Constraining the onset of viscous hydrodynamics

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Flow and dissipation in ultrarelativistic Heavy Ion Collisions
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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

H-QM Helmholtz Research School Quark Matter Studies

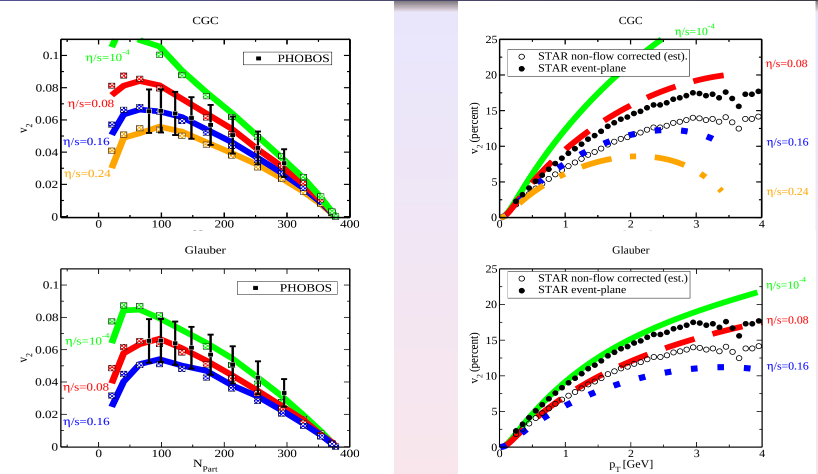
FIAS Frankfurt Institute for Advanced Studies

What are the necessary model parameters for hydrodynamical simulations?

- Thermalization time τ_0 is fixed by hand.
- Initial energy density profile and eccentricity: CGC or Glauber.
- Decoupling freeze-out temperature fixed by hand to reproduce $< p_\perp >$
- Initial value of shear stress tensor and bulk pressure.
- Transport coefficient values
- $\ldots$

$\Longrightarrow$ After fixing these, only getting v_2 right is non-trivial (parameter-free)

Extraction of $\eta/\mathcal{S}$ from experiments

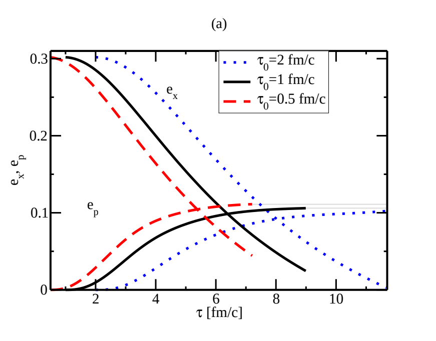


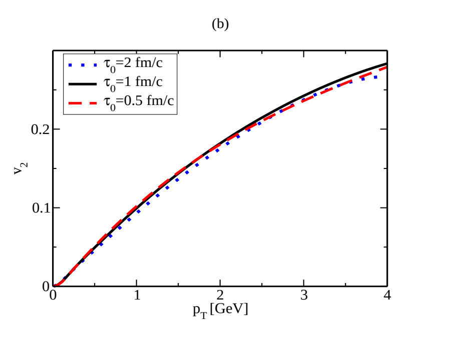
M. Luzum and P. Romatschke, Phys.Rev.C78:034915,2008.

$\eta/\mathcal{S} \lesssim$0.5 but its value has large uncertainties.

$\Longrightarrow$ By studying systematic studies of the impact parameter dependence of the eccentricity scaled elliptic flow v_2/ϵ can help to reduce these uncertainties, (see U. Heinz's talk). U. Heinz et. al. arXiv:0908.2617

The initial time τ_0 is not completely constrained from hydro simulations





M. Luzum and P. Romatschke, Phys.Rev.C78:034915,2008.

Thermalization and viscous hydro in parton cascade models

- Gyulassy, Pang and Zang (1997): comparison between kinetic theory and Navier-Stokes results.
  Nucl.Phys.A626:999-1018,1997.
- Z. Xu and C. Greiner (2007): studies on thermalization of gluon matter by calculating the transport rates within BAMPS.
  Phys.Rev.C76:024911,2007.
- Huovinen and Molnar (2008): use kinetic theory to determine the validity of causal IS eqs. and Navier-Stokes theory for relativistic heavy ion collisions.
  Phys.Rev.C79:014906,2009.

