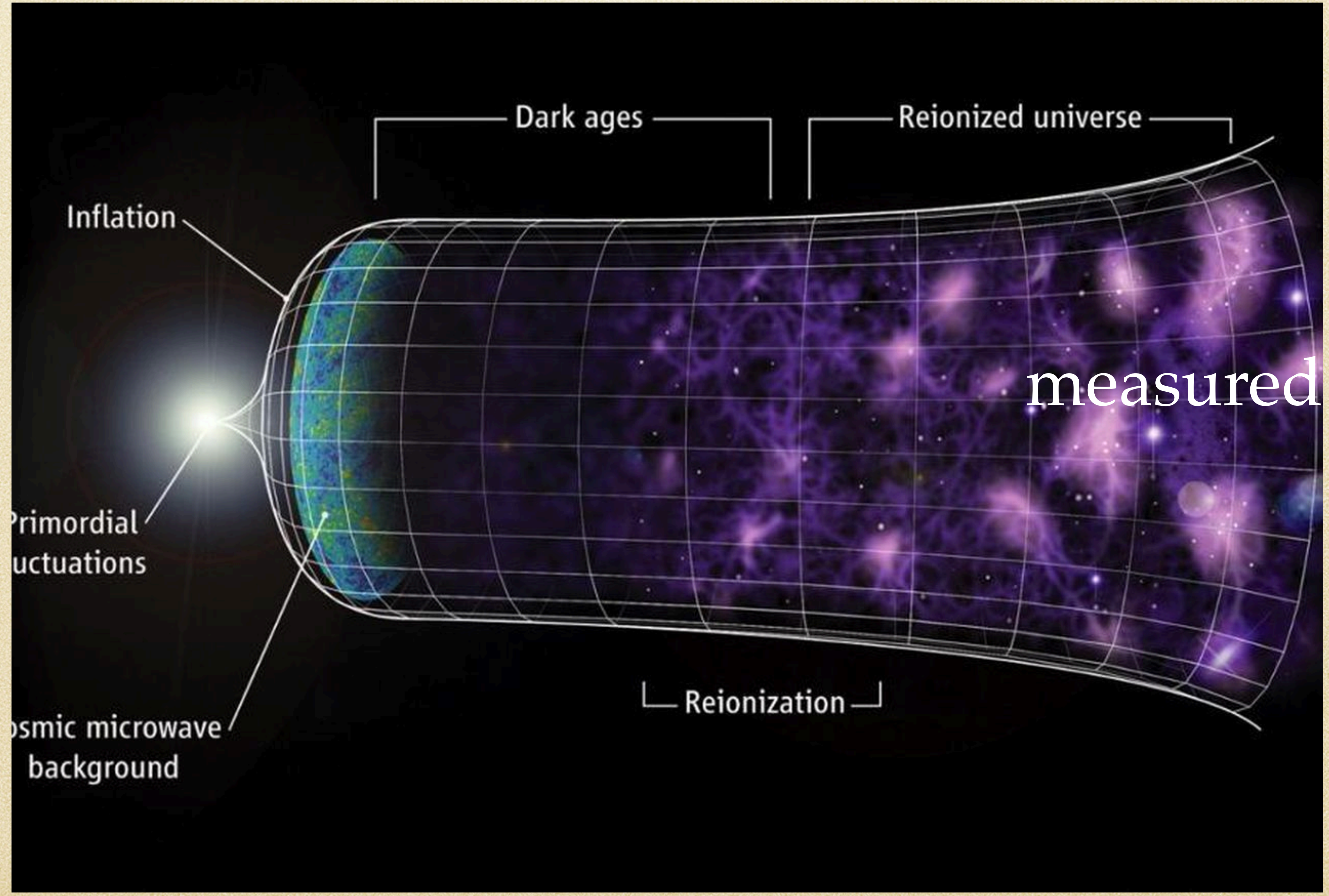
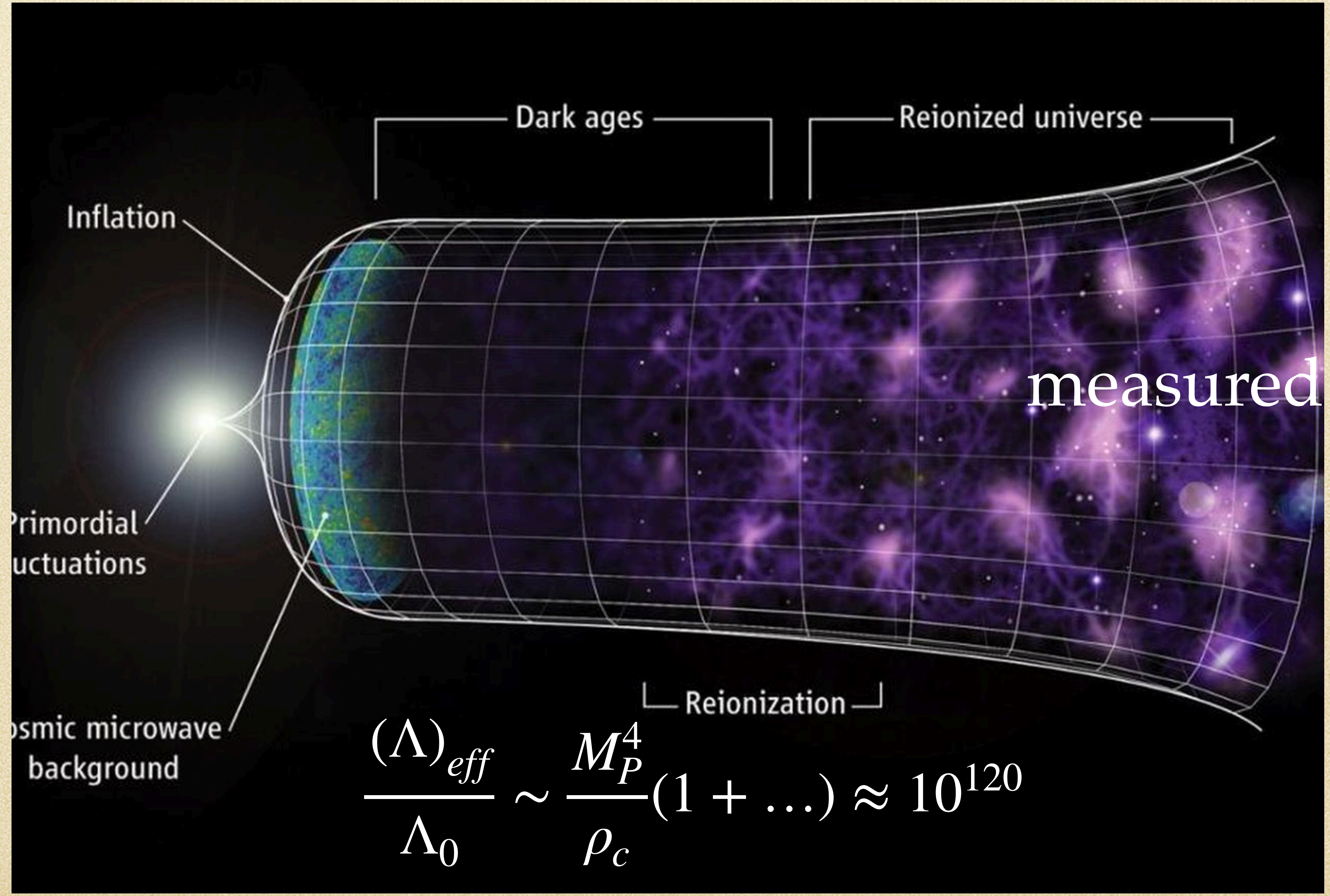


Evolving Universe Issue



$$\frac{(\Lambda)_{eff}}{\Lambda_0} \sim \frac{M_P^4}{\rho_c}(1 + \ldots) \approx 10^{120}$$

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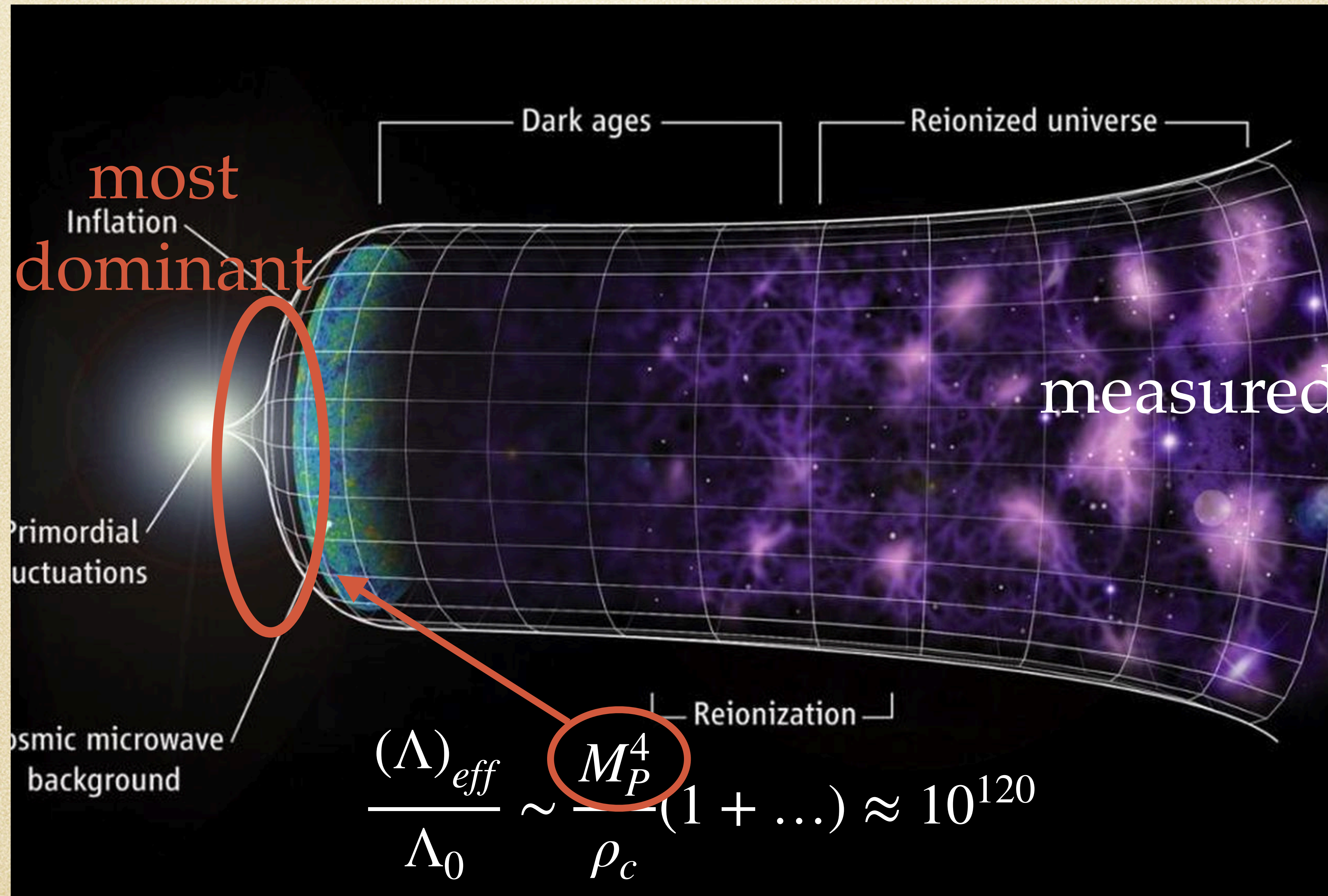

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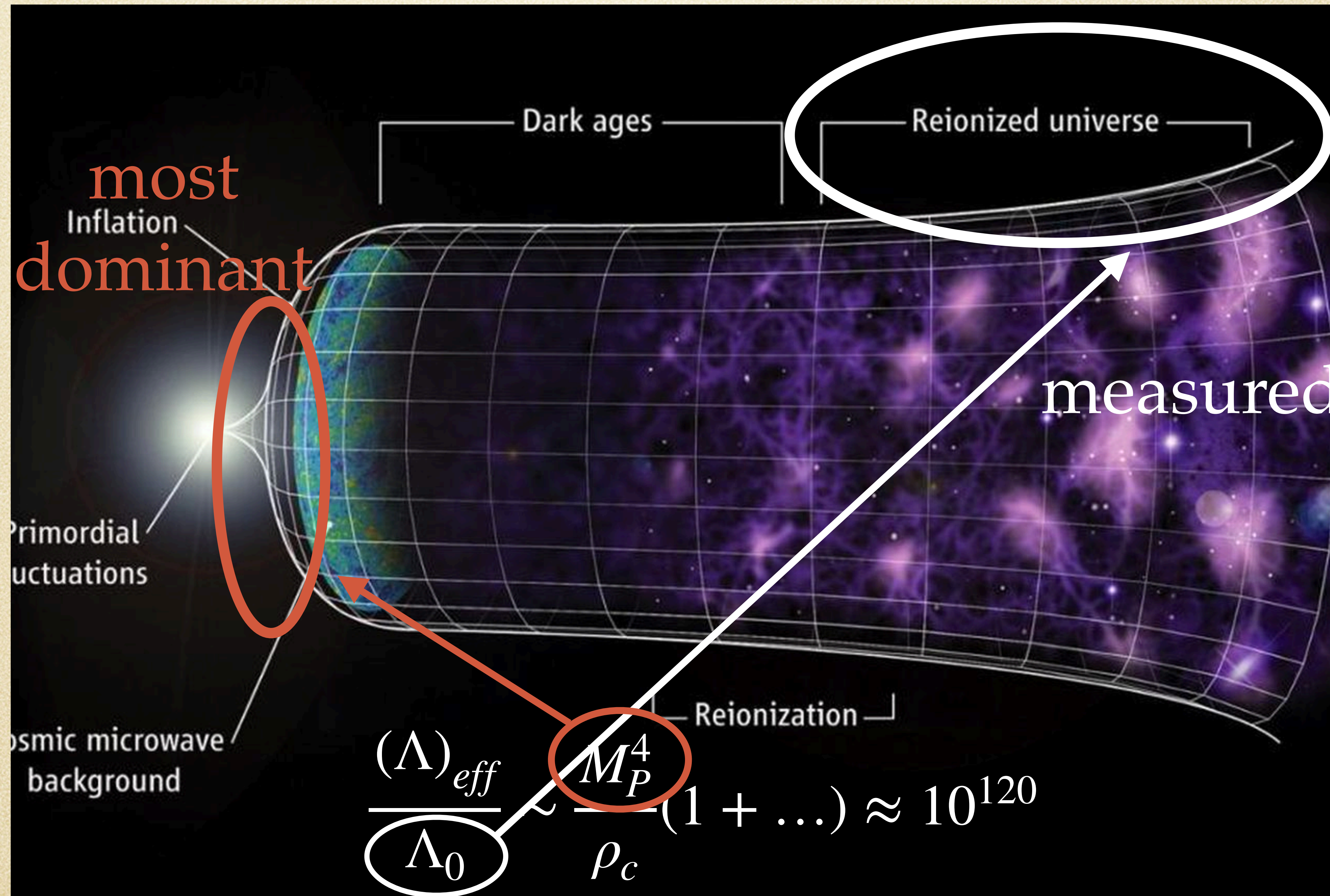

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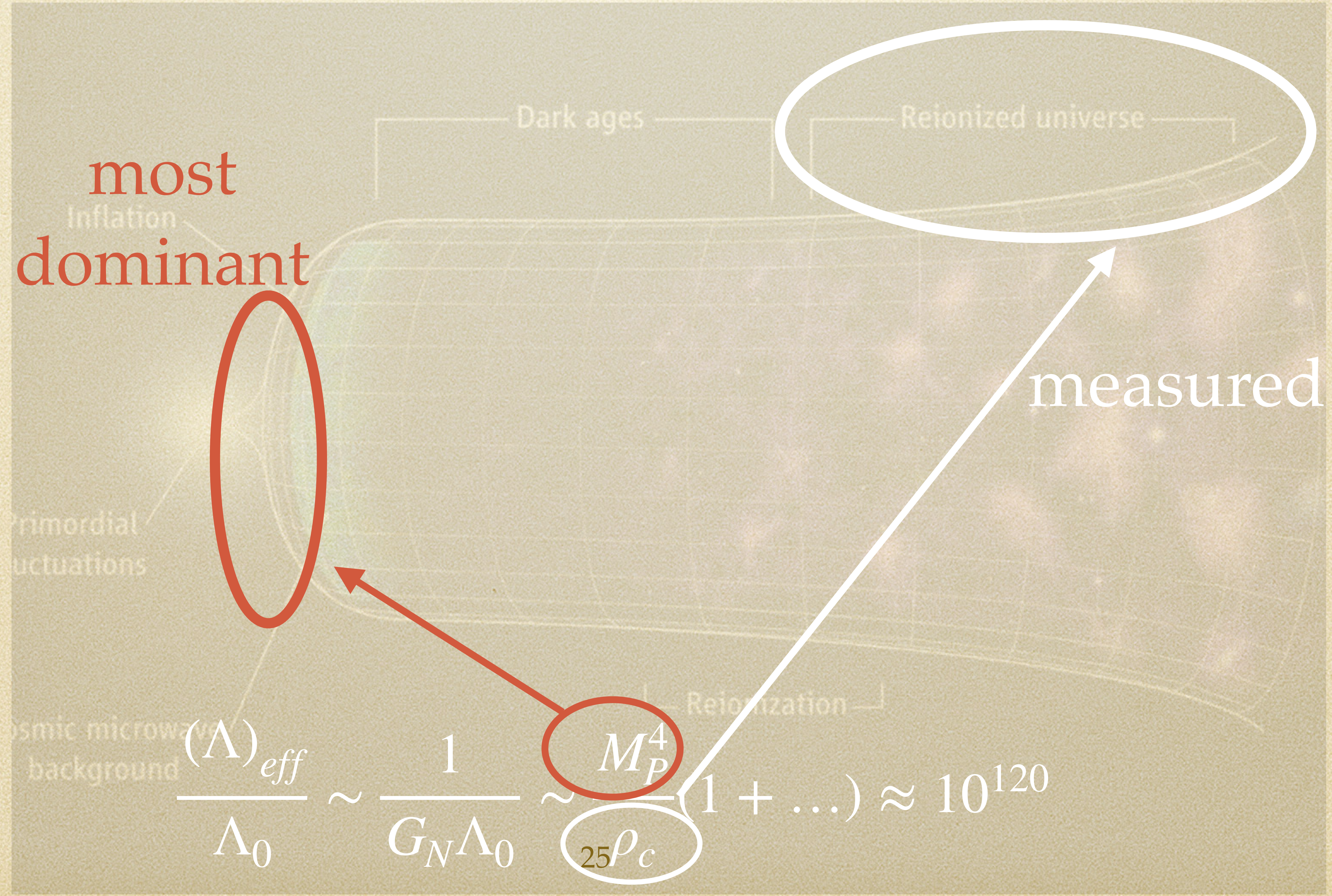

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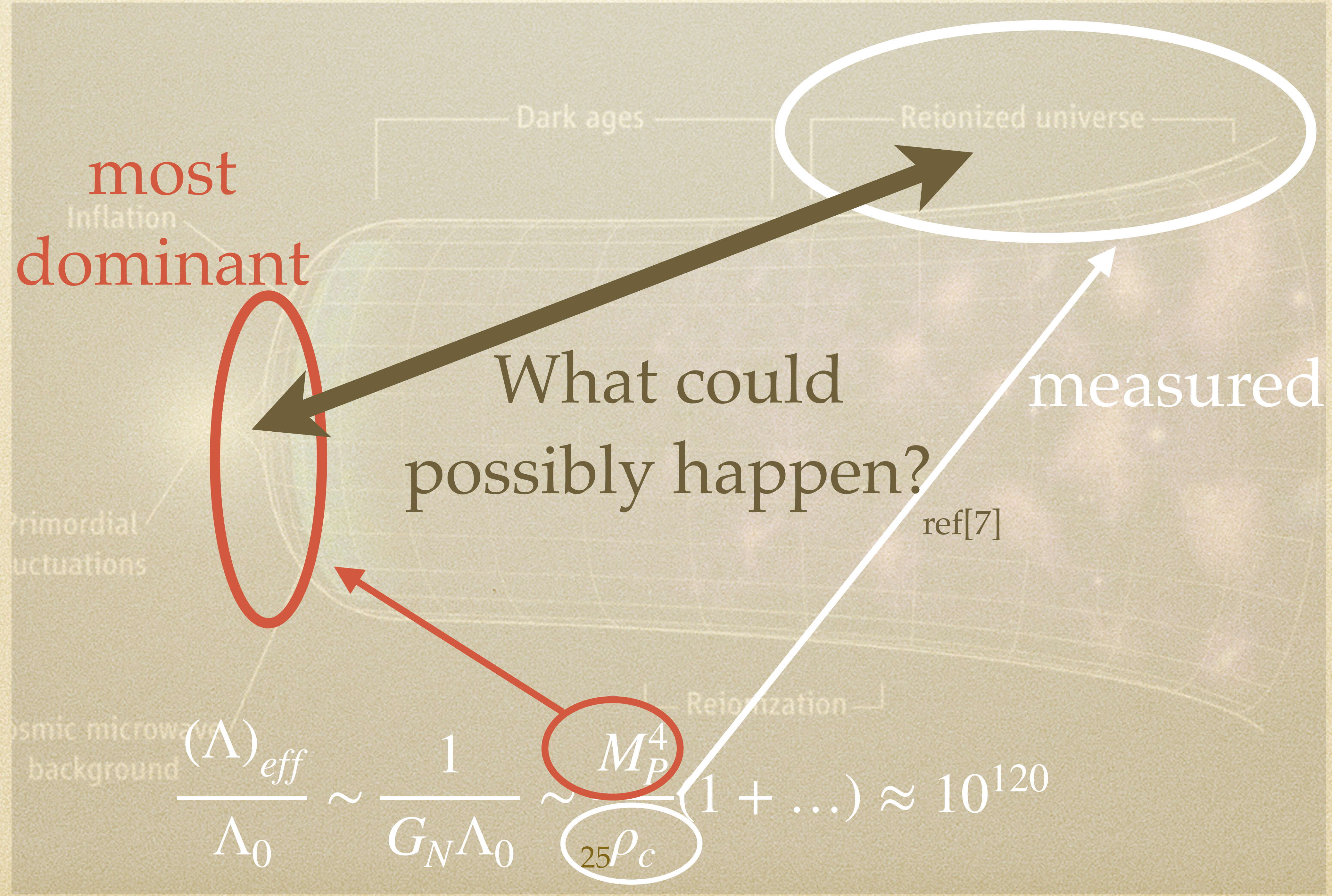

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Scale Dependent Framework

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Gravity as classical theory

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$$S = \int d^4x \sqrt{-g} \left(\frac{R}{G_N} - 2\frac{\Lambda_0}{G_N} \right)$$

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$$\Gamma_k = \int d^4x\sqrt{-g}\left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k}\right) + \ldots$$

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Yes, but …

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Could still be predictive QFT
(Asymptotic Safety)

Asymptotic Safety in a nutshell

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- Idea: works if non trivial UV-fixed points for finite number of couplings (S.W)

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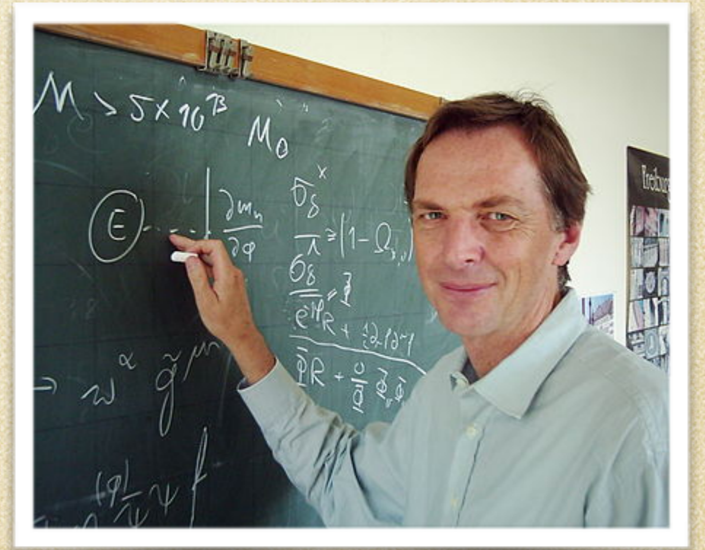
- Tool: Functional renormalization group equation



C. Wetterich

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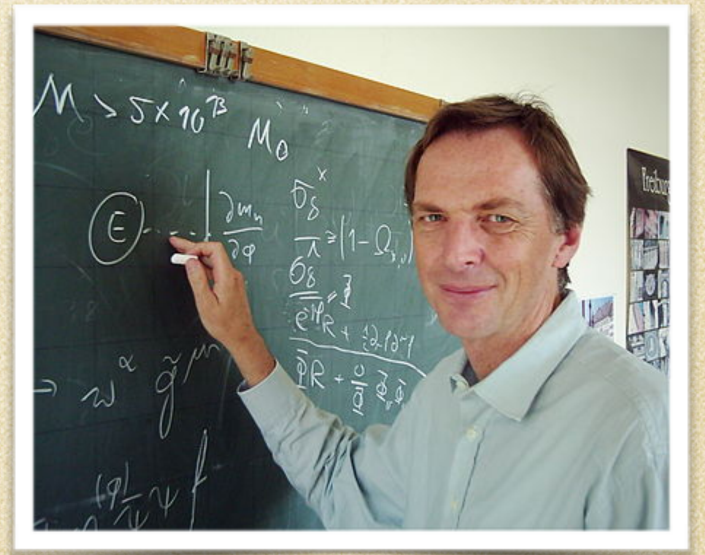

