Matching Stages In Heavy Ion Collision Models

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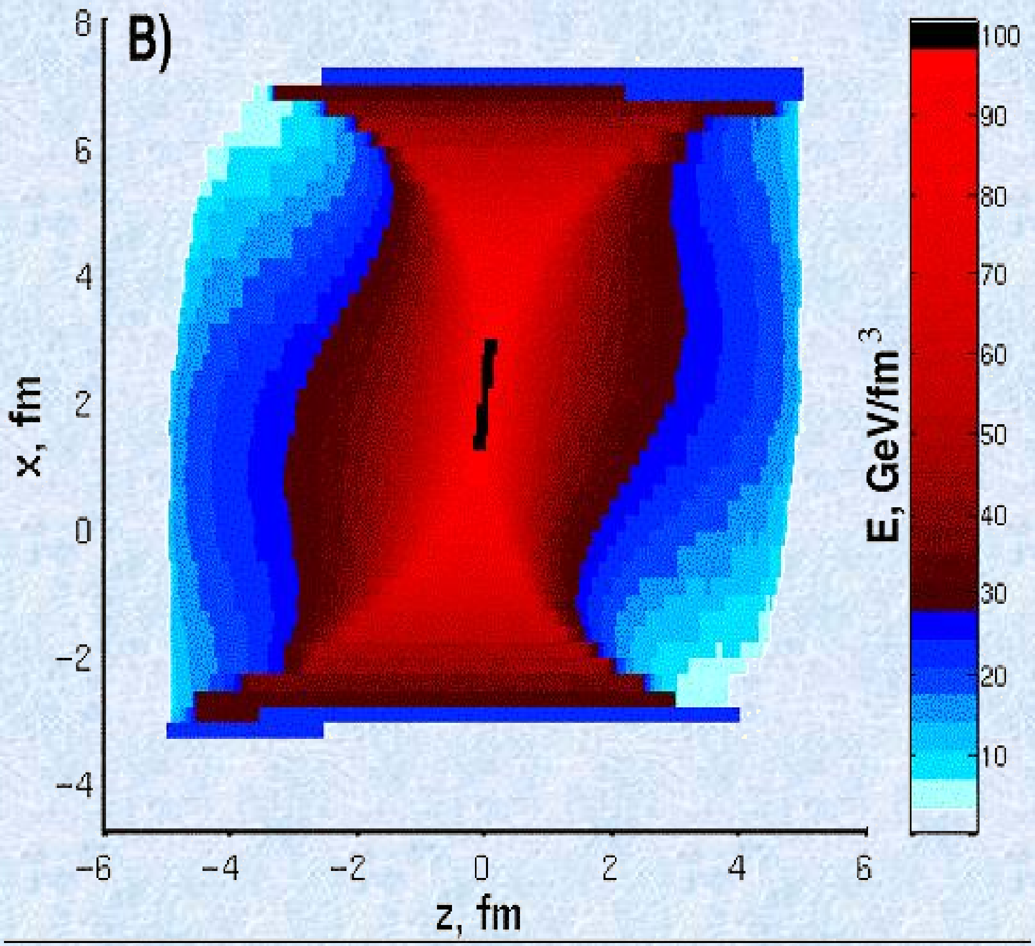
Outline

- Introduction→ Multi-Module model
- Numerical Extraction of FO hyper-surface
- Cooper-Frye Description (space-like FO ?)
- Anisotropic distribution (cut-Juttner, cancelling Juttner)
- Simple covariant solution
- Summary

[Y. Cheng, et al., Phy. Rev. C81, 064910 (2010)]

