Shear viscosity and resonance lifetimes in the hadron gas

presented by Jean-Bernard Rose

with D. Oliinychenko, J. Torres-Rincon, A. Schäfer, H. Petersen

based on arxiV:1709.03826 and arxiV:1709.00369

CRC-TR 211 Transport Meeting, Frankfurt, Germany, January 25th, 2018

Outline

1. Introduction: Viscosity of the hadron gas
2. Transport
   – SMASH
3. Methodology
   – Viscosity considerations
     • Green-Kubo formalism
     • Test case #1: Constant isotropic cross-section
     • Test case #2: Energy-dependent cross-section
   – Entropy considerations
4. Results
   – Full hadron gas viscosity
   – Comparison & discussion
5. Conclusion

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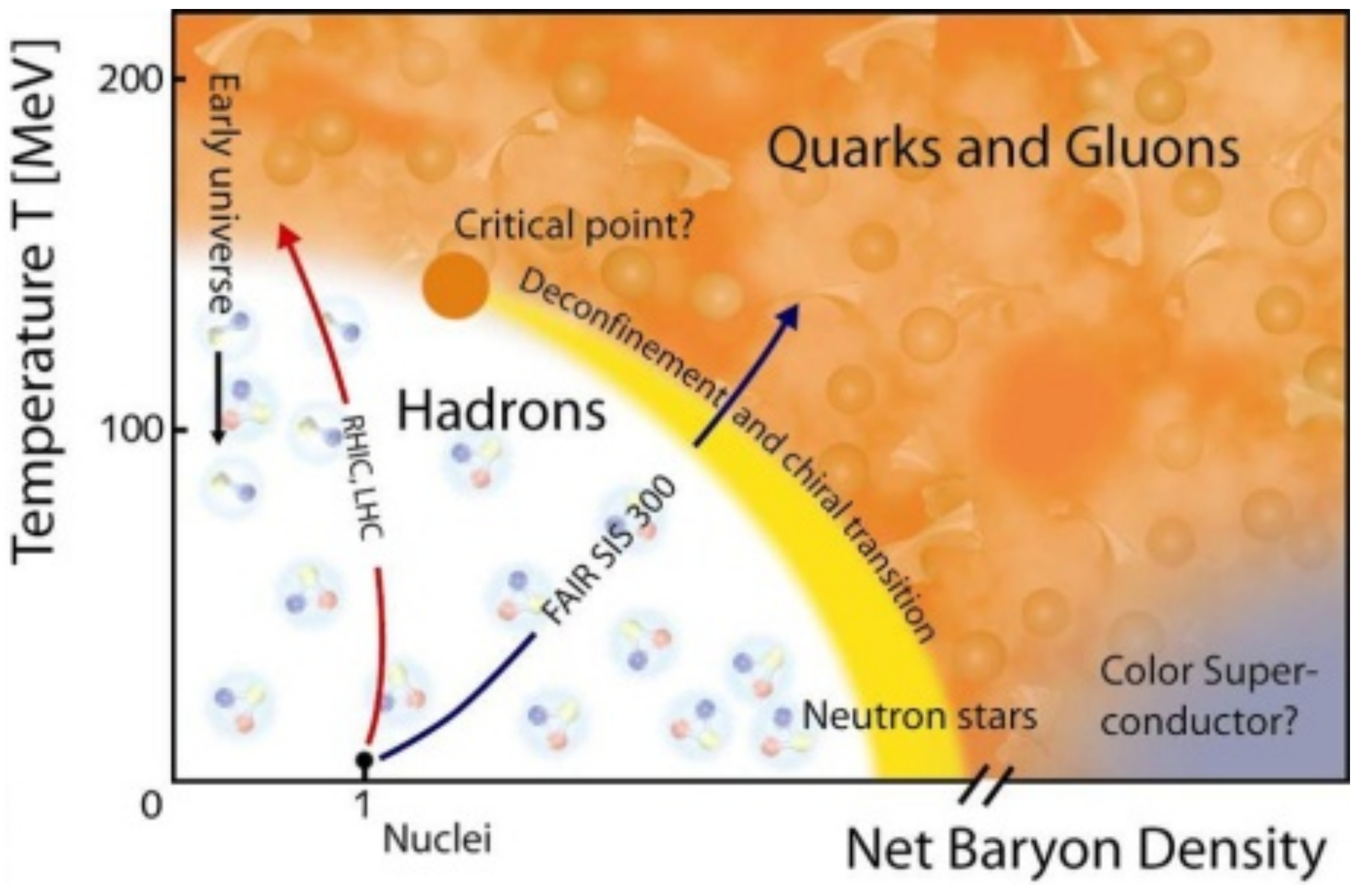
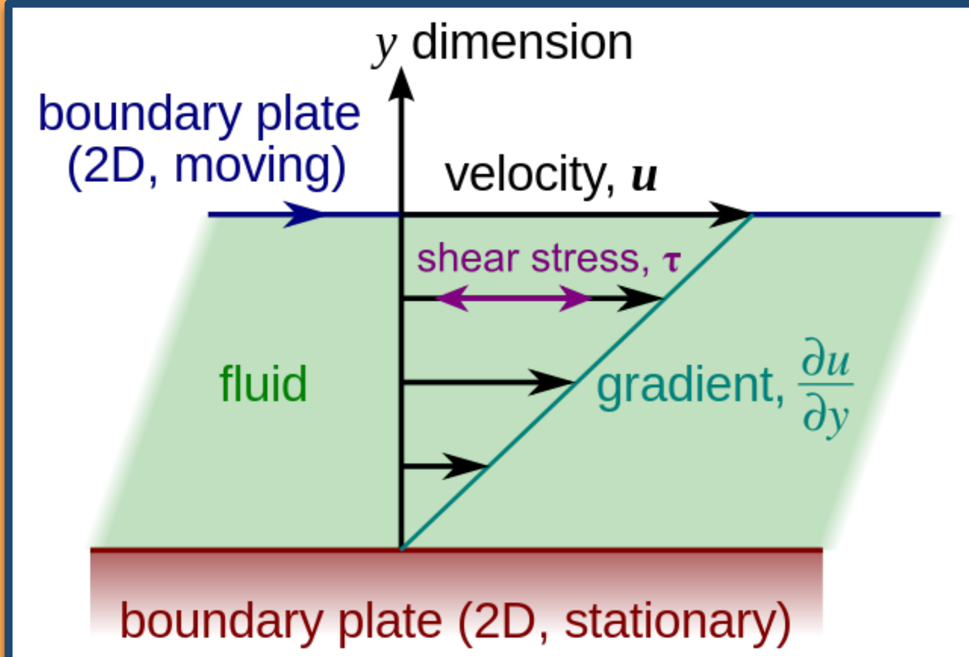
What is the hadron gas?



What is viscosity?

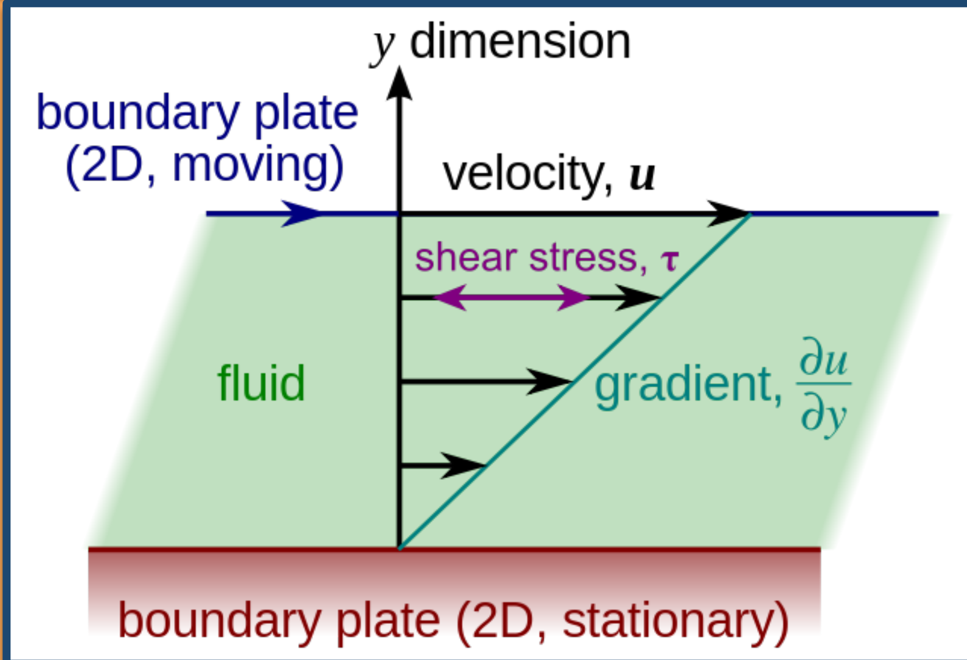
Viscosity is a measure of the friction between layers of a fluid



Wikipedia-Viscosity

...and why do we need it?

Viscosity is a measure of the friction between layers of a fluid



Wikipedia-Viscosity

- Hydrodynamics is conservation laws:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0$$

- With 1st order dissipative corrections (Navier-Stokes):

$$T^{\mu\nu} = eu^\mu u^\nu - (p + \zeta\theta)\Delta^{\mu\nu} + 2\eta\sigma^{\mu\nu}, \quad N^\mu = nu^\mu + \varkappa\partial^\mu \frac{\mu}{T}$$

...and why do we need it?

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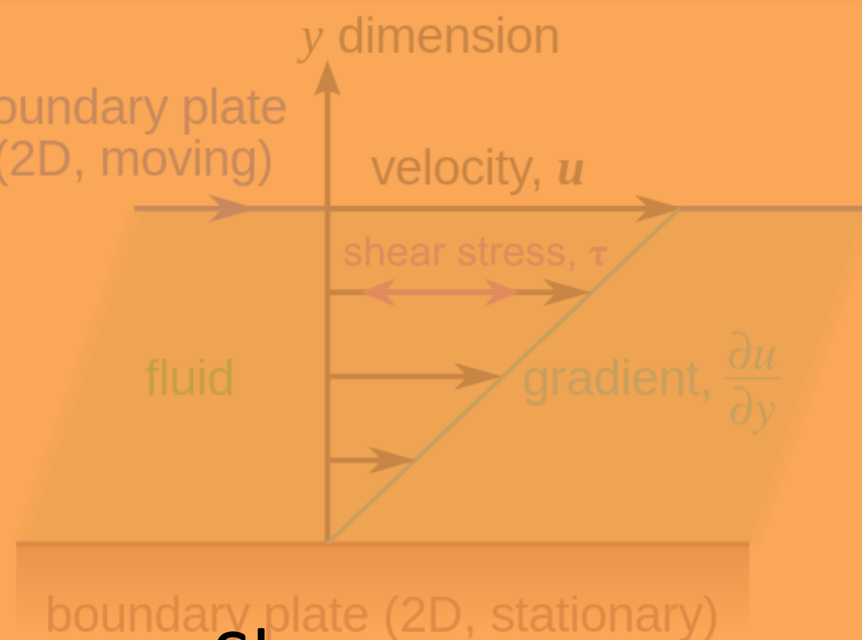


- Hydrodynamics is conservation laws:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0$$

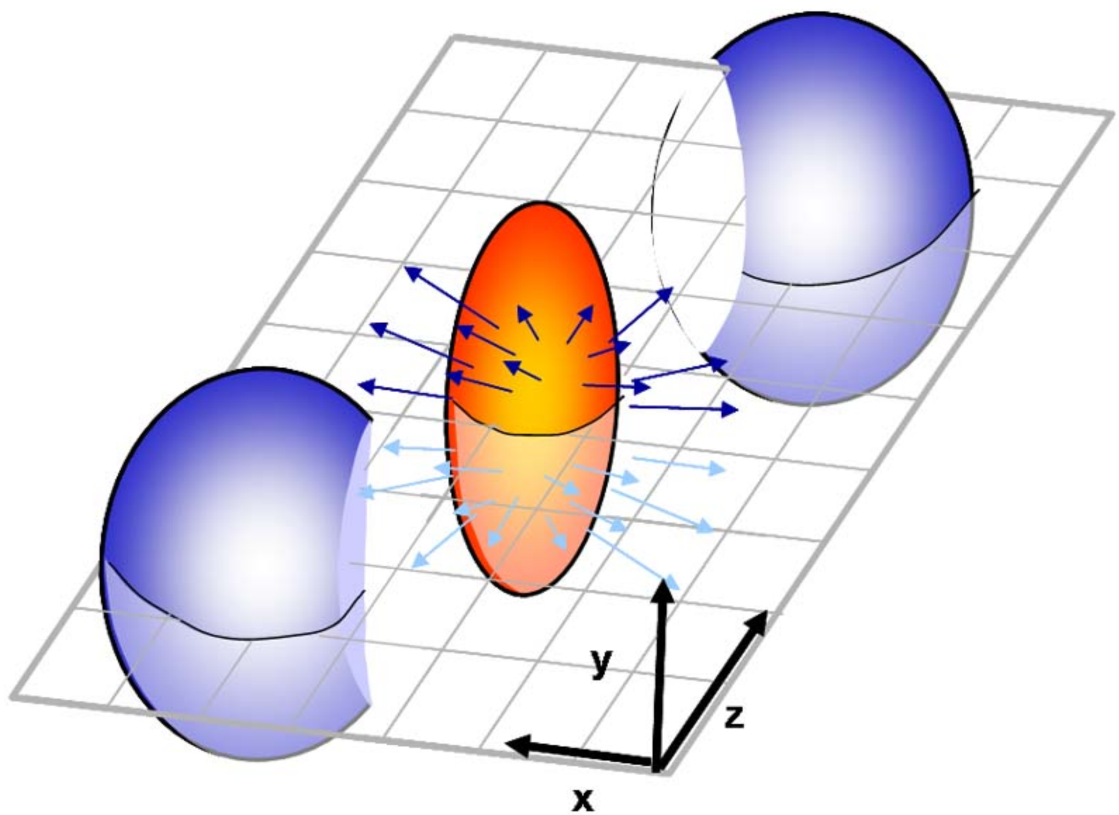
- With 1[st] order dissipative corrections (Navier-Stokes):

Bulk (ζ) — Shear (η)

$$T^{\mu\nu} = e u^\mu u^\nu - (p + \zeta\vartheta)\Delta^{\mu\nu} + 2\eta\sigma^{\mu\nu}, \quad N^\mu = n u^\mu + \varkappa \partial^\mu \frac{\mu}{T}$$

Viscosity in heavy ion collisions

- RHIC and LHC measured large elliptic flow at the high energies corresponding to what is thought to be QGP



http://www.quantumdiaries.org/wp-content/uploads/2011/02/FlowPr.jpg

Viscosity in heavy ion collisions

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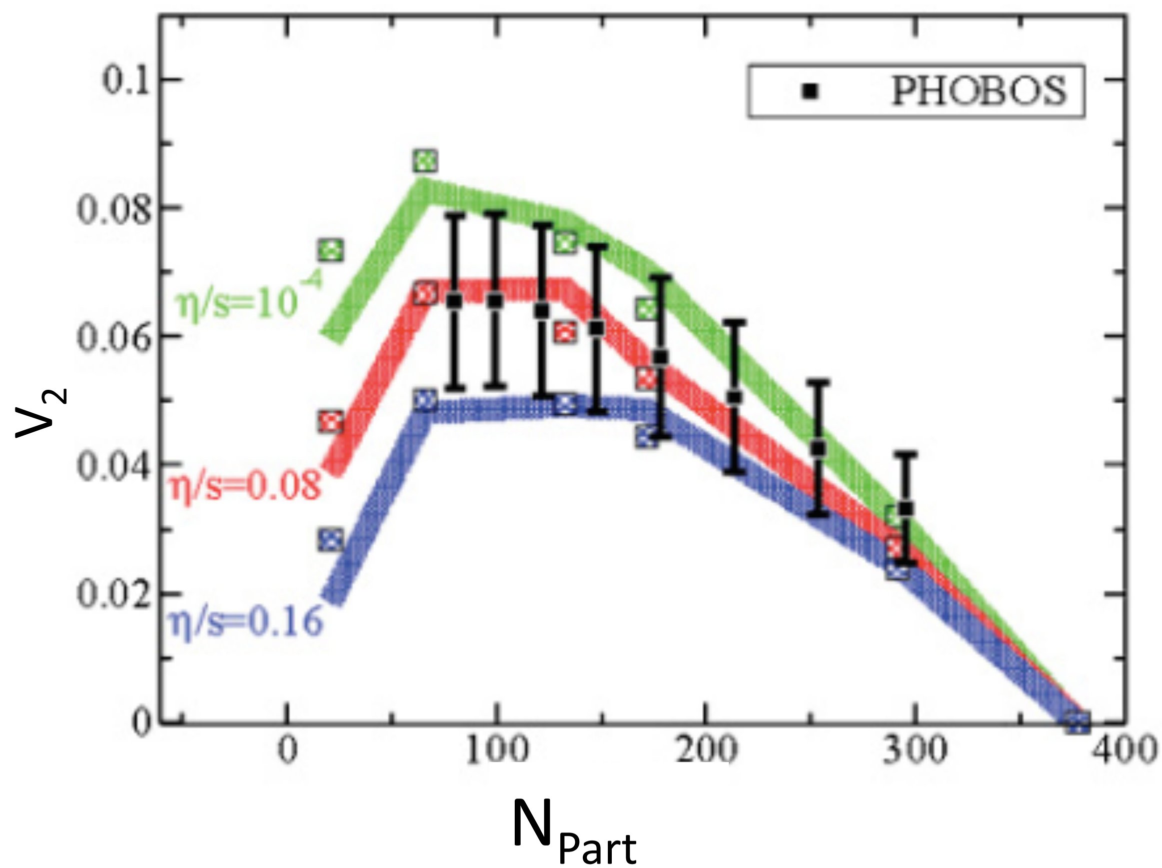
- Hydrodynamics relatively successful at explaining this with small *η/s*