Requirements over the solutions of viscous hydrodynamics

1. Weak constraint: The effective longitudinal pressure to be positive during the simulated time $\mathcal{P}_L \geq 0$.
2. Strong constraint: Requiring that $\pi \ll \mathcal{P}$ during the simulated time.

$$\Downarrow$$

- τ_0 is non-trivially related with $\pi^{\mu\nu}(\tau_0)$ and $\mathcal{E}(\tau_0)$.
- This relation imposes lower bounds over τ_0.

Basic setup: Fluid equations

The energy momentum-tensor for a relativistic fluid in the presence of shear viscosity is:

$$T^{\mu\nu} = \mathcal{E}\, u^\mu\, u^\nu - \mathcal{P}\Delta^{\mu\nu} + \pi^{\mu\nu},$$

From the conservation laws of energy and momentum, $D_\mu T^{\mu\alpha} = 0$:

$$\begin{aligned}
(\mathcal{E}+\mathcal{P})Du^\mu &= \nabla^\mu\mathcal{P} - \Delta^\mu_\alpha D_\beta \pi^{\alpha\beta}\,,\\
D\mathcal{E} &= -(\mathcal{E}+\mathcal{P})\nabla_\mu u^\mu + \frac{1}{2}\pi^{\mu\nu}\nabla_{\langle\nu}u_{\mu\rangle}\,,
\end{aligned}$$

Basic setup: Conformal viscous hydrodynamics

By demanding conformal invariance of the energy-momentum tensor of the fluid, it imposes the constraint

$$\pi^{\mu\nu} \to \bar{\pi}^{\mu\nu} = e^{6\,w(x)}\pi^{\mu\nu}$$

For a conformal fluid, the equation of motion for $\pi^{\mu\nu}$ up to second order in the gradient expansion is

$$\begin{aligned}
\pi^{\mu\nu} \quad = \quad & \eta\nabla^{\langle\mu}u^{\nu\rangle} - \tau_\pi\left[\Delta^\mu_\alpha\Delta^\nu_\beta D\pi^{\alpha\beta} + \frac{4}{3}\pi^{\mu\nu}(\nabla_\alpha u^\alpha)\right] \\
& +\kappa\left[\frac{1}{2}R^{<\mu\nu>} + u_\alpha R^{\alpha<\mu\nu>\beta}u_\beta\right] \\
& -\frac{1}{2\eta^2}\,\lambda_1\,\pi^{<\mu}{}_\lambda\pi^{\nu>\lambda} + \frac{1}{2\eta}\,\lambda_2\pi^{<\mu}{}_\lambda\omega^{\nu>\lambda} - \frac{1}{2}\,\lambda_3\omega^{<\mu}{}_\lambda\omega^{\nu>\lambda},
\end{aligned}$$

R. Baier et. al., JHEP 04, 100 (2008), 0712.245; S. Bhattacharyya et. al., JHEP 02, 045 (2008),0712.2456.

Basic setup: transport coefficients

Trans. coefficient	Weakly-coupled QCD	Strongly-coupled $\mathcal{N}=4$ SYM
$\bar{\eta}\equiv\eta/\mathcal{S}$	$\sim 1/(g^4\log g^{-1})$	$1/(4\pi)$
τ_π	$6\bar{\eta}/T$	$(2-\log 2)/(2\pi T)$
λ_1	$(-2.2\to -2.0)\,\bar{\eta}^2\mathcal{S}/T$	$2\,\bar{\eta}^2\mathcal{S}/T$

The coefficients $\tau_\pi \sim T^{-1}$ and $\lambda_1 \sim \bar{\eta}^2/T$. We parametrize these coefficients as ($\bar{\eta}\equiv\eta/\mathcal{S}$):

$$\tau_\pi = \frac{c_\pi}{T}\,\bar{\eta} = \frac{c_\pi\,\bar{\eta}}{\gamma\,\mathcal{E}^{1/4}}\,,$$

$$\lambda_1 = c_{\lambda_1}\bar{\eta}^2\left(\frac{\mathcal{S}}{T}\right) = \frac{4}{3\gamma^2}\,c_{\lambda_1}\,\bar{\eta}^2\,\mathcal{E}^{1/2}\,,$$

γ is a parameter related with the eqn. of state and the degrees of freedom of the system.

