G̈bungen zau Qacntenmechanik I
Blatt?
(P3) $I_{\varepsilon}\left(x_{0}\right)=\int_{-\infty}^{\infty} d x \frac{1}{\sqrt{\pi} \varepsilon} \operatorname{uxp}\left(-\frac{x^{2}}{\varepsilon^{2}}\right)\left(x-x_{0}\right)^{n}$
Definieve erzengende Fanktion

$$
\begin{aligned}
F_{\varepsilon}(a) & =\int_{-\infty}^{\infty} d x \frac{1}{\sqrt{11} \varepsilon} \text { uxp }\left[-\frac{x^{2}}{\varepsilon^{2}}+a\left(x-x_{0}\right)\right] \\
& =\frac{1}{\sqrt{\pi} \varepsilon} \sqrt{\pi} \varepsilon \operatorname{mps}\left(\frac{a^{2} \varepsilon^{2}}{4}-a x_{0}\right) \\
& =\operatorname{Mrs}\left(\frac{a^{2} \varepsilon^{2}}{4}-a x_{0}\right)
\end{aligned}
$$

Find $\varepsilon \rightarrow 0$

$$
\begin{aligned}
& \text { Fir } \varepsilon \rightarrow 0 \\
& F_{\varepsilon \rightarrow 0}(a)=\operatorname{MxP}\left(-a x_{0}\right) \\
& \lim _{\varepsilon \rightarrow \infty} I_{\varepsilon}\left(x_{0}\right)=\left.\frac{d^{n}}{d a^{n}} F_{\varepsilon \rightarrow 0}(a)\right|_{a=0}=\left(-x_{0}\right)^{n}
\end{aligned}
$$

qed.

P4 Wenn do Behanptung gilt, folgt

$$
\begin{aligned}
f(x) & =; \int_{-\infty}^{\infty} \frac{d r}{2 \pi} \tilde{f}(r) \operatorname{Mxp}(i h x) \\
& =\frac{-}{-\infty} \int_{-\infty}^{\infty} \frac{d r}{2 \pi} \int_{-\infty}^{\infty} d x^{\prime} f\left(x^{\prime}\right) \operatorname{AxP}\left[i r\left(x-x^{\prime}\right)\right]
\end{aligned}
$$

Wenn wir also zeige hínnee, anß im Sinne verallgen einerter Funktionch gilt

$$
\int_{-\alpha}^{\infty} \frac{d r}{2 \pi} \operatorname{Mrr}\left[: r\left(x-x^{\prime}\right)\right]=\delta\left(x-x^{\prime}\right)
$$

häben wor dse Un kelrformel becriesen.
liv setzer der Finfech heit helber $x=0$ and zeigen also
GI. (8) auf dem B/att
Es gilt

$$
\begin{aligned}
& \text { Es gilt } \\
& \tilde{\delta}_{\varepsilon}(\mu)=\int_{-\infty}^{\infty} d x \delta_{\varepsilon}(x) \operatorname{upp}(-i r x) \\
&=\int_{-\infty}^{\infty} d x \frac{1}{\sqrt{\pi} \varepsilon} \operatorname{up}\left(-\frac{x^{2}}{\varepsilon^{2}}-i r x\right) \\
&=\frac{P_{2}}{\sqrt{\pi} \varepsilon} \sqrt{\pi} \varepsilon \operatorname{uxp}\left(-\frac{\varepsilon^{2} r^{2}}{4}\right) \\
&=\operatorname{uxp}\left(-\frac{\varepsilon^{2} r^{2}}{4}\right)
\end{aligned}
$$

Príten wiv num die Uumkehrformel wach

$$
\begin{aligned}
& \int_{-\infty}^{\infty} \frac{d r}{2 \pi} \widetilde{\delta}_{\varepsilon}(h) \operatorname{Ans}(i r x)=\int_{-\infty}^{\infty} \frac{d r}{2 \pi} \operatorname{uxp}\left(-\frac{\varepsilon r^{2}}{4}+i h x\right) \\
& \stackrel{P_{2}}{=} \frac{1}{2 \pi} \sqrt{\frac{4 \pi}{\varepsilon^{2}}} \operatorname{mps}\left(-\frac{x^{2}}{\varepsilon^{2}}\right)=\frac{1}{\sqrt{\pi} \varepsilon} \operatorname{sxp}\left(-\frac{x^{2}}{\varepsilon^{2}}\right) \\
& =\delta_{\varepsilon}(x) \quad q\left(d_{1}\right.
\end{aligned}
$$