Relativistic fluid dynamical model



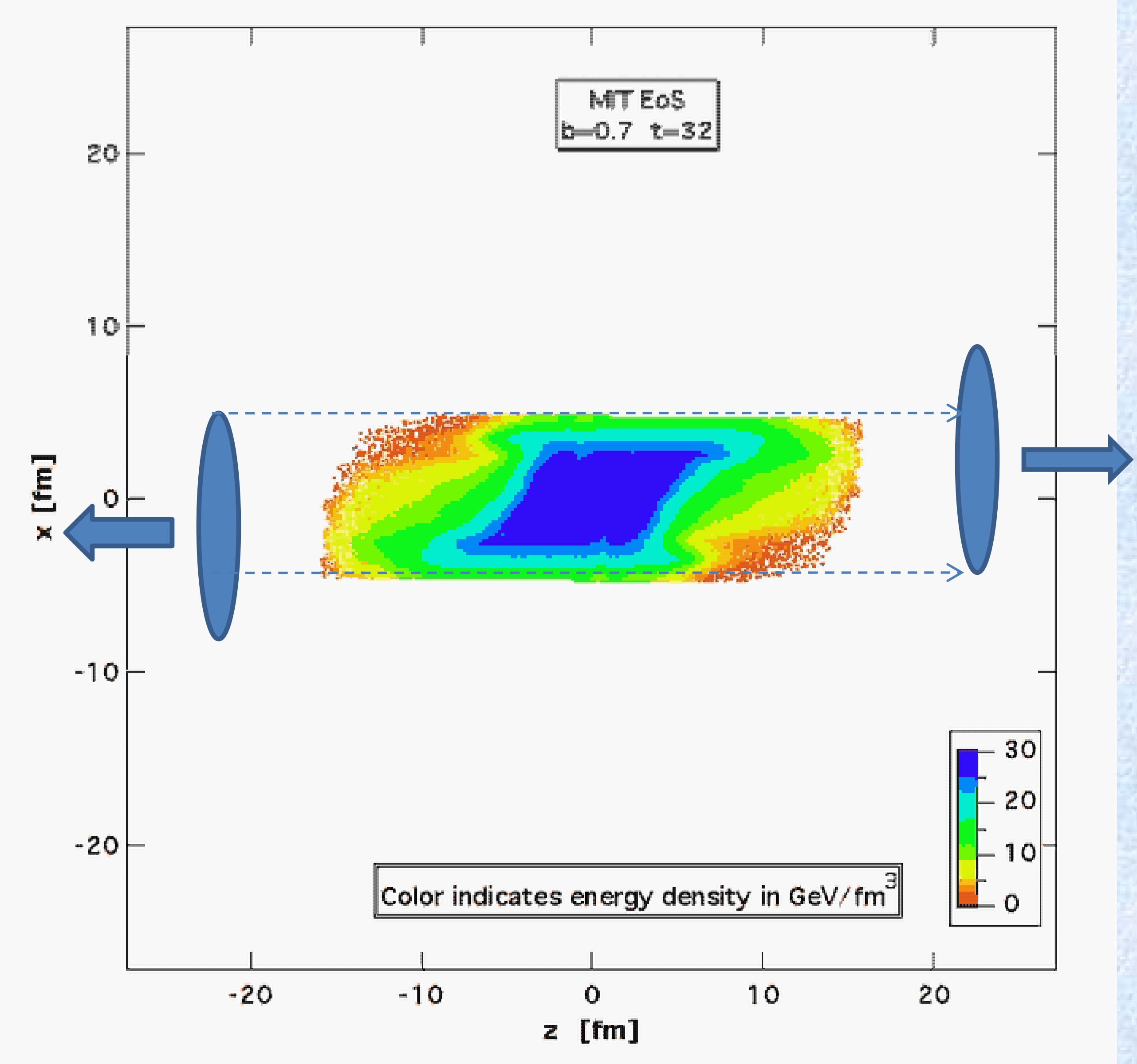
Multi Module Modeling