Strong coupling: R. Baier et. al., JHEP 04, 100 (2008), 0712.245; S. Bhattacharyya et. al., JHEP 02, 045 (2008),0712.2456.

Weak coupling: M. York and G. Moore, Phys.Rev.D79:054011,2009.

Basic setup: 0+1 dim. conformal viscous hydrodynamical equations

For 0+1 dimensional expansion which is boost invariant

Results II: Convergence line ($\pi/\mathcal{P} \leqslant 1/3$)

Strong coupling case

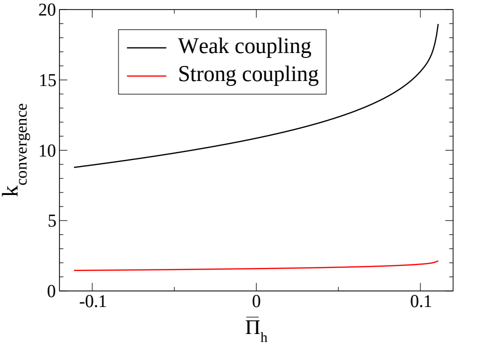
$k_{\text{convergence}}(\pi_{\text{h}} = 0) = 1.58.$

$T_{\text{h}} = 350$ MeV $\Rightarrow \tau_{\text{h}} > 0.48$ fm/c.

Weak coupling case

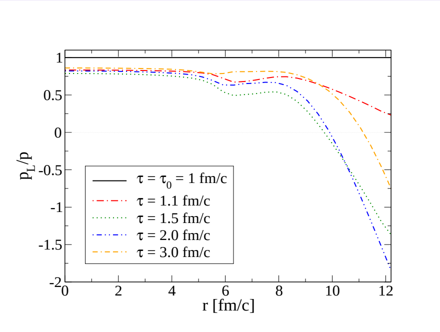
$k_{\text{convergence}}(\pi_{\text{h}} = 0) = 13.8.$

$T_{\text{h}} = 350$ MeV $\Rightarrow \tau_{\text{h}} > 4.2$ fm/c.

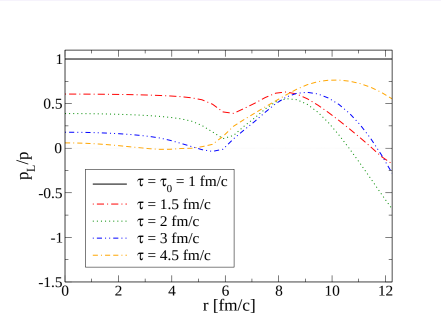


M. Martinez and M. Strickland, Phys. Rev. C 79, 044903 (2009)

Are there implications for higher dimensional hydrodynamical simulations?



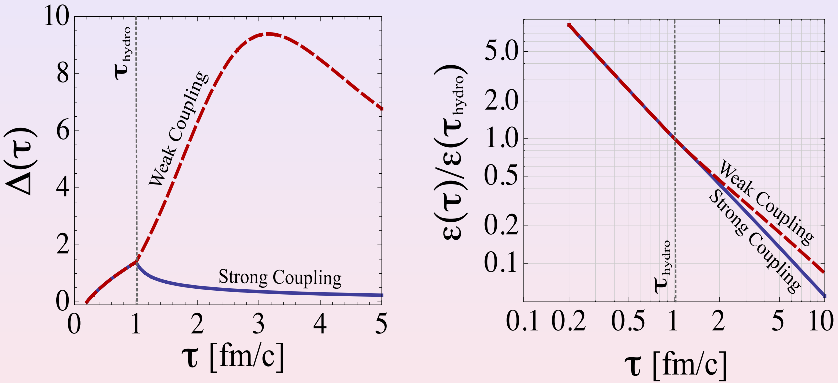
Strong Coupling



Weak Coupling

$\lambda_1 = 0, T_0 = 350$ MeV, $\tau_0 = 1$ fm/c and $\Pi^{\nu}_{\mu}(\tau_0) = 0$.

Matching pre-equilibrium dynamics and viscous hydrodynamics



Matching the values of every component of the stress-energy tensor $T^{\mu\nu}(\tau = \tau_{\text{h}})$ using Landau matching conditions allows to know the initial conditions for viscous hydro from early-time pre-equilibrated QGP.