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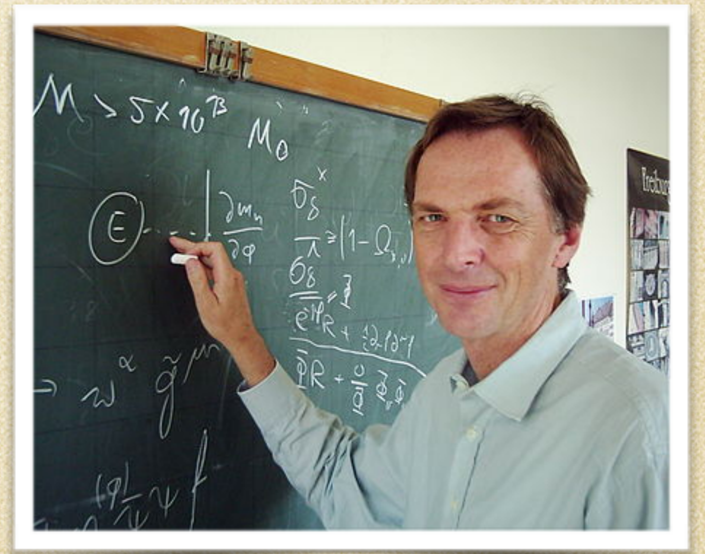

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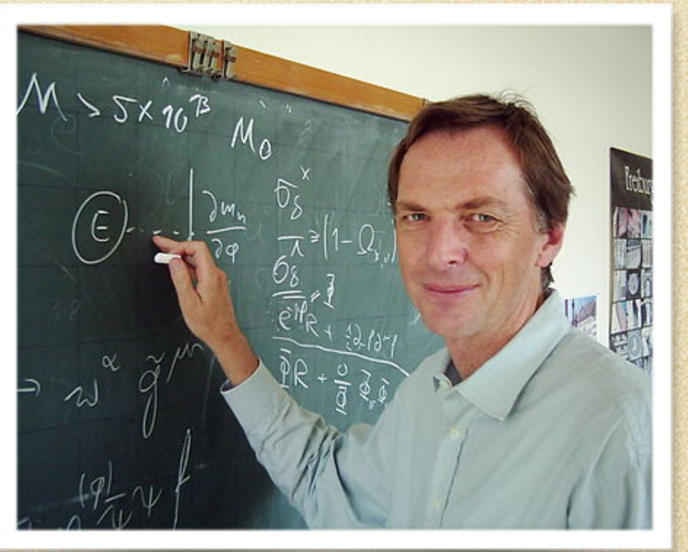
two point function



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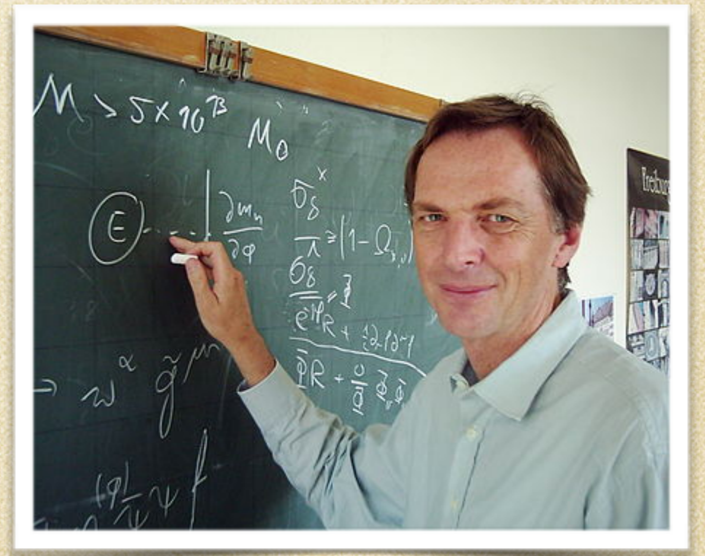
regulator



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$$\Lambda_k = \lambda_k k^2$$

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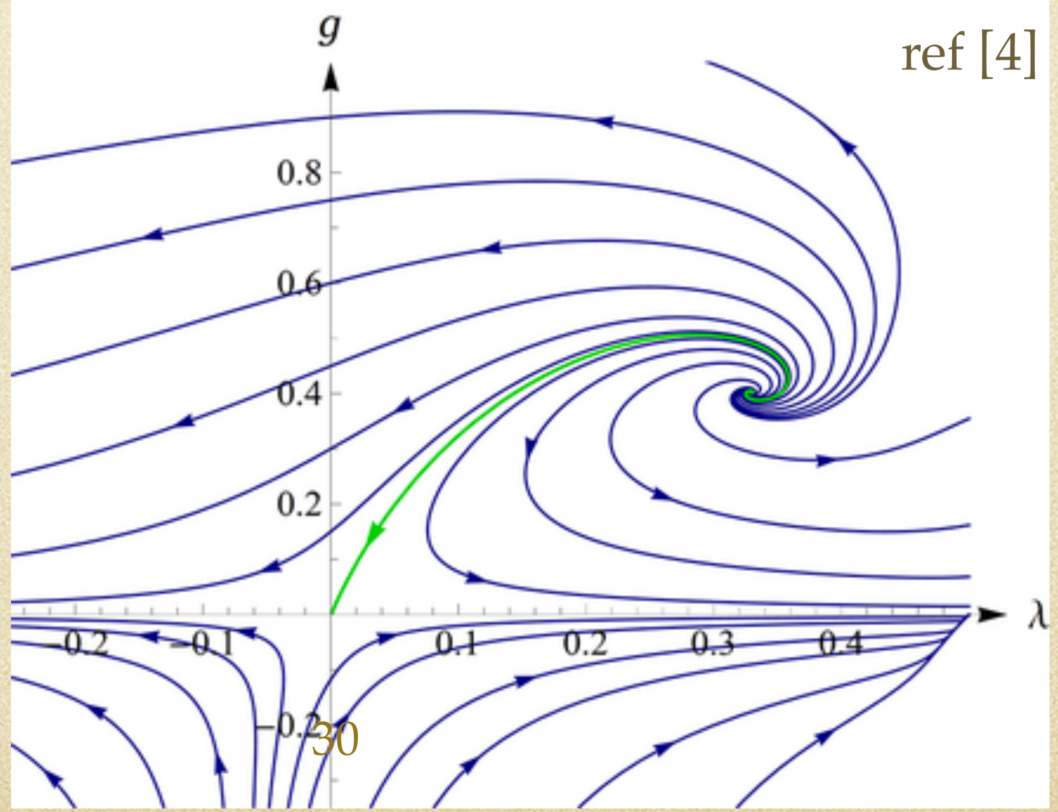

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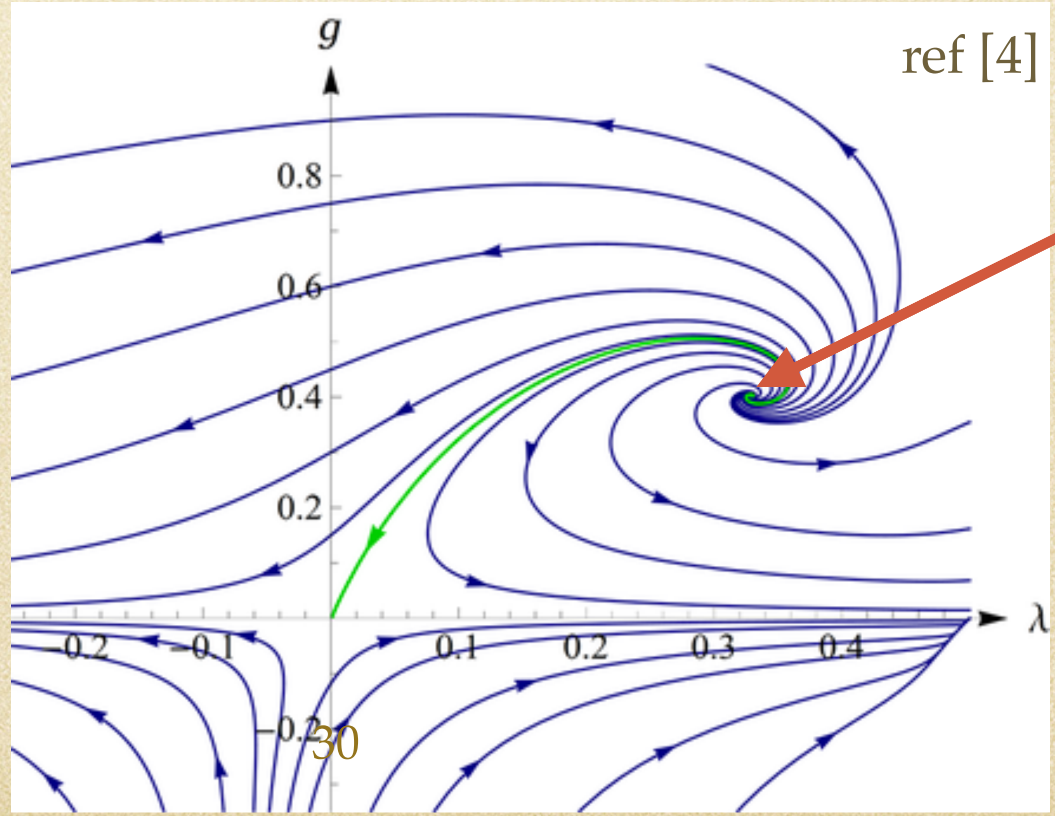

UV FP

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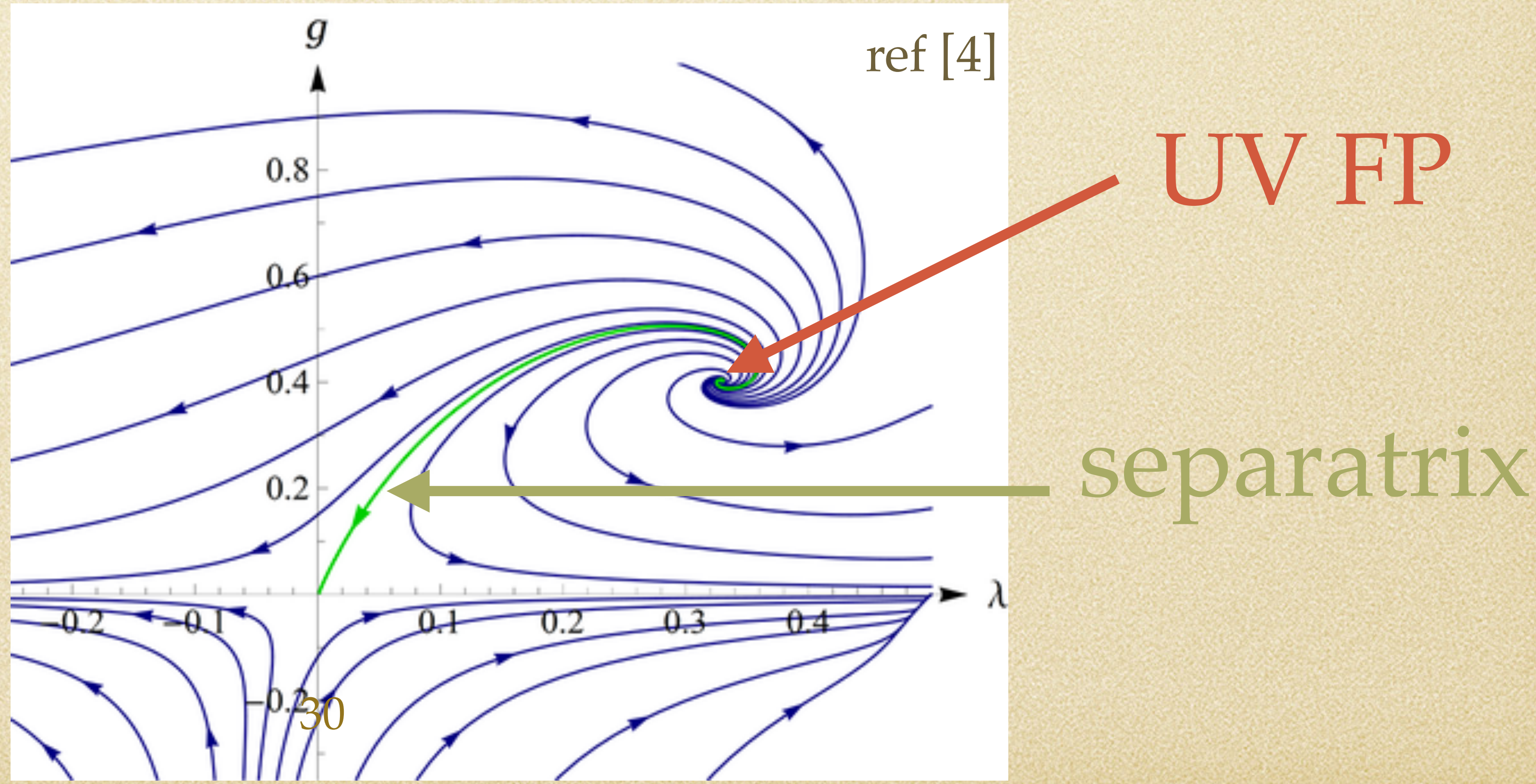

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Even far below M_{Pl}:
Effect of non local operators

Scale Dependent Framework

OK, assume $$\Gamma_k = \int d^4x \sqrt{-g} \left(\frac{R}{G_k} - 2\frac{\Lambda_k}{G_k} \right)$$

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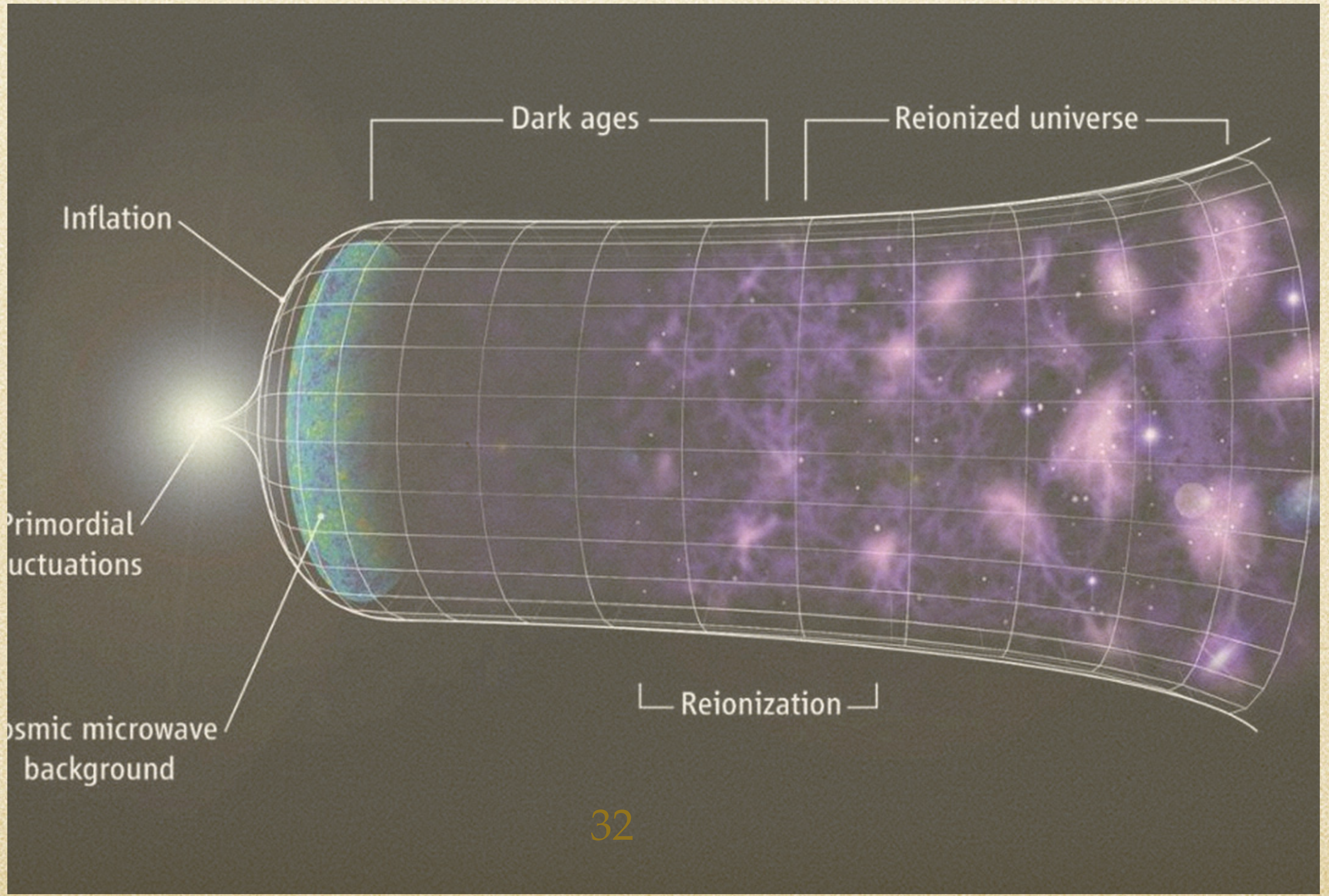

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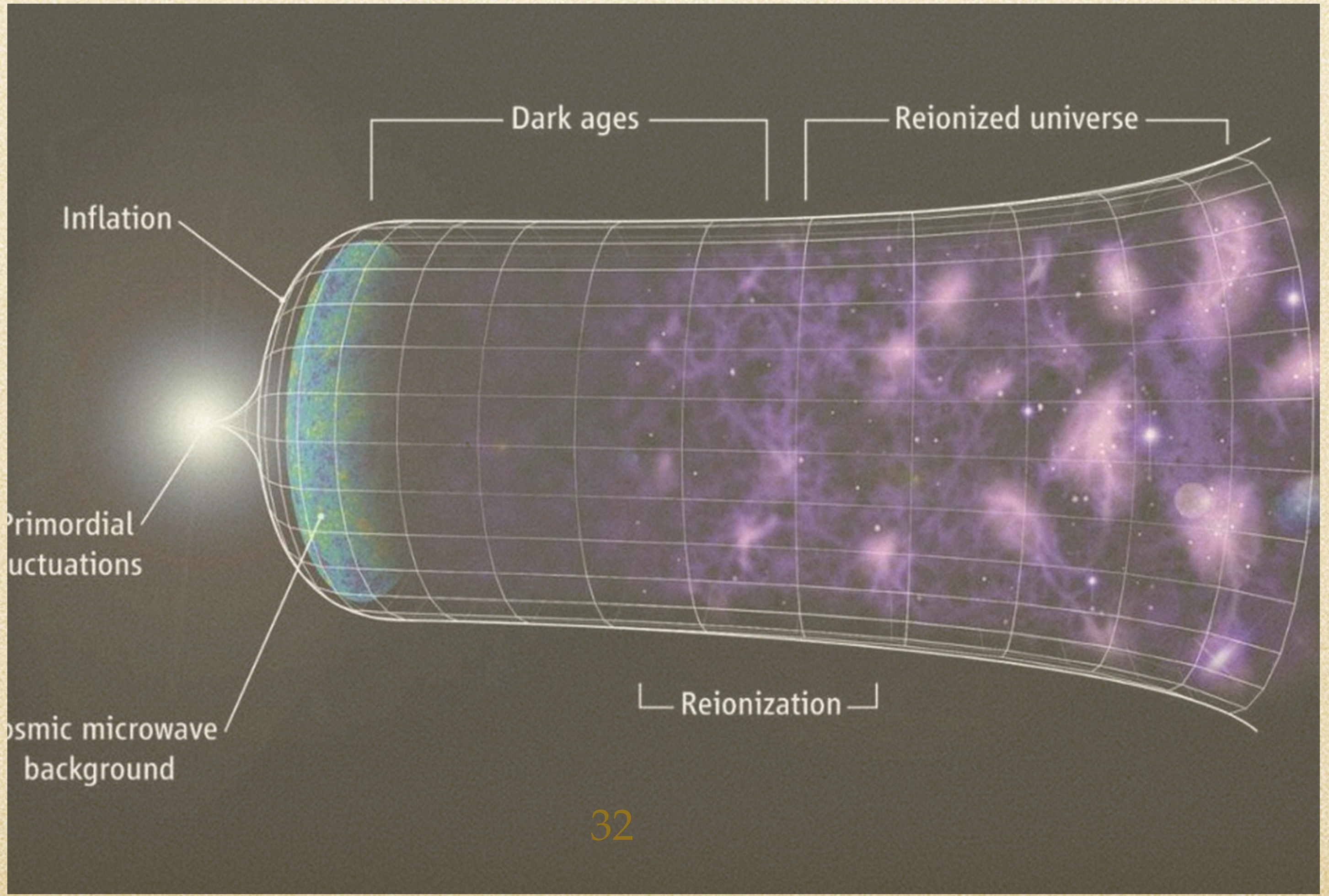

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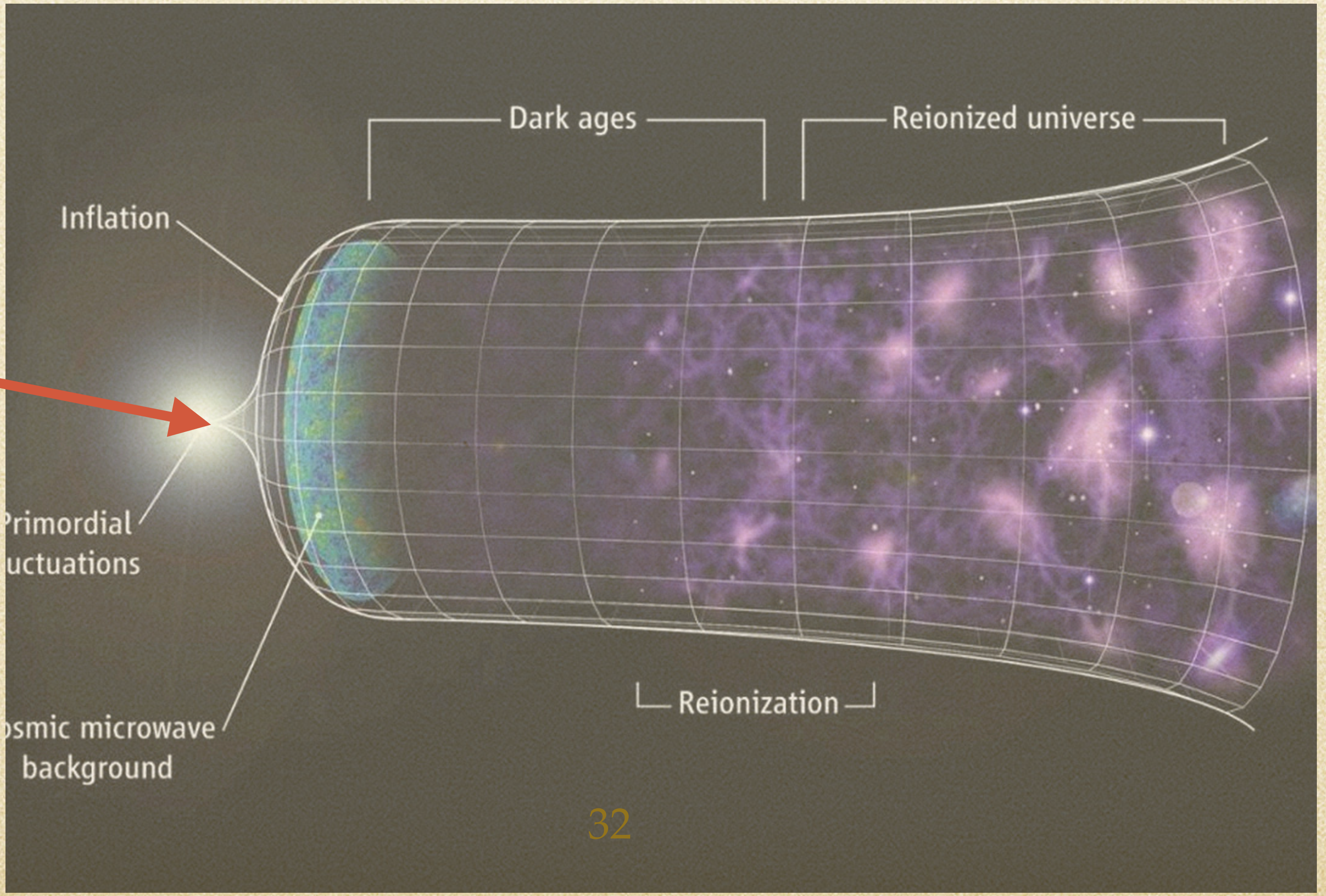