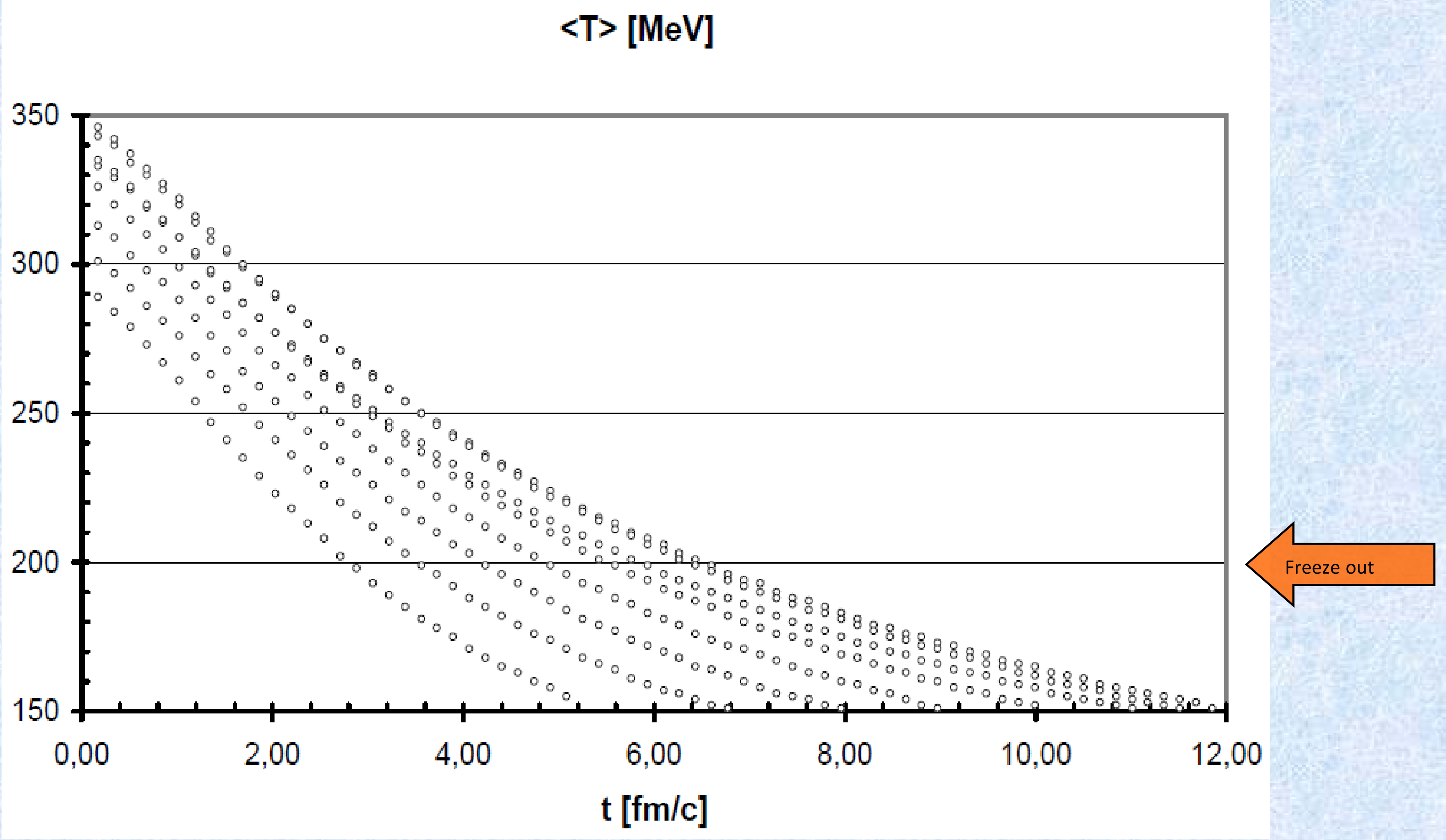
- Initial state. YM flux tube
- Fluid dynamics
- Freeze out or Transition to MD
- NCQ scaling