Luzum & Romatschke 10.1103/Phys. Rev. C 78.034915

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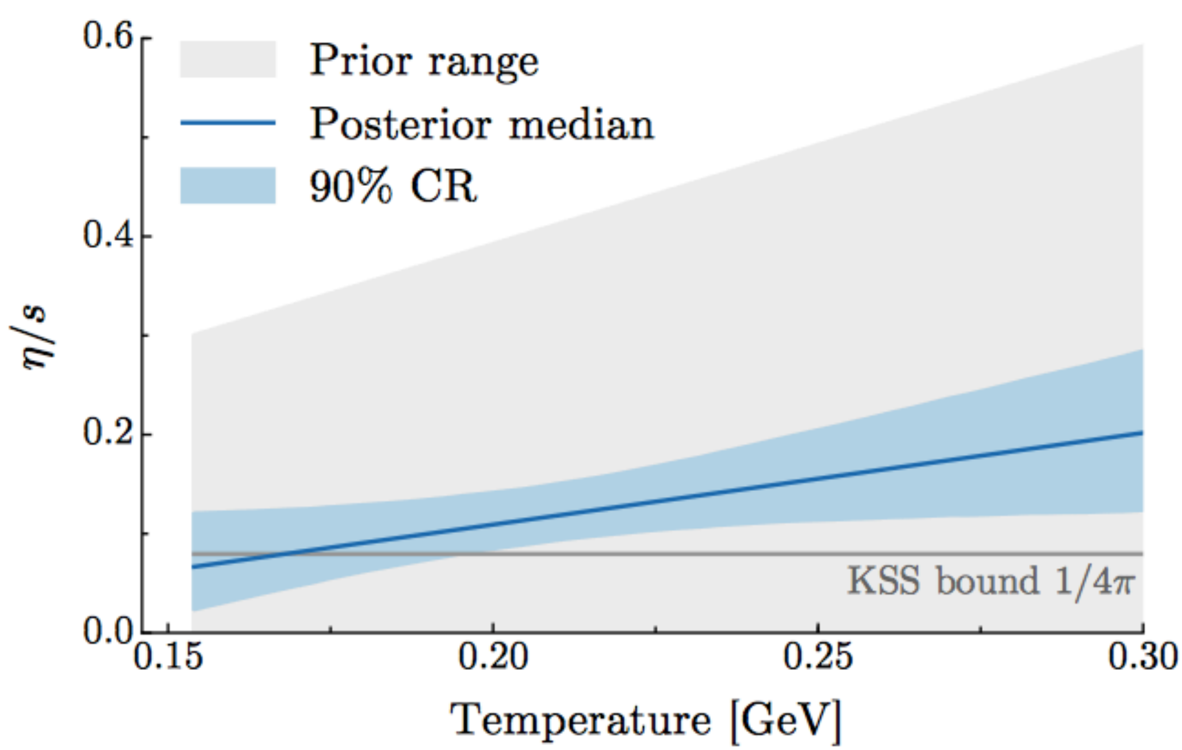
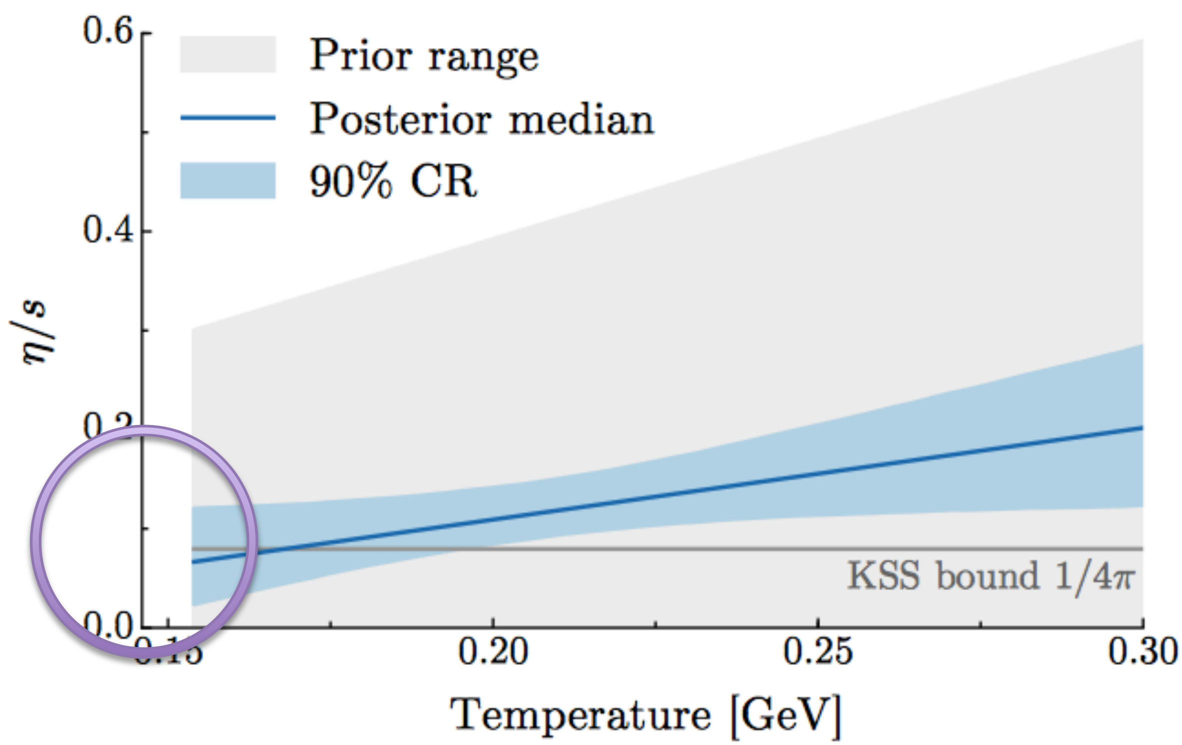


Bernhardt, Moreland, Bass, Liu & Heinz Phys. Rev. C94 no. 2 (2016) 024907

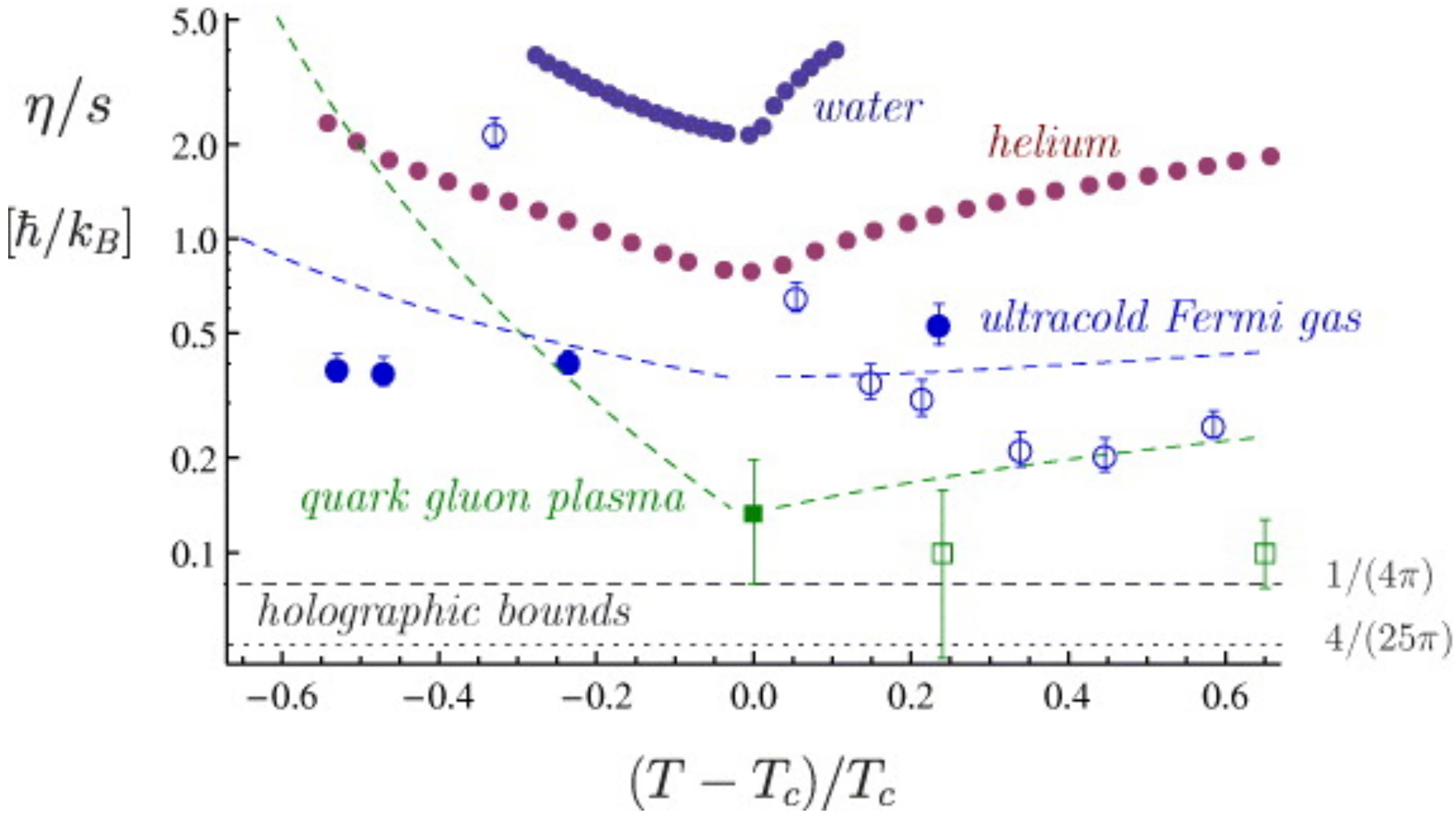
Viscosity in heavy ion collisions

- RHIC and LHC measured large elliptic flow at the high energies corresponding to what is thought to be QGP
- Hydrodynamics relatively successful at explaining this with small η/s
- Still not clear what the behavior of η/s is at low energies (FAIR, late stage RHIC/LHC)



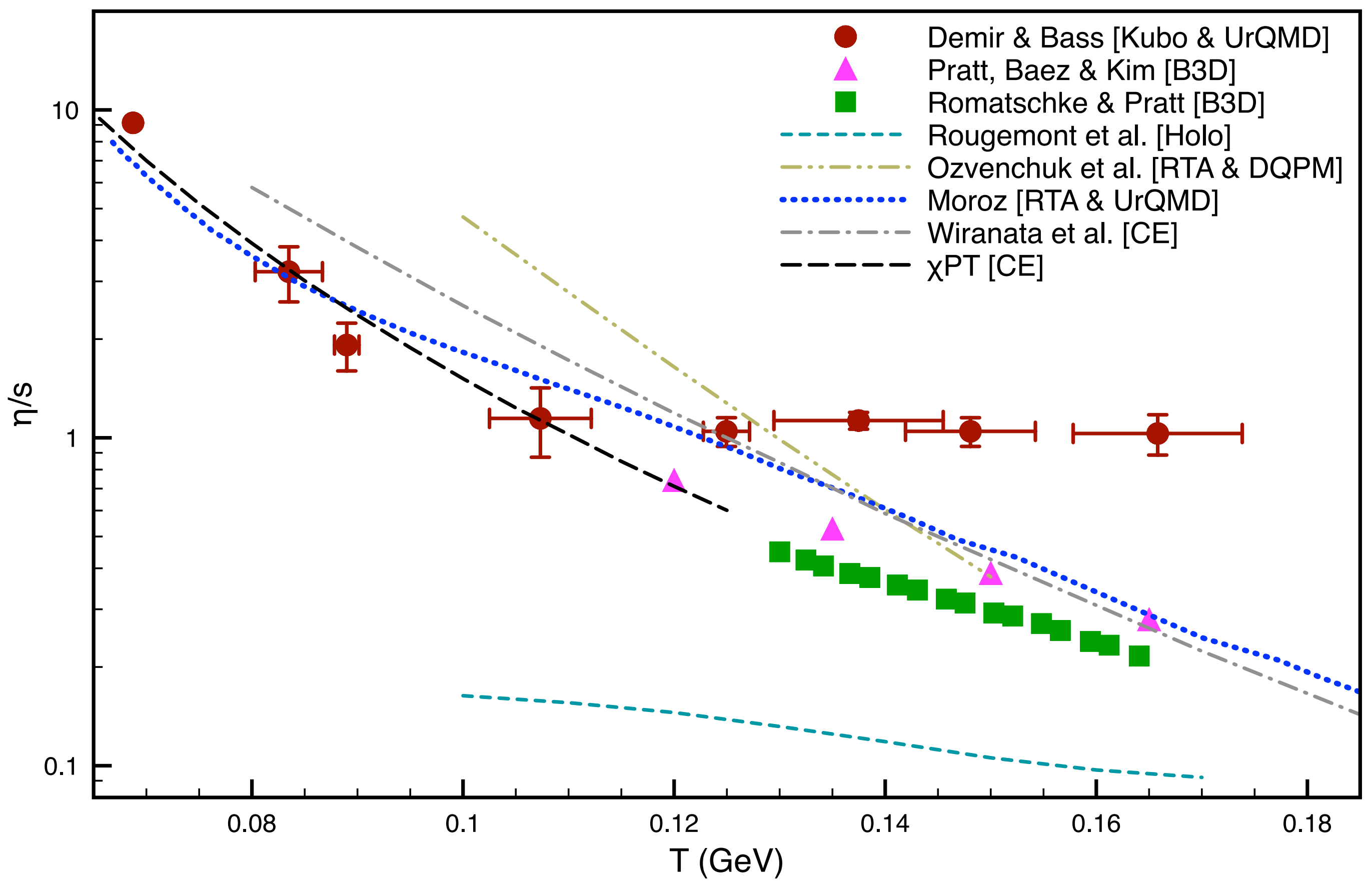
Bernhardt, Moreland, Bass, Liu & Heinz Phys. Rev. C94 no. 2 (2016) 024907

How low is low?



A. Adams, L. D. Carr, T. Schäfer, P. Steinberg, J E Thomas, New J. Phys. 15 (2013) 045022

Previous HG viscosity calculations



-Demir & Bass, Phys.Rev.Lett. 102 (2009) 172302
-Pratt, Baez & Kim, Phys. Rev. C95 (2017) 024901
-Romatschke & Pratt, arXiv:1409.0010v1
-Rougemont et al., arXiv:1704.05558
-Ozvenchuk et al., Phys. Rev. C87 (2013) 064903
-Moroz, arXiv:1301.6670
-Wiranata et al., Phys. Rev. C88 (2013) 044917
-Torres-Rincon, arXiv:1205.0782

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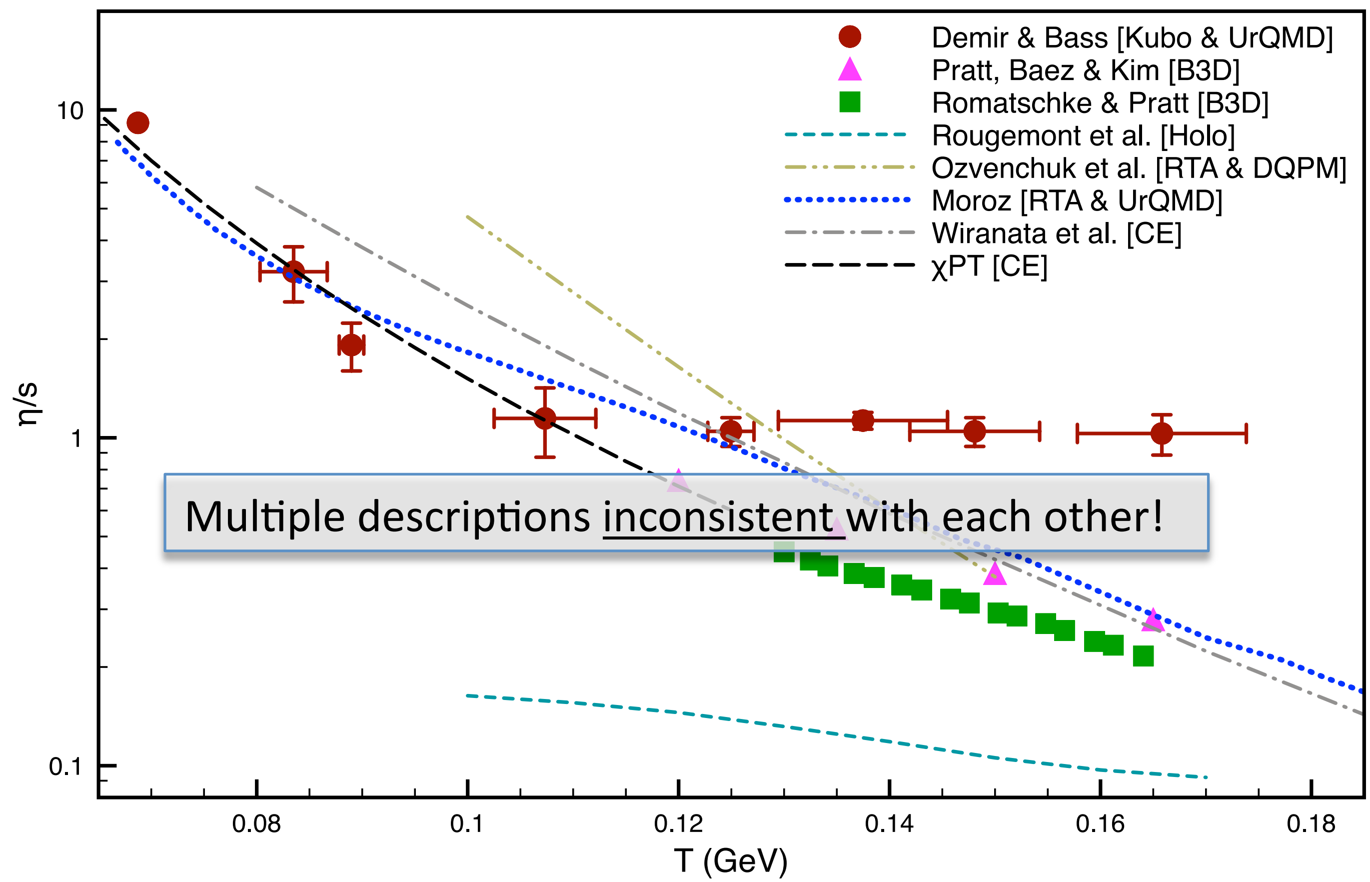

Multiple descriptions inconsistent with each other!

- Demir & Bass, Phys.Rev.Lett. 102 (2009) 172302
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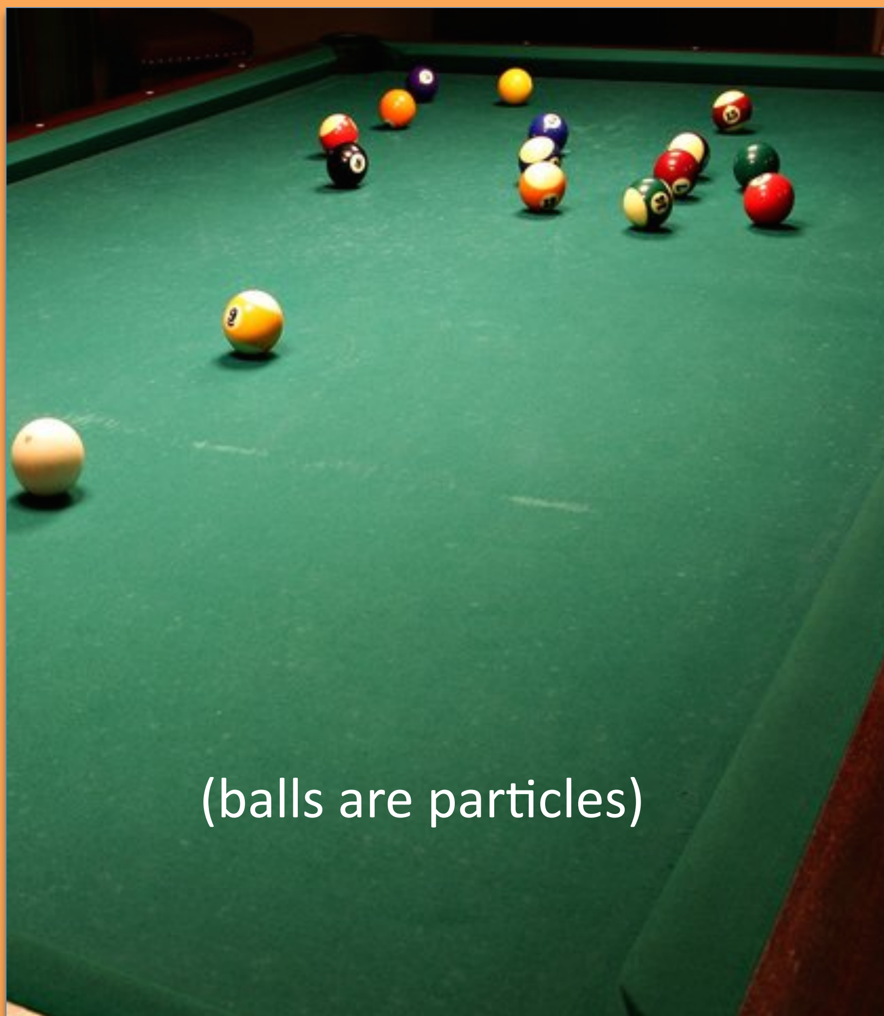
Transport approaches



- **Transport models are 3D billiard tables**
- **But...**
  - Balls do not see each other as being the same size
  - Balls can annihilate
  - Balls can decay
  - Balls can become other balls on collision

Transport approaches

- **More fundamentally, transport *effectively* solves the Boltzmann equation:**

$$p^{\mu}\partial_{\mu} f_i(x,p) + m_i F^{\alpha} \partial^{p}_{\alpha} f_i(x,p) = C^{i}_{coll}$$

where $f_i(x,p)$ is the one-particle distribution function, F^{α} the force experienced by particles and C^{i}_{coll} is the collision term.