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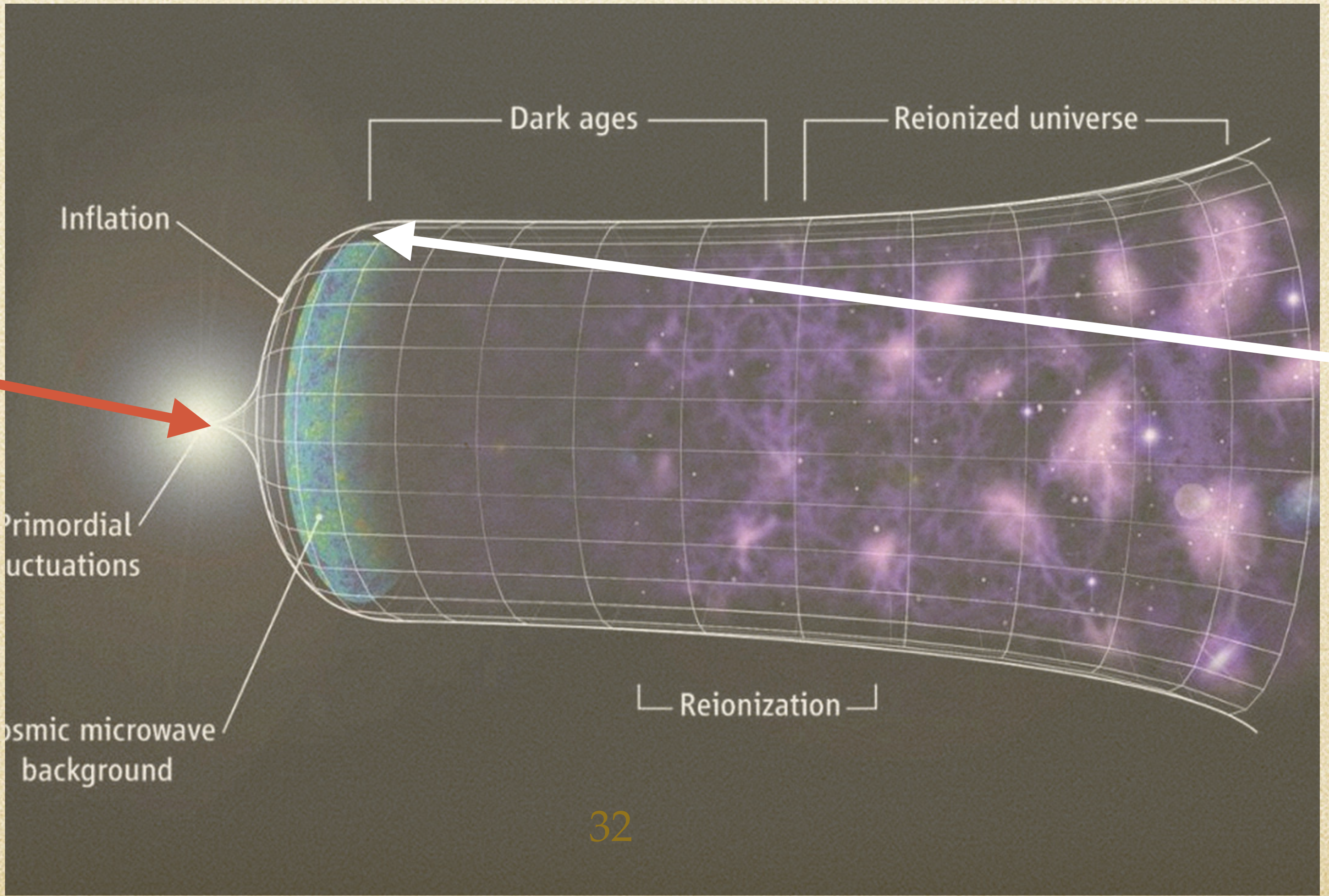

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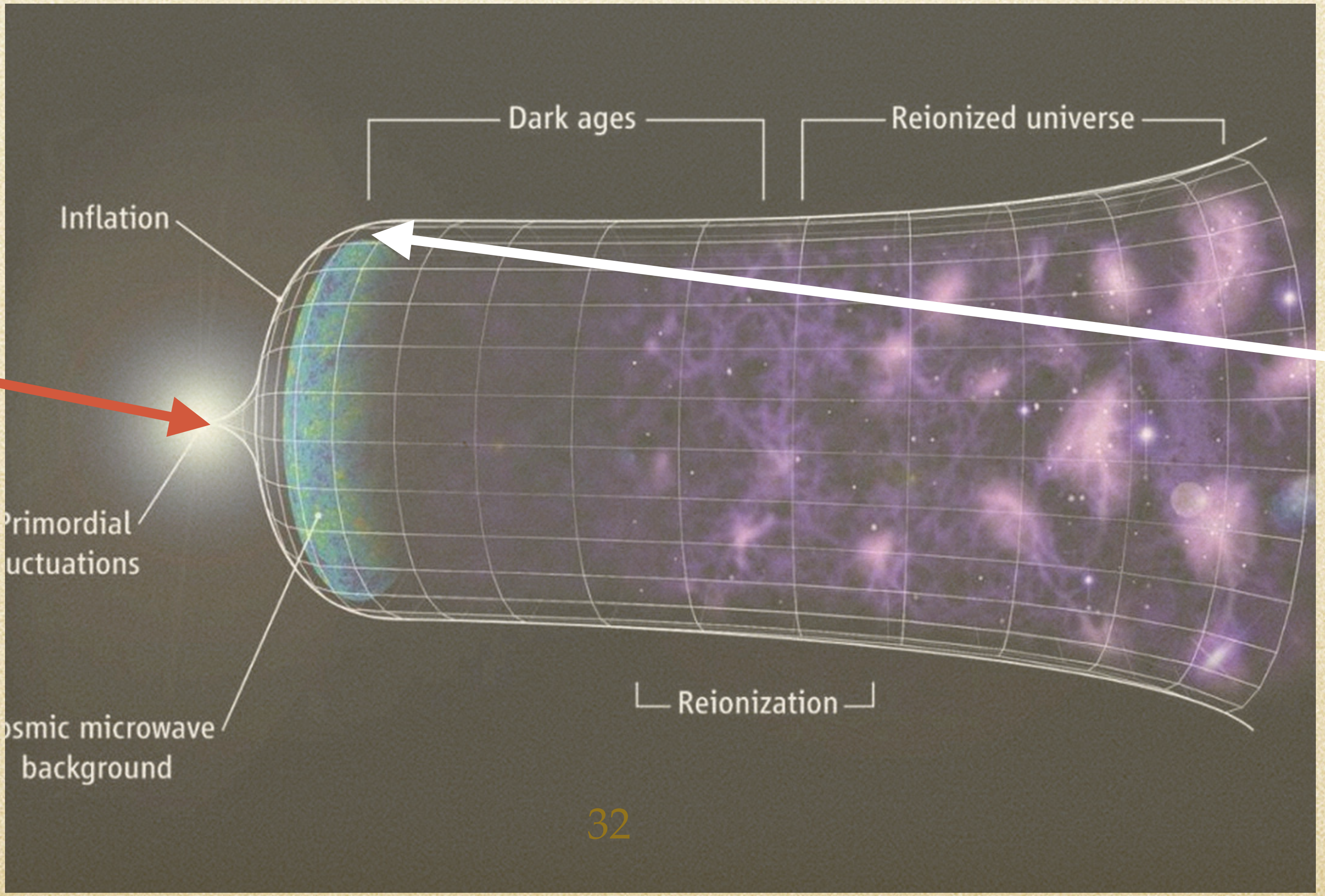

$$\frac{1}{G_{k'} \cdot \Lambda_{k'}} \neq 1$$

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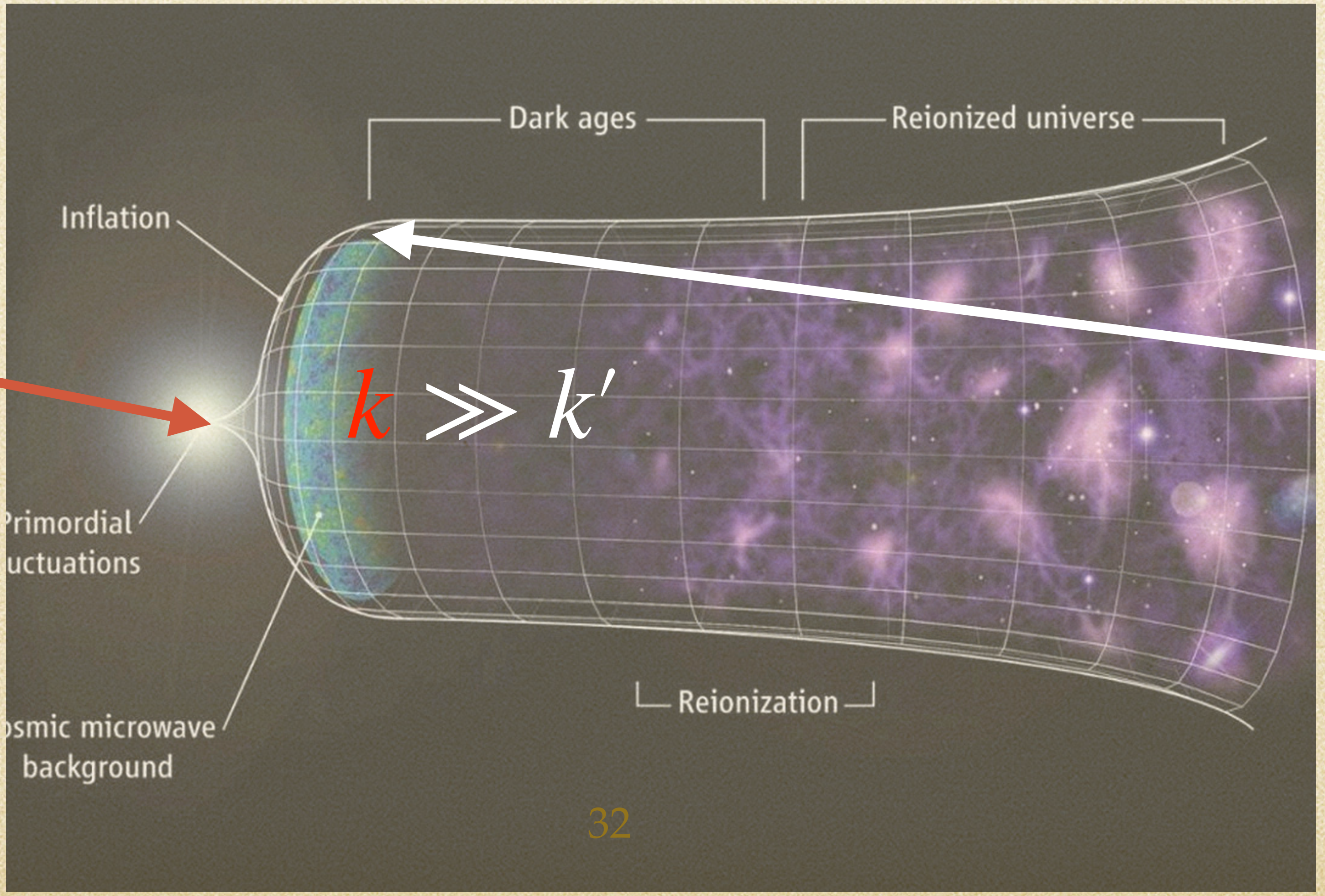

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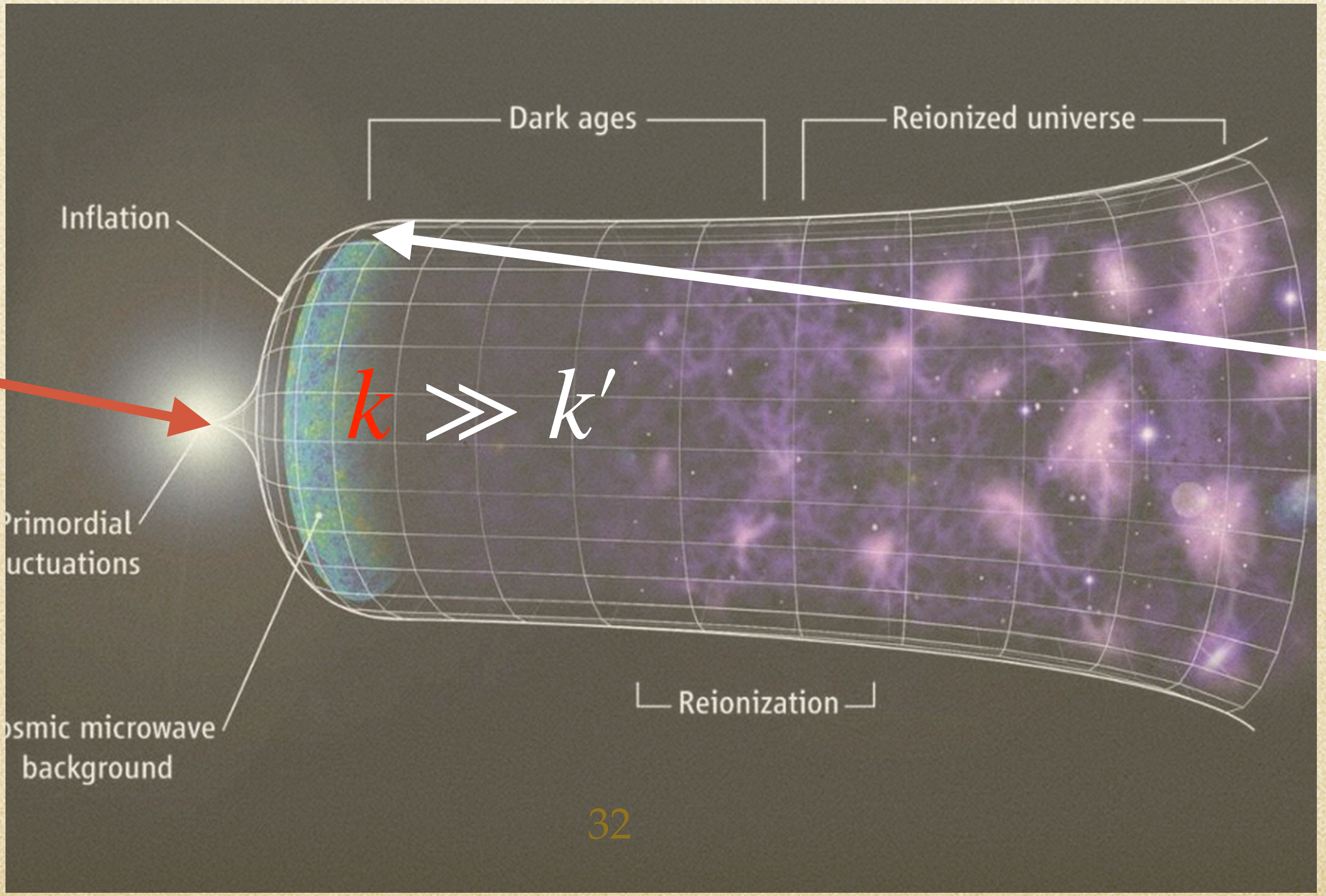

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lets see:

Deflation During Inflation

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$$G_{\mu\nu} = -\Lambda_k g_{\mu\nu} - \Delta t_{\mu\nu}$$

with

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not constant!

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Assume homogenous background

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$$ds^2 = -dt^2 + a(t)\left(\frac{1}{1-\kappa r^2}dr^2 + r^2 d\Omega_2^2\right)$$

spatial curvature

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Gap equations

$$\left(\frac{\dot a}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda_k}{3} = \frac{1}{3}\rho_{SD}$$

$$2\frac{\ddot a}{a} + \left(\frac{\dot a}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda_k = -\,p_{SD}$$

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Since k=k(t) $\Rightarrow$ $G_k = G(t)$ & $\Lambda_k = \Lambda(t)$

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Note:

Deflation During Inflation

Gap equations

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Note:

$$\frac{d\Gamma_k}{dk} \equiv 0$$

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Ensures Bianchi identities & diff. inv. ref [8]

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Back to G(t) …

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Problem:

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Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda(t)}{3} = \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \Lambda(t) = -2\left(\frac{\dot{G}}{G}\right)^2 + \frac{\ddot{G}}{G} + 2\left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right)$$

Problem: 2 equations

Deflation During Inflation

Gap equations

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Deflation During Inflation

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Solution:

Deflation During Inflation

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Problem: 2 equations

3 unknown functions: a(t), G(t), $\Lambda(t)$

Solution: Impose energy condition!

Deflation During Inflation

Deflation During Inflation

Null Energy Condition (NEC):

Deflation During Inflation

Null Energy Condition (NEC):

$$\Delta t_{\mu\nu}\ell^{\mu}\ell^{\nu} = 0$$

Deflation During Inflation

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Deflation During Inflation

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$$\frac{d\ell^{\mu}}{dt} + \Gamma^{\mu}_{\alpha\beta} \ell^{\alpha} \ell^{\beta} = 0$$

Deflation During Inflation

Null Energy Condition (NEC):

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$$\frac{d\ell^{\mu}}{dt} + \Gamma^{\mu}_{\alpha\beta}\ell^{\alpha}\ell^{\beta} = 0 \;\; \rightarrow$$

Deflation During Inflation

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Deflation During Inflation

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thus

$$-2\left(\frac{\dot{G}}{G}\right)^2 + \left(\frac{\ddot{G}}{G}\right) - \left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right) = 0$$

Deflation During Inflation

Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda(t)}{3} = \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right)$$

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$$+\left(\frac{\ddot{G}}{G}\right) - \left(\frac{\dot{G}}{G}\right)\left(\frac{\dot{a}}{a}\right) = 2\left(\frac{\dot{G}}{G}\right)^2$$

Deflation During Inflation

Gap equations

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{\kappa}{a^2} - \frac{\Lambda(t)}{3} = \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{G}}{G}\right) \qquad \text{2 gap}$$

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3 unknowns, 3 equations

Deflation During Inflation

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3 unknowns, 3 equations $\Rightarrow$ solve!

Deflation During Inflation

Solution:

$$a(t) = a_i e^{\frac{t}{\sqrt{\Lambda_0/3}}}$$

$$G(t) = \frac{G_0}{1 + \xi a(t)}$$

$$\Lambda(t) = \Lambda_0 \left[\frac{1 + 2\xi a(t)}{1 + \xi a(t)} \right]$$

Deflation During Inflation

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$$a(t) = a_i e^{\frac{t}{\sqrt{\Lambda_0/3}}}$$ still inflation

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controls SD

Deflation During Inflation

Solution:

$$\lim_{\xi \to 0} a(t) = a_i e^{\frac{t}{\sqrt{\Lambda_0/3}}}$$

$$\lim_{\xi \to 0} G(t) = G_0$$

$$\lim_{\xi \to 0} \Lambda(t) = \Lambda_0$$

controls SD

3 integration constants: G_0, Λ_0, ξ

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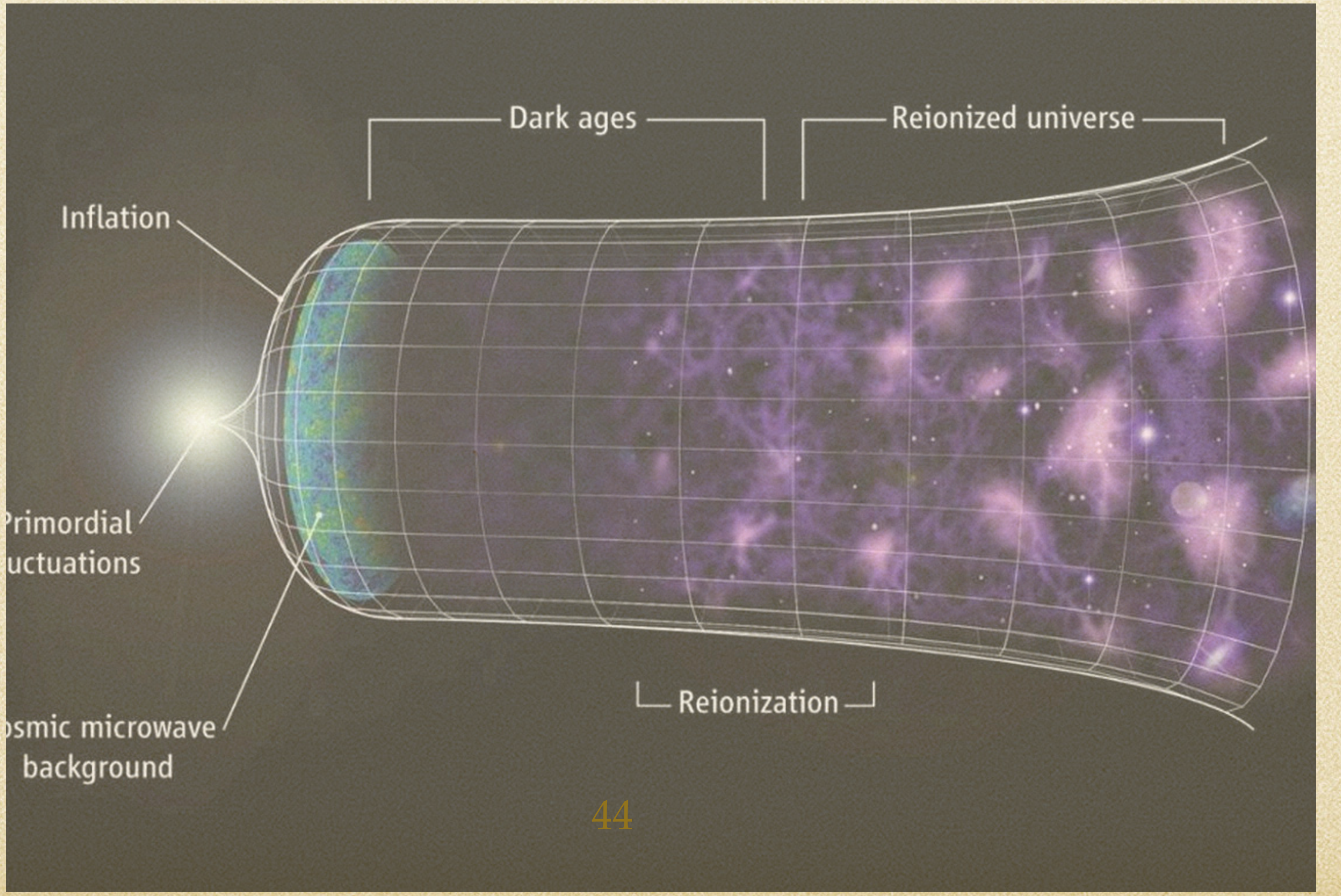
What does this mean for the CCP?

Deflation During Inflation

- What does this mean for the CCP?

Deflation During Inflation

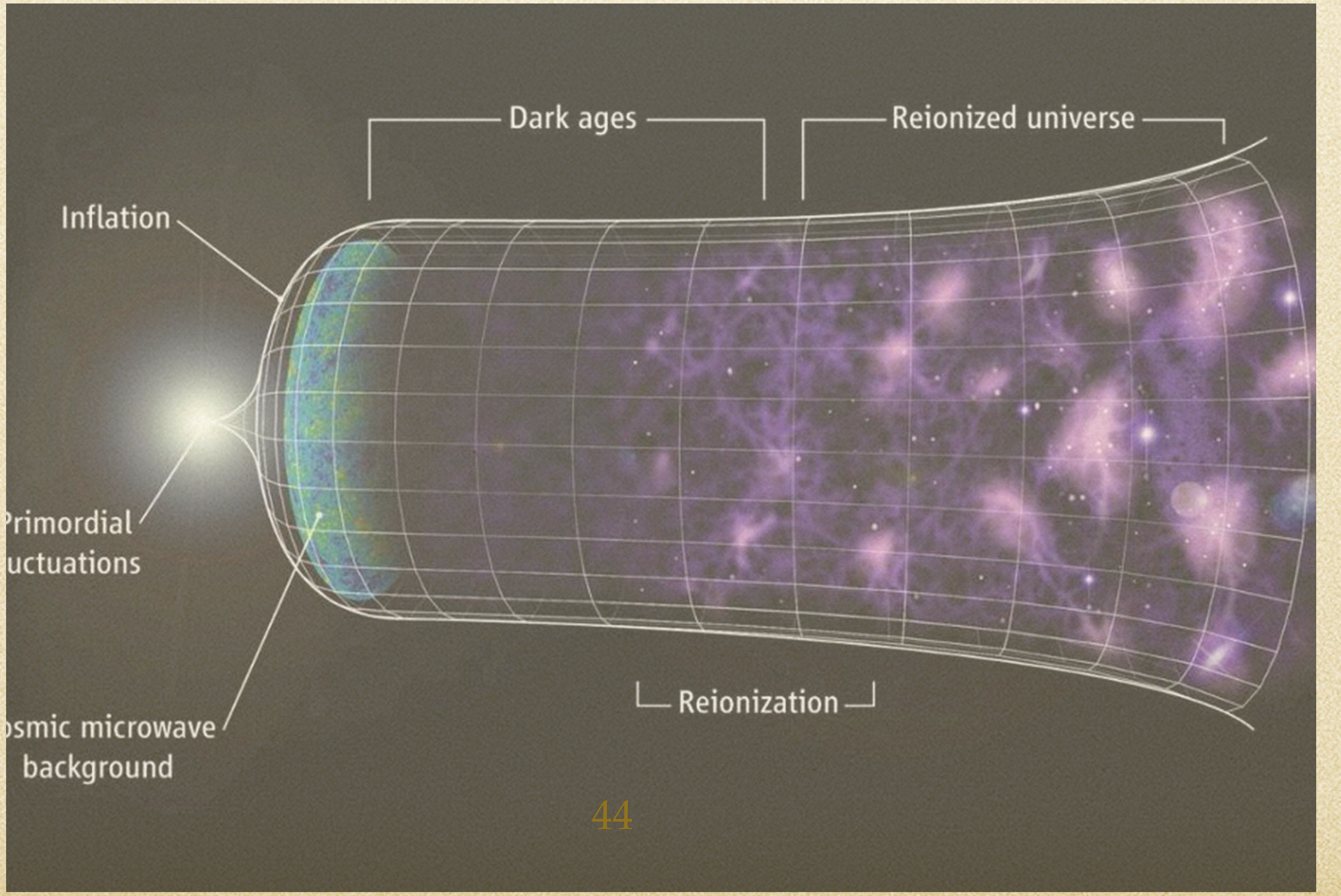
What does this mean for the CCP?