Initial state by V. Magas, L.P. Csernai and D. Strottman

Phys. Rev. C64 (01) 014901

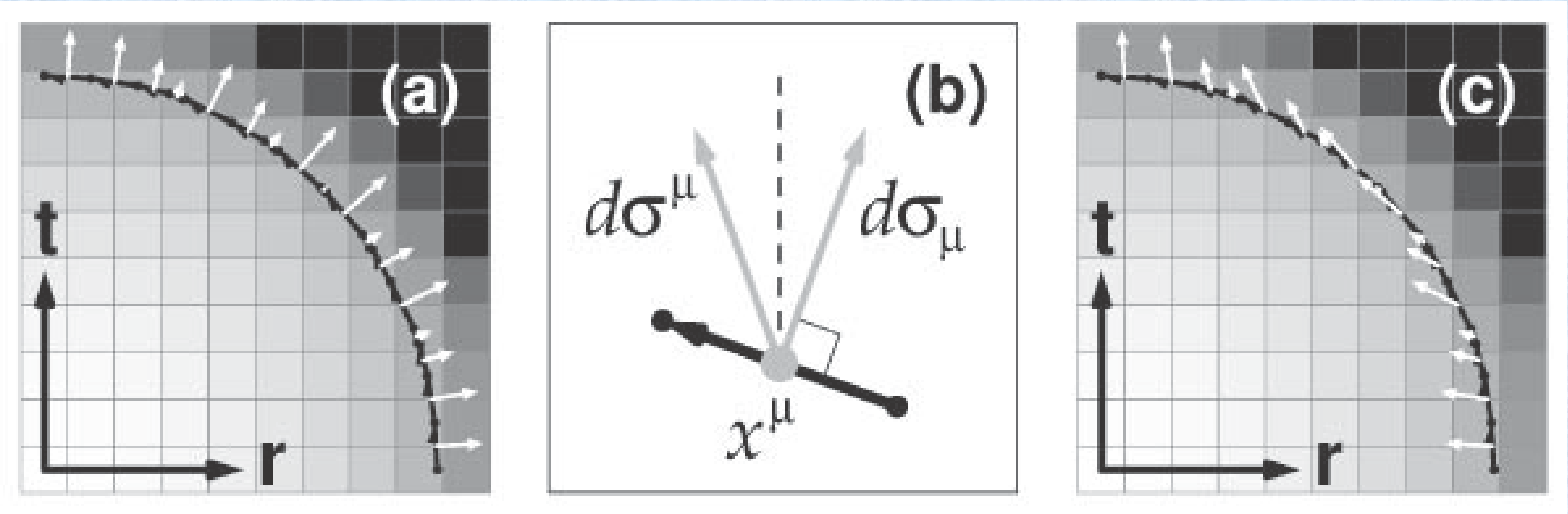
- Au+Au 65+65 A GeV, b= 70 % of b_max
- Lagrangian fluid cells, moving, ~ 5 mill.
- MIT Bag model EoS
- FO at T ~ 200 MeV, but calculated much longer, until pressure is zero for 90% of the cells.
- Structure and asymmetries of initial. state are maintained in nearly perfect expansion.
- Spatially tilted at FO, 3rd Flow component!