In schescher Somne gilt als.

$$
\begin{aligned}
& \text { schwachel } \operatorname{sinhc} \text { gilt als. } \\
& \left.\delta(x)=\lim _{\varepsilon \rightarrow 0^{+}} \delta_{\varepsilon} x\right)=\lim _{\varepsilon \rightarrow 0^{+}} \int_{-\infty}^{\infty} \frac{d r}{2 \pi} \widetilde{\delta}_{\varepsilon}(r / \operatorname{un}(i r x) \\
& =\int_{-\infty}^{\infty} \frac{d r}{2 \pi} \lim _{\varepsilon \rightarrow 0} \widetilde{\delta_{\varepsilon}}(h) \operatorname{man}(i r x)=\int_{-\infty}^{\infty} \frac{d r}{2 \pi} \text { vaplirx) }
\end{aligned}
$$

PST( $(1)$ Mit

$$
\psi(x, t)=\int_{-\infty}^{\infty} \frac{d r}{2 \pi} \bar{\psi}(h, t) \operatorname{uxp}(i r x)
$$

folg $t$

$$
\begin{array}{r}
\int_{-\infty}^{\infty} \frac{d h}{2 \pi}\left[i \hbar \partial_{t} \bar{\psi}(r, t)\right] \operatorname{uxp}(i h x)=\int_{-\infty}^{\infty} \frac{d h}{2 \pi} \bar{\psi}(h, t) \\
\left(+\frac{\hbar_{1}^{2} r^{2}}{2 m}\right) \operatorname{Nap}(i h x)
\end{array}
$$

Da die Fonviertrafo um kelve bar ist, ma $\beta$ also

$$
i \hbar \partial_{t} \tilde{\psi}(h, t)=\frac{\hbar^{2} r^{2}}{2 \mu} \tilde{\psi}(r, t)
$$

$$
\begin{aligned}
& \text { gelten. } \\
& \Rightarrow \partial_{t} \tilde{\psi}(h, t)=-\frac{i \hbar r^{2}}{2 m} \tilde{\psi}(r, t) \\
& \text { "Bt soch durch "Trec }
\end{aligned}
$$

gelten.


$$
\begin{aligned}
& \quad \frac{d \tilde{\psi}}{\tilde{\psi}}=-d t \frac{i \hbar r^{2}}{2 m} \\
& \Rightarrow \tilde{\psi}(k, t)=\frac{\tilde{\psi}(t=0, r)}{\tilde{q}_{0}(r)} \operatorname{NPp}\left(-\frac{i t_{1} r^{2} t}{2 m}\right)
\end{aligned}
$$

Die allgenewne Lósung, lantet also

$$
\begin{aligned}
& \text { Ngencone Lösung lantet alis } \\
& \psi(x, t)=\int_{-\infty}^{\infty} \frac{d h}{2 \pi} \tilde{\psi}_{0}(h) \operatorname{Mxp}\left(-\frac{i \hbar h^{2} t}{2 m}+i h x\right) \\
& \text { Dc die Sclvódingengler- }
\end{aligned}
$$

phy sikatische Inter pretation: Dc die Sclvò dingenglerchung von 1. Ordunng in dev Zut ist, benòtigt man eine Anfangsbedingung

$$
\begin{aligned}
& \psi\left(x_{1},=0\right)=\psi_{0}(x)=\int_{-\infty}^{1} \frac{d r}{2 \pi} \widetilde{\psi}_{0}(r) \operatorname{uxp}(i r x) \\
& \Leftrightarrow \tilde{\psi}_{0}(H)=\int_{-\infty}^{\infty} d x \psi_{0}(x) \operatorname{uxp}(-i r x)
\end{aligned}
$$