M. Martinez and M. Strickland, arXiv:0909.0264 [hep-ph]

Pre-equilibrated phase of the QGP: $\tau_0 \leq \tau \leq \tau_{\rm h}$

From kinetic theory, the energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \delta T^{\mu\nu} \equiv \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p^0} f_{\rm iso}\left(\sqrt{\mathbf{p^2} + \xi(\mathbf{p}\cdot\hat{\mathbf{n}})^2}, p_{\rm hard}\right).$$

The microscopical anisotropy parameter is

$$\xi = \frac{\langle p_T^2 \rangle}{2\,\langle p_L^2 \rangle} - 1 = \left(\frac{\tau}{\tau_0}\right)^\delta - 1 \Longrightarrow \delta = \begin{cases} 2 & \text{Free streaming expansion} \\ 2/3 & \text{Collisional-broadening} \end{cases}$$

From this function, the components of the E-M tensor are:

$$\begin{aligned} T^{00} &= \mathcal{R}(\xi)\,\mathcal{E}_{\rm iso}(p_{\rm hard})\,, \\ T^{xx} = T^{yy} &= \frac{3}{2\xi}\left(\frac{1+(\xi^2-1)\mathcal{R}(\xi)}{\xi+1}\right)\mathcal{P}_T^{\rm iso}(p_{\rm hard})\,, \\ T^{zz} &= \frac{3}{\xi}\left(\frac{(\xi+1)\mathcal{R}(\xi)-1}{\xi+1}\right)\mathcal{P}_L^{\rm iso}(p_{\rm hard})\,, \\ \mathcal{R}(\xi) &= \frac{1}{2}\left[\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right], \end{aligned}$$

M. Martinez and M. Strickland, arXiv:0909.0264 [hep-ph].

Matching initial conditions

The Landau matching conditions are

$$\begin{aligned} \mathcal{E}(T) &= u_\mu T^{\mu\nu}_{(0)} u_\nu \,, & (1)\\ u_\mu \delta N^\mu &= 0\,, & (2)\\ u_\mu \delta T^{\mu\nu} u_\nu &= 0\,, & (3) \end{aligned}$$

From Eqn. (1):

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3 p^0} (u\cdot p)^2 \exp\left[-\sqrt{\mathbf{p^2}+\xi\, p_z^2}\,/\,p_{\rm hard}\right] = \int \frac{d^3\mathbf{p}}{(2\pi)^3 p^0} (u\cdot p)^2 \exp\left[-p/T\right].$$

$$\Longrightarrow p_{\rm hard} = \big(\mathcal{R}(\xi)\big)^{-1/4}\, T$$

To match the initial condition of π, we study the anisotropy in momentum-space:

$$\tau_0 \le \tau$$

$$\Delta(\tau) = \frac{(T^{xx}+T^{yy})}{2\,T^{zz}} - 1$$

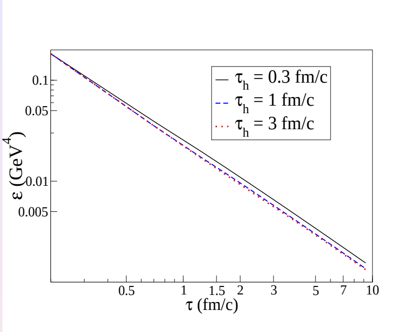
$$\tau = \tau_{\rm h} \qquad \Longleftrightarrow$$

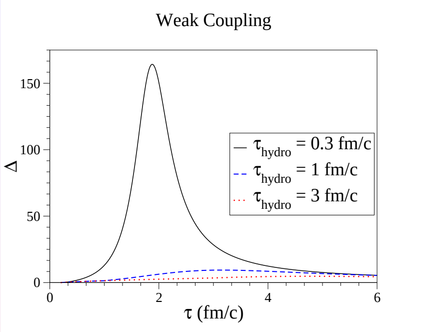
$$\tau_{\rm h} \le \tau$$

$$\Delta(\tau) = \frac{9}{2}\left(\frac{\pi}{\mathcal{E}-3\pi}\right)$$

M. Martinez and M. Strickland, arXiv:0909.0264 [hep-ph].

Pre-equilibrated+viscous phase model evolution





Using this evolution model, we study entropy generation as a function of τ_h. See details in arXiv:0909.0264 [hep-ph].