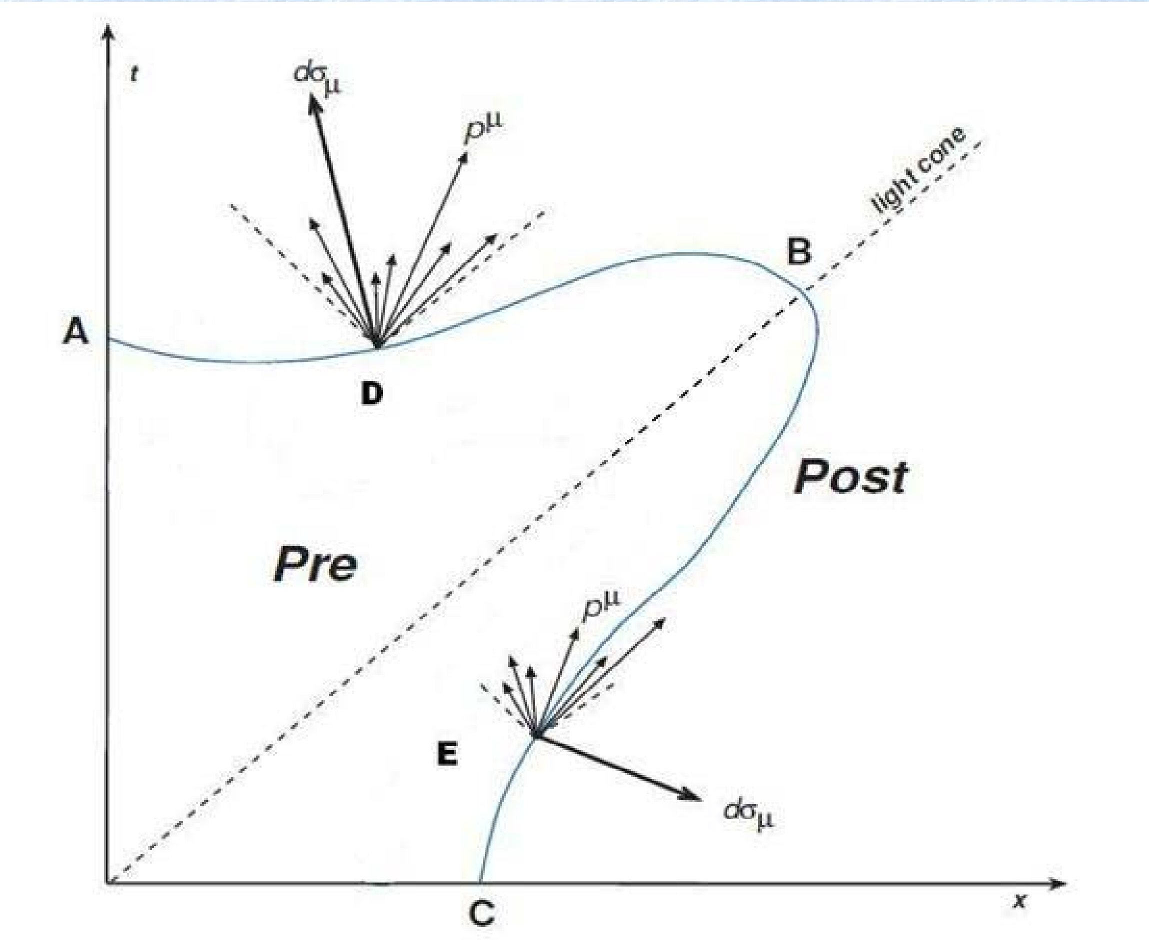
Average temperature versus time in Au+Au collisions at 65+65 AGeV, for impact parameters, b = 0, 0.1, 0.2, … 0.7 b_{max} from the top (0.0) down (0.7).

Numerical Extraction of hyper-surface



(a) gray-level → temperature evolution, contra-variant FO contour vector x^μ (black) and covariant normal vector $d\sigma_\mu$ (white)
(b) Covariant and contra-variant normal vectors $d\sigma_\mu$ and $d\sigma^\mu$
(c) Contra-variant FO contour vector x^μ (black) , and contra-variant normal vector $d\sigma^\mu$(white)→ Not all contra-variant normal vectors point to the exterior direction.

Conservation laws across hyper-surface



$$[N^\mu d\sigma_\mu] = 0;$$
$$[T^{\mu\nu} d\sigma_\mu] = 0;$$
$$[S^\mu d\sigma_\mu] \geqslant 0,$$

Equilibrate states:

Pre FO → Post FO

$$j = N^\mu d\sigma_\mu$$

$$A^\mu = T^{\mu\nu} d\sigma_\nu$$

FD to MD Transition

- From Fluid dynamical model to Molecular dynamical model

Cooper-Frye formula:

$$E\frac{dN}{d^3p} = \int_\sigma f(x,p)\, p^\mu d\sigma_\mu$$

Generating the phase space distribution of non-interacting particles

In local equilibrium

When time-like FO surface $p^\mu d\sigma_\mu > 0$

? When space-like FO surface $p^\mu d\sigma_\mu > 0$

Need an anisotropic distribution :

$f(x,p) \rightarrow \Theta(p^\upsilon \Lambda_\upsilon)\ f(p) > 0$

Enforcing the conservation laws

Anisotropic distribution in post FO

- Juttner distribution

RFG (Rest Frame of Gas)