Deflation During Inflation

What does this mean for the CCP?

$$G_k \cdot \Lambda_k = G(t) \cdot \Lambda(t)$$

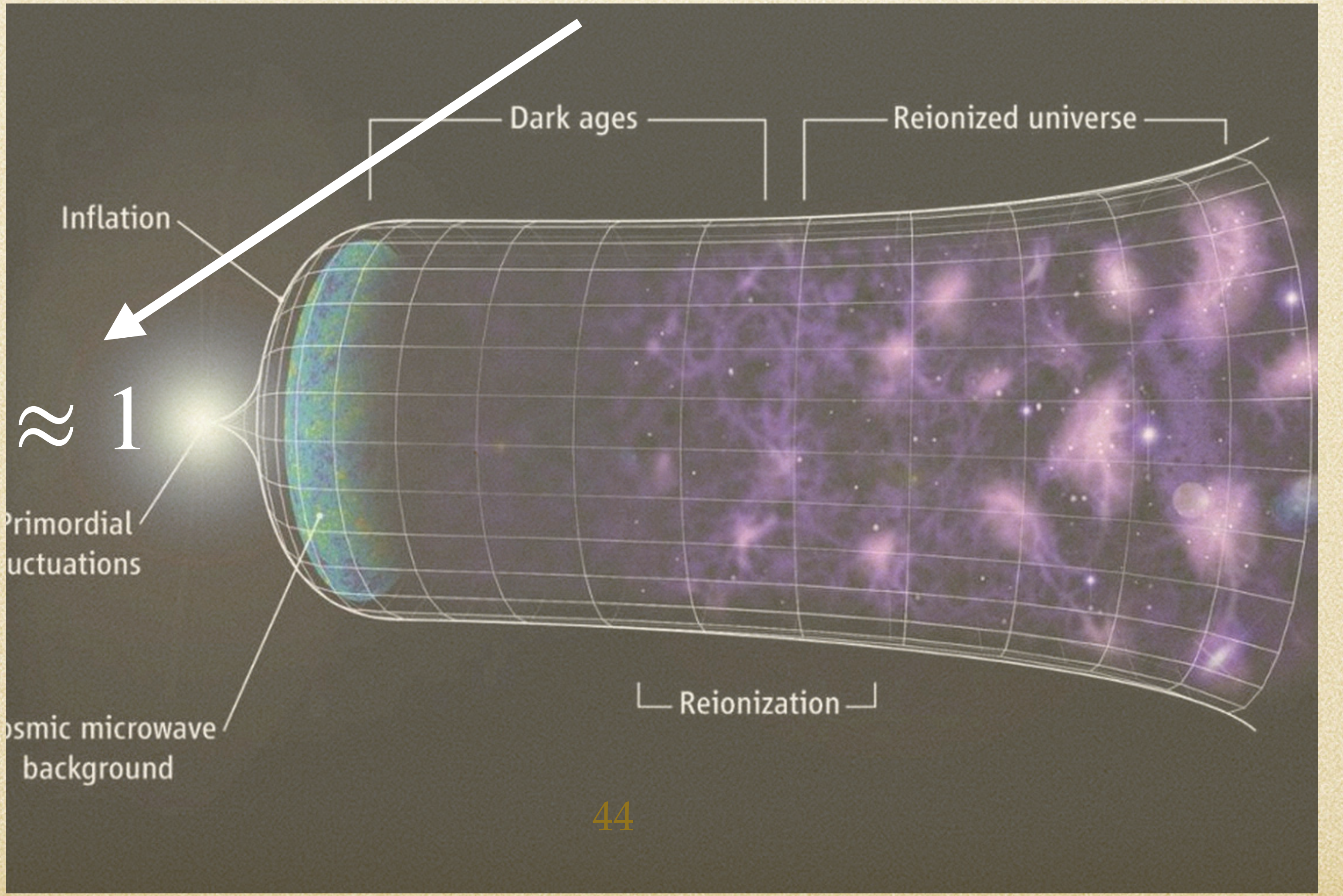


Deflation During Inflation

What does this mean for the CCP?

$$G_k \cdot \Lambda_k = G(t) \cdot \Lambda(t)$$

$$G(t_i) \cdot \Lambda(t_i) \approx 1$$



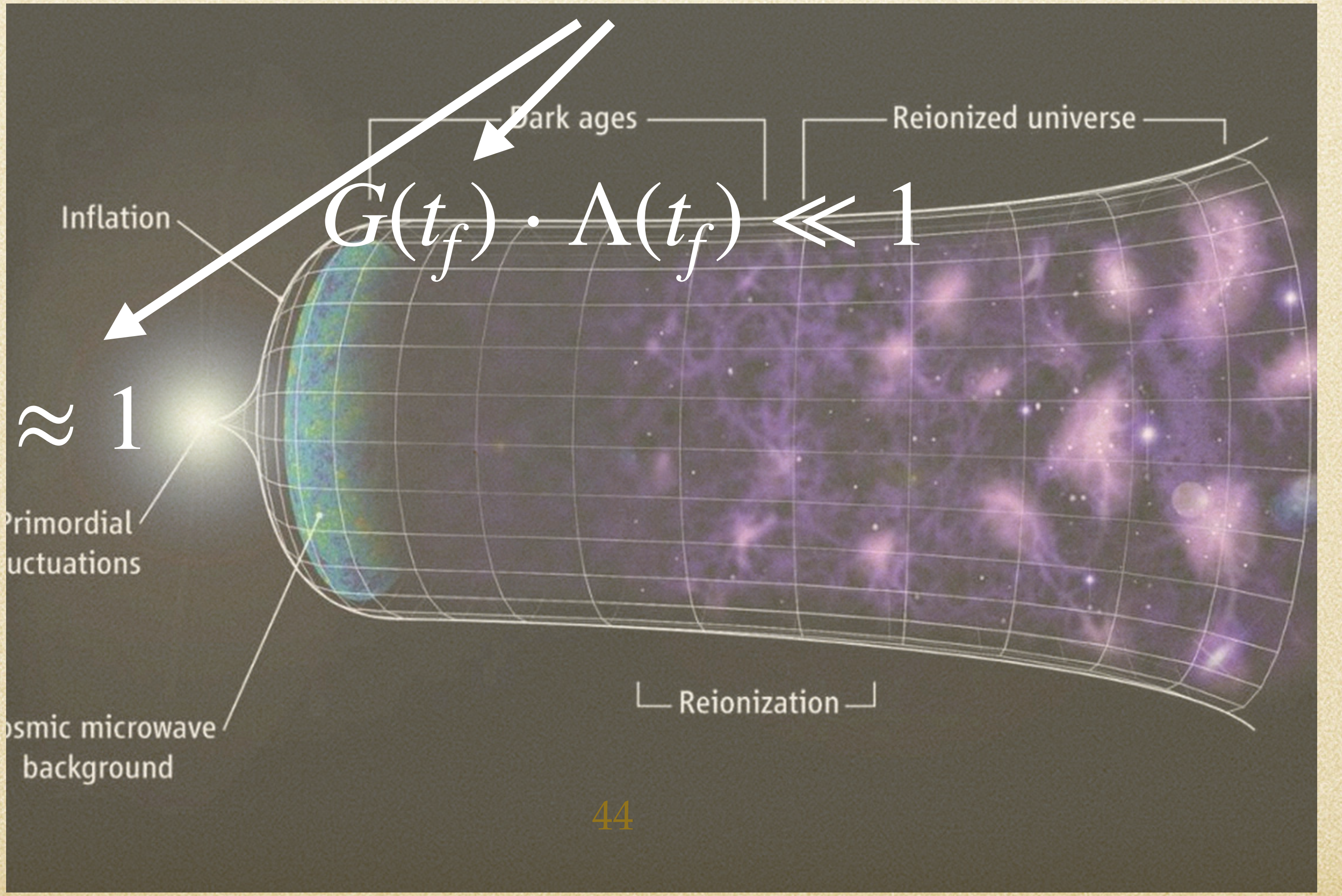
Deflation During Inflation

What does this mean for the CCP?

$$G_k \cdot \Lambda_k = G(t) \cdot \Lambda(t)$$

$$G(t_f) \cdot \Lambda(t_f) \ll 1$$

$$G(t_i) \cdot \Lambda(t_i) \approx 1$$



Deflation During Inflation

What does this mean for the CCP?

Deflation During Inflation

What does this mean for the CCP?

$$G(t) \cdot \Lambda(t) = \frac{G_0}{1 + \xi a(t)} \cdot \Lambda_0 \left[\frac{1 + 2\xi a(t)}{1 + \xi a(t)} \right]$$

Deflation During Inflation

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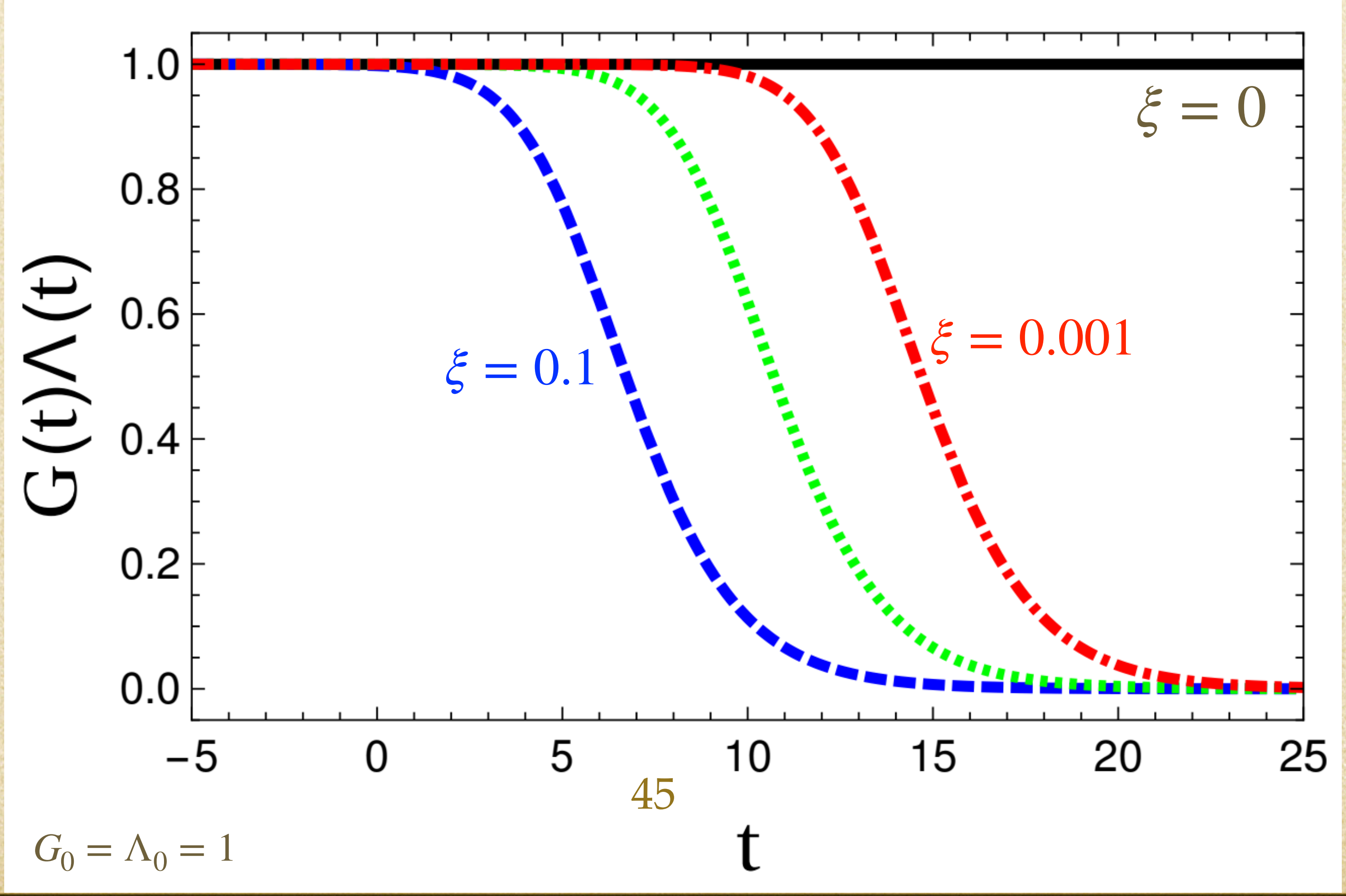

Deflation During Inflation

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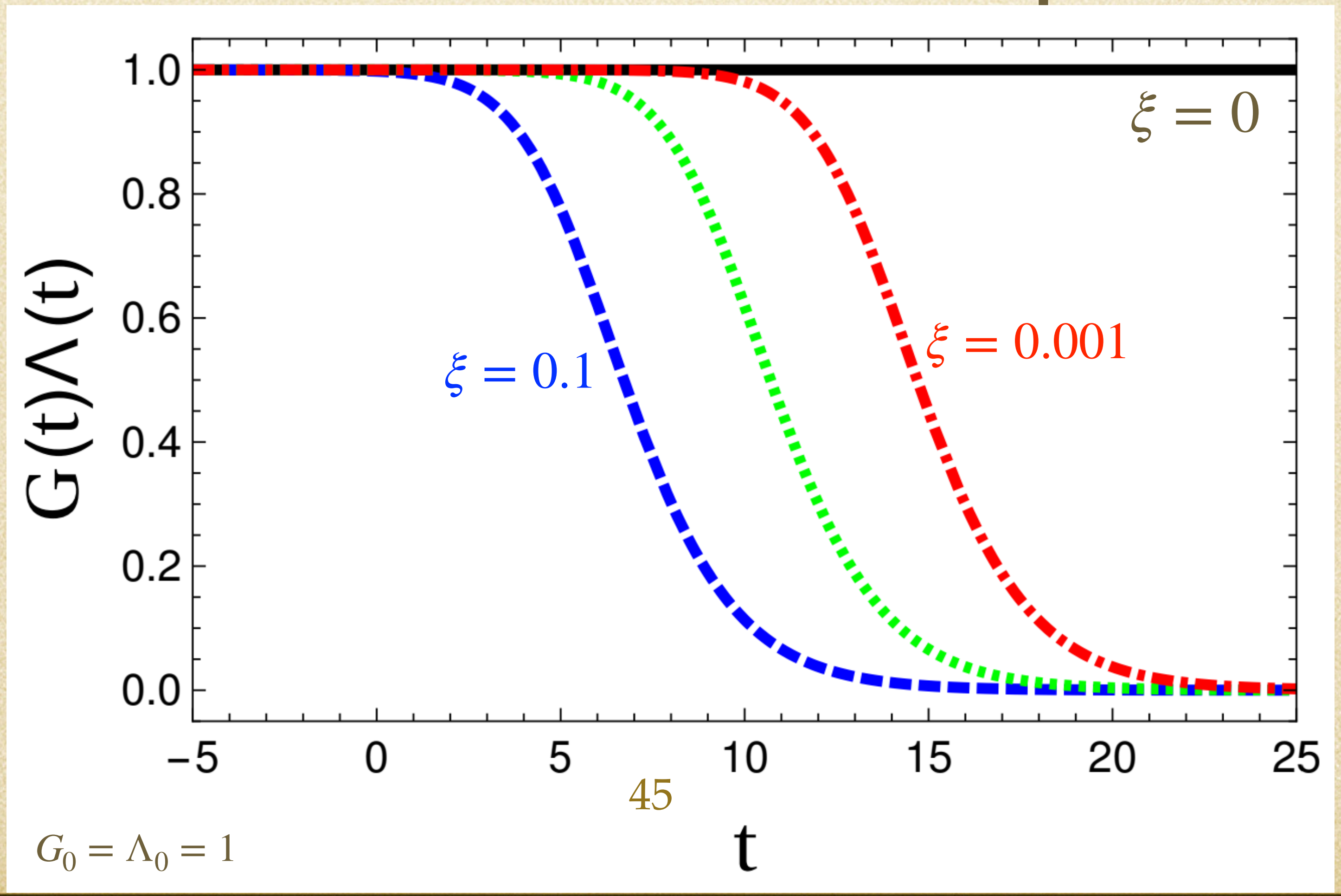

Looks good, conditions?

Deflation During Inflation

CCP conditions on parameters

Deflation During Inflation

CCP conditions on parameters

- Initial a
- Initial CCP
- Final G
- Final CCP
- Flatness

Deflation During Inflation

CCP conditions on parameters

- Initial a $a(t_i) = 1$
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- Final G
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Deflation During Inflation

CCP conditions on parameters

- Initial a $a(t_i) = 1$
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Deflation During Inflation

CCP conditions on parameters

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Deflation During Inflation

CCP conditions on parameters

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- Final CCP $G(t_f) \cdot \Lambda(t_f) = 10^{-(120\pm5)}$
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Deflation During Inflation

CCP conditions on parameters

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- Final G $G(t_f) = G_N$
- Final CCP $G(t_f) \cdot \Lambda(t_f) = 10^{-(120\pm5)}$
- Flatness $N_e \geq 60; \quad t_f - t_i = N_e\sqrt{\Lambda_0/3}$

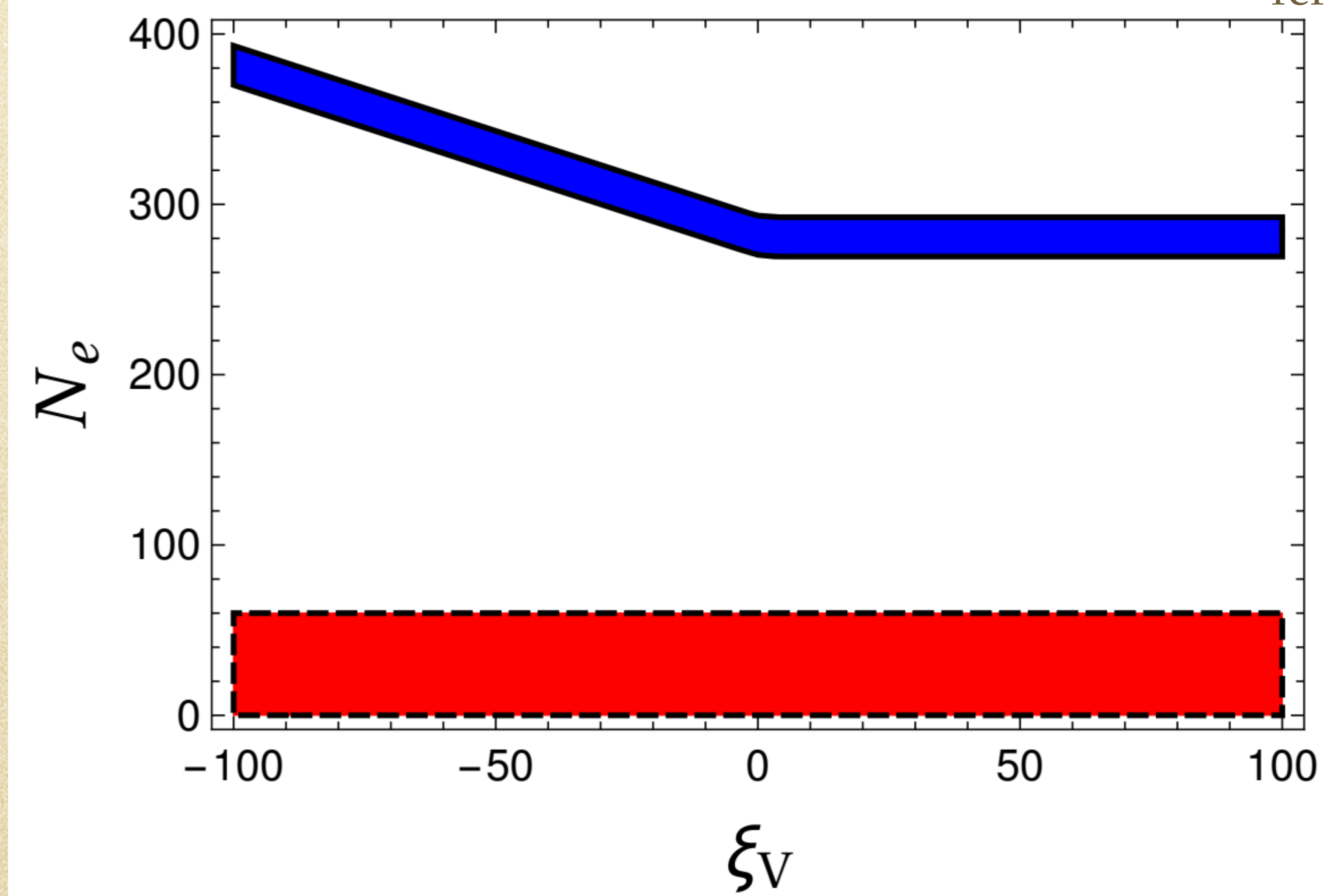
Deflation During Inflation

CCP conditions on parameters

Deflation During Inflation

CCP conditions on parameters

ref [0]



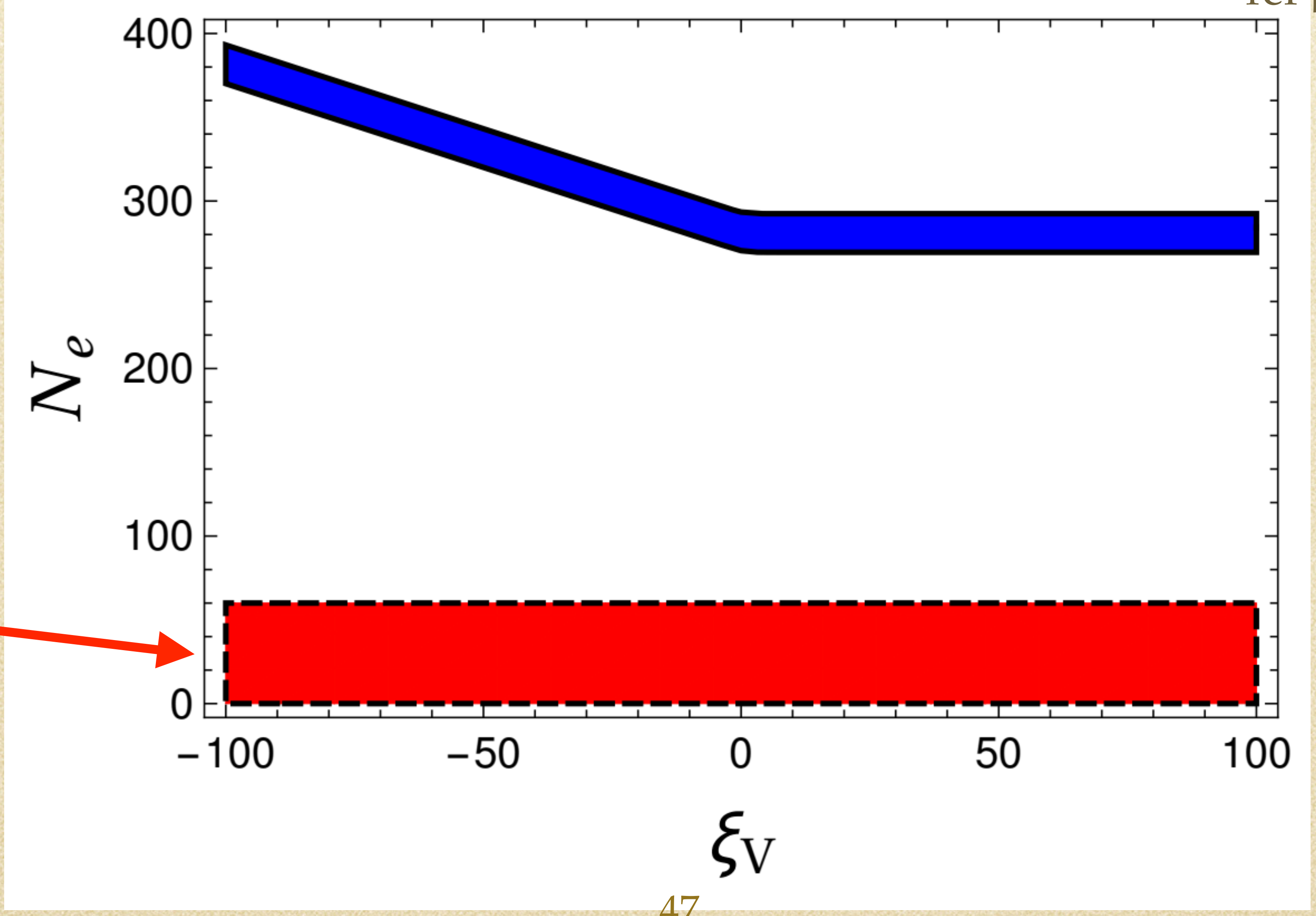
$\xi = e^{\xi_V}$

Deflation During Inflation

CCP conditions on parameters

ref [0]