The transport code: SMASH

*Not the official logo. Definitely never will be.

- **Transport *effectively* solves the Boltzmann equation:**

$$p^{\mu}\partial_{\mu} f_i(x,p) + m_i F^{\alpha} \partial_{\alpha}^{p} f_i(x,p) = C_{coll}^{i}$$

- **Particles represented by gaussian wave packets**
- **Geometric collision criterion:**

$$d_{trans} < d_{int} = \sqrt{\frac{\sigma_{tot}}{\pi}}$$

- **Each particles species is represented with point-like test particles**

$$\sigma \rightarrow \sigma \cdot N_{test}^{-1}$$
$$N \rightarrow N \cdot N_{test}$$

Weil et al., Phys. Rev. C94 (2016) no.5, 054905

SMASH: General setup

- Timestepless evolution
  - All future possible actions stored
  - Propagate to next action
  - Update list of possible actions
  - And so on...
- Mean-field potentials

$$U = a\left(\frac{\varrho}{\varrho_0}\right) + b\left(\frac{\varrho}{\varrho_0}\right)^{\tau} \pm 2\, S_{pot}\, \frac{\varrho_{I3}}{\varrho_0}$$

- Modii:
  - Collider, Sphere, **Infinite matter**, Afterburner

SMASH: Degrees of freedom

N	Δ	Λ	Σ	Ξ	Ω
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^-_{1672}
N_{1462}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1532}	Ω^-_{2252}
N_{1515}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}	
N_{1535}	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}	
N_{1655}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}	
N_{1675}	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}	
N_{1685}	Δ_{1930}	Λ_{1800}	Σ_{1915}		
N_{1700}	Δ_{1950}	Λ_{1810}	Σ_{1940}		
N_{1710}		Λ_{1820}	Σ_{2030}		
N_{1720}		Λ_{1830}	Σ_{2250}		
N_{1875}		Λ_{1890}			
N_{1900}		Λ_{2100}			
N_{1990}		Λ_{2110}			
N_{2000}		Λ_{2350}			
N_{2190}					
N_{2220}					
N_{2250}					

Unflavored				Strange
π_{138}	$f_{0\,980}$	$f_{2\,1275}$	$\pi_{2\,1670}$	K_{494}
π_{1300}	$f_{0\,1370}$	$f'_{2\,1525}$		K^*_{892}
π_{1800}	$f_{0\,1500}$	$f_{2\,1950}$	$\rho_{3\,1690}$	$K_{1\,1270}$
	$f_{0\,1710}$	$f_{2\,2010}$		$K_{1\,1400}$
η_{548}		$f_{2\,2300}$	$\phi_{3\,1850}$	K^*_{1410}
η'_{958}	$a_{0\,980}$	$f_{2\,2340}$		$K_0^*{}_{1430}$
η_{1295}	$a_{0\,1450}$		$a_{4\,2040}$	$K_2^*{}_{1430}$
η_{1405}		$f_{1\,1285}$		K^*_{1680}
η_{1475}	ϕ_{1019}	$f_{1\,1420}$	$f_{4\,2050}$	$K_{2\,1770}$
	ϕ_{1680}			$K_3^*{}_{1780}$
σ_{800}		$a_{2\,1320}$		$K_{2\,1820}$
	$h_{1\,1170}$			$K_4^*{}_{2045}$
ρ_{776}		$\pi_{1\,1400}$		
ρ_{1450}	$b_{1\,1235}$	$\pi_{1\,1600}$		
ρ_{1700}				
	$a_{1\,1260}$	$\eta_{2\,1645}$		
ω_{783}				
ω_{1420}		$\omega_{3\,1670}$		
ω_{1650}				

- Isospin symmetry
- Perturbative treatment of non-hadronic particles (photons, dileptons)

SMASH: Resonances

- Particles stable if decay width smaller than 10 keV
- Spectral functions of resonances are described by relativistic Breit-Wigner functions

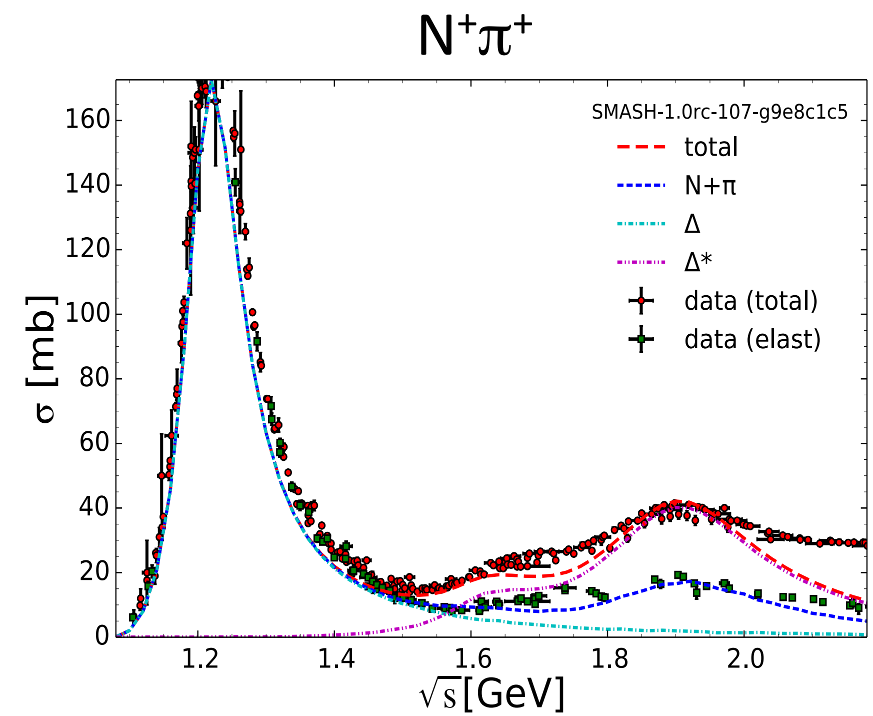
$$A(m) = \frac{2N}{\pi} \frac{m^2\,\Gamma(m)}{\left(m^2 - M_0^2\right)^2 + m^2\,\Gamma(m)^2}$$

- Manley et al. treatment for partial widths $\Gamma_{R\to ab}$
- Resonance lifetime for 1→2

$$\tau_{res} = \frac{1}{\Gamma(m)}$$

- Cross-sections for 2→1:

$$\sigma_{ab\to R} = \frac{2J_R+1}{(2J_a+1)(2J_b+1)} S_{ab} \frac{2\pi^2}{p_i^2} \Gamma_{ab\to R}(s)\, A_R(\sqrt{s})$$



Weil et al., Phys. Rev. C94 (2016) no.5, 054905

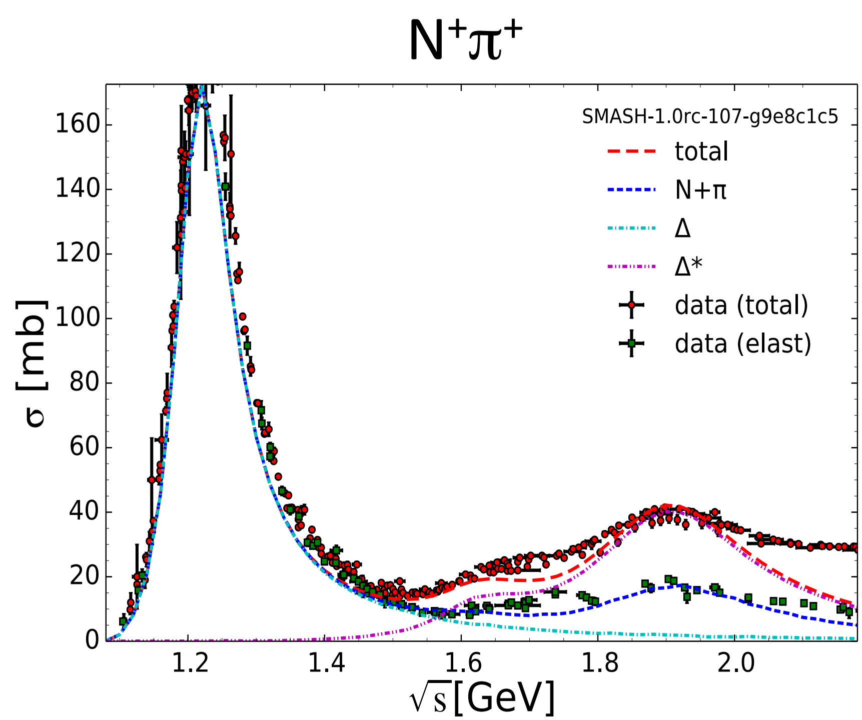
Manley et al., Phys. Rev. D45 (1992), 4002

SMASH: Scatterings

- Elastic scatterings parameterized for NN reactions; all other elastic scatterings assumed to go through resonances
- Inelastic scatterings implemented:

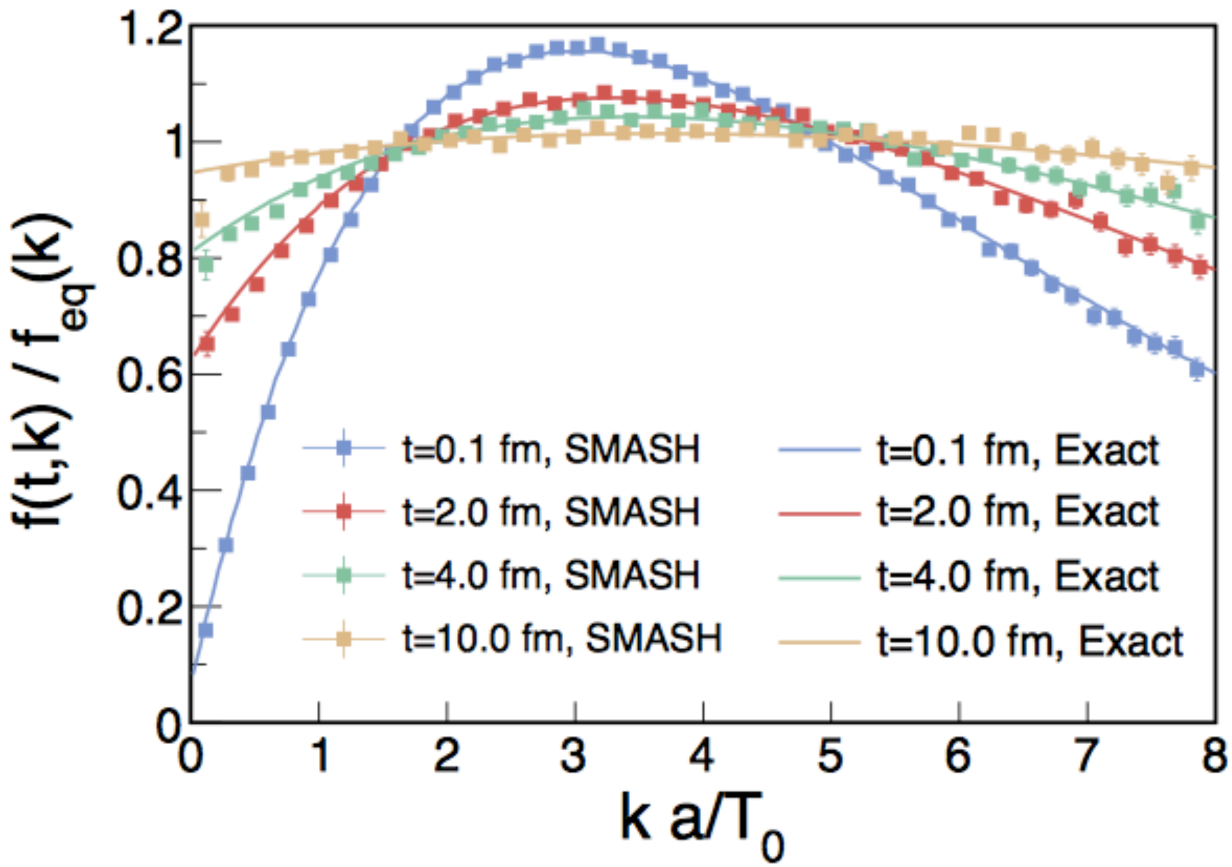
$$\sigma_{ab\to Rc}(s) = \frac{(2J_R+1)(2J_c+1)}{s|p_i|} \cdot \sum \left(C^I_{ab} C^I_{Rc}\right)^2 \frac{|M|^2_{ab\leftrightarrow Rc}(s,I)}{16\pi} \cdot \int_{m_R^{\min}}^{\sqrt{s}-m_c} dm\, A_R(m) \cdot |p_f|(\sqrt{s}, m, m_c)$$

- Currently include
  - NN⟷NR, NN⟷ ΔR
  - KN⟷KN, KN⟷πH
  - +antiparticles
- Strings (turned off)



Weil et al., Phys. Rev. C94 (2016) no.5, 054905

SMASH: Does it all work?

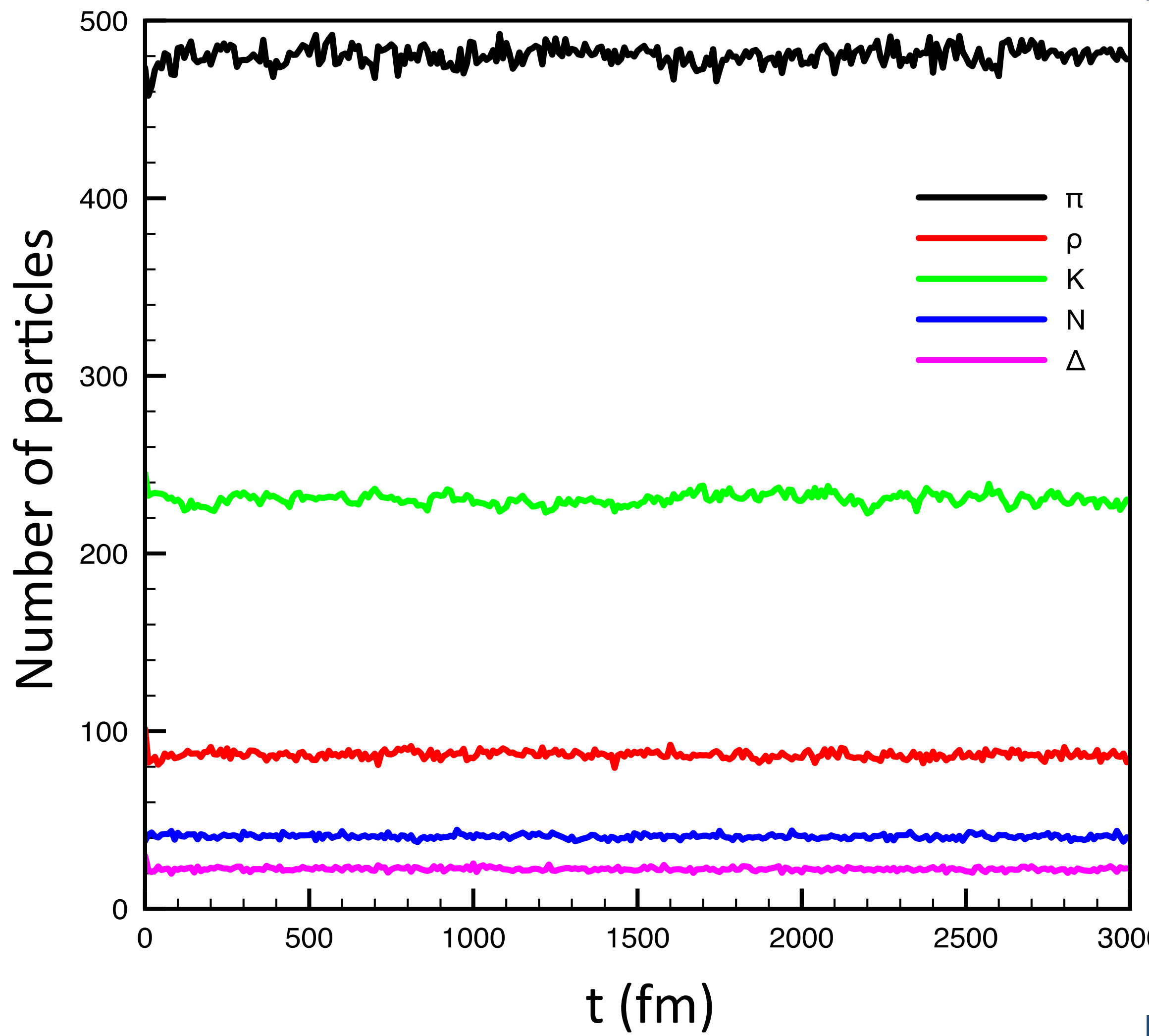


Tindall et al., Phys.Lett. B770 (2017) 532-538

Exact solution of Boltzmann equation in expanding universe

Viscosity in SMASH

- **Box calculations simulating infinite matter to apply the Green-Kubo procedure**
- **MUST have thermal & chemical equilibrium**
  - Baryon/antibaryon annihilation implemented to conserve detailed balance via an average decay to 5π