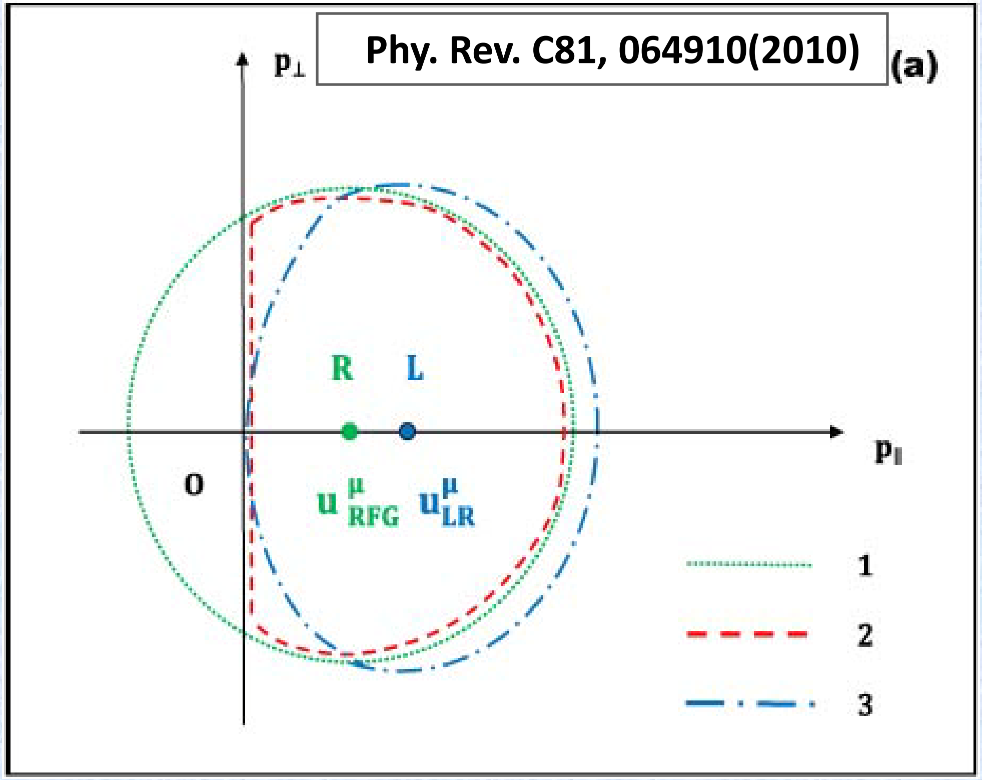
→ starting→ "R" u^{μ}_{RFG}

- Cut /Canceling Juttner distribution

[Tamosiunas ,2004]

LR (Local Rest Frame)