flatness excluded



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Deflation During Inflation

- CCP conditions on parameters

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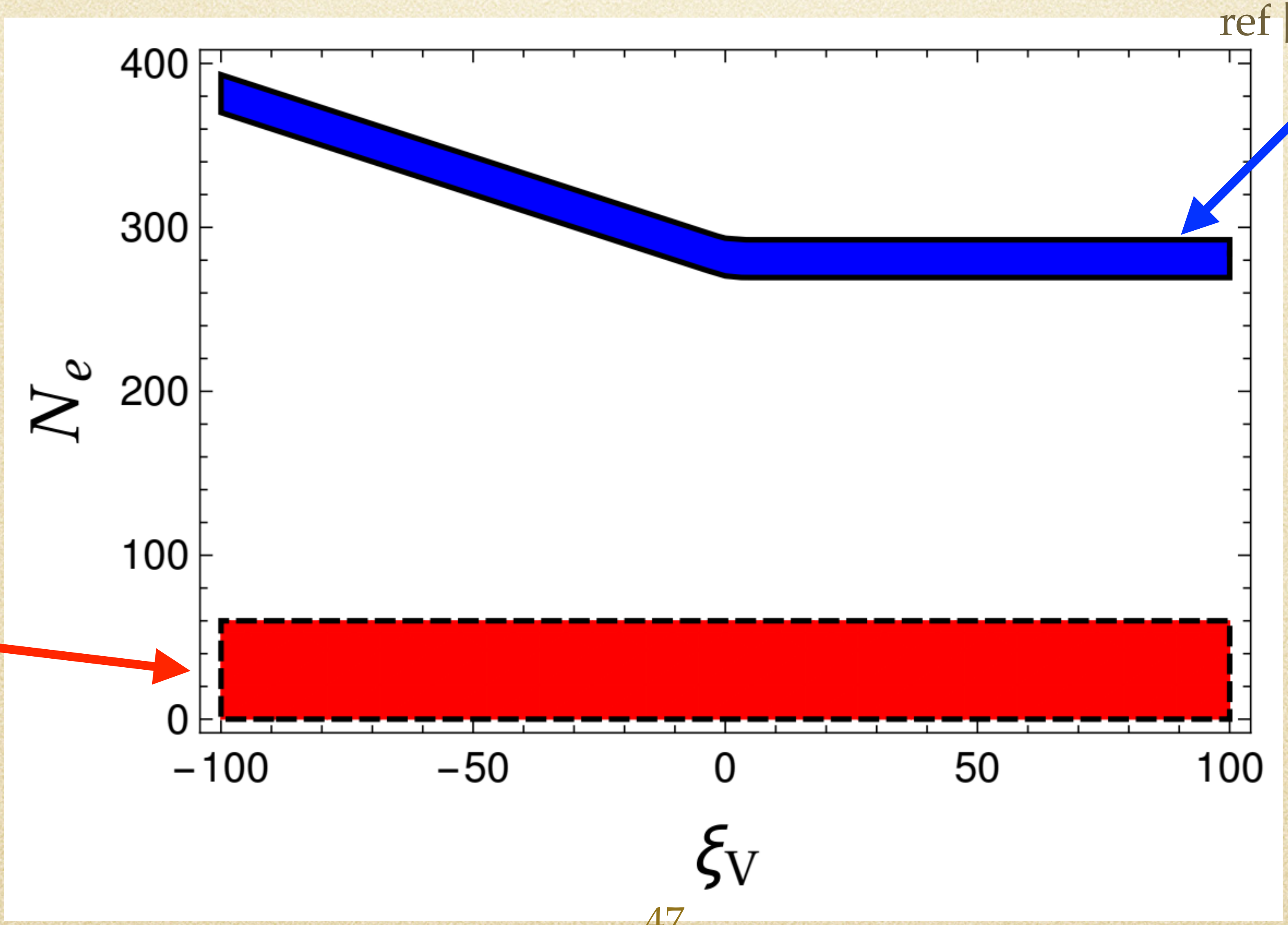

solves CCP

flatness excluded

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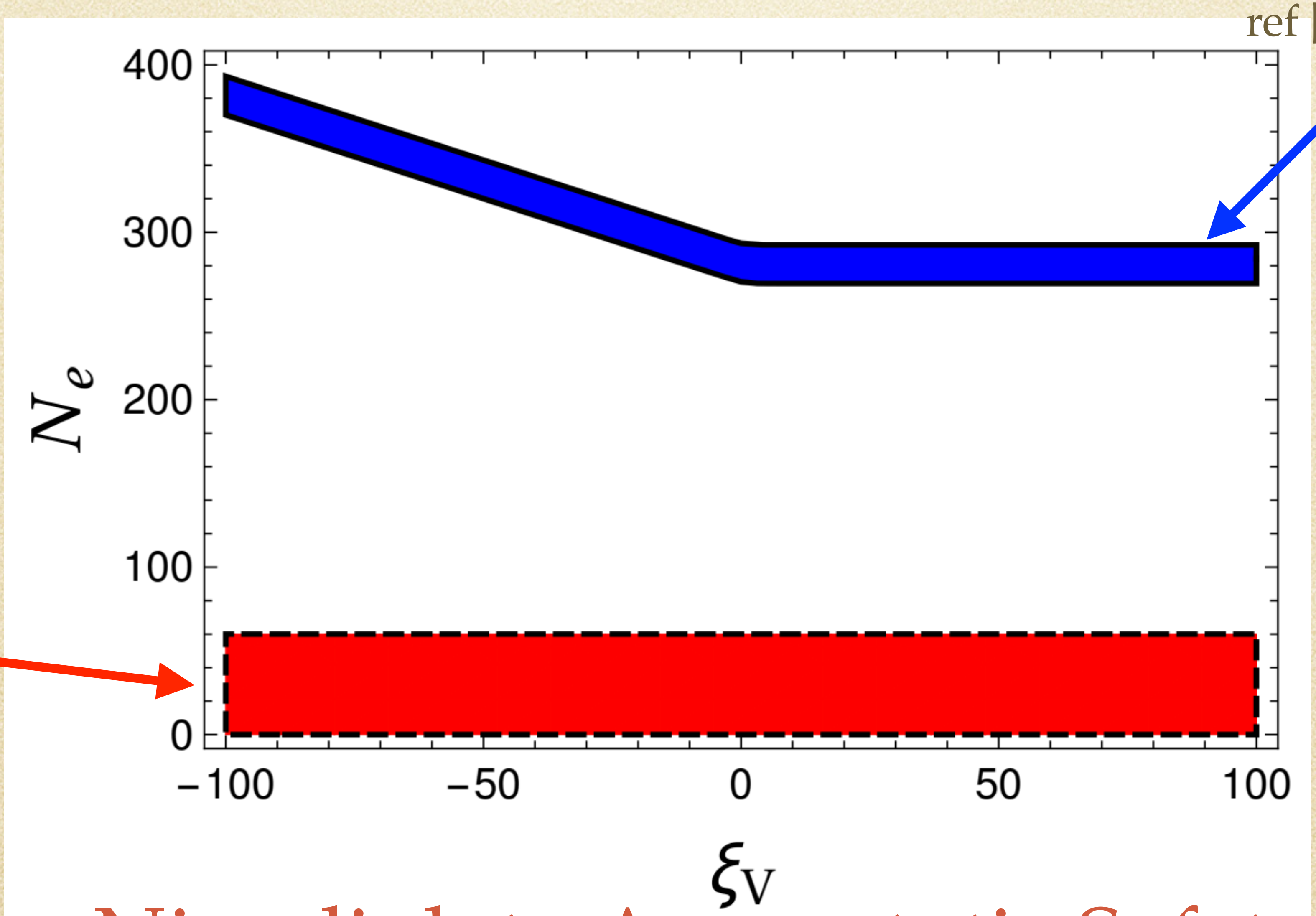
Deflation During Inflation

CCP conditions on parameters

ref [0]



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Nice, link to Asymptotic Safety?

Link to AS?

Link to AS?

Similar thoughts
In the late Universe

Link to AS?

Similar thoughts
In the late Universe

Graviton fluctuations erase the cosmological constant

C. Wetterich*

Universität Heidelberg, Institut für Theoretische Physik, Philosophenweg 16, D-69120 Heidelberg

arXiv:1704.08040v2

Link to AS?

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Now our idea coming from the UV:

Link to AS?

Link to AS?

Remember:

Link to AS?

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$$G_k = \frac{\hat{g}_k}{k^2}$$

$$\Lambda_k = \hat{\lambda}_k k^2$$

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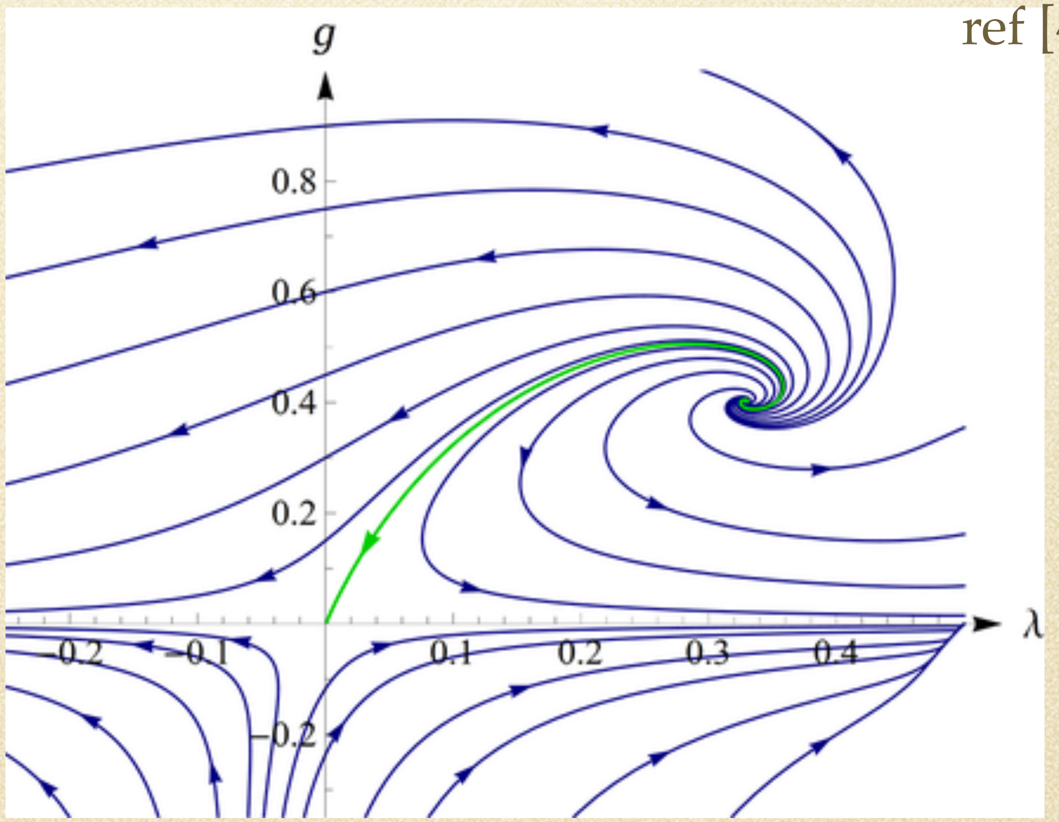
ref [4]



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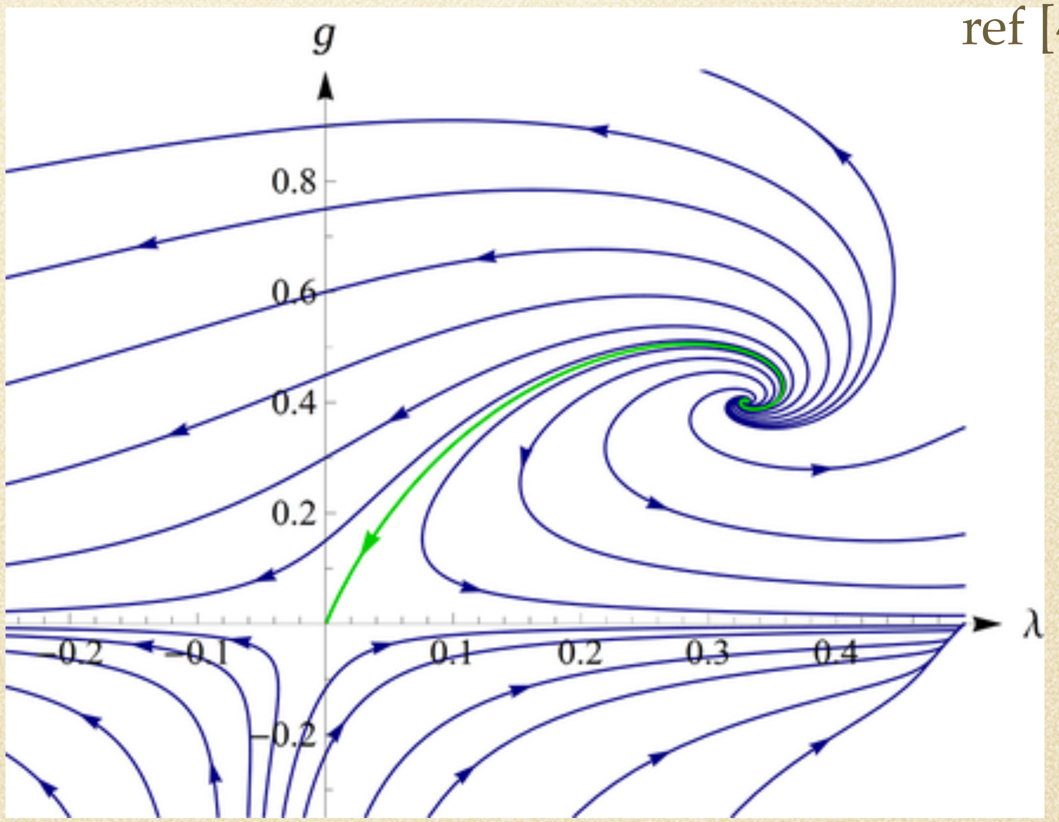

ref [4]

UV FP

Link to AS?

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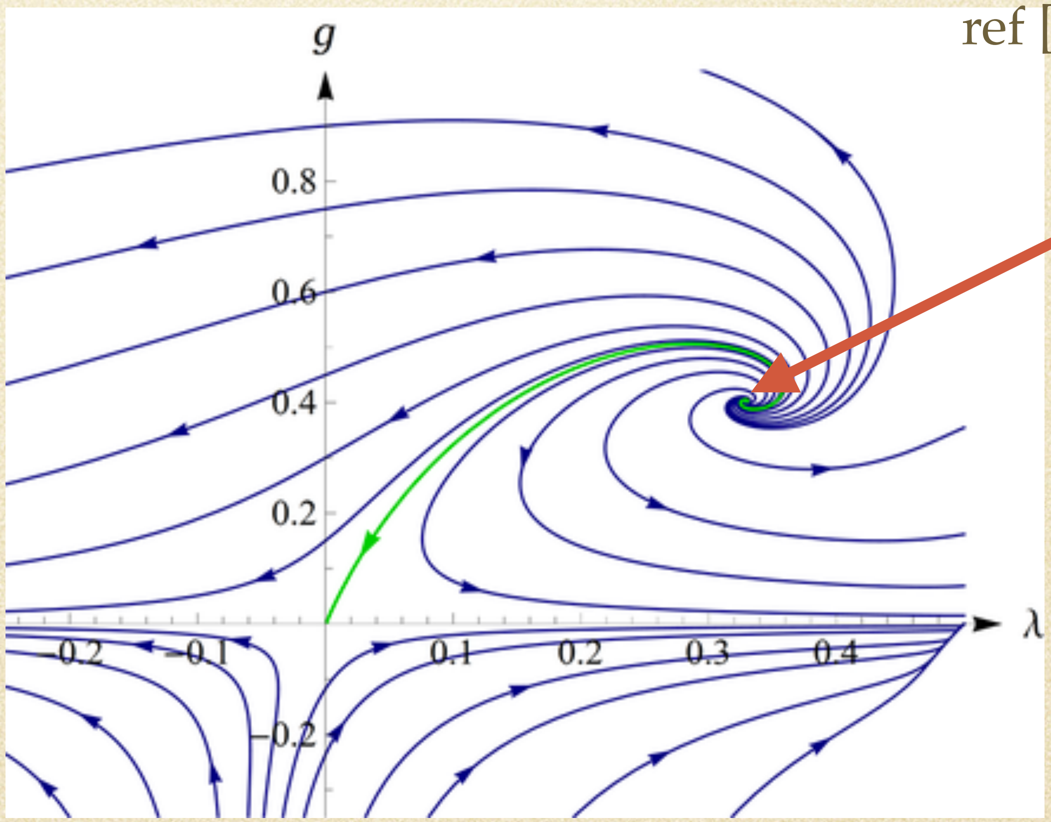

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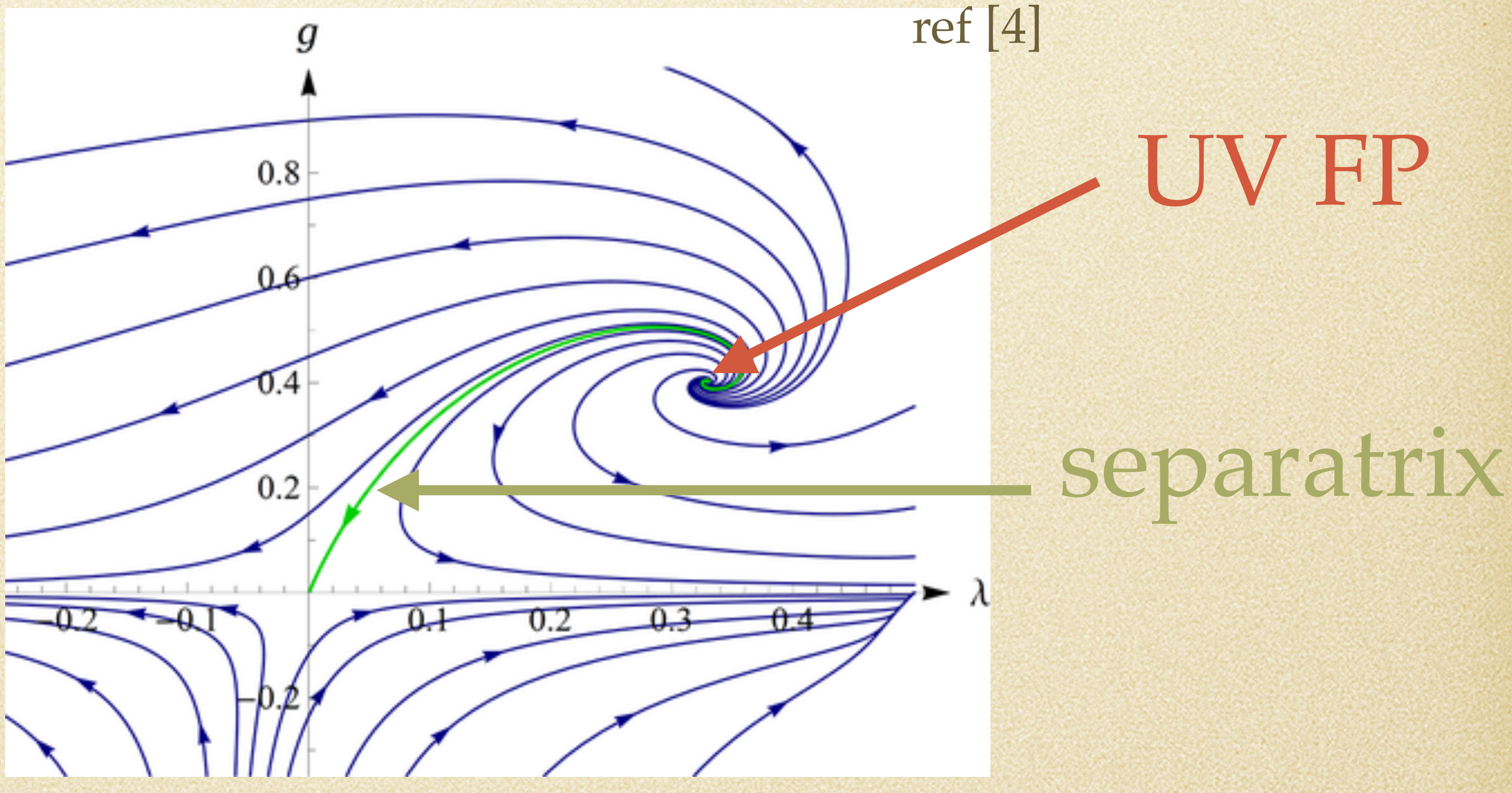

Link to AS?

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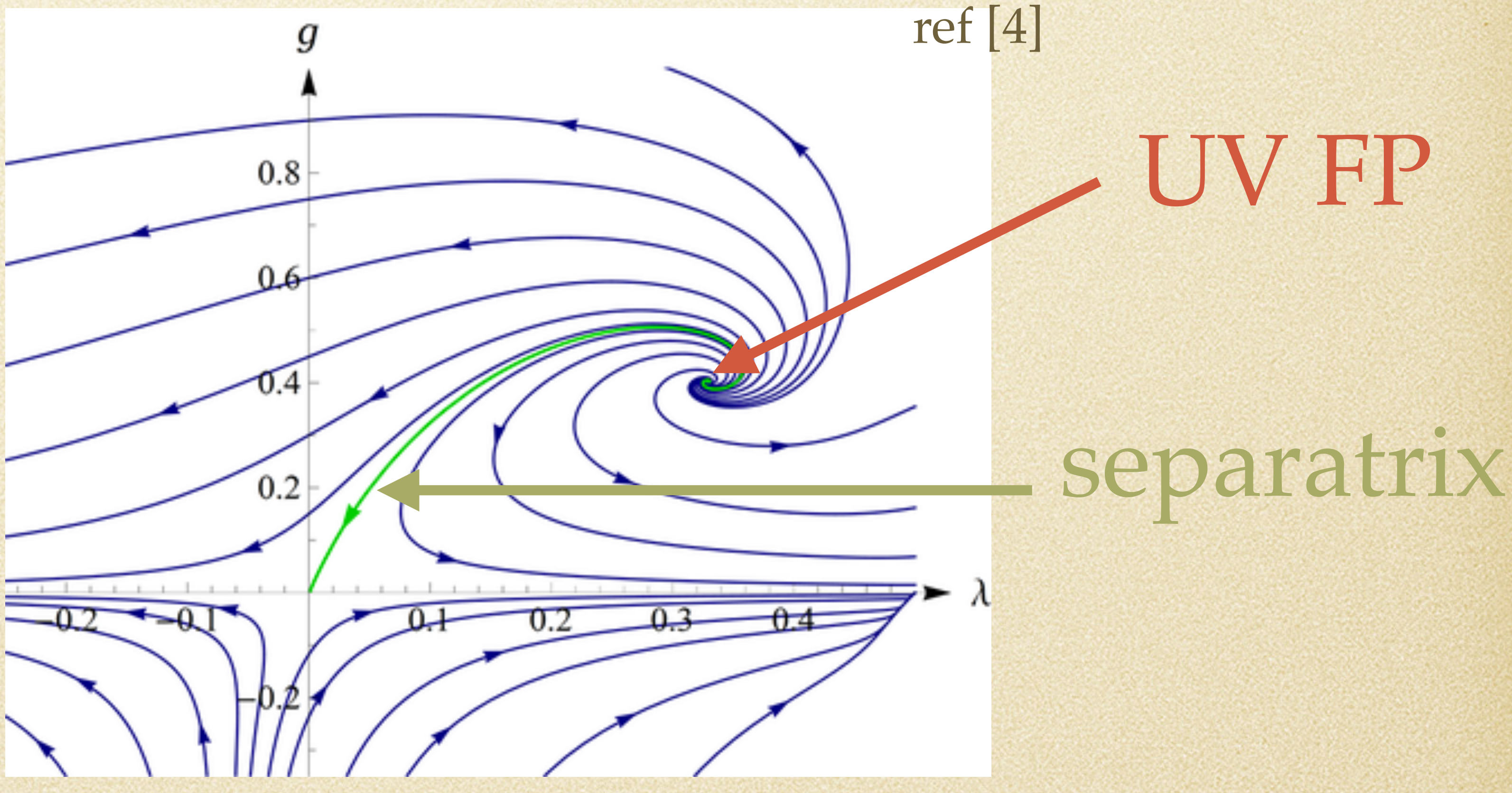


$$G_k = \frac{\hat{g}_k}{k^2}$$

$$\Lambda_k = \hat{\lambda}_k k^2$$

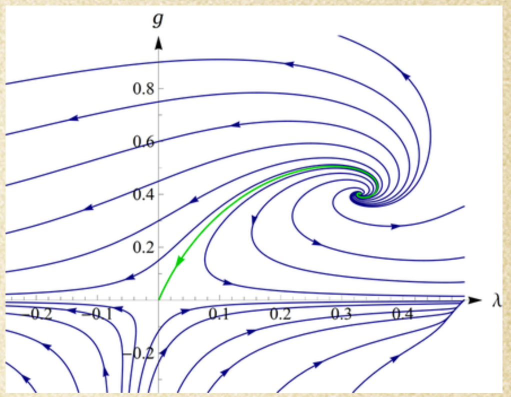
with

ref [5]

$$\hat{g}(\hat{t}) = \frac{g_0 e^{2\hat{t}}}{1 + g_0\left(e^{2\hat{t}} - 1\right)/g^*} \qquad \hat{\lambda}(\hat{t}) = \frac{g^*\lambda_0 + e^{-2\hat{t}}\left(e^{4\hat{t}} - 1\right) g_0 \lambda^*}{1 + g_0\left(e^{2\hat{t}} - 1\right)/g^*}$$

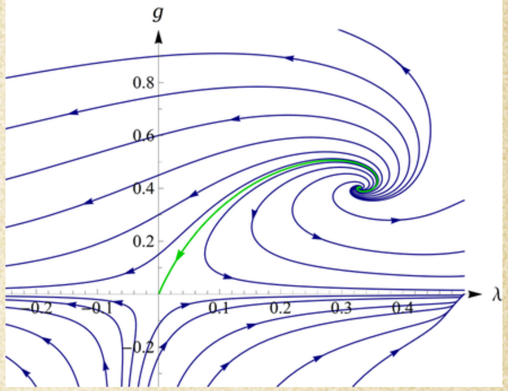
$$\hat{t} = \log(k/k_0)$$

Link to AS?



For CCP need:

Link to AS?



For CCP need:

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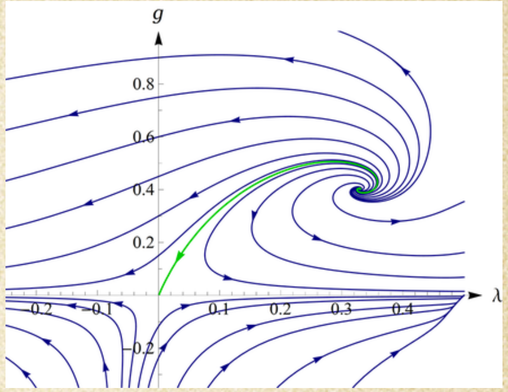
Link to AS?



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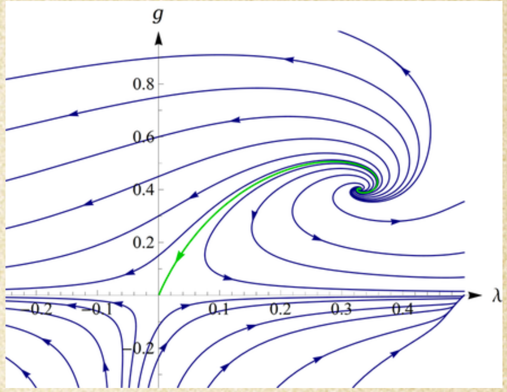
insert & plot

Link to AS?