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Green-Kubo Formalism

The shear viscosity is calculated from

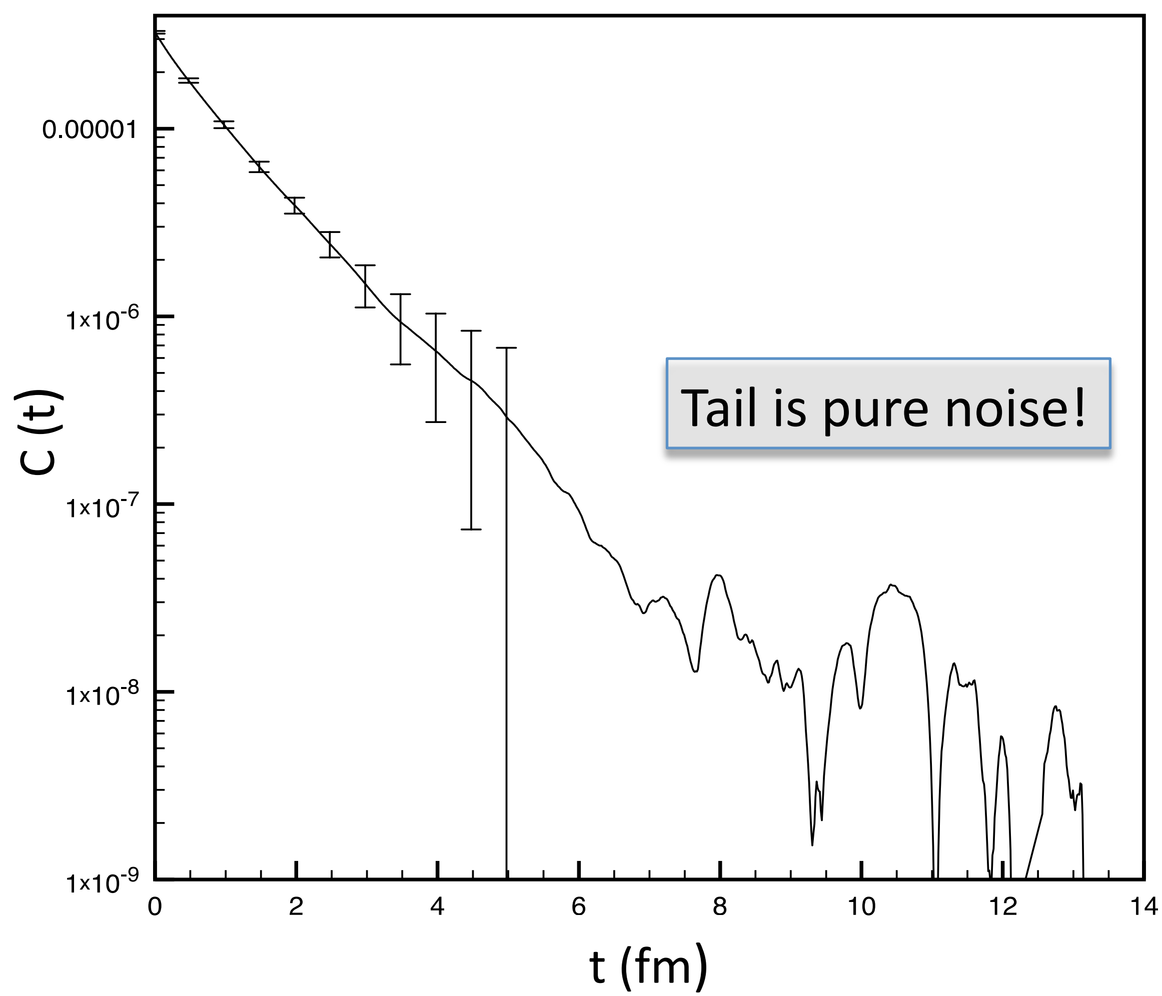
$$\eta = \frac{V}{T}\int_0^\infty C^{xy}(t)\,dt$$

where

$$C^{xy}(t) = \frac{1}{N}\sum_s^N T^{xy}(s)\,T^{xy}(s+t)$$

and

$$T^{\mu\nu} = \frac{1}{V}\sum_i^{N_{part}} \frac{p_i^\mu p_i^\nu}{p_i^0}$$



N is the number of time steps, and N_{part} the number of particles

Green-Kubo Formalism

It has been shown that the correlation

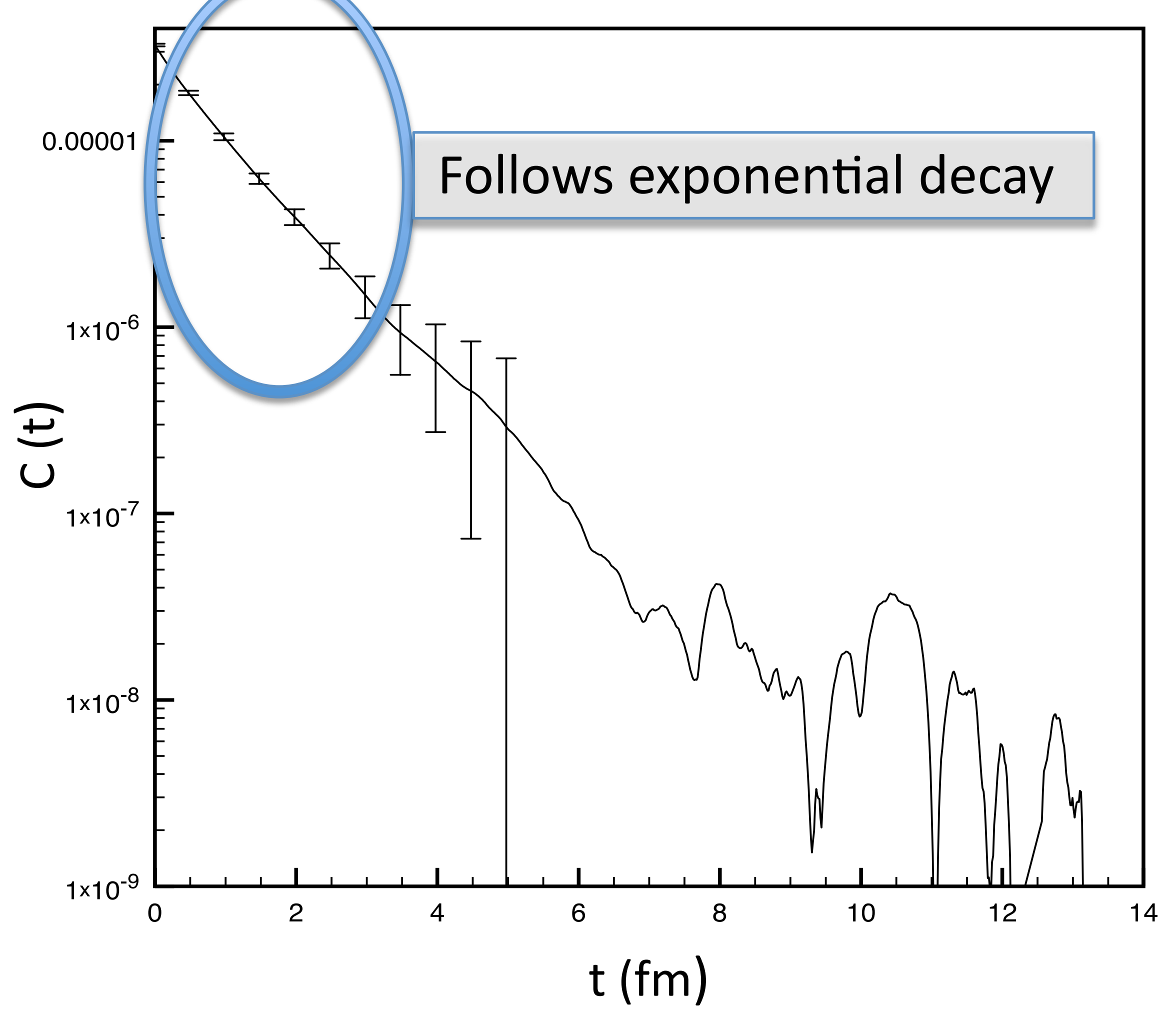
$$\eta = \frac{V}{T} \int_0^{\infty} C^{xy}(t) dt$$

Follows

$$C^{xy}(t) = C^{xy}(0) \exp\left(-\frac{t}{\tau}\right)$$

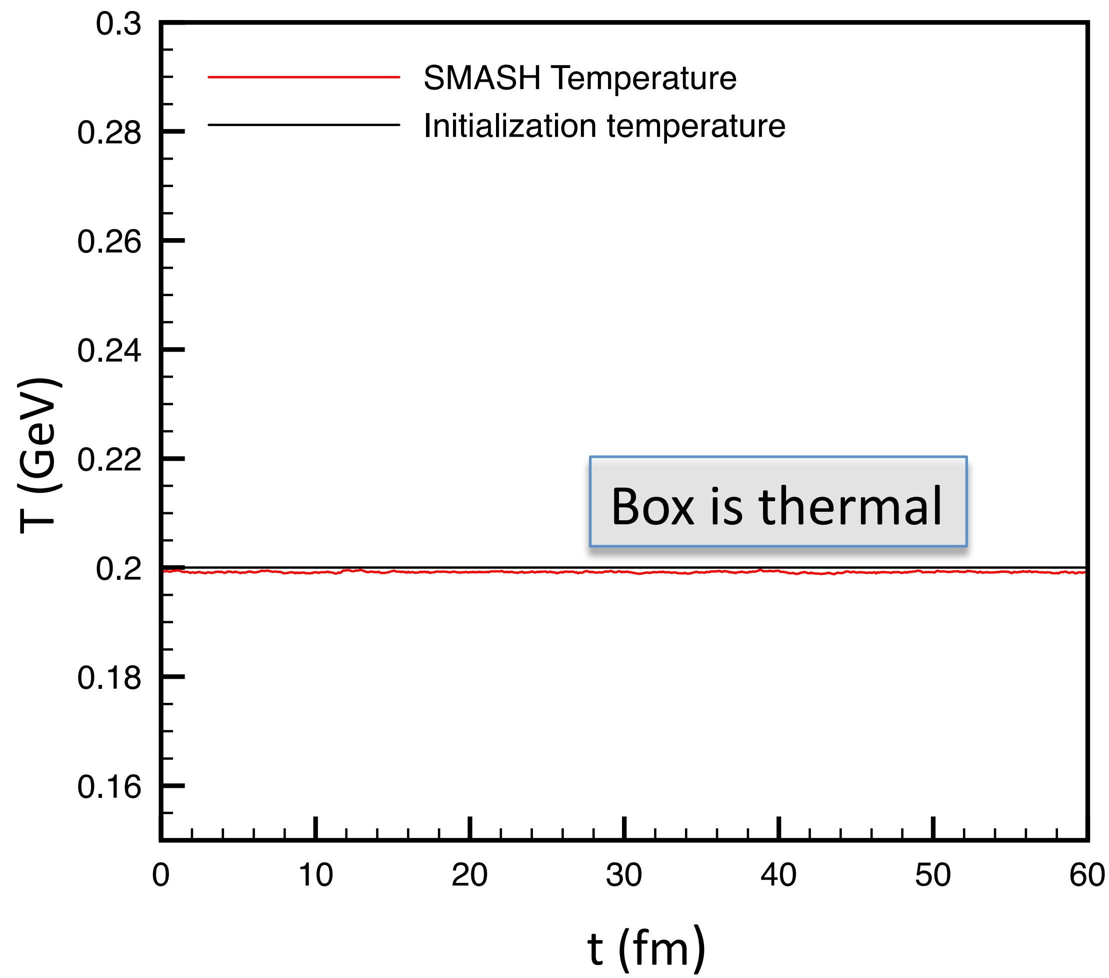
So that

$$\eta = \frac{V C^{xy}(0) \tau}{T}$$



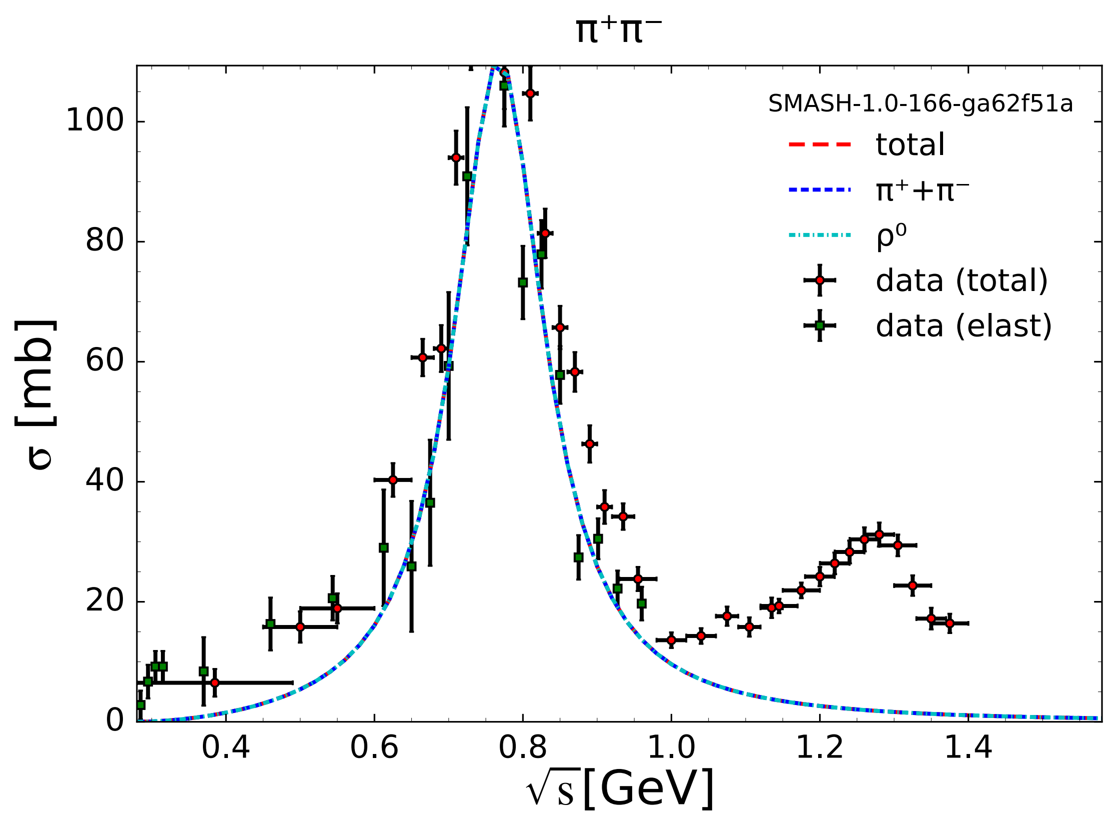
Test case #1: Constant σ

- Pions in a (20 fm)^3 box simulating infinite matter
- Constant, isotropic σ
- Runs for t_{max}=200 fm
- Initialized with initial densities consistent with Boltzmann ideal gas



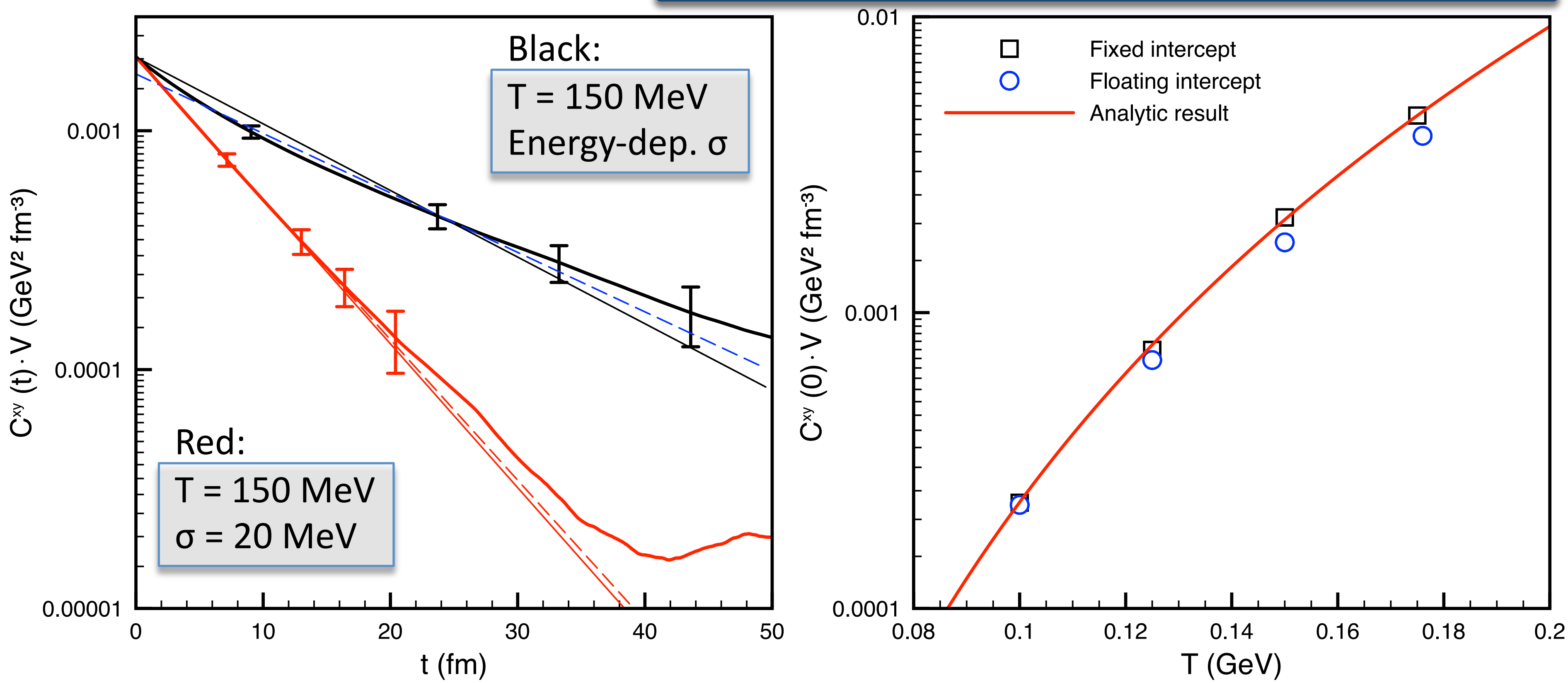
Test case #2: Energy-dependent σ

- Pions in a $(20\ fm)^3$ box simulating infinite matter
- Cross-section uses ρ resonance
- Runs for t_{max}=200 fm
- Initialized with initial densities consistent with Boltzmann ideal gas



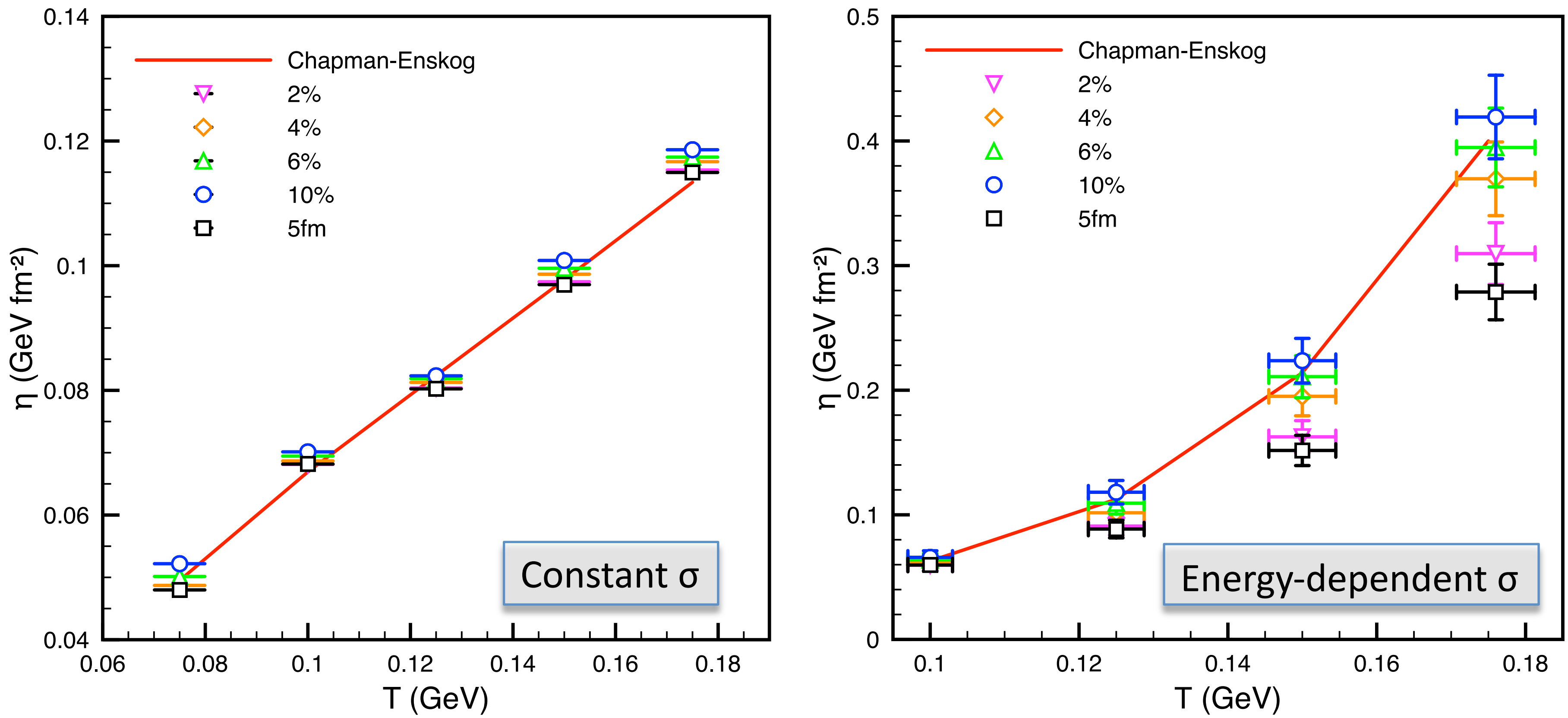
How to fit?