M. Martinez and M. Strickland, arXiv:0909.0264 [hep-ph].

Conclusions

- We have derived a two criteria that can be used to assess the applicability of viscous hydrodynamics:
  (a) Requiring the longitudinal pressure to be positive during the simulated time.
  (b) Requiring that $|\pi| < \mathcal{P}/3$ during the simulated time.
- These requirements lead to a non-trivial relation between possible initial simulation time τ_0, energy density $\mathcal{E}_0$, and the initial value of the fluid shear tensor π, for a 0+1 dim. viscous plasma.
- The constraints provide guidance for where one might expect 2nd-order viscous hydrodynamics to apply in higher-dimensional cases.
- Maybe it would be necessary to include non-conformal terms in the expansion (D. Rischke's talk) or higher order (3rd.) corrections (A. Muronga and A. El's talk).
- We model the pre-equilibrated phase of the QGP to find the initial conditions of the energy density and shear tensor as a function of the as a function of the lifetime of the pre-equilibrated phase for a 0+1 dimensional expansion.

Future perspectives

- It is possible to match all components of the energy momentum tensor and fluid four-velocity by using simple analytical models such as 3d free streaming or 3d collisionally-broadened expansion.
- It is necessary to specify information about the transverse expansion during the pre-equilibrium period and how this impacts the anisotropy at early times:
  1. 3d parton cascade models: Z. Xu and C. Greiner
  2. 3d Boltzmann-Vlasov-Yang-Mills simulations: Dumitru, Nara, Strickland and Schenke.

Dimensionless equations for 0+1 viscous hydrodynamics

Using the ideal eqn. of state, we can remove the dimensionful scales and rewrite the fluid equations for 0+1 dim. viscous plasma as:

$$\bar{\tau}\,\partial_{\bar{\tau}}\bar{\epsilon} + \frac{4}{3}\,\bar{\epsilon} - \overline{\Pi} = 0$$

$$\overline{\Pi} + \frac{c_\pi}{\gamma\,k\,\bar{\epsilon}^{1/4}}\left[\partial_{\bar{\tau}}\overline{\Pi} + \frac{4}{3}\frac{\overline{\Pi}}{\bar{\tau}}\right] - \frac{16\,\bar{\eta}}{9\,\gamma\,k}\frac{\bar{\epsilon}^{3/4}}{\bar{\tau}} + \frac{3\,c_{\lambda_1}}{8}\frac{\overline{\Pi}^2}{\bar{\epsilon}} = 0\,,$$

where: $\bar{\epsilon} \equiv \epsilon/\epsilon_0, \overline{\Pi} \equiv \Pi/\epsilon_0$ and $\bar{\tau} \equiv \tau/\tau_0$.

Notation and conventions

- The metric for a Minkowski space in the curvilinear coordinates (τ, x, y, ζ) is $g_{\mu\nu} = \mathrm{diag}(g_{\tau\tau}, g_{xx}, g_{yy}, g_{\zeta\zeta}) = (1, -1, -1, -\tau^2)$
- $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is a projector orthogonal to the fluid velocity, $u_\mu \Delta^{\mu\nu} = 0$.
- The comoving time derivative: $D \equiv u^\alpha D_\alpha$.
- The comoving space derivative: $\nabla^\mu \equiv \Delta^{\mu\alpha} D_\alpha$.
- The brackets $\langle\ \rangle$ denote an operator that is symmetric, traceless, and orthogonal to the fluid velocity:

$$A_{\langle\mu}B_{\nu\rangle} = \left(\Delta^\alpha_\mu \Delta^\beta_\nu + \Delta^\alpha_\nu \Delta^\beta_\mu - \frac{2}{3}\Delta^{\alpha\beta}\Delta_{\mu\nu}\right) A_\alpha B_\beta,$$

- The symmetric and anti-symmetric operators:

$$\begin{aligned} A_{(\mu}B_{\nu)} &= \frac{1}{2}\left(A_\mu B_\nu + A_\nu B_\mu\right) \\ A_{[\mu}B_{\nu]} &= \frac{1}{2}\left(A_\mu B_\nu - A_\nu B_\mu\right). \end{aligned}$$