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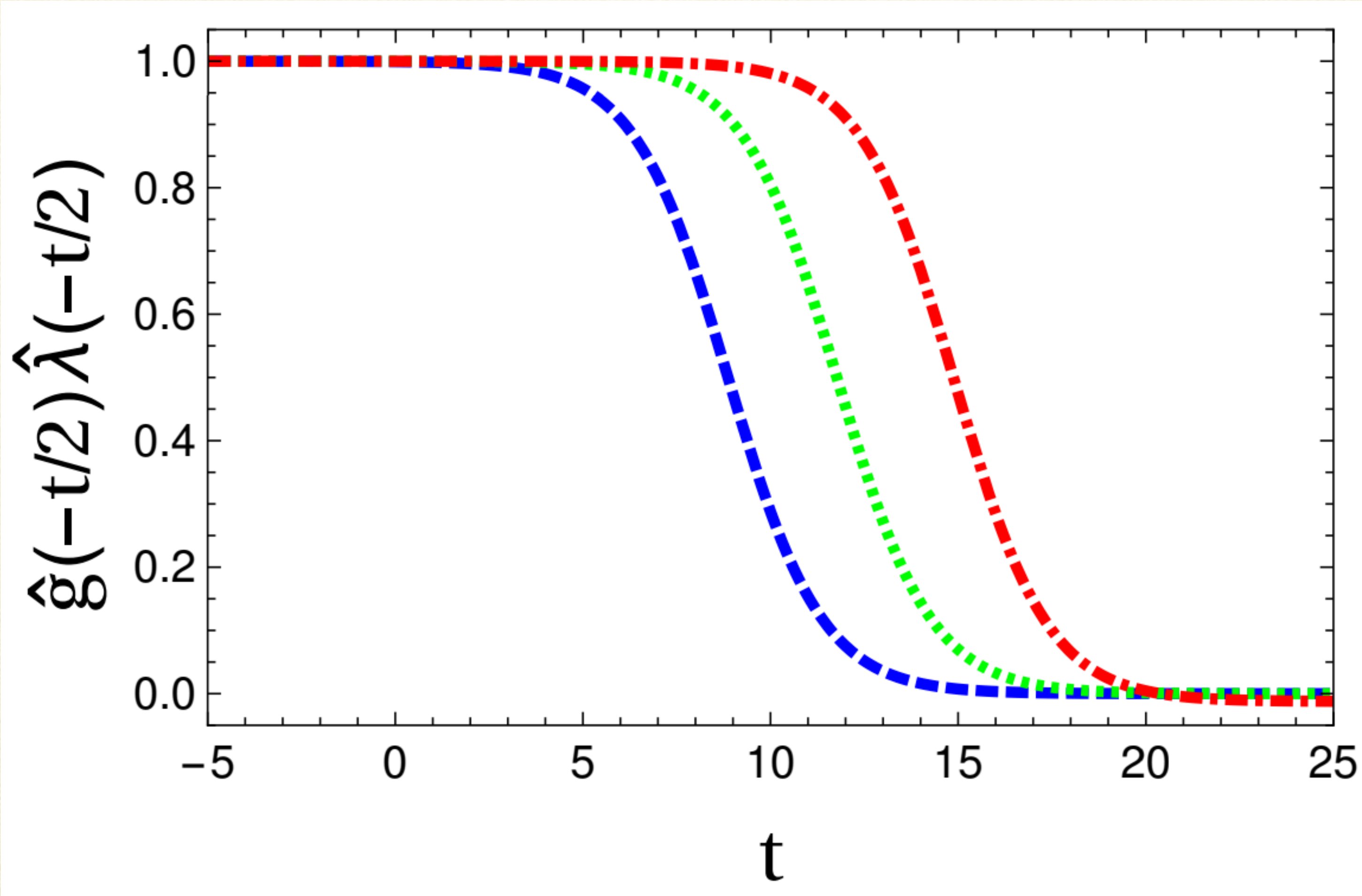

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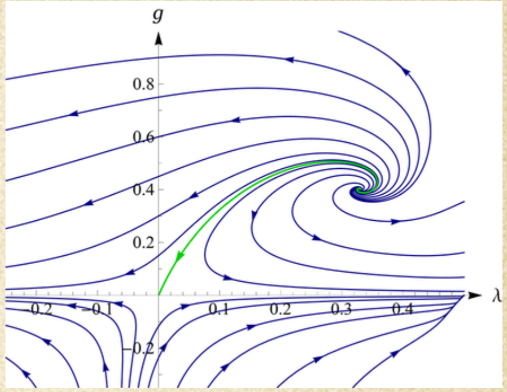

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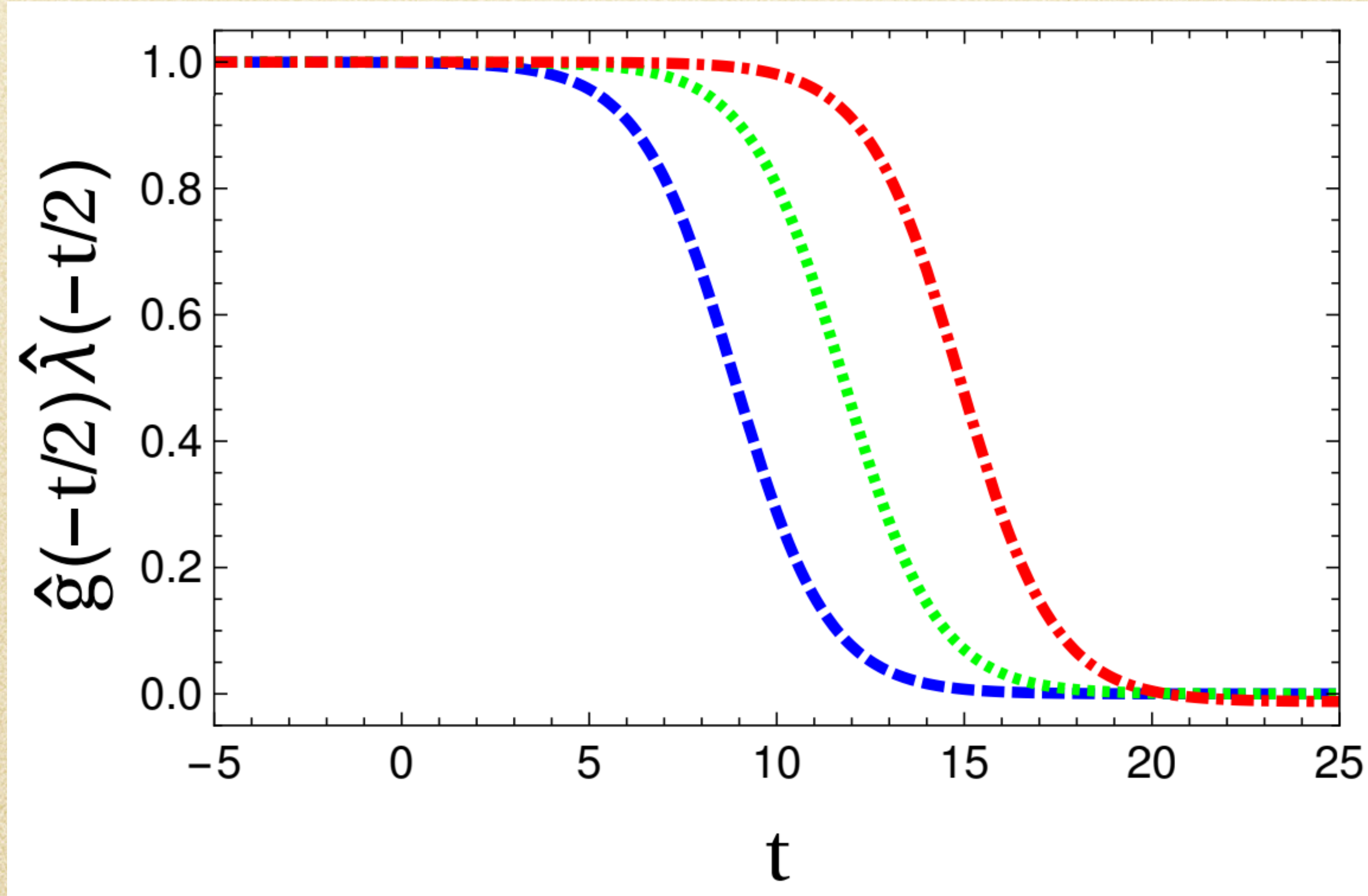
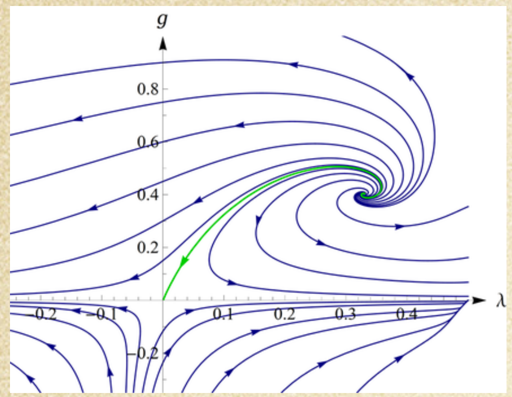


looks familiar?



Link to AS?

AS renormalization flow

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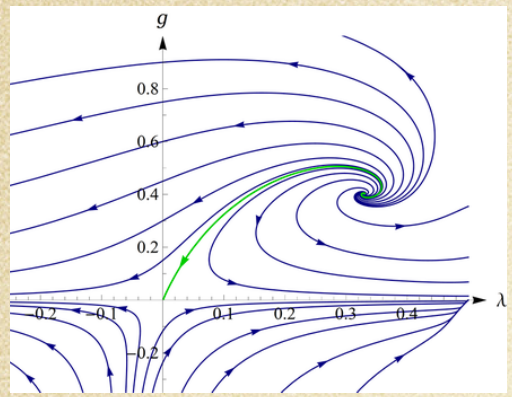

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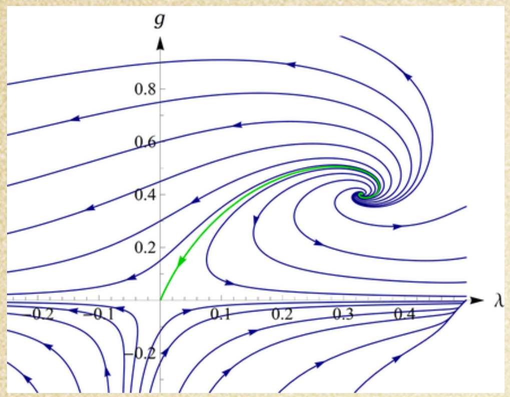
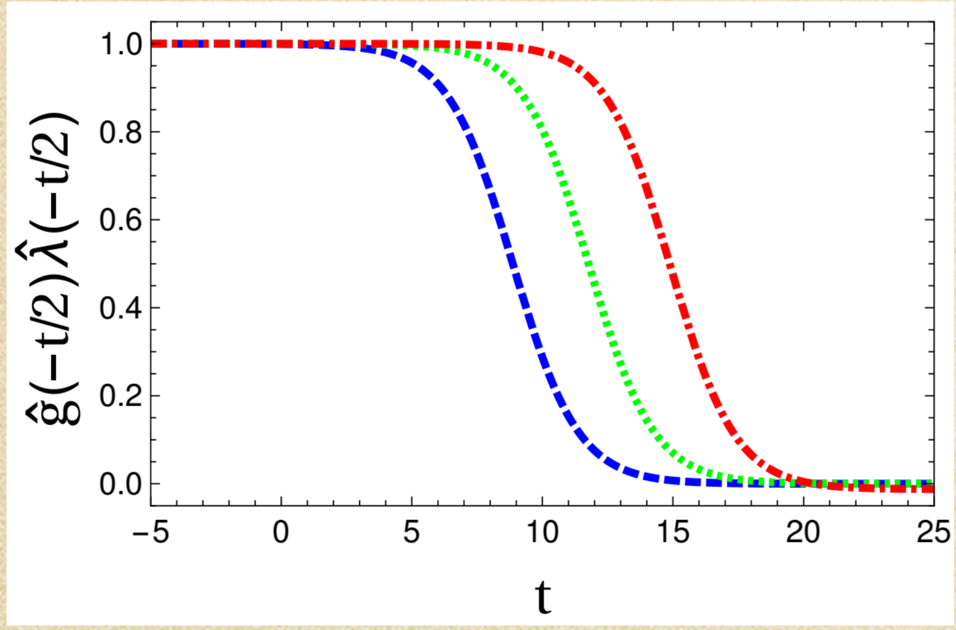
SD & NEC

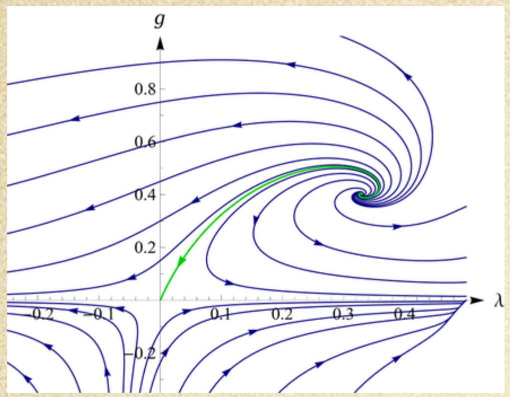
$$G(t) \cdot \Lambda(t)$$



Link to AS?

AS renormalization flow	SD & NEC
$G_k \cdot \Lambda_k = \frac{\hat{g}_k}{k^2} k^2 \hat{\lambda}_k = \hat{g}_k \cdot \hat{\lambda}_k$	$G(t) \cdot \Lambda(t)$





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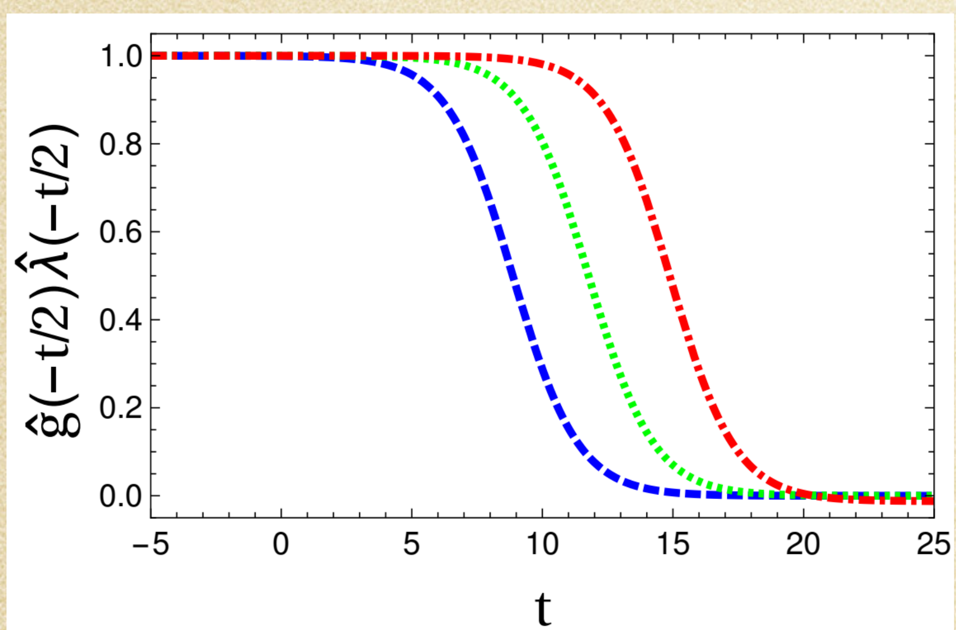

SD & NEC

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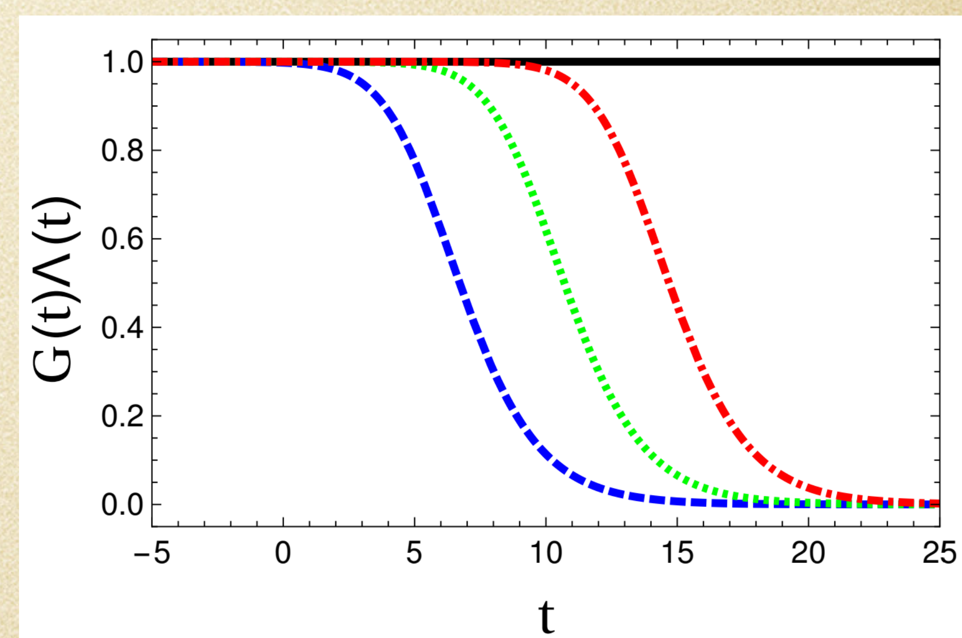

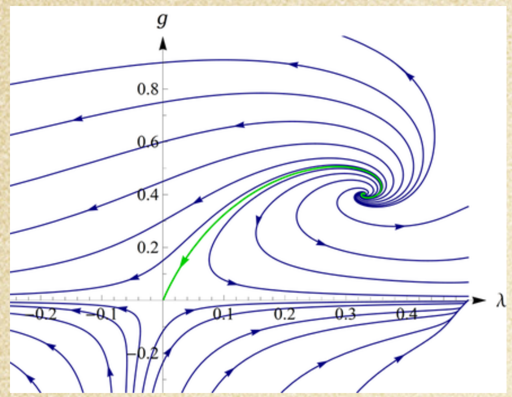

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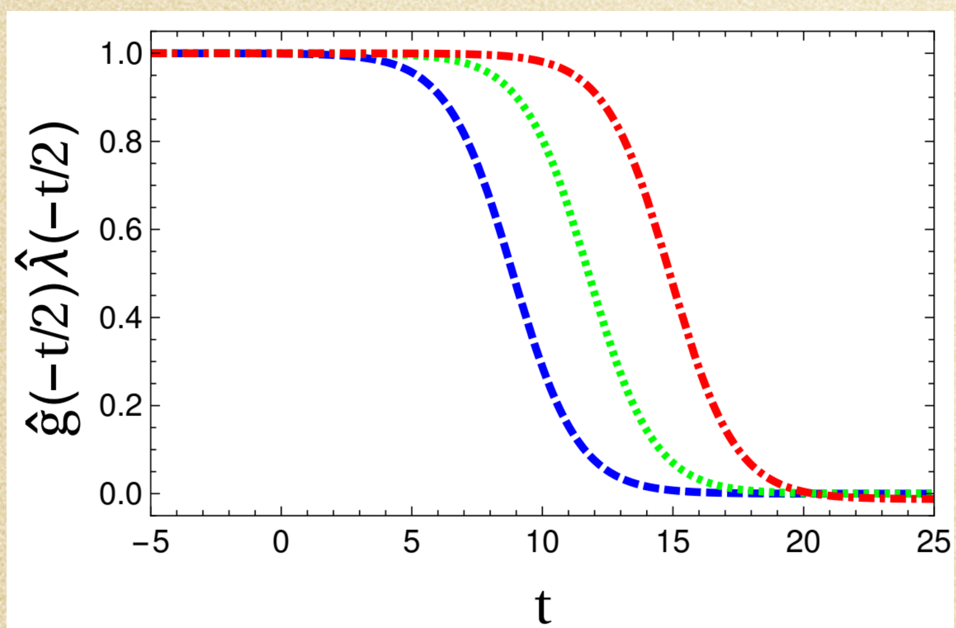

looks familiar!

SD & NEC

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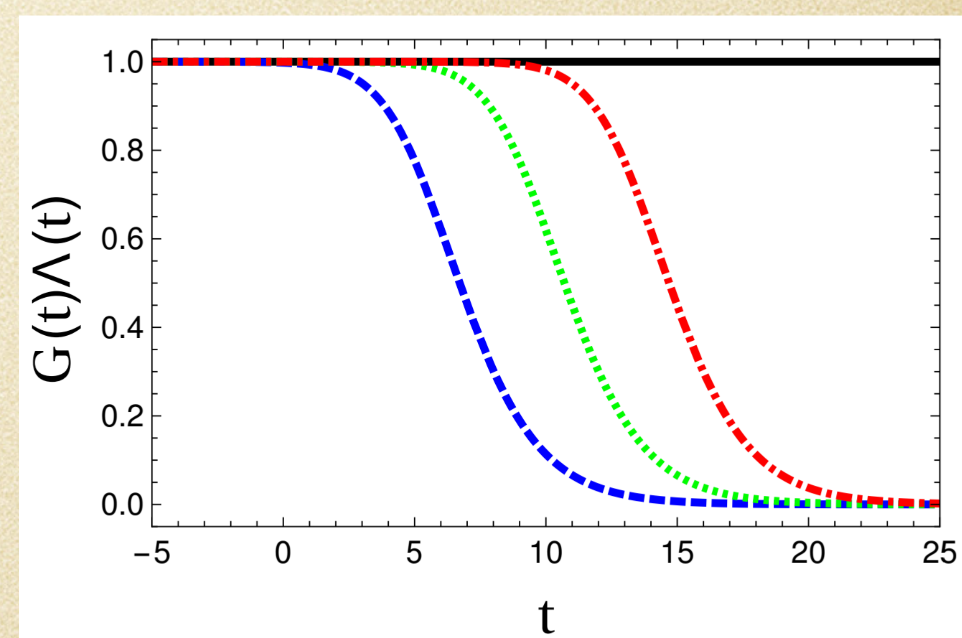

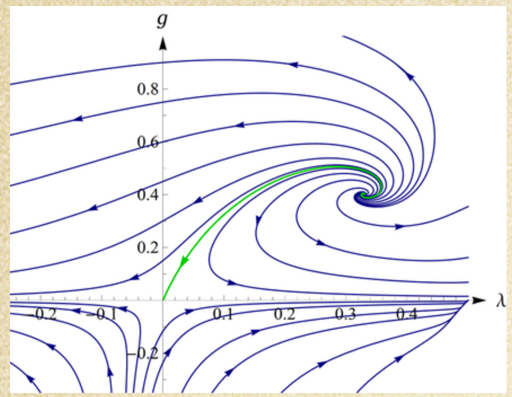

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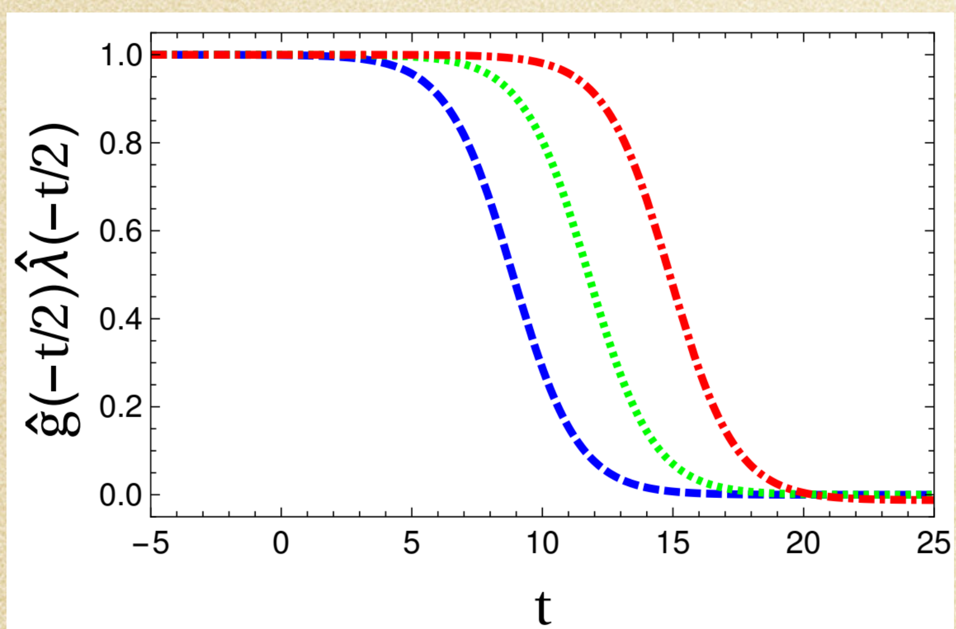

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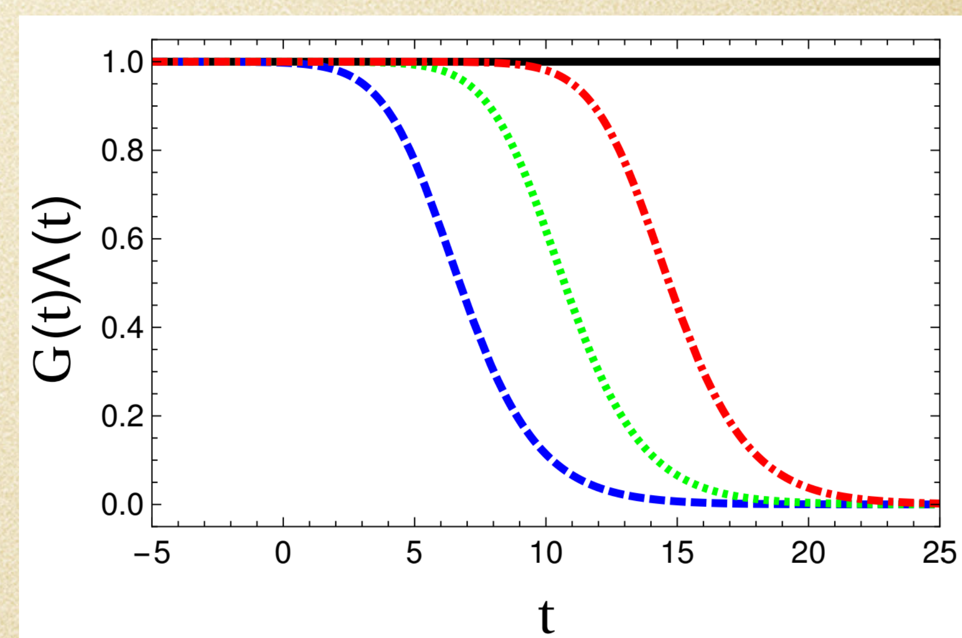

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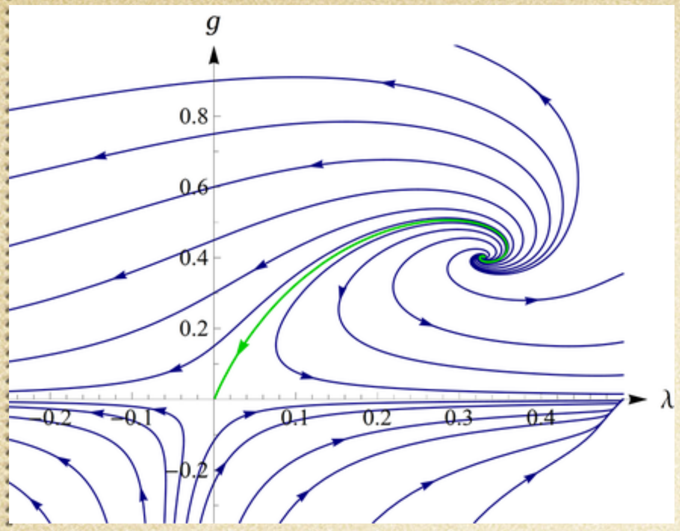
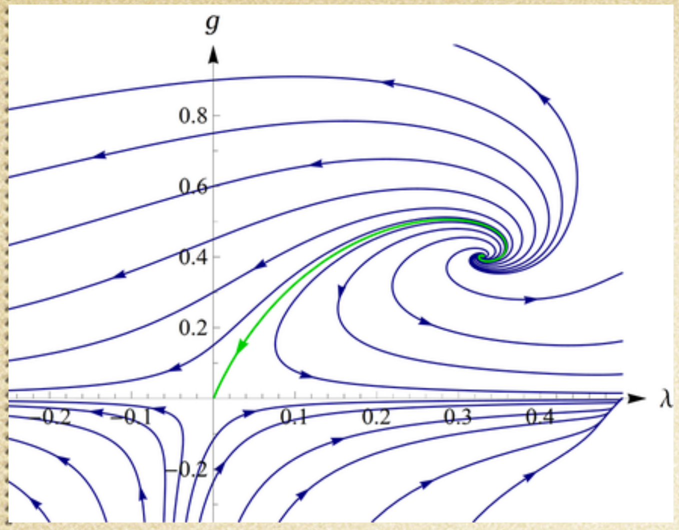
why, how?