$$C^{xy}(0) = \frac{g\exp\left(\frac{\mu}{T}\right)}{30\pi^2 V}\int_0^\infty dp\ \frac{p^6}{m^2+p^2}\ \exp\left(-\frac{\sqrt{m^2+p^2}}{T}\right)$$

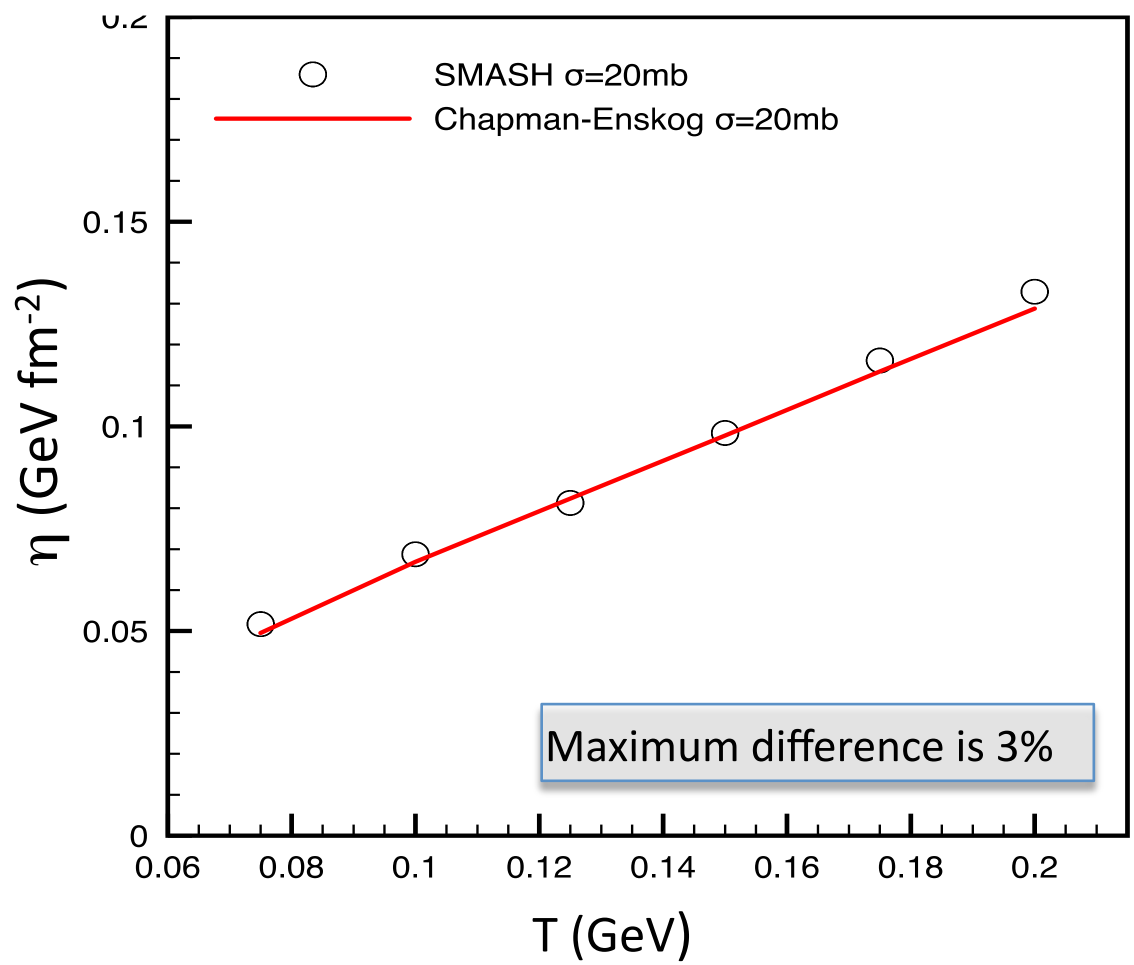


Where to stop?

J. Torres-Rincon, PhD dissertation (2012), *Hadronic Transport Coefficients from Effective Field Theories*

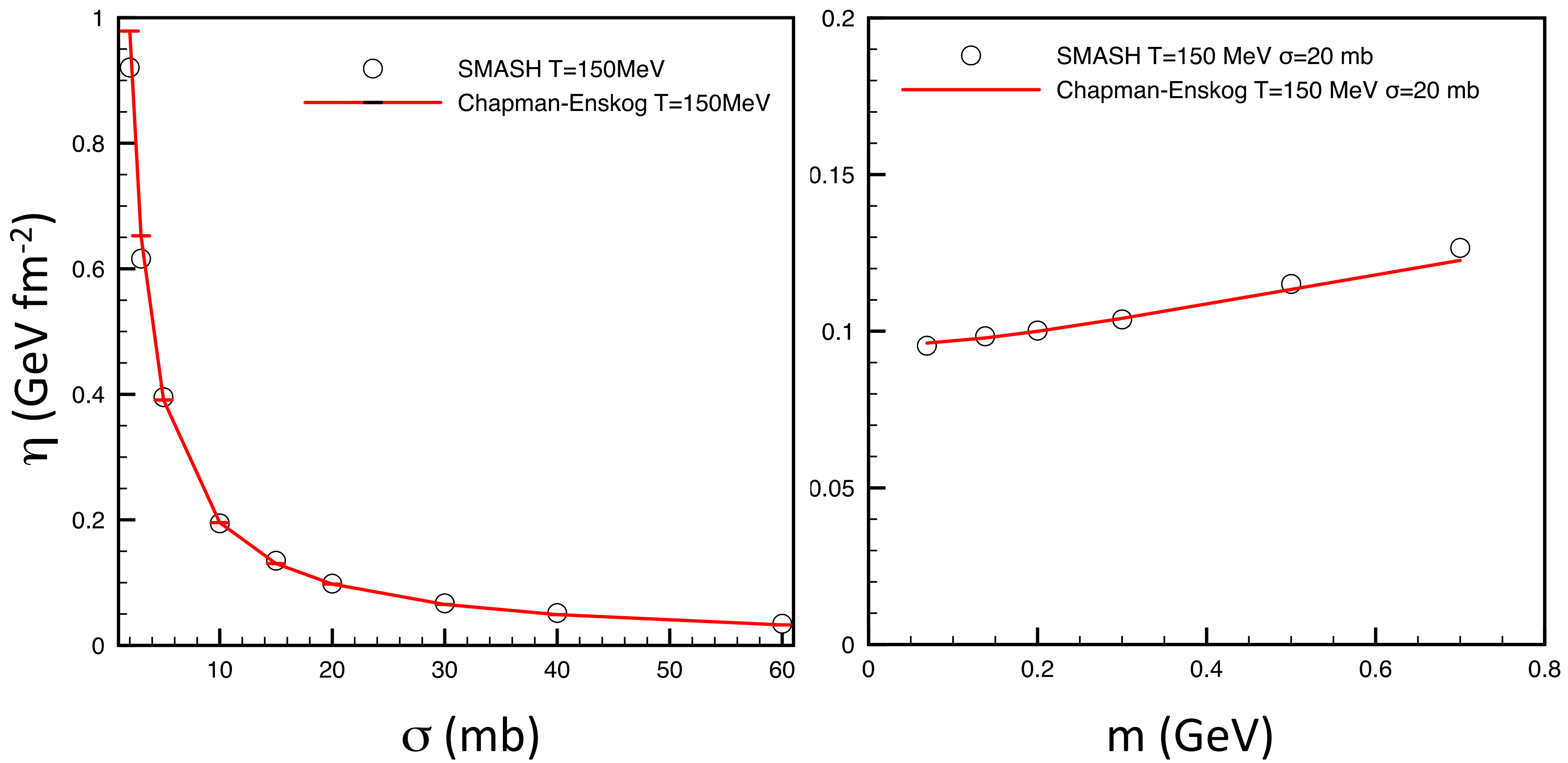


Constant σ: Temperature dependence



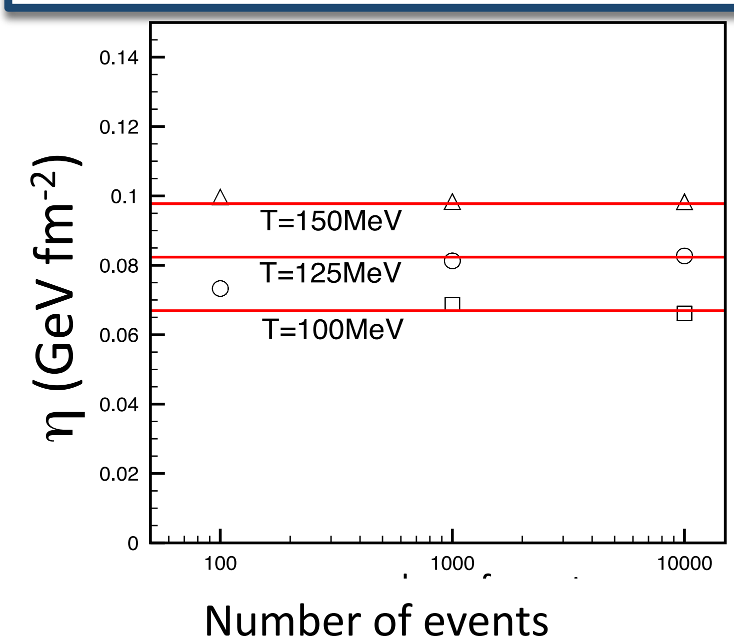
J. Torres-Rincon, PhD dissertation (2012), *Hadronic Transport Coefficients from Effective Field Theories*

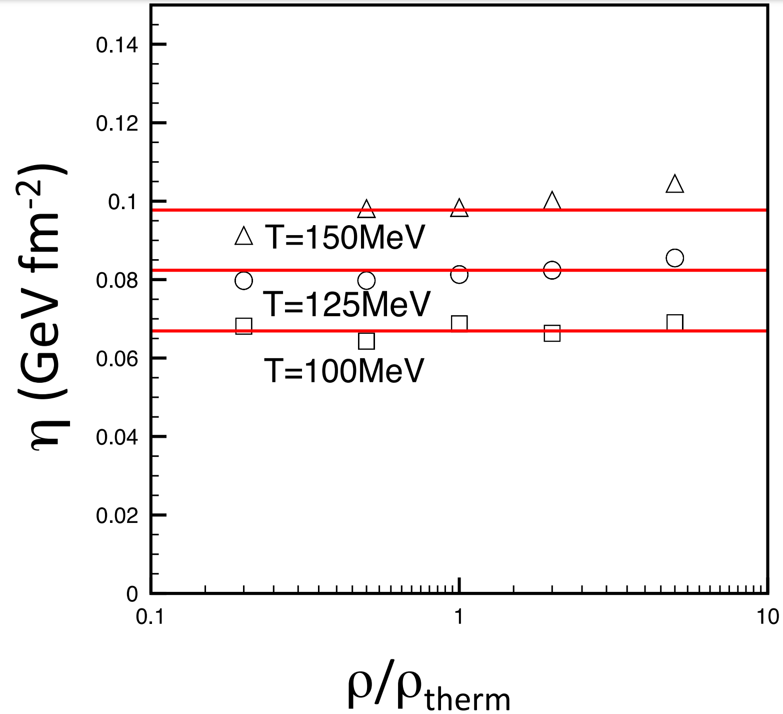
Constant σ: Cross-section/mass dependence

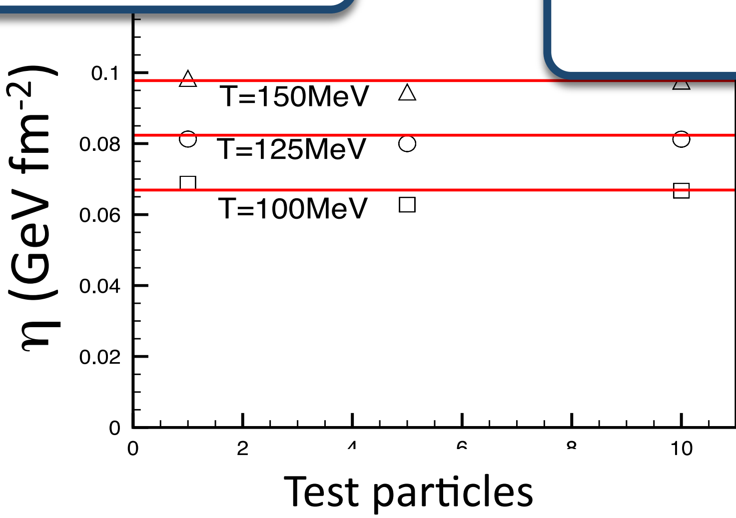


Very good agreement with analytical calculations!

Constant σ: Systematics





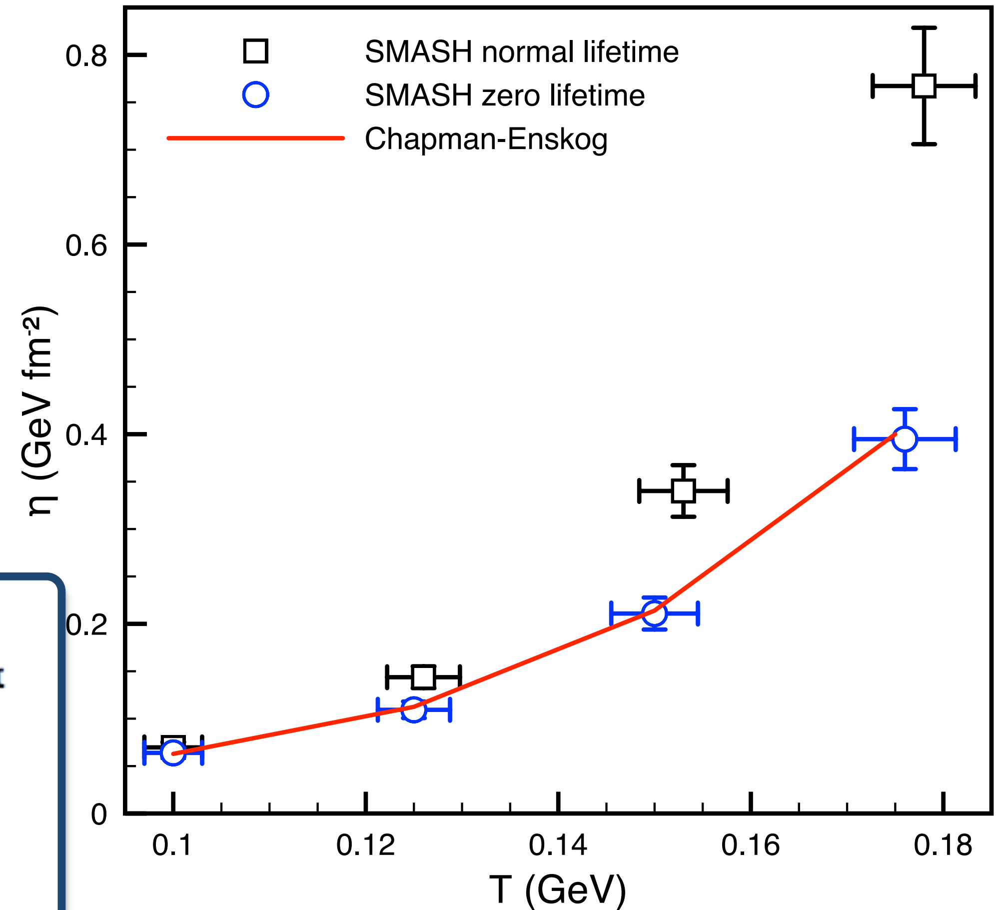


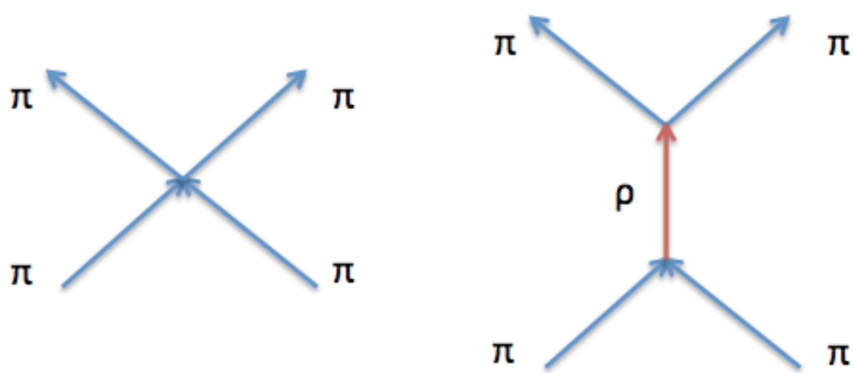
Main take-away:

The method is relatively inelastic to variations of most parameters; maximum error is less than 10%

Energy-dependent σ: Resonance lifetimes

- Normal SMASH run does not coincide directly with Chapman-Enskog
    - Resonance lifetimes





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What about entropy?

The entropy density can be calculated from the Gibbs formula:

$$s = \frac{e + p - \mu n}{T} = \frac{w - \mu n}{T}$$

where the energy density and pressure can be taken from the average shear-stress tensor according to:

$$T^{\mu\nu} = diag(e, p, p, p)$$

Assuming a nearly ideal gas, one can fit the temperature and chemical potential with momentum distributions:

$$\frac{dN}{dp} = \frac{g}{2\pi^2} V p^2 \exp\left(-\frac{\sqrt{p^2 + m^2} - \mu}{T}\right)$$

What about entropy?

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When is this correct?