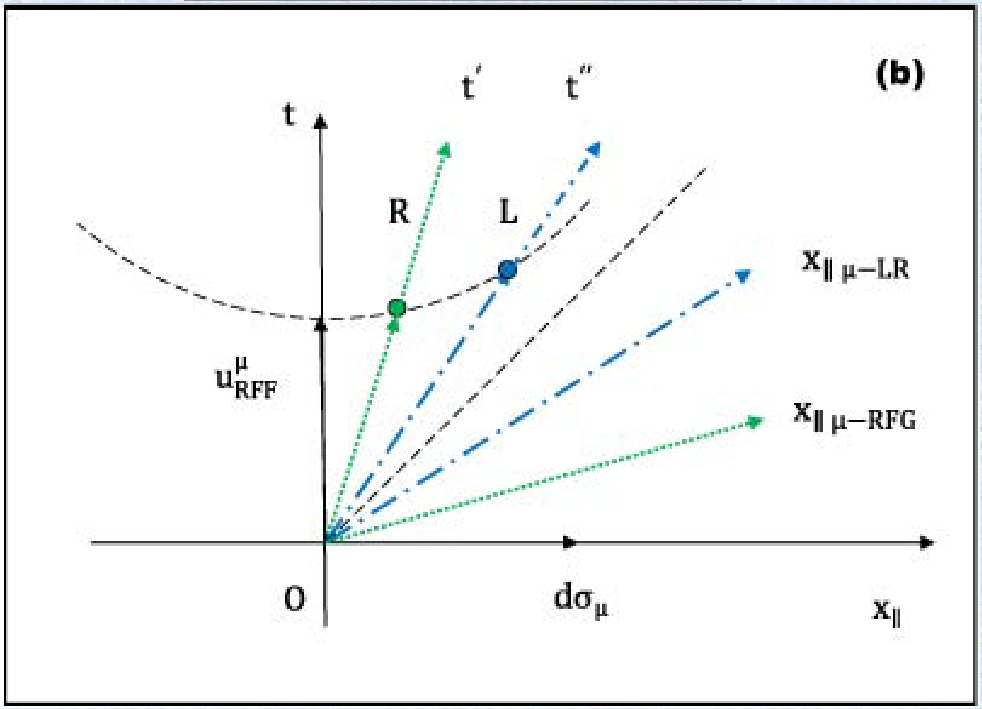
→ final → "L" u^{μ}_{LR}



Boost in the orthogonal direction of v_RFG leads to the spatial anisotropic distribution

Anisotropic distribution in post FO

Phy. Rev. C81, 064910(2010)



Space-time coordinate

Rest Frame of Front (RFF) → front of cut/canceling → orthogonal to the normal surface $d\sigma_\mu$

Rest Frame of Gas (RFG)→ "R"→ Starting Isotropic/uncut→ "t′ "

Local Rest Frame (LR) → "L"→ Final cut/canceling → "t′′ "

Basic conventions

- Energy momentum tensor, for perfect fluid:

$$T^{\mu\nu} = (e+P)u^\mu u^\nu - Pg^{\mu\nu}$$

- FO hyper-surface normal vector:

$$d\sigma^\mu = (\sigma_t, \sigma_x, \sigma_y, \sigma_z) = (\sigma_t, \sigma) \ \text{and} \ d\sigma_\mu = (\sigma_t, -\sigma_x, -\sigma_y, -\sigma_z) = (\sigma_t, -\sigma)$$

$$d\hat{\sigma}^\mu = \gamma_\sigma(1, s_x, s_y, s_z) = \gamma_\sigma(1, \mathrm{s}) \ \text{and} \ d\hat{\sigma}_\mu = \gamma_\sigma(1, -s_x, -s_y, -s_z) = \gamma_\sigma(1, -\mathrm{s})$$

$$d\hat{\sigma}^\mu \equiv d\sigma^\mu / D \qquad \gamma_\sigma^2 = \pm\frac{1}{1-\mathrm{s}^2}$$

- Conserved energy momentum current, baryon current:

$$\mathbf{A}^\mu = T^{\mu\nu} d\sigma_\nu = w\, u^\mu u^\nu \, d\sigma_\nu - P\, g^{\mu\nu}\, d\sigma_\nu = w\, \gamma^2 (\sigma_t - (\mathbf{v}\cdot\sigma))\mathbf{v} - P\,\sigma$$

$$j \equiv N^\mu d\sigma_\mu = n\, u^\mu d\sigma_\mu = n\, \gamma\, (\sigma_t - (\mathbf{v}\cdot\sigma))$$

- A^μ → Taking its norm, taking its projection to normal vector:

↓ $A^\mu A_\mu$ ↓ $A^\mu d\sigma_\mu$

Simple solution for equilibrated final state

Simple solution for the final state in equilibrium:

$$\mathbf{A}^\mu \mathbf{A}_\mu = w^2 (u^\mu d\sigma_\mu)^2 + P^2 (d\sigma^\mu d\sigma_\mu) - 2Pw(u^\mu d\sigma_\mu) u_\mu g^{\mu\nu} d\sigma_\nu$$

$$= w(e-P)(u^\mu d\sigma_\mu)^2 + P^2 (d\sigma^\mu d\sigma_\mu) \ ,$$

$$\mathbf{A}^\mu d\sigma_\mu = w(u^\mu d\sigma_\mu)^2 - P(d\sigma^\mu d\sigma_\mu) \ .$$

$$A^\mu A_\mu = (e-P) A^\mu d\sigma_\mu + e\, P\, (d\sigma^\mu d\sigma_\mu)$$

+ EoS

Special case for baryon free matter with EoS as: P=e/3

$$d\hat{\sigma}^\mu d\hat{\sigma}_\mu\, e^2 + 2\, a^\mu d\hat{\sigma}_\mu\, e - 3\, a^\mu a_\mu = 0$$