Link to AS?

Remember:



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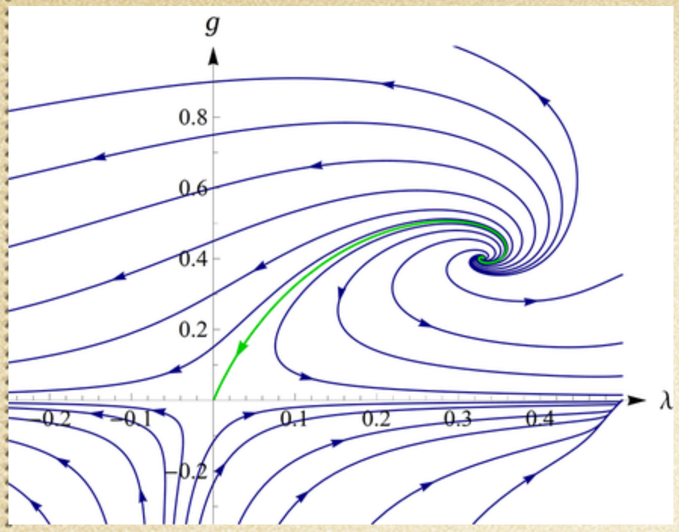
$$\hat{g}(\hat{t}) \cdot \hat{\lambda}(\hat{t}) = \frac{g_0 e^{2\hat{t}}}{1 + g_0\left(e^{2\hat{t}} - 1\right)/g*} \cdot \frac{g*\lambda_0 + e^{-2\hat{t}}\left(e^{4\hat{t}} - 1\right) g_0 \lambda*}{1 + g_0\left(e^{2\hat{t}} - 1\right)/g*}$$



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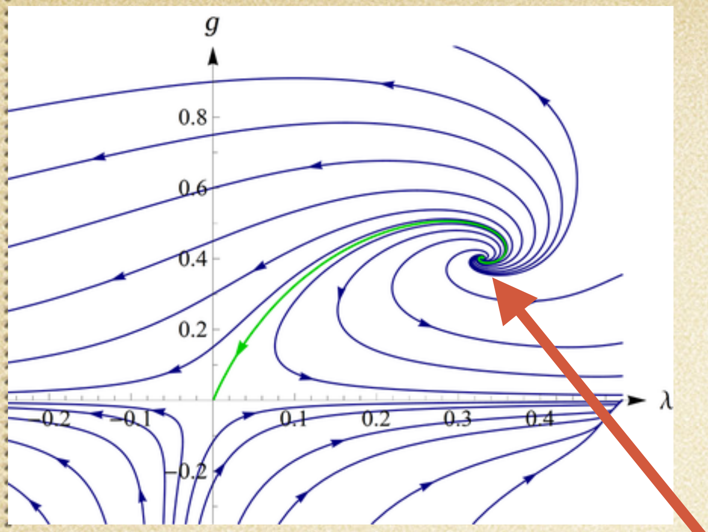
Approximate to UV FP & separatrix



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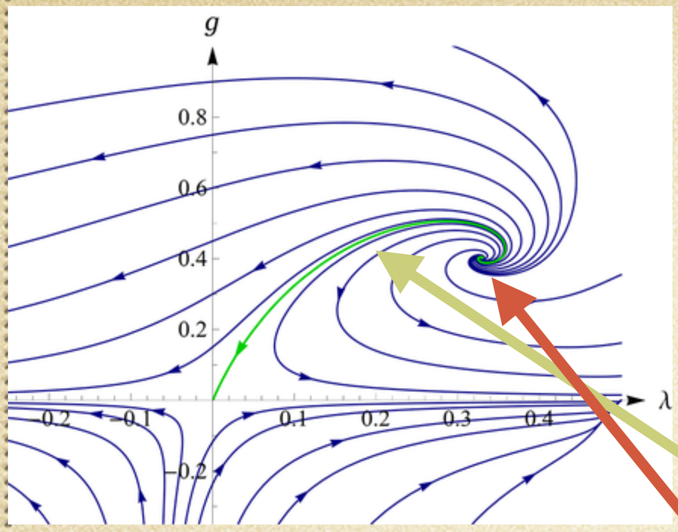

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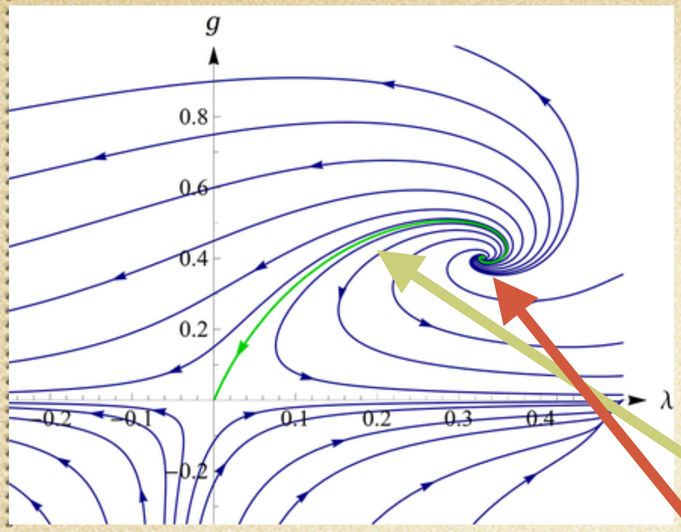

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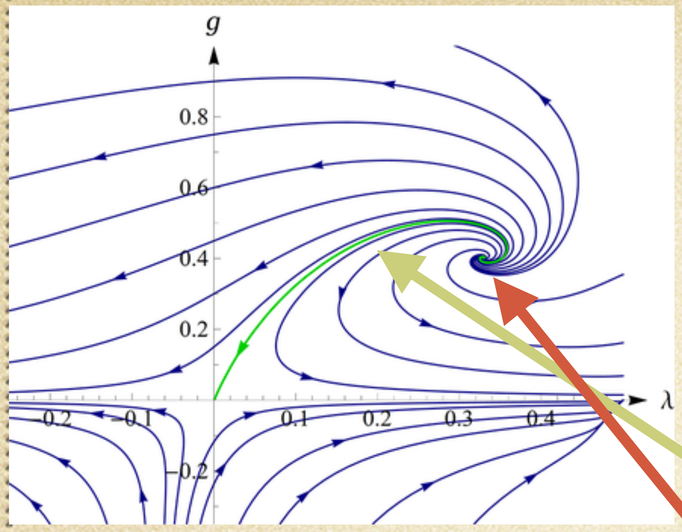

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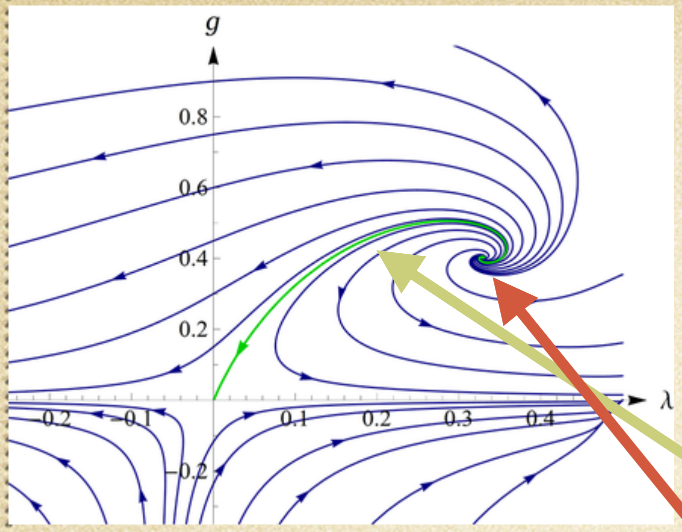

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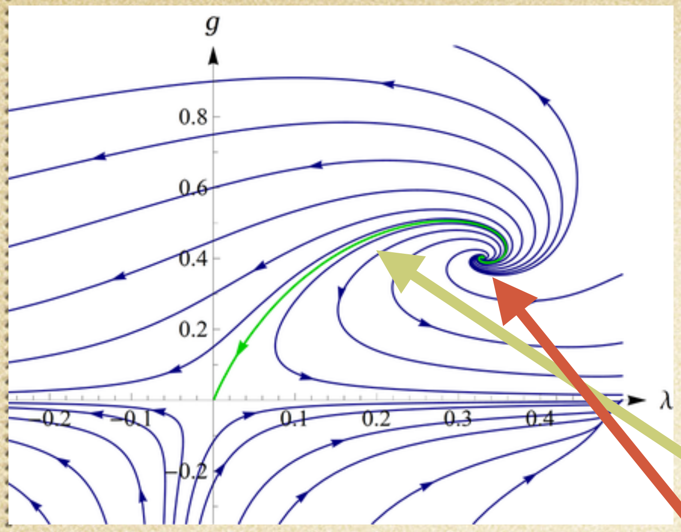

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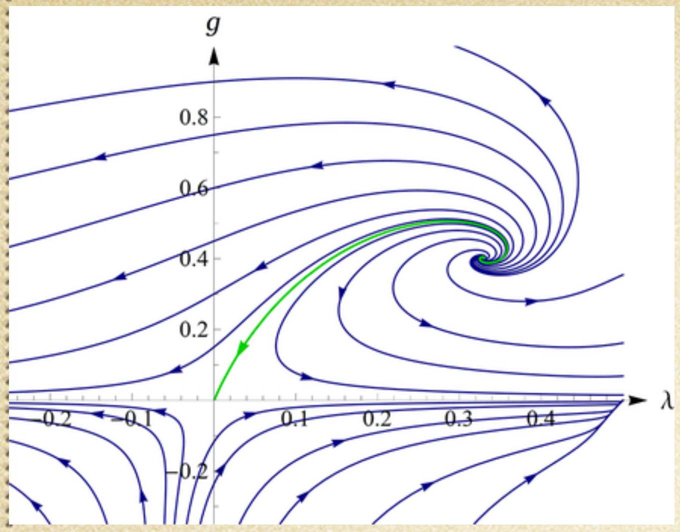

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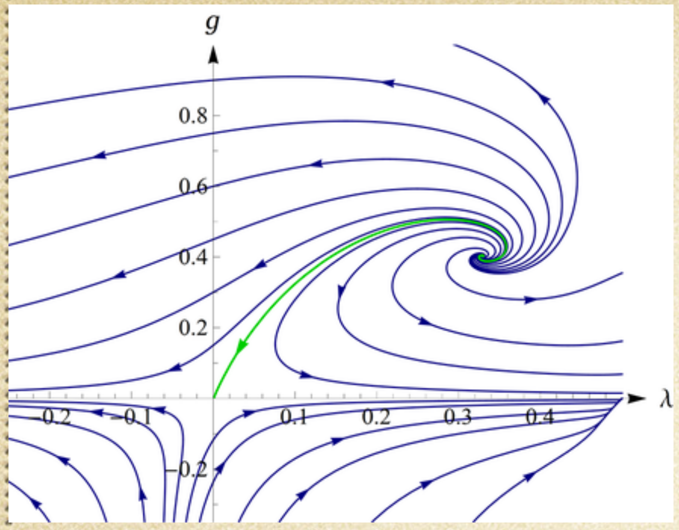

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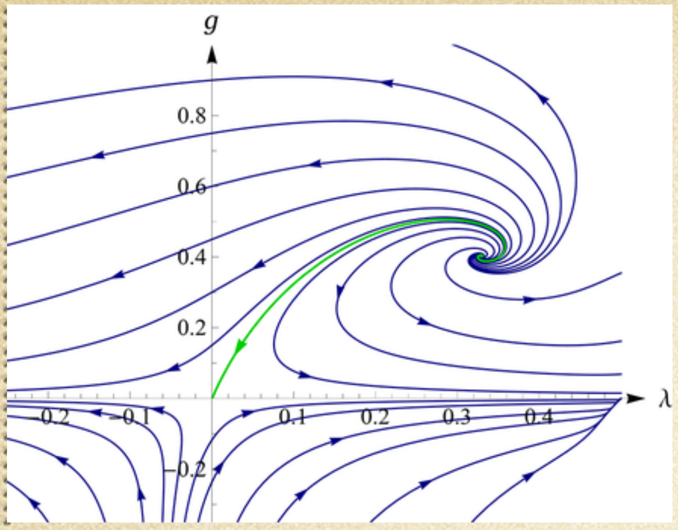

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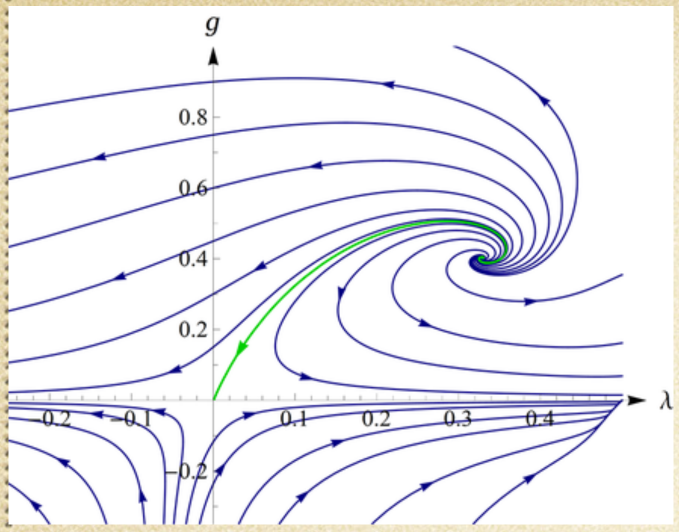

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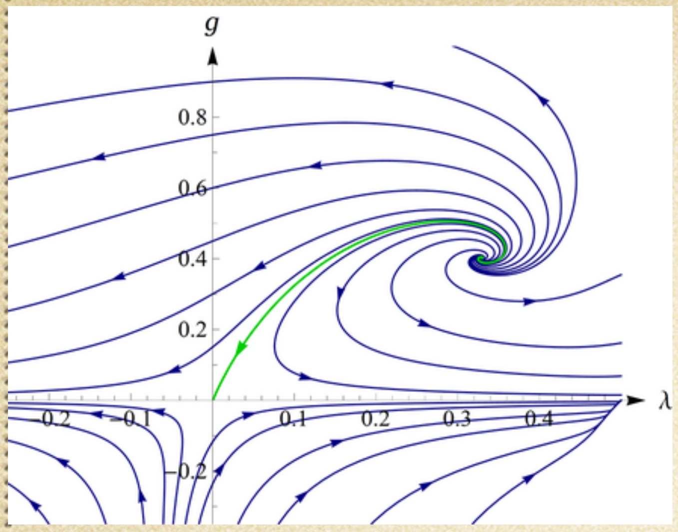

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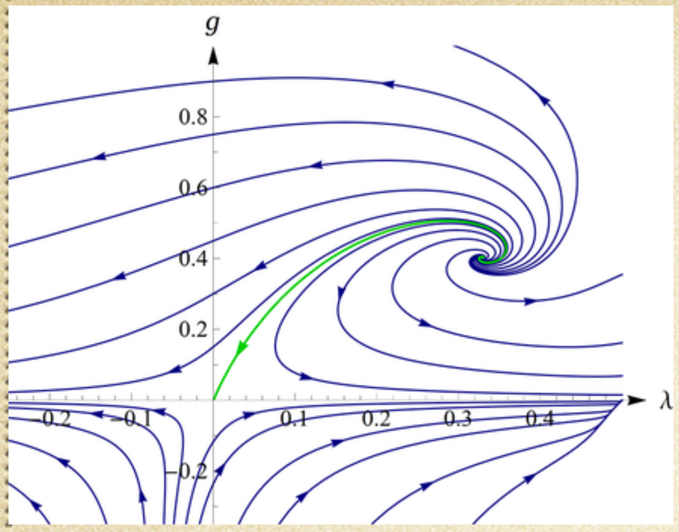
SD & NEC



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SD & NEC

- Non trivial "coincidence"
- Works for many flow truncations
- UV FP @ inflation makes sense
- Separatrix special flow trajectory
- scale setting makes sense $\frac{k}{k_0} = e^{-t/(2\tau)}$

Concluding Comments

- CCP 3.0: watch out SD during inflation
- Showed: with SD CCP3 is solved
- Beautiful matching between AS & SD
- Outlook: Post inflation?

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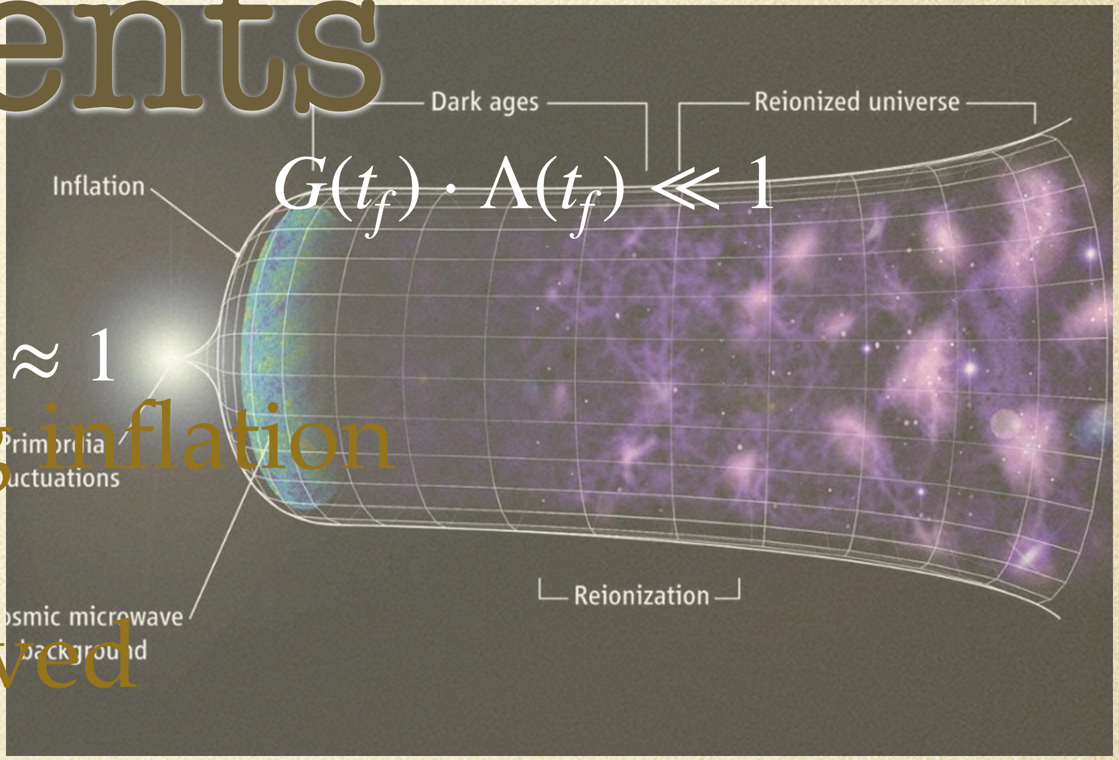

$G(t_f) \cdot \Lambda(t_f) \ll 1$

$G(t_i) \cdot \Lambda(t_i) \approx 1$

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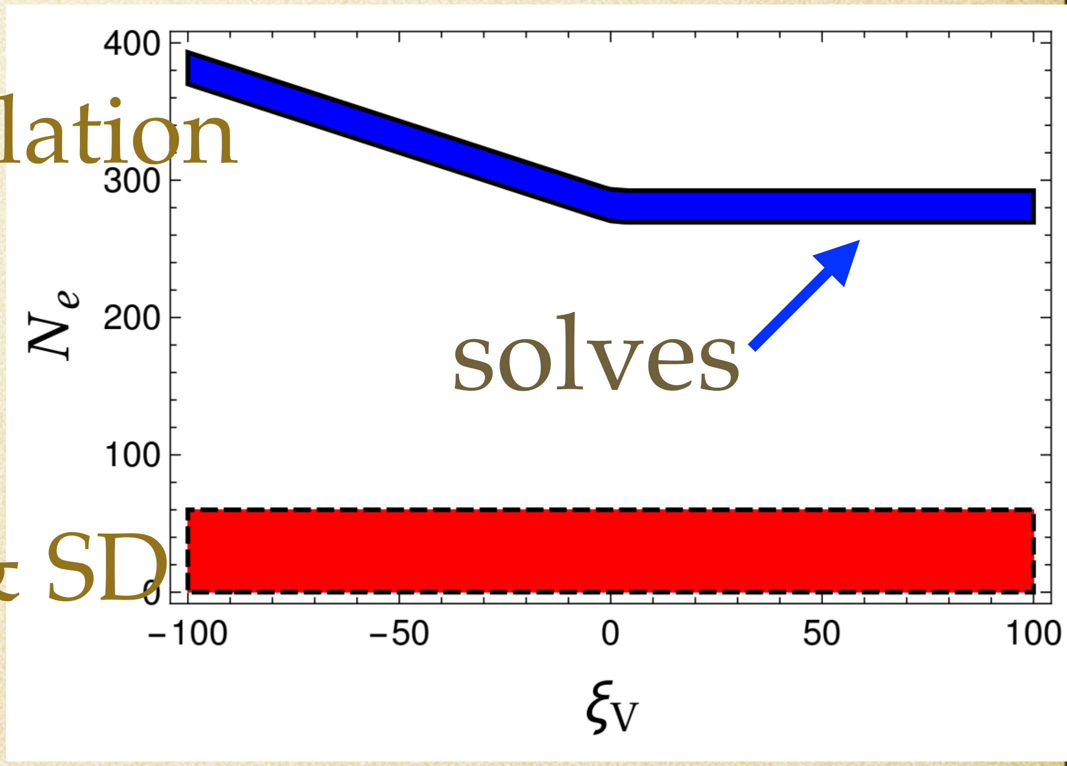