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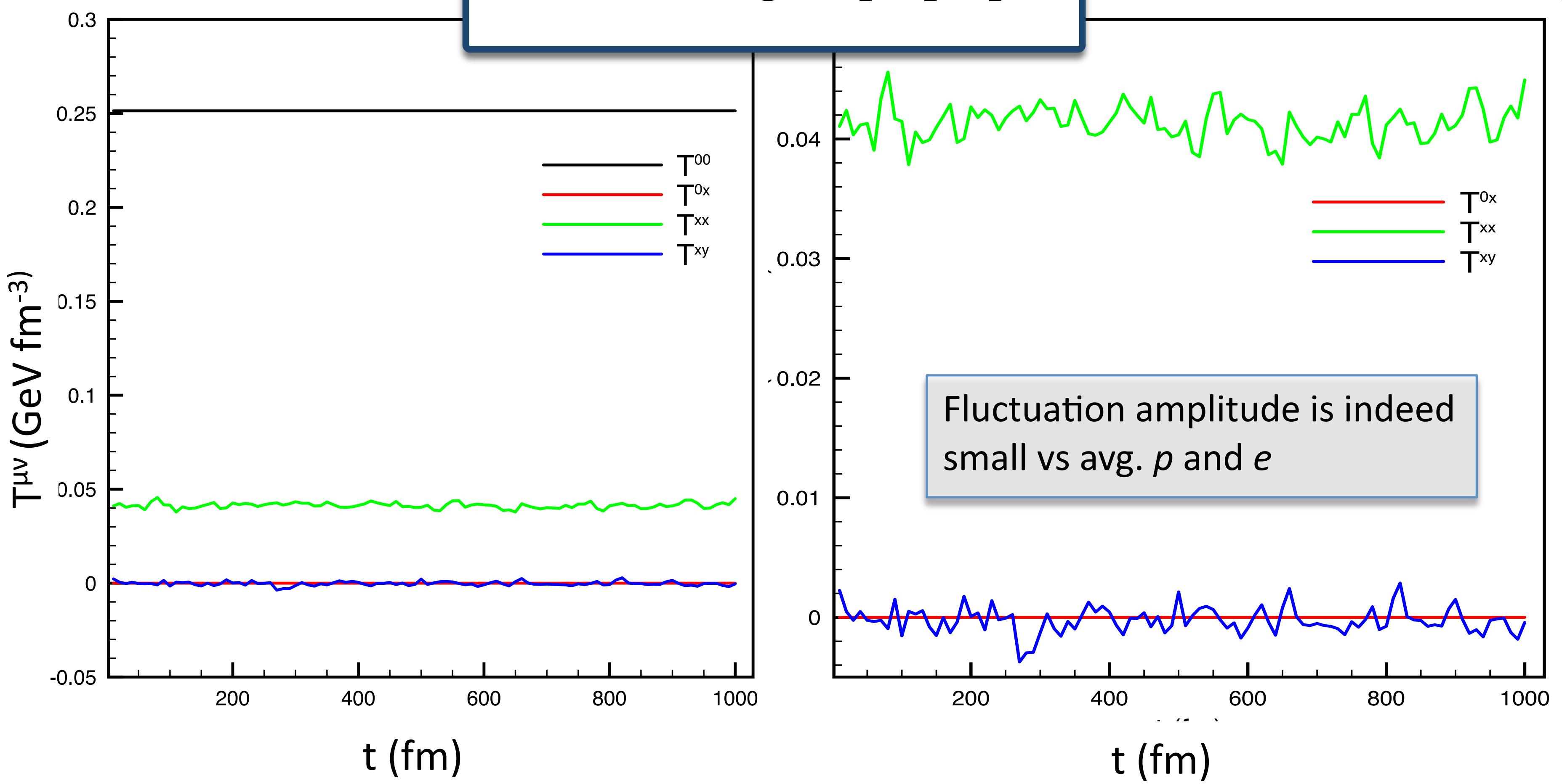
Energy density and pressure

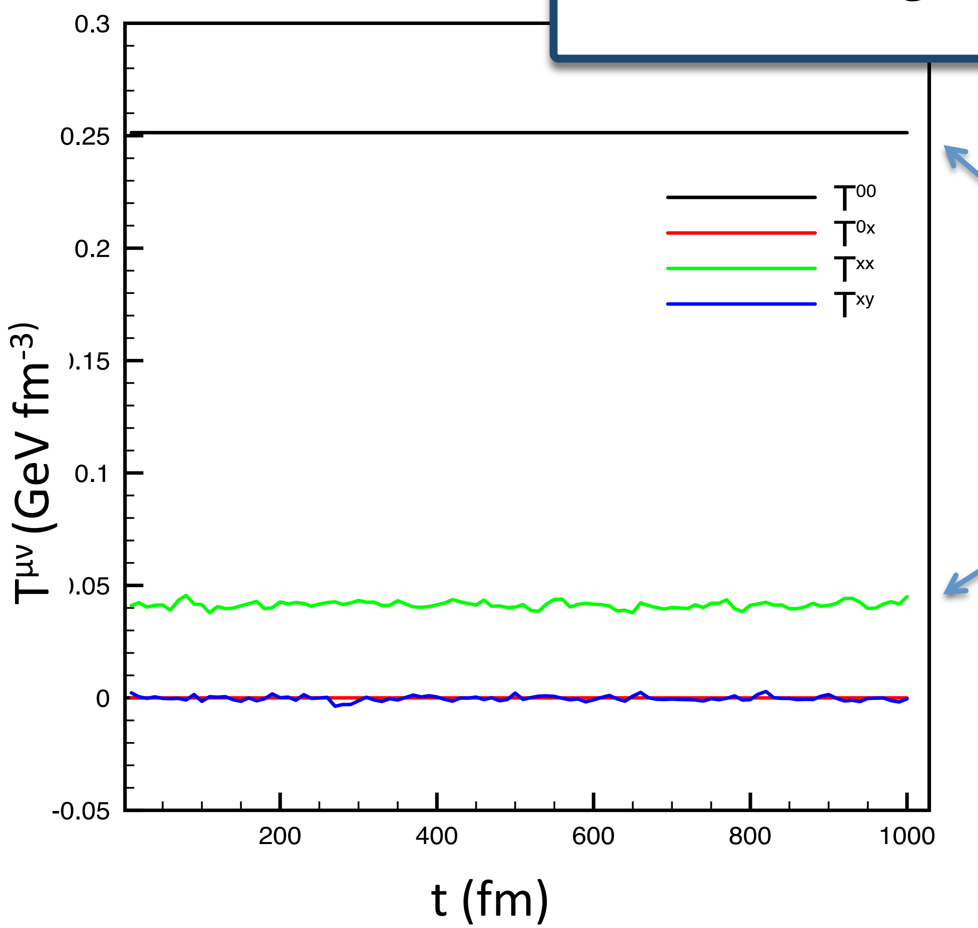
$$T^{\mu\nu} = diag(e, p, p, p)$$



Fluctuation amplitude is indeed small vs avg. *p* and *e*

Energy density and pressure

$$T^{\mu\nu} = diag(e, p, p, p)$$



$$\frac{p}{e} \sim \frac{1}{6}$$

In the range of typical hadron gas equation of state

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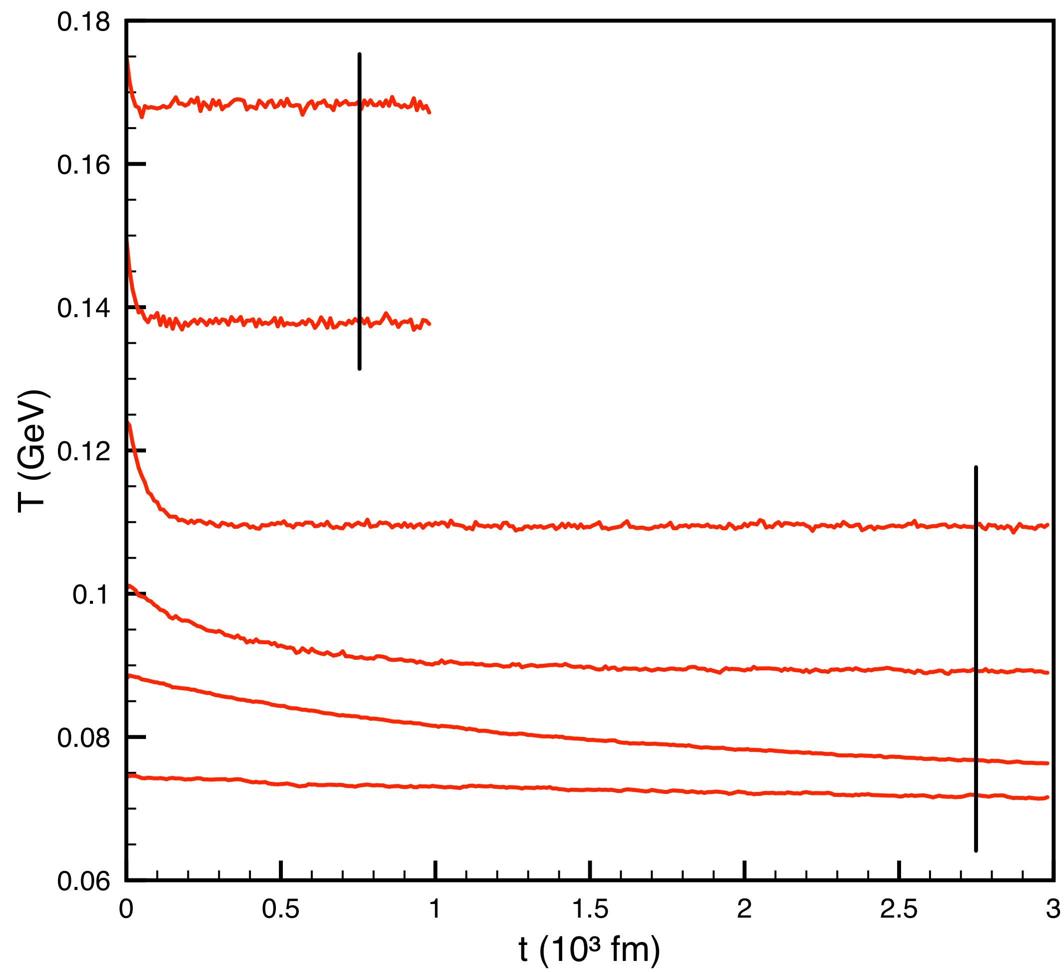
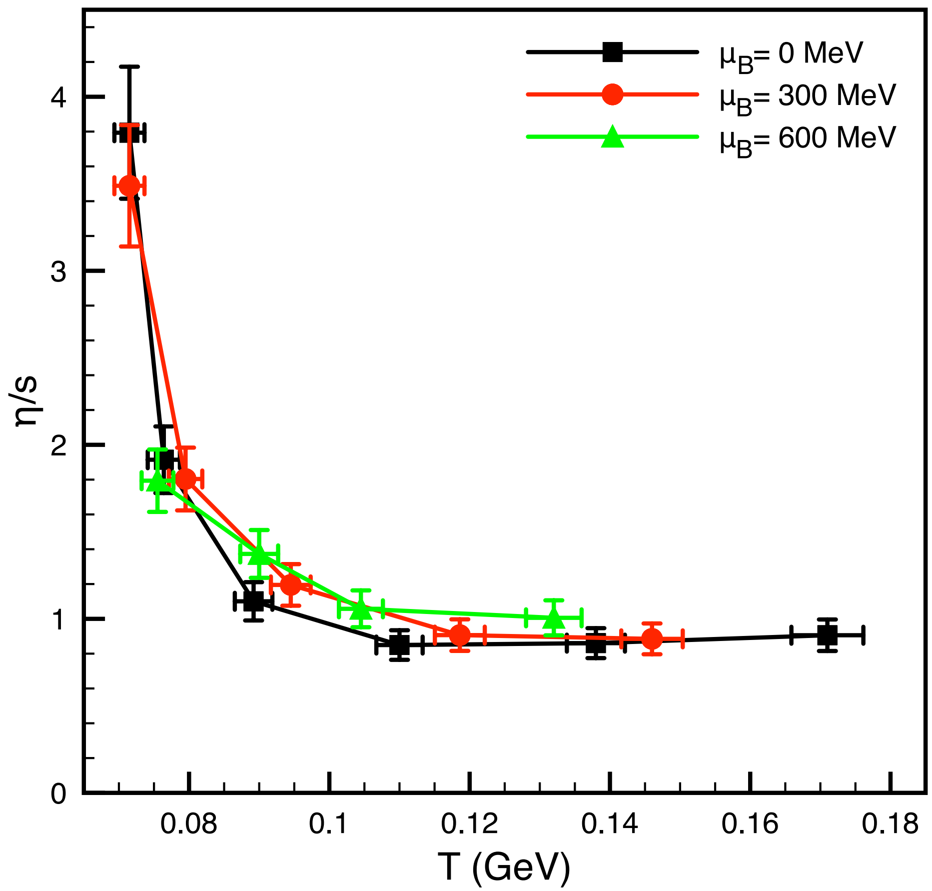
Hadron Gas (HG)

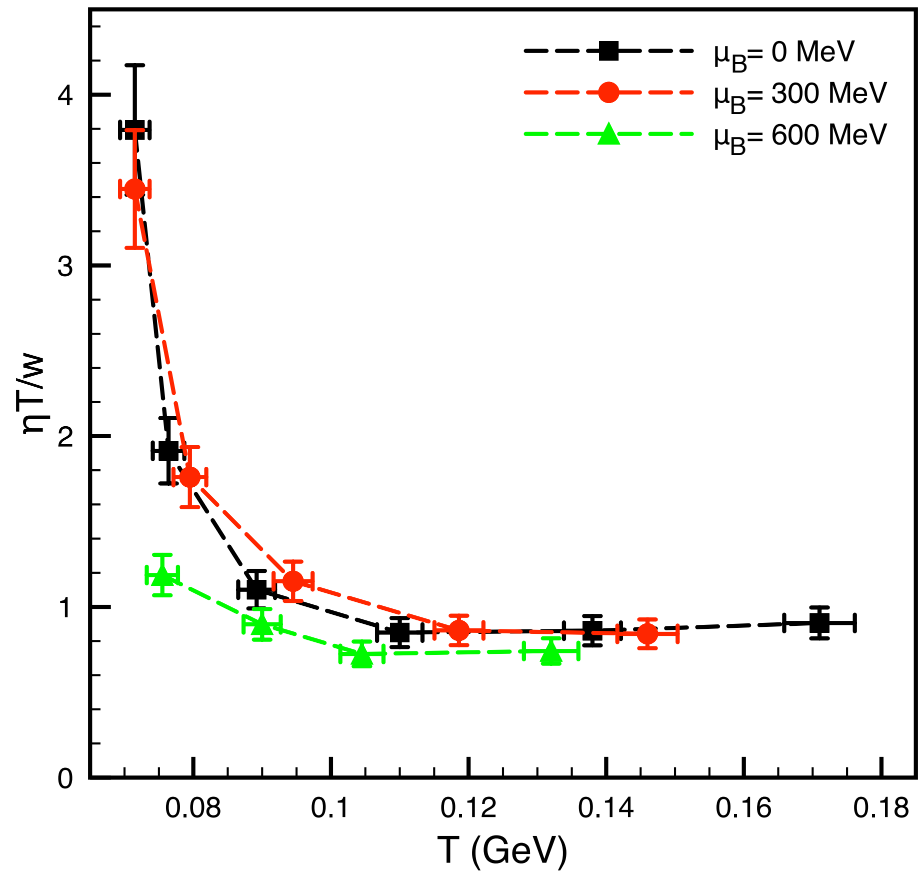
- All particles and resonances initialized to thermal multiplicities
- Must wait for equilibration and compute *T*, μ once in equilibrium from most abundant particles
  - T fitted from weighted momentum spectra of *π, K & N*
  - μ_B obtained from *N* / anti-*N* ratio



η/s and ηT/w



μB= 0 MeV
μB= 300 MeV
μB= 600 MeV
η/s
T (GeV)
0
1
2
3
4
0.08
0.1
0.12
0.14
0.16
0.18



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ηT/w
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1
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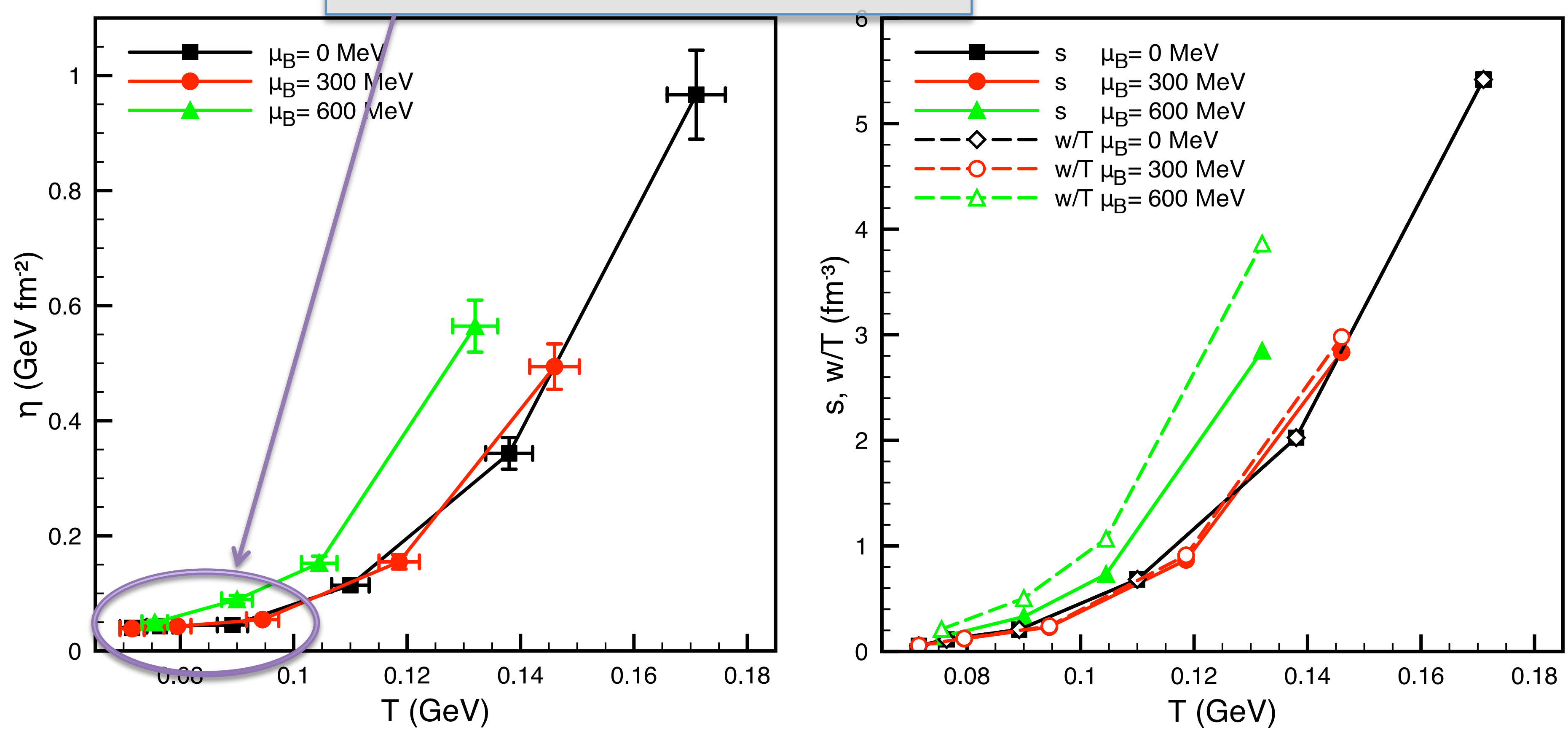
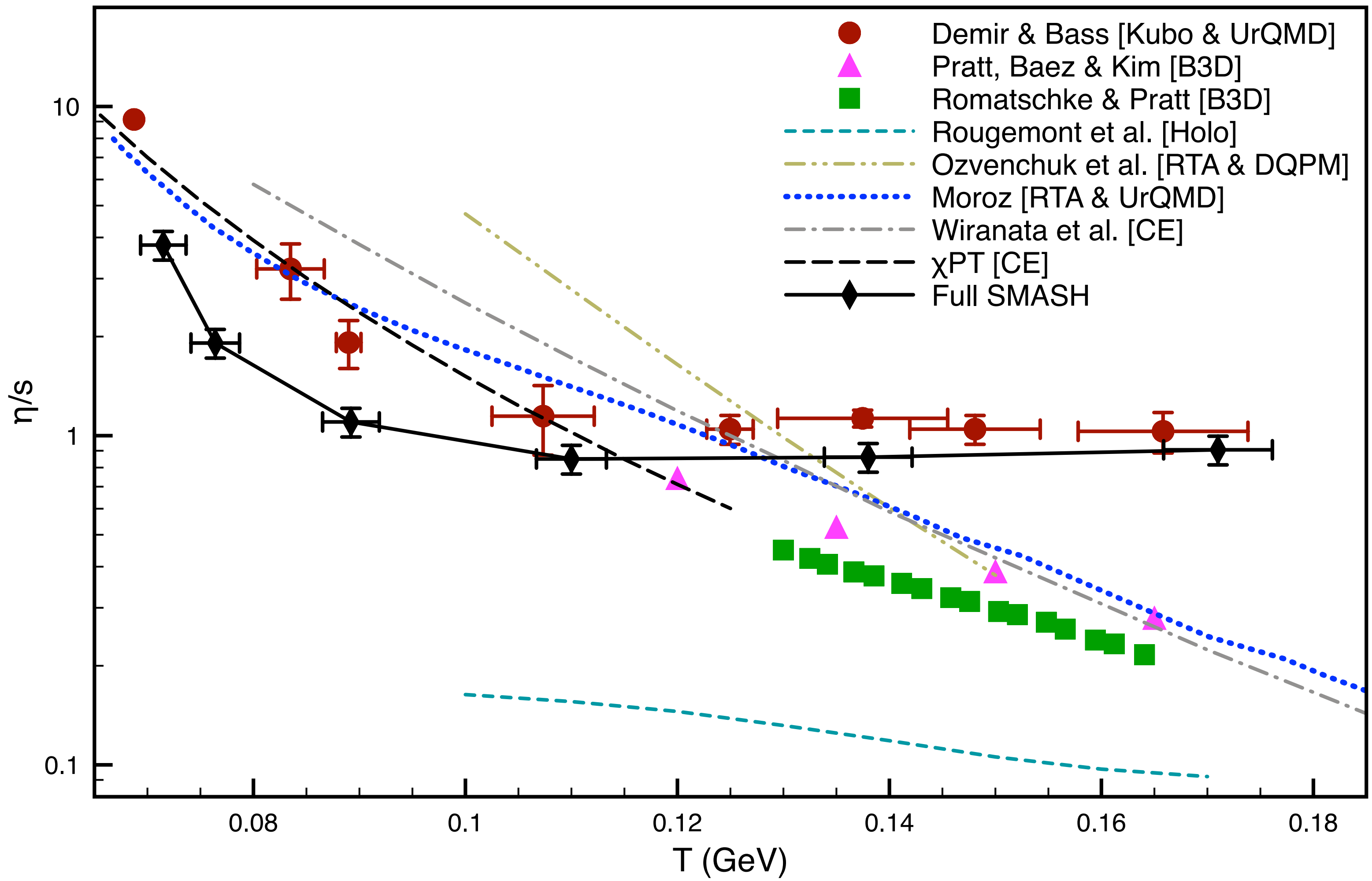
0.12
0.14
0.16
0.18