Bei vorgegebecier Anfangsbedingang ("Pra'paration") ist die Uellenfe. Lition zt allen zuitan dard die Solvodingerglacelang ein dentig bestinnent.
(b)

$$
\begin{aligned}
& N(t)=\int_{-\infty}^{\infty} d x|\psi(x, t)|^{2} \\
&=\int_{-\infty}^{\infty} d x \psi^{*}(x, t) \psi(x, t) \\
&=\int_{-\infty}^{\infty} d x \int_{-\infty}^{\infty} \frac{d r}{2 \pi} \tilde{\psi}^{*}(h, t) \operatorname{kx}(-i h x) \\
& \int_{-\infty}^{\infty} \frac{d r^{\prime}}{2 \pi} \widetilde{\psi}\left(h_{1}^{\prime}, t\right) \operatorname{man}\left(i^{\prime}(x)\right.
\end{aligned}
$$

$$
\begin{aligned}
= & \int_{-\infty}^{\infty} \frac{d r}{2 \pi} \int_{-\infty}^{\infty} \frac{d r^{\prime}}{2 \pi} \tilde{\psi}^{*}(r, t) \tilde{\psi}\left(r^{\prime}, t\right) \\
& \int_{-\infty}^{\infty} d x \operatorname{urr}\left[i\left(h^{\prime}-r\right) x\right]
\end{aligned}
$$

$\stackrel{p 4}{=} \int_{-\infty}^{\infty} \frac{d r}{2 \pi} \int_{-\infty}^{\infty} \frac{d r^{\prime}}{2 \pi} \tilde{\psi}^{*}(r, r) \tilde{\psi}\left(r^{\prime}, t\right), 2 \pi \delta\left(r-r^{\prime}\right)$

$$
\begin{aligned}
N(t) & =\int_{-\infty}^{\infty} \frac{d h}{2 \pi} \tilde{\psi}^{x}(h, t) \tilde{\psi}(h, t) \\
& =\int_{-\infty}^{\infty} \frac{d h}{2 \pi}|\tilde{\psi}(r, t)|^{2}
\end{aligned}
$$

Da

$$
\left.\tilde{\psi}(k, k)=\tilde{\psi}_{0}(k) \operatorname{uxp}^{( }-\frac{\hbar g^{2}}{2 \mu_{n}} t\right)_{1}
$$

$$
\begin{aligned}
& |\bar{\psi}(h, t)|^{2}=\left|\tilde{\psi}_{0}(h)\right|^{2} \Rightarrow N(t)=\text { const. }
\end{aligned}
$$

folg $t$
(c)

$$
\begin{aligned}
\frac{d N(t)}{d t}= & \int_{-\infty}^{\infty} d x \frac{\partial}{\partial t}\left[\psi^{*}(x, t) \psi(x, t)\right] \\
= & \int_{-\infty}^{\infty} d x\left[\frac{\partial \psi^{*}(x, t)}{\partial t} \psi(x, t)+\psi^{*}(x, t) \frac{\partial \psi(x, t)}{\partial t}\right] \\
= & \int_{-\infty}^{\infty} d x\left[-\frac{i \hbar}{2 \mu} \frac{\partial^{2} \psi^{*}(x, t)}{\partial x^{2}} \psi(x, t)\right. \\
& \left.+\psi^{*}(x, t) \frac{i \hbar}{2 m} \frac{\partial^{2} \psi(x, t)}{\partial x^{2}}\right] \\
= & -\frac{i \hbar}{2 \mu} \int_{-\infty}^{\infty} d x\left[\frac{\partial^{2} \psi^{x}(x, t)}{\partial x^{2}} \psi(x, t)-\psi^{*}(x, t) \frac{\partial^{2} \psi(x, t)}{\partial x^{2}}\right]
\end{aligned}
$$

$2 \times$ partiell integriver

$$
=-\frac{i \hbar}{2 v} \int_{-\infty}^{\infty} d x\left[\psi^{y}(x, t) \frac{\partial^{2} \psi(x, t)}{\partial x^{2}}-\psi^{y}(x, t) \frac{\left.\left.\partial^{2} f(x, t)\right)\right]}{\partial x^{2}}\right]
$$

$$
=0 \quad q u d
$$