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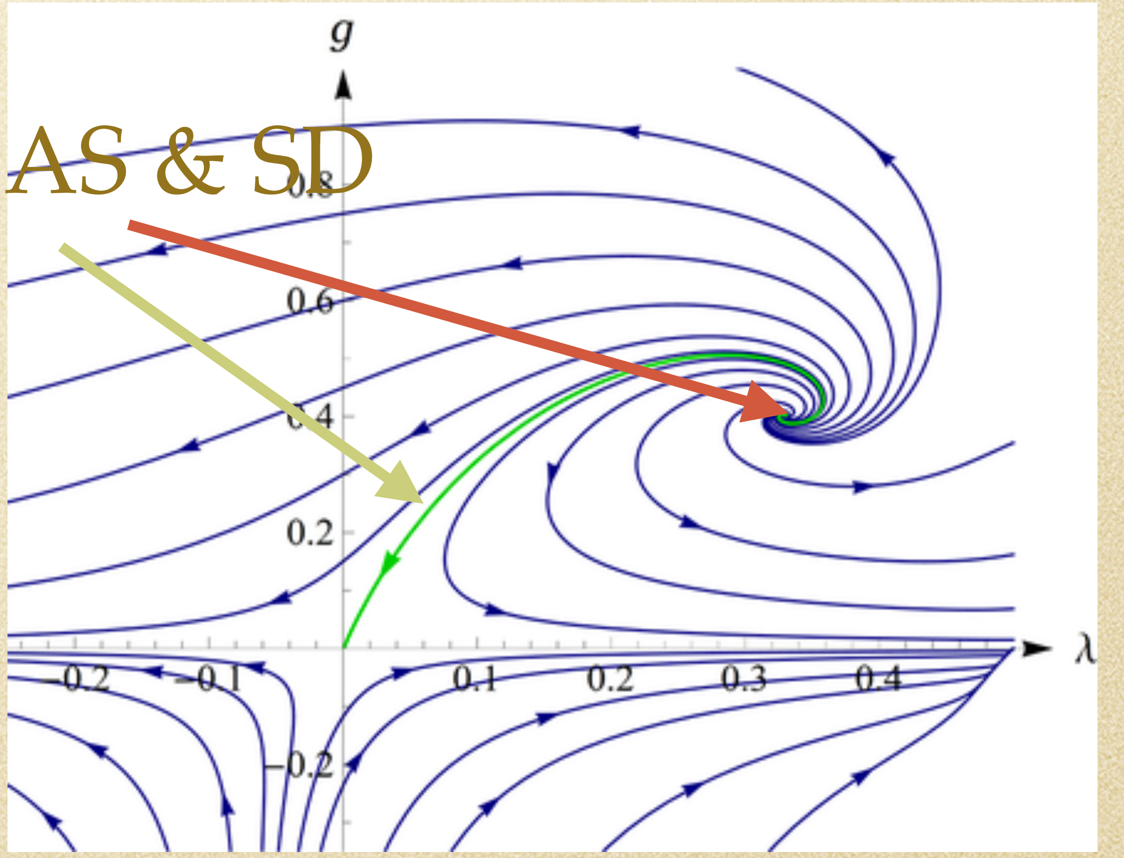

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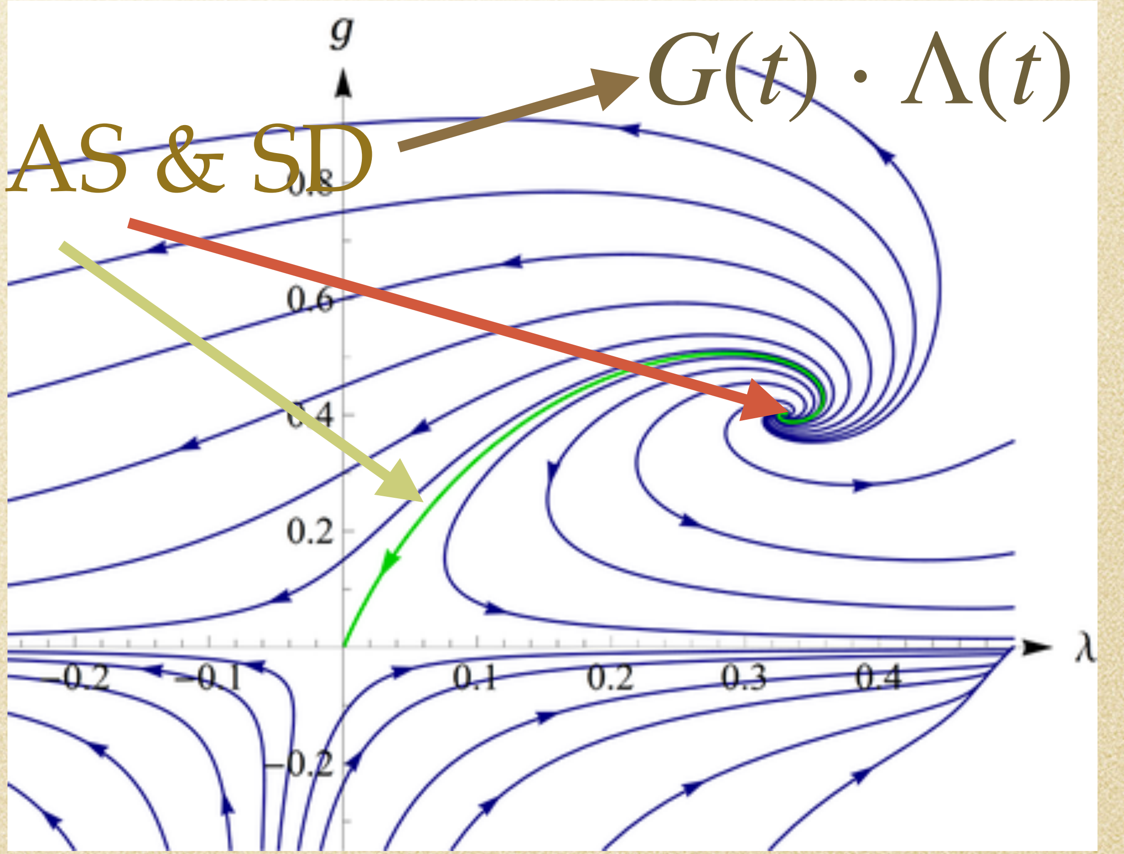

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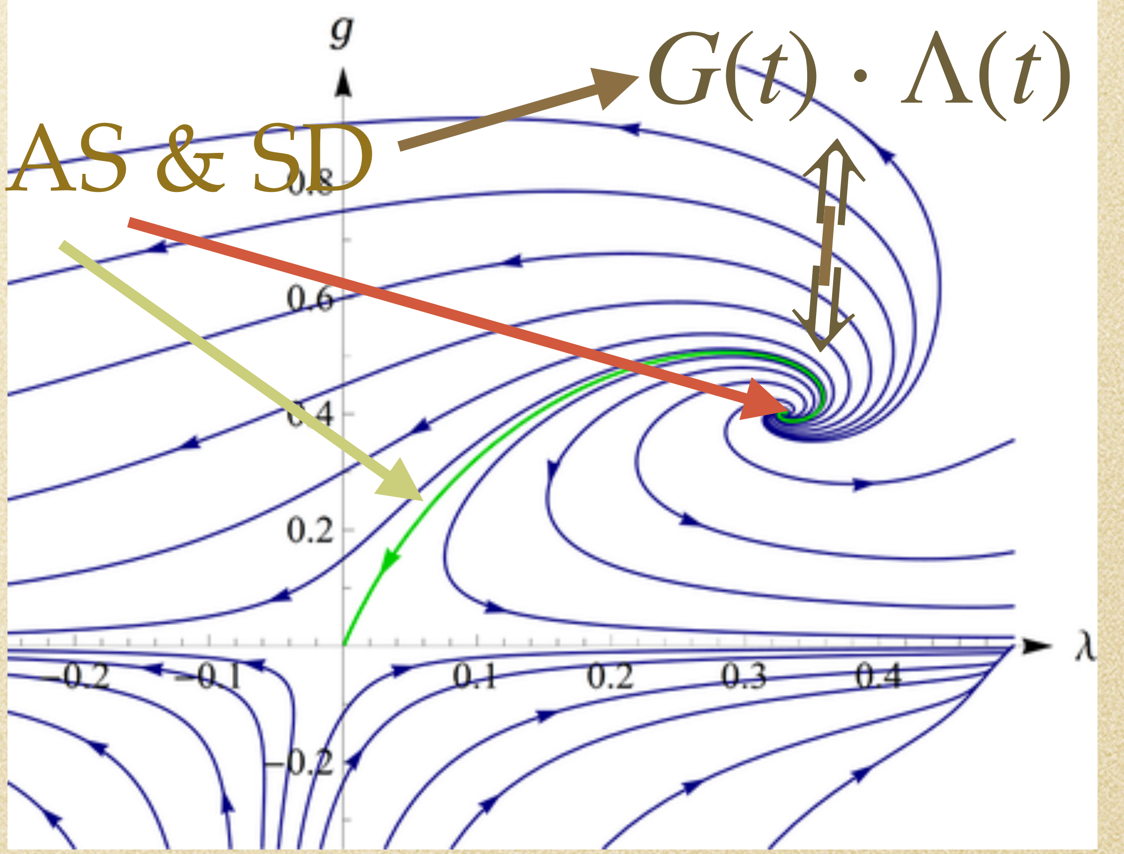

Thank You!

Outlook

Outlook

Post inflation:

Outlook

Post inflation:

More couplings, more complicated

Outlook

Post inflation:

More couplings, more complicated

Simplifying assumption:

Outlook

Post inflation:

More couplings, more complicated

Simplifying assumption:

Matter couplings don't run (so much)

Outlook

Post inflation:

Outlook

Post inflation:

$$H_0^2\Omega_\Lambda(t) + H_0^2\Omega_{rad,0}a^{-4}g(t) + H_0^2\Omega_{mat,0}a^{-3}g(t) = H^2(t) - H(t)\frac{\dot{g}}{g},$$

$$-\frac{H_0^2\Omega_{rad,0}g}{a^4} + 3H_0^2\Omega_\Lambda(t) - H^2 + \frac{2H\dot{g}}{g} - \frac{2\dot{g}^2}{g^2} + \frac{\ddot{g}}{g} = 2\frac{\ddot{a}}{a}$$

$$\dot{g}(g\dot{a} + 2a\dot{g}) = ag\ddot{g}$$

Outlook

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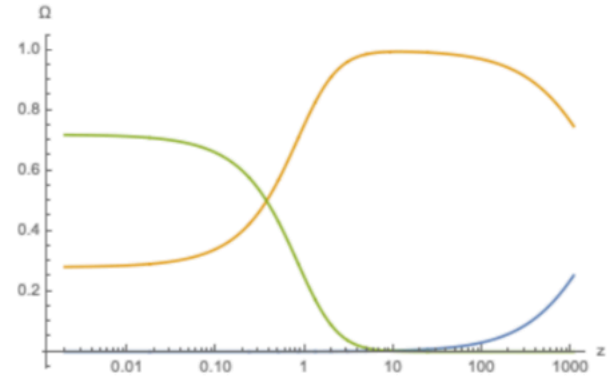
Solve Numerically and compare to NON-SD

Outlook

Post inflation:

Outlook

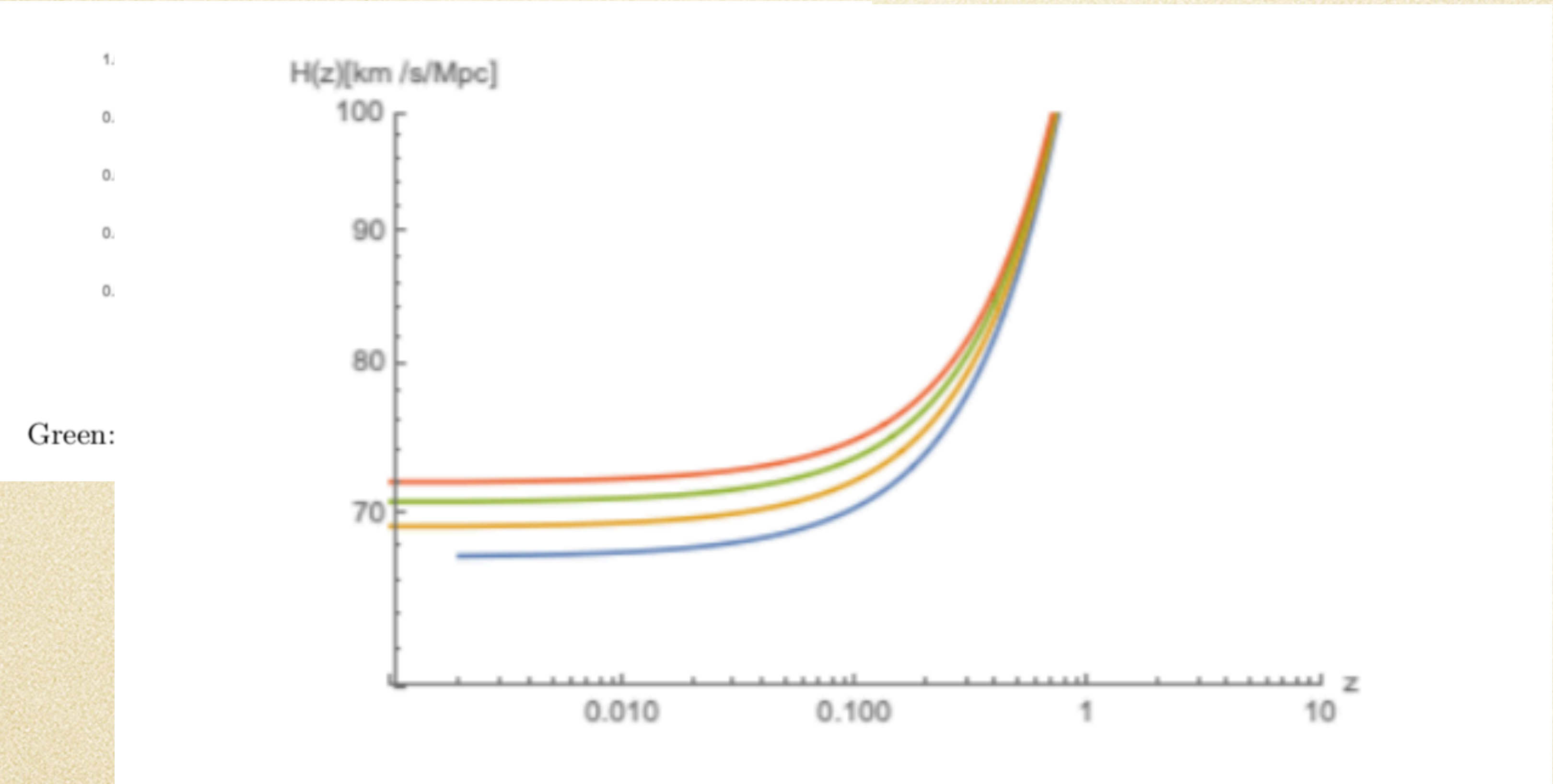
Post inflation:



Green: Cosmological term, Orange: Matter, Blue: Radiation.

Outlook

Post inflation:



Blue: Classical curve from ΛCDM Colors: SD solution

Outlook

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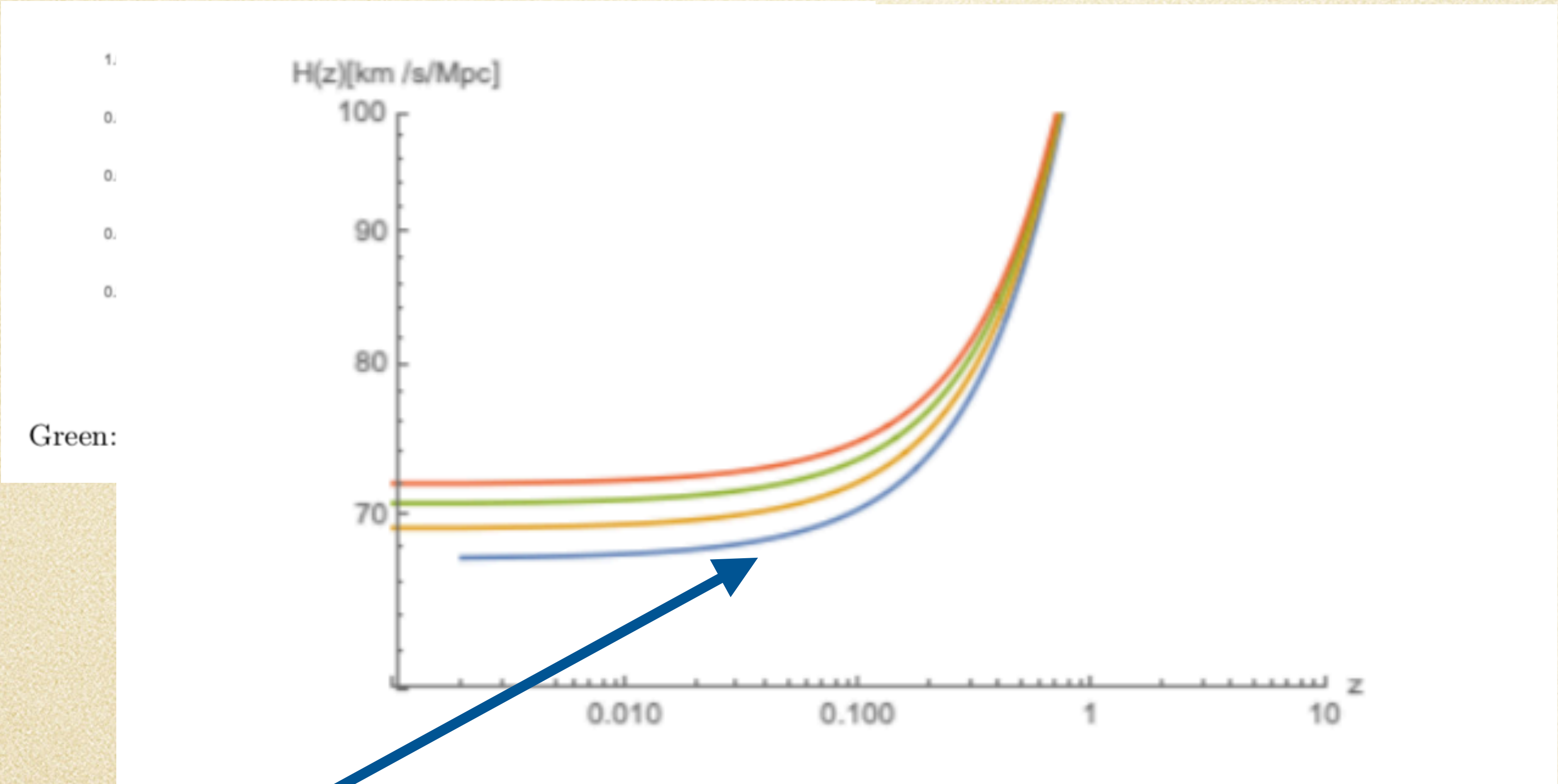

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Λ–CDM interpretation of CMB

Outlook

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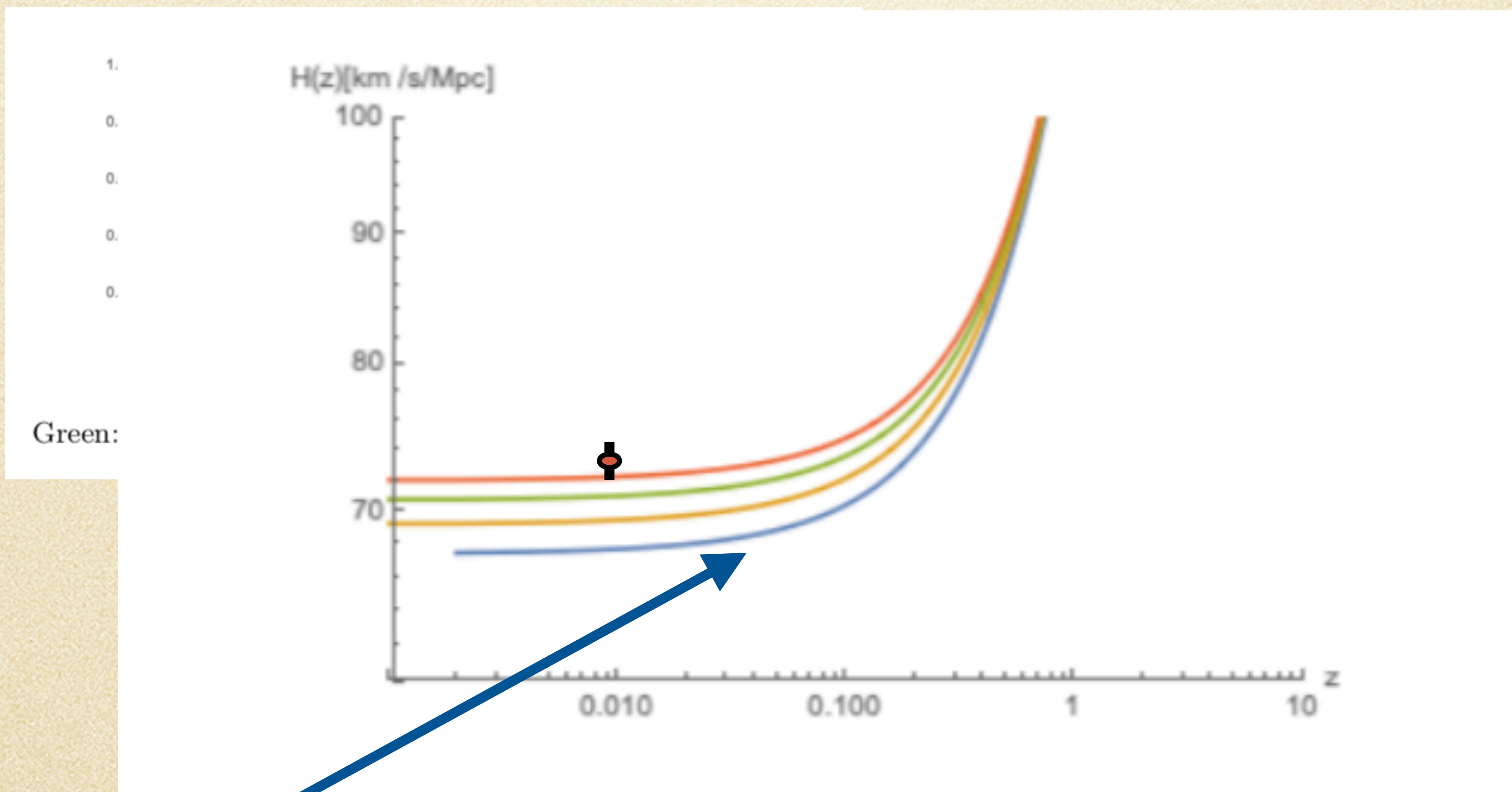

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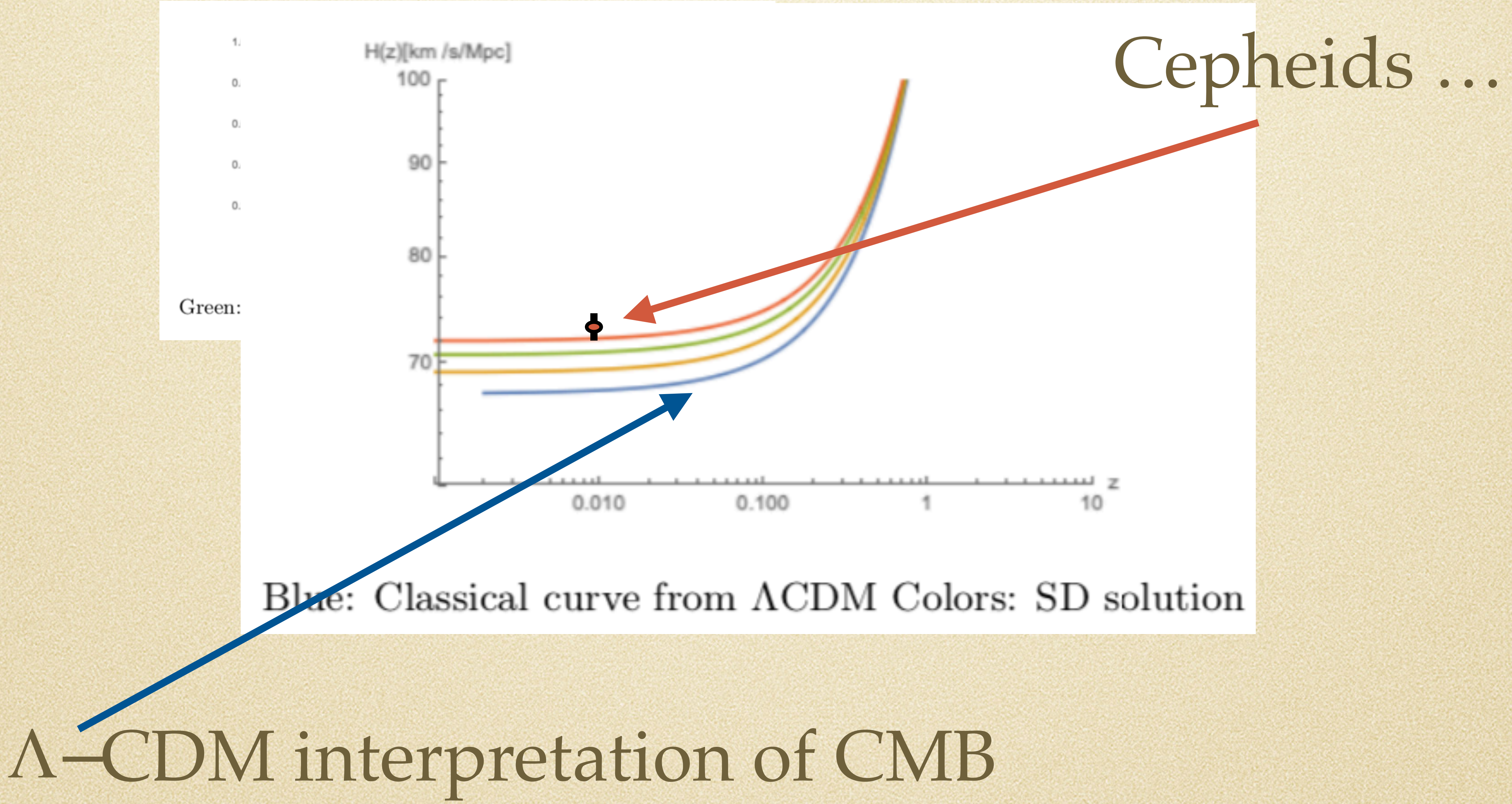

Literature

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5) e.g. M. Niedermaier and M. Reuter, Living Rev. Rel. 9, 5 (2006)
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