η, s and w/T

Viscosity decreases slower at small temperatures; explains rise of *η/s*



HG: Viscosity Comparison



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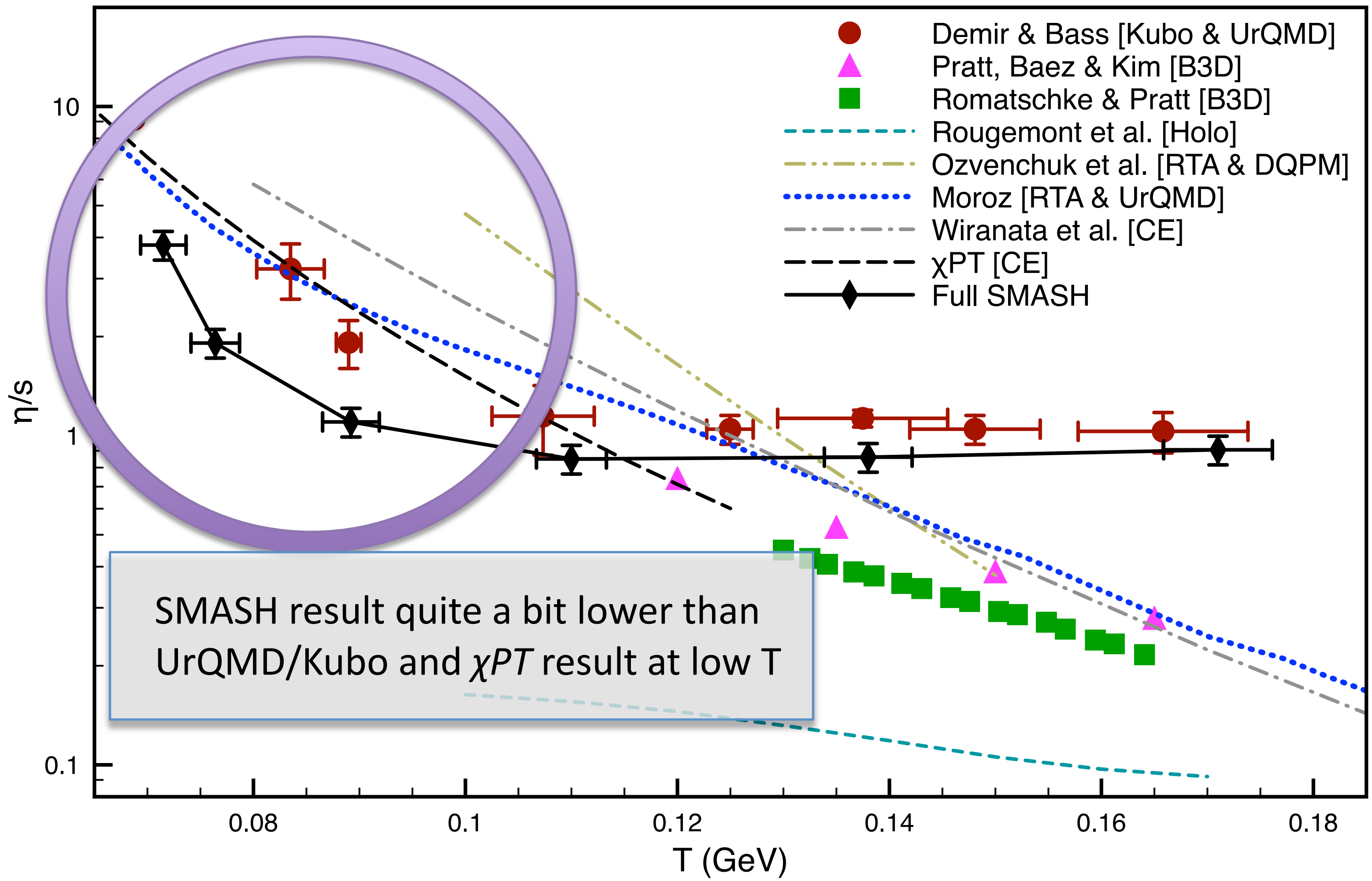
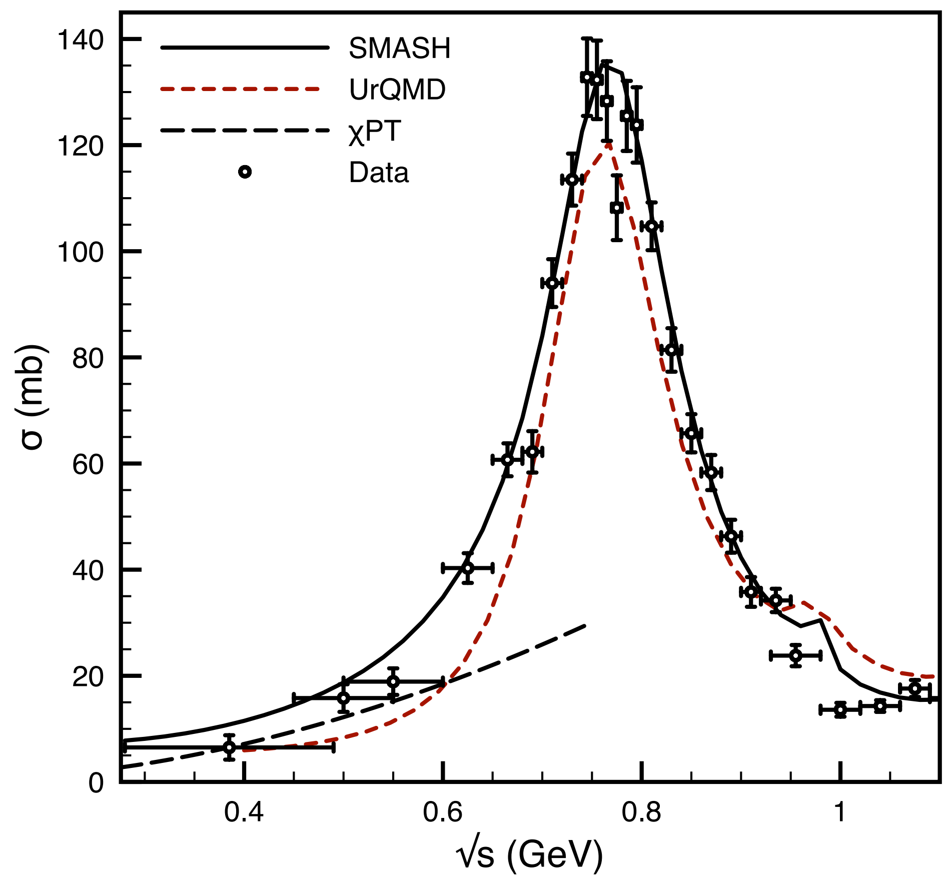


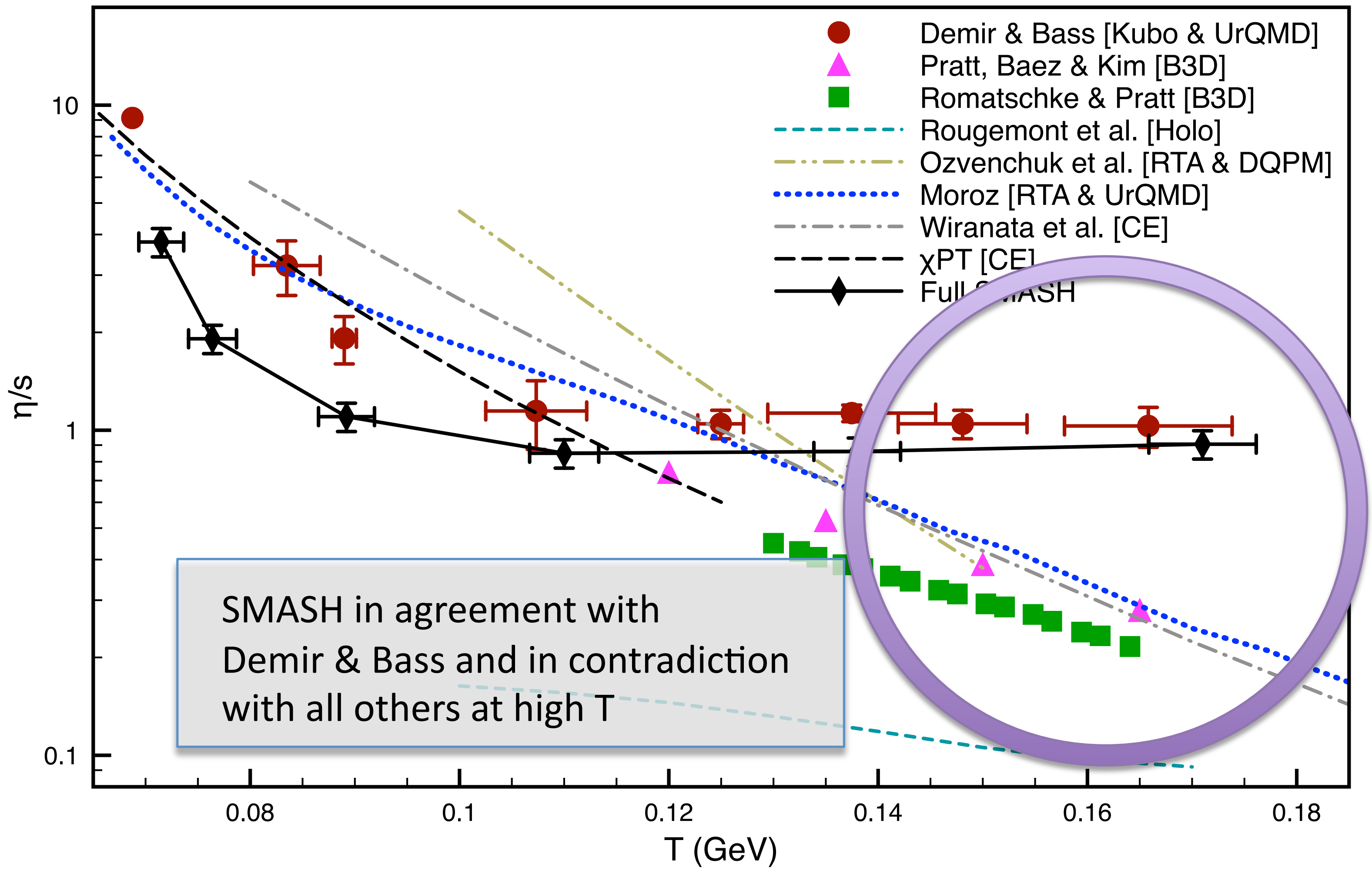
SMASH result quite a bit lower than UrQMD/Kubo and *χPT* result at low T

Low temperature η/s

- Low temperature hadron gas is composed almost exclusively of pions
- π-π cross-section is then most relevant
  - At very low energy, SMASH much higher than UrQMD/χPT
  - χPT includes angular dependence, UrQMD&SMASH don't; increases viscosity by factor up to 5/3 for ρ resonance

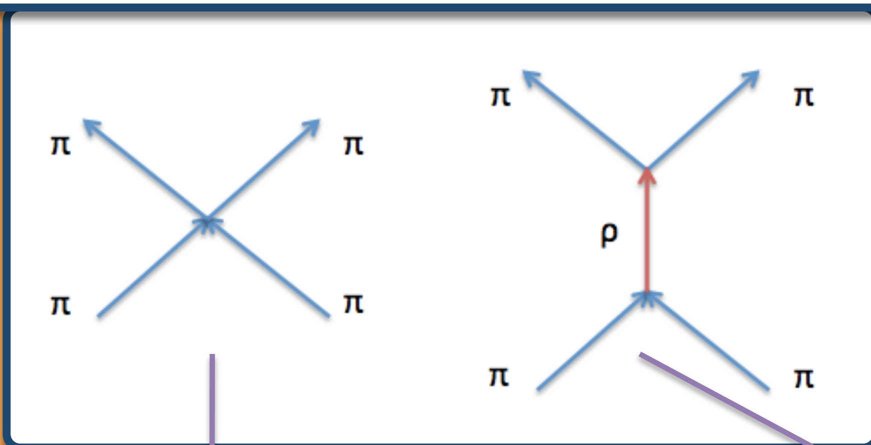


HG: Viscosity Comparison



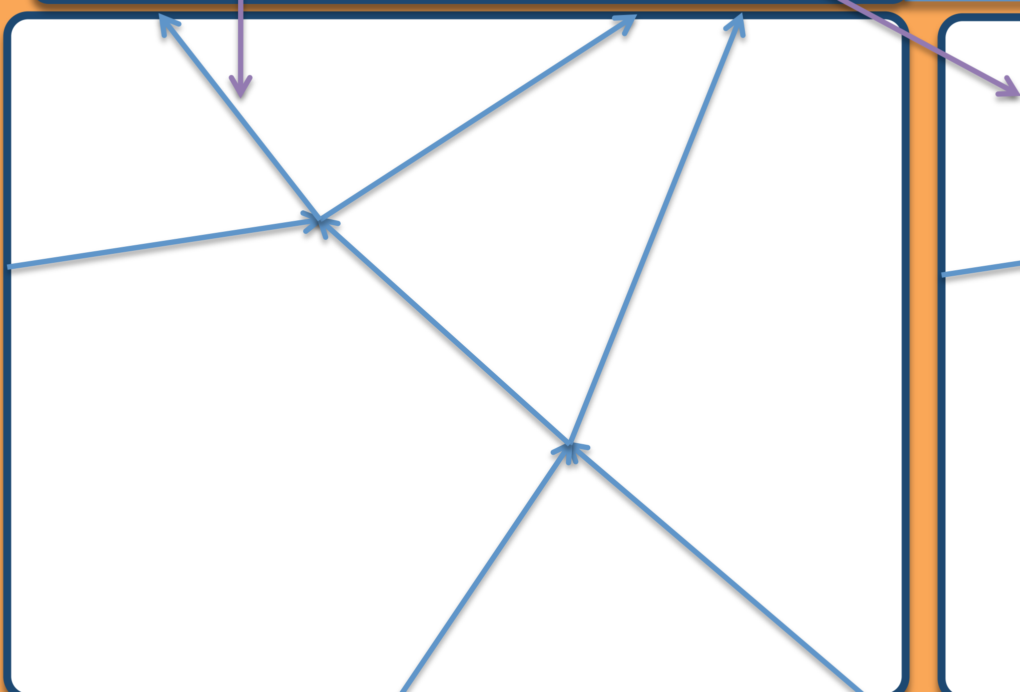
SMASH in agreement with Demir & Bass and in contradiction with all others at high T

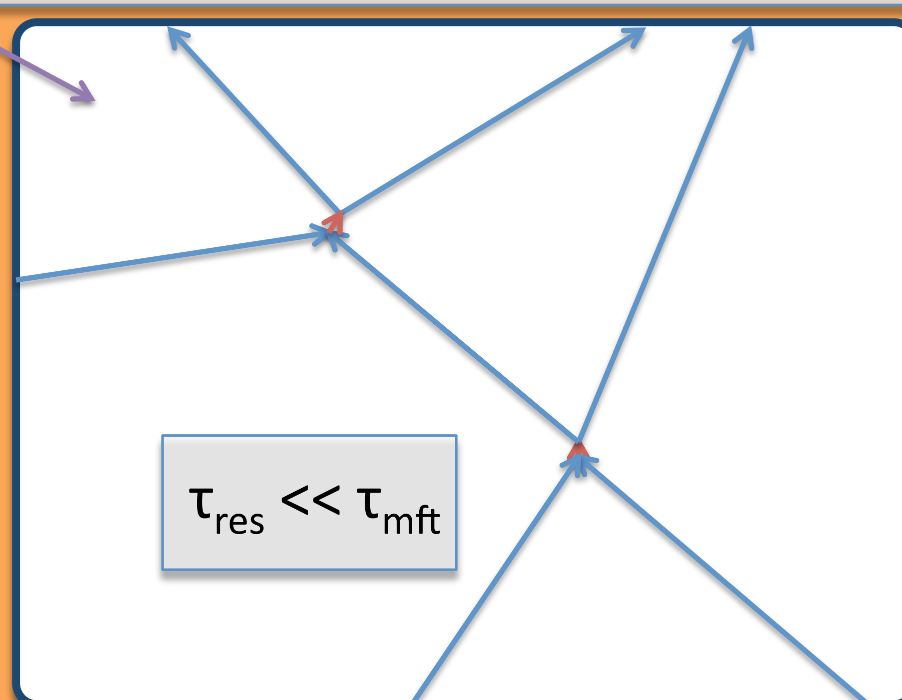
High temperature η/s : Resonance lifetimes



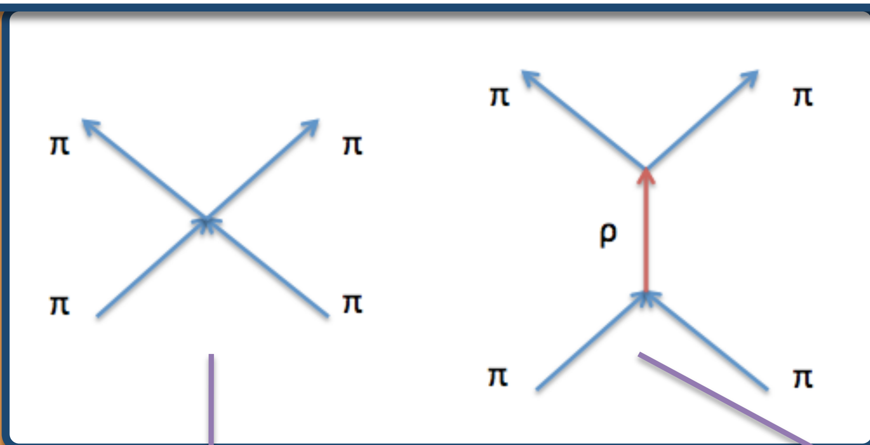
Must look at the microscopic picture from different descriptions