$$u^\mu = d\hat{\sigma}^\mu \qquad a^\mu = A^\mu / D$$

Simple solution for equilibrated final state

Case for finite baryon current, and with an EoS as: P=P(n,e)

$$A^\mu A_\mu = (e-P)A^\mu d\sigma_\mu + e\,P\,(d\sigma^\mu d\sigma_\mu)$$

$$j = n(u^\mu d\sigma_\mu)$$

$$\mathbf{A}^\mu d\sigma_\mu \;=\; w(u^\mu d\sigma_\mu)^2 - P(d\sigma^\mu d\sigma_\mu)$$

$$\frac{e+P}{n^2} = \frac{1}{j^2}\;[\mathbf{A}^\mu d\sigma_\mu + P(d\sigma^\mu d\sigma_\mu)]$$

This equation is the same as the generalized Rayleigh line, [Taub, 1949 & Csernai 1987] (and Taub adiabat):

$$j^2 = \frac{[P](d\sigma^\mu d\sigma_\mu)}{[X]} \qquad [P] = \frac{[(e+P)X]}{X_1 + X_0} \qquad X = \frac{e+P}{n^2}$$

Space-like FO is weak at RHIC /LHC, but important at FAIR!

Simple solution for anisotropic final state

- Anisotropic distribution → anisotropic pressure tensor → pressure components are non-identical

$$T^{\mu\nu} = diag\,(e, P, P, P)\,|_{LR} \longrightarrow T^{\mu\nu} = diag\,(e, P_{\|}, P_{\perp}, P_{\perp})\,|_{LR}$$

Isotropic momentum dist. → anisotropic momentum dist.

Perfect $T^{\mu\nu}$ ⟶ $T^{\mu\nu} = e\, u^{\mu}_{LR} u^{\nu}_{LR} - P_{\perp} \Delta^{\mu\nu}_{LR} + (P_{\|} - P_{\perp}) \hat{F}^{\mu} \hat{F}^{\nu}$

Orthogonal projector to u_{LR}^{μ} $\Delta^{\mu\nu}_{LR} = g^{\mu\nu} - u^{\mu} u^{\nu}$

Unit vector projection of $d\sigma^{\mu}$
Orthogonal to u_{LR}^{μ} $\hat{F}^{\mu} = C \Delta^{\mu\nu} d\hat{\sigma}_{\mu}$

Simple solution for anisotropic final state

$$T^{\mu\nu} = e\, u^\mu_{\rm LR} u^\nu_{\rm LR} - P_\perp \Delta^{\mu\nu}_{\rm LR} + (P_\| - P_\perp)\hat{F}^\mu \hat{F}^\nu$$

$$\begin{aligned} A_0^\mu A_{0\mu} &= w_\perp (e - P_\perp)(u^\mu d\sigma_\mu)^2 + (P_\perp)^2 (d\sigma^\mu d\sigma_\mu) \\ &- (P_\| - P_\perp)(P_\| + P_\perp)(\hat{F}^\nu\, d\sigma_\nu)^2 , \\ A_0^\mu d\sigma_\mu &= w_\perp (u^\mu d\sigma_\mu)^2 - P_\perp (d\sigma^\mu d\sigma_\mu) \\ &+ (P_\| - P_\perp)(\hat{F}^\nu\, d\sigma_\nu)^2 . \end{aligned}$$

Escape probability relative to degree of anisotropic

$$A_0^\mu A_{0\mu} = (e - P_\perp) A_0^\mu d\sigma_\mu + e\, P_\perp\, (d\sigma^\mu d\sigma_\mu) - (e + P_\|)(P_\| - P_\perp)(\hat{F}^\nu\, d\sigma_\nu)^2 .$$

Simple covariant solution ← Eos +

Summary

- Initial state is important
- Anisotropic distribution, more realistic especially for space-like transitions
- Enforcing the conservation laws
- Transparent and covariant method
- Connecting PIC-hydro and PACIAE is in progress

Thanks for your attention!