At low T and density:





High temperature η/s : Resonance lifetimes



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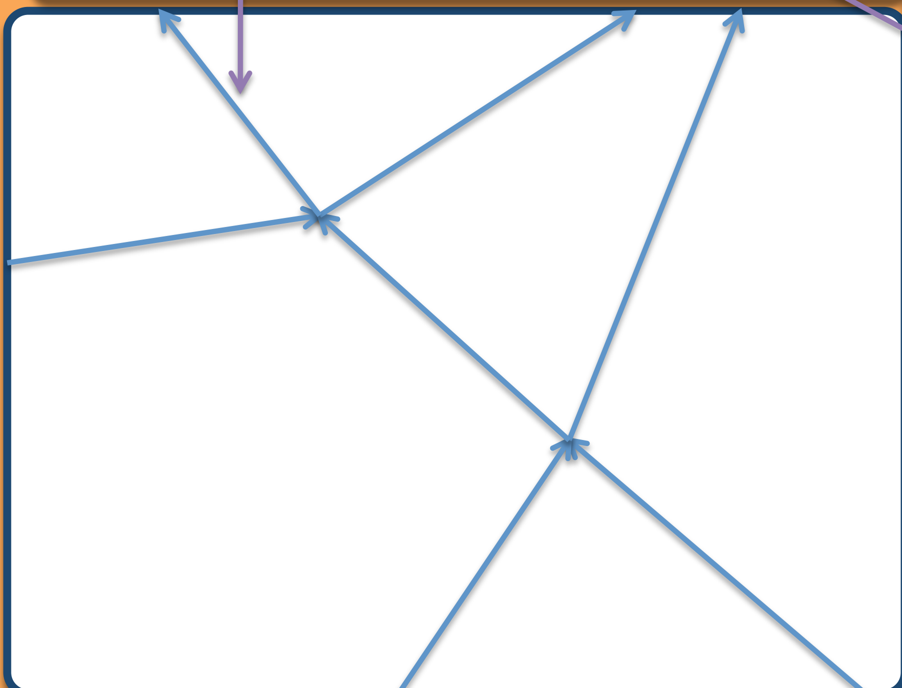

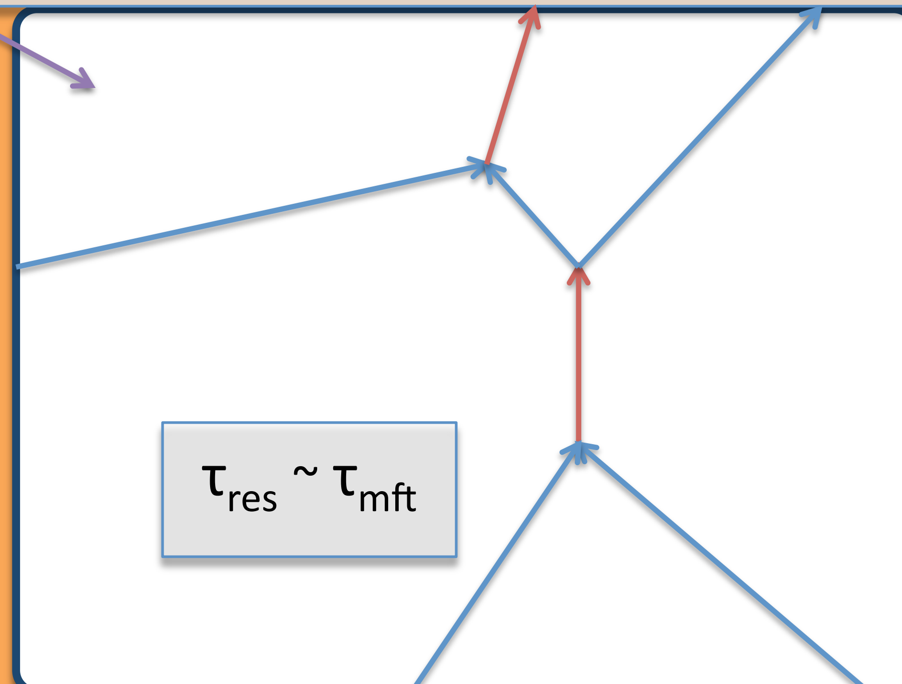

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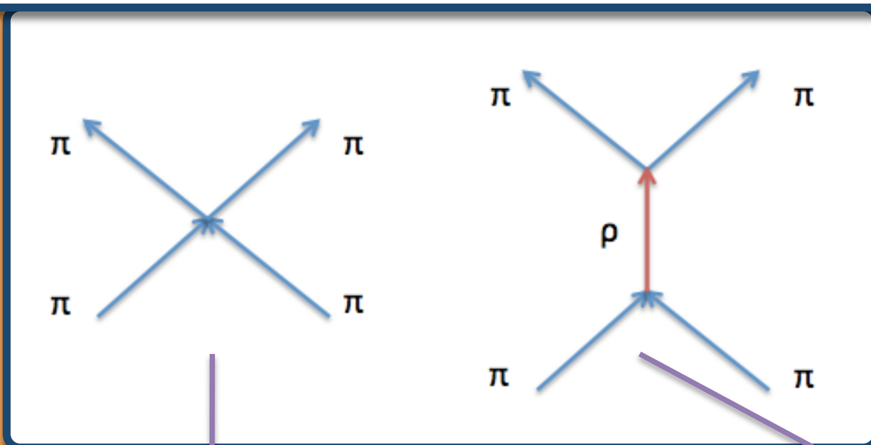

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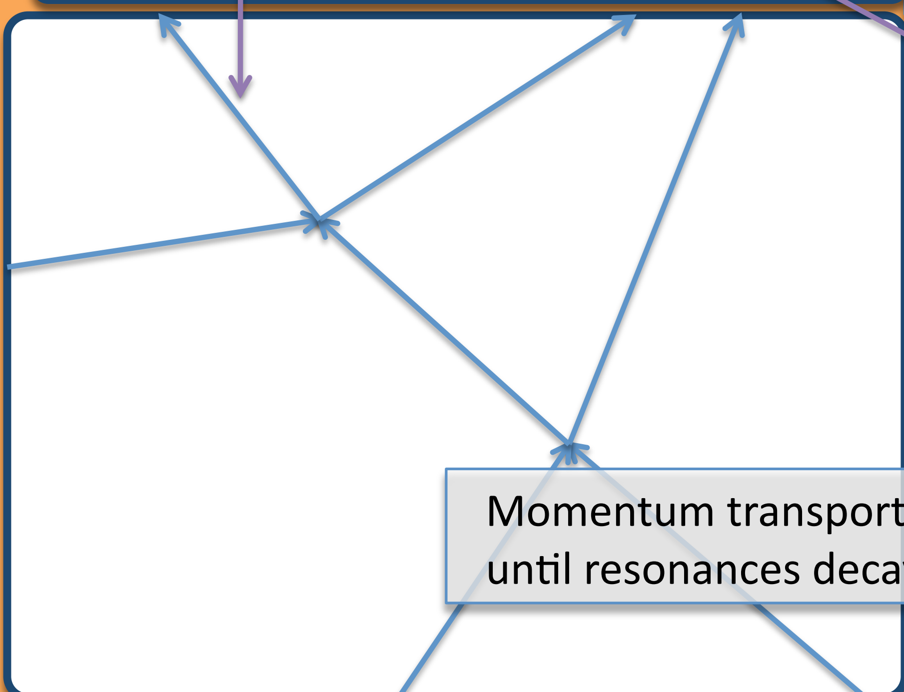
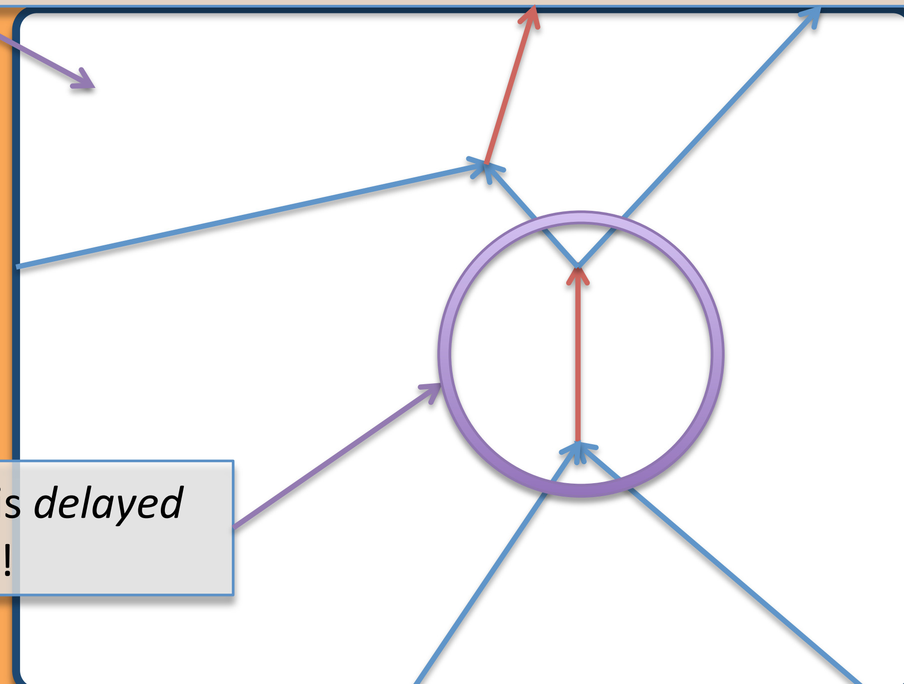
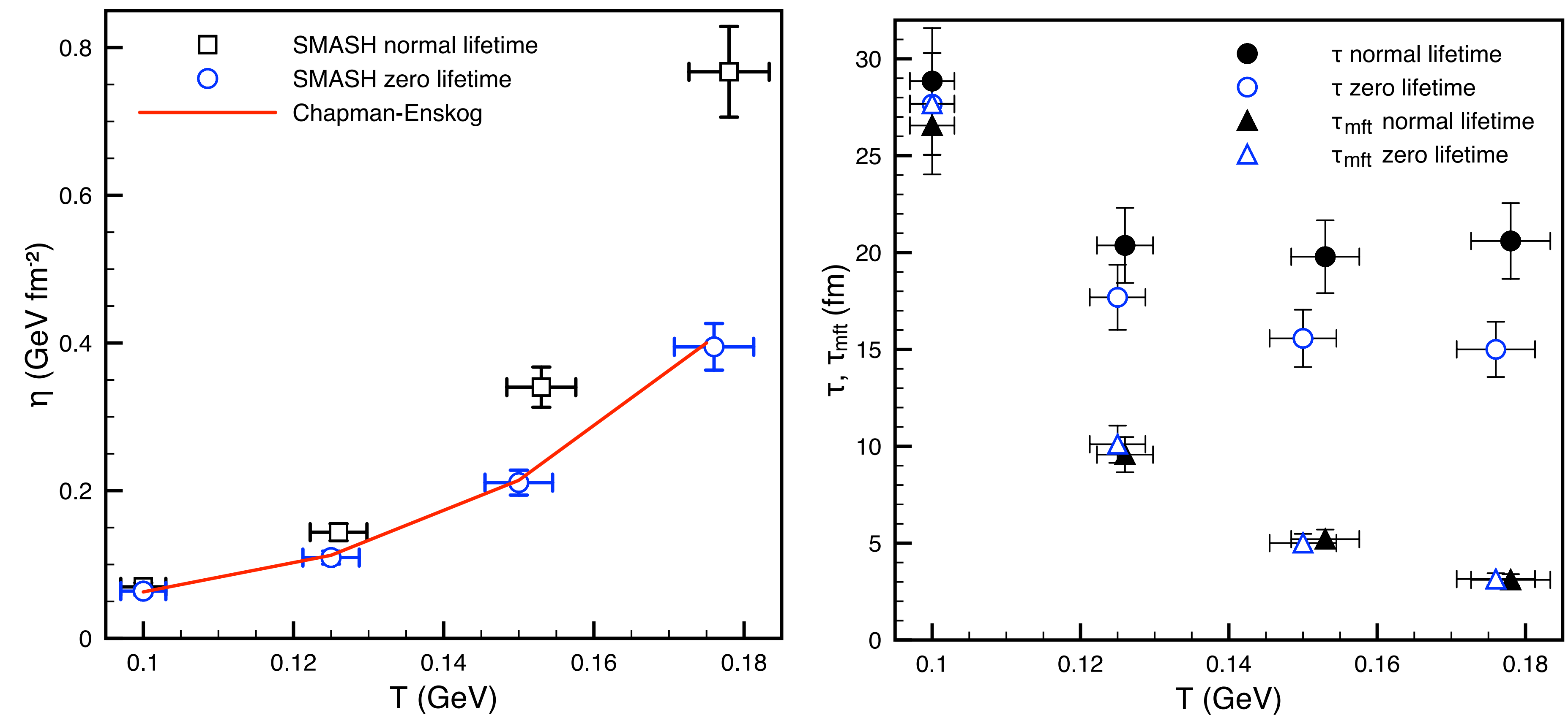




Momentum transport is *delayed* until resonances decay!

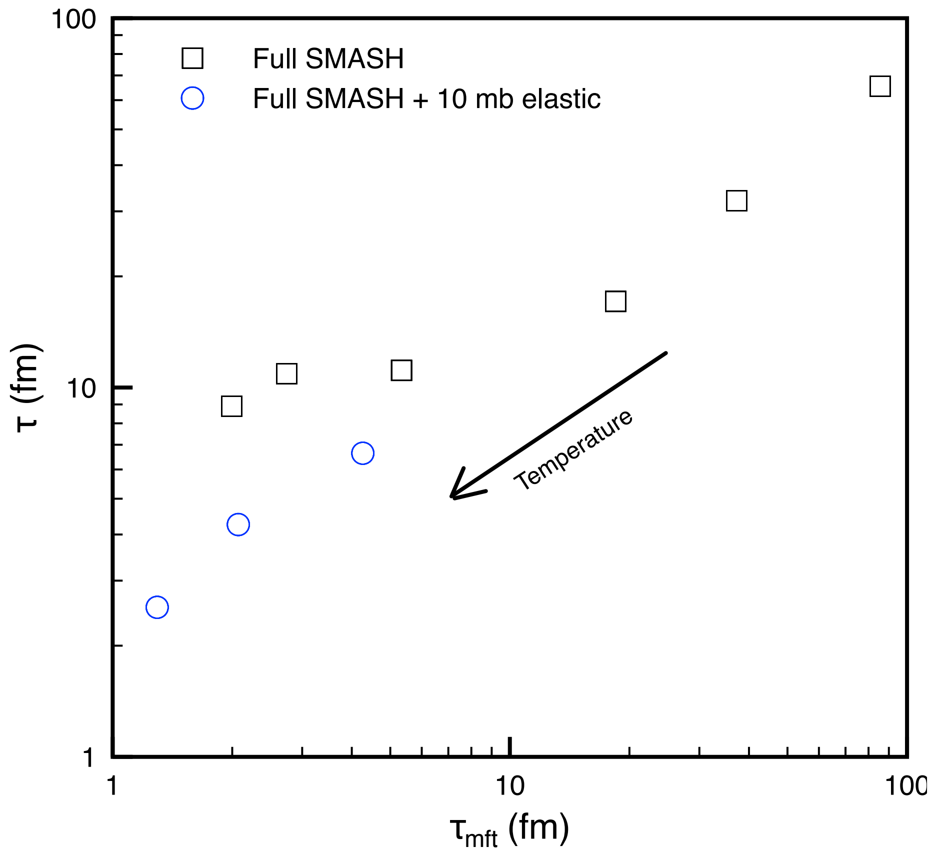
π-ρ: Zero lifetimes vs relaxation time

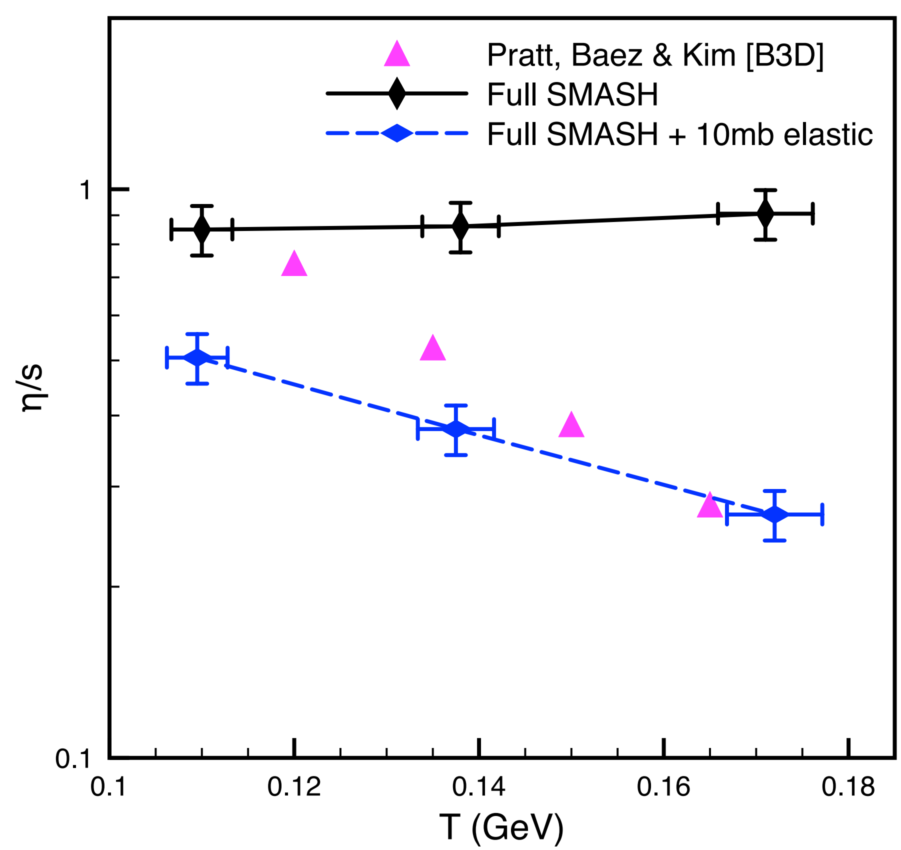
Large part of the difference explained from eliminating lifetimes



Effect of many non-resonant interactions

Introduce a constant elastic cross-section between all particles to add many non-resonant interactions





Outline

1. Introduction: Viscosity of the hadron gas
2. Transport
   – SMASH
3. Methodology
   – Viscosity considerations
     • Green-Kubo formalism
     • Test case #1: Constant isotropic cross-section
     • Test case #2: Energy-dependent cross-section
   – Entropy considerations
4. Results
   – Full hadron gas viscosity
   – Comparison & discussion
5. **Conclusion**

Summary & Outlook

- **Investigated temperature, cross-section and mass dependence of the shear viscosity in an elastic pion box**
  - Very good agreement with Chapman-Enskog approximation (within 3%)
  - Systematics show that method is robust to variation of technical parameters

- **Full hadron gas *η/s* calculated**
  - Slightly lower than other calculations at low T because of large π-π cross-section
  - High T discrepancy explained by looking at microscopic details of resonance modelling; finite lifetime increases viscosity
  - Could be used to constrain the treatment of resonances

- **Outlook:**
  - More rigorous analysis of the dependence between τ and τ_{res} needed
  - Investigation of angular dependent interactions on viscosity
  - At temperatures close to the phase transition, inclusion of multi-particle interaction will probably play a role, and needs to be investigated
  - Other transport coefficients (electrical conductivity, bulk viscosity, etc.)