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The statistical hadronization model and the microcanonical ensemble of the hadron gas.

- Introduction
- The general approach
- Importance of conservation laws

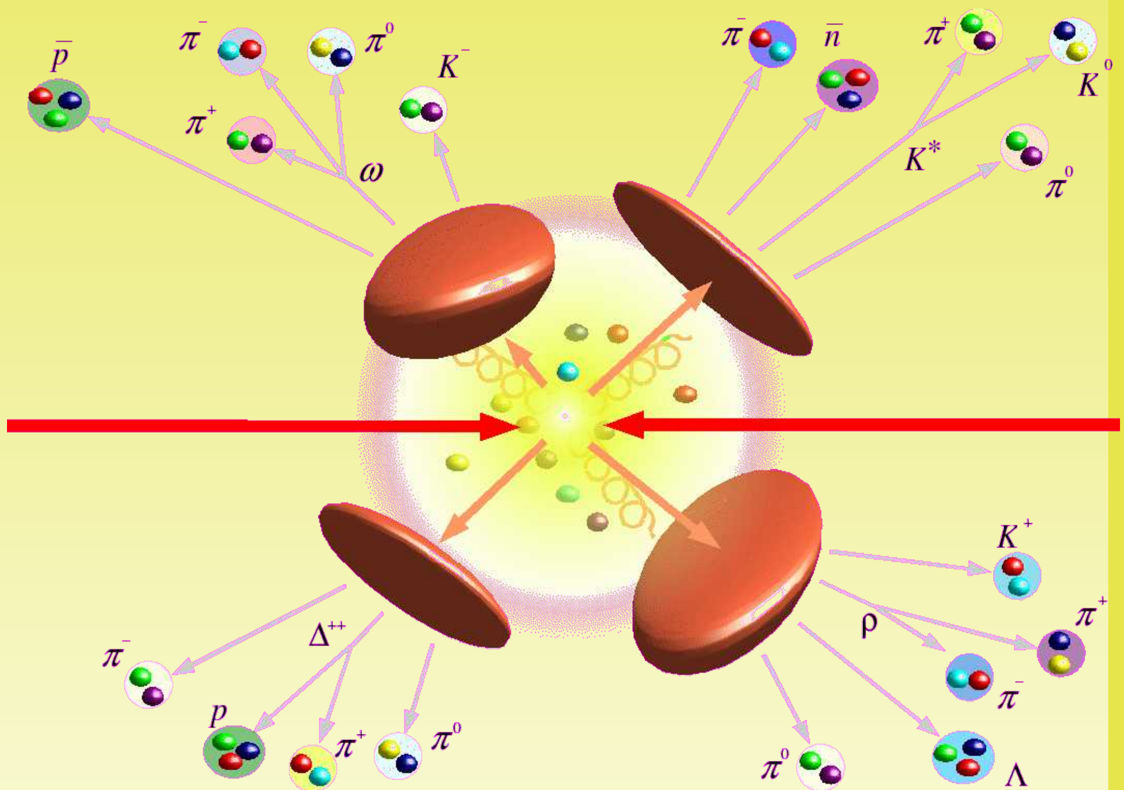
  - Production ratios of exclusive channels in low-energy e^+e^- collisions

The statistical hadronization picture

- As a result of an high energy collision a set of colorless clusters are produced in the pre-hadronization stage. Their distribution **cannot** be predicted within the model.

- Each cluster decays into the final hadrons in a purely statistical fashion: Each multihadronic state compatible with the quantum numbers of the cluster is equally likely.

- The microcanonical ensemble is the natural ensemble to describe a cluster.



Statistical ensembles

Cluster size

Difficulty

$$Z_{GC}(\beta,\vec{\mu})=\sum_{\vec{Q}}\exp[\beta\vec{\mu}\vec{Q}]Z_{C}(\beta,\vec{Q})=\sum_{states}\exp[-\beta(E_{state}-\vec{\mu}\vec{Q}_{state})]$$

$$Z_{C}(\beta,\vec{Q})=\int dE\exp[-\beta E]\Omega(E,\vec{Q})=\sum_{states}\exp[-\beta E_{state}]\delta_{\vec{Q}-\vec{Q}_{state}}$$

$$\Omega(E,\vec{Q})=\sum_{states}\delta\left(E-E_{state}\right)\delta_{\vec{Q}-\vec{Q}_{state}}$$

One has to remember that the equivalence between statistical ensembles olds only for the first moments.

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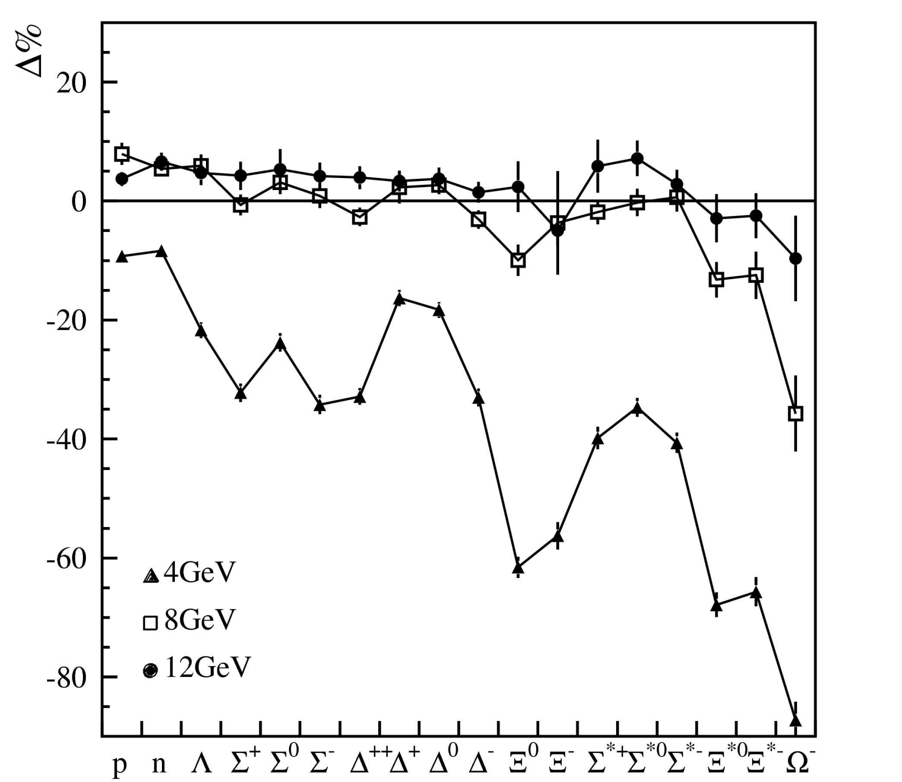
Microcanonical vs Canonical
(completely neutral cluster)

•Microcanonical: Fixed energy-momentum and abelian charges

•Canonical: Fixed abelian charges

$$\frac{\langle N_j \rangle_{micro} - \langle N_j \rangle_{can}}{\langle N_j \rangle_{can}}$$



[redacted]density: 0.4 GeV/fm^3

F. Becattini, LF, Eur.Phys.J.C38:225-246,2004

Parenthesis: How much energy is used to produce particles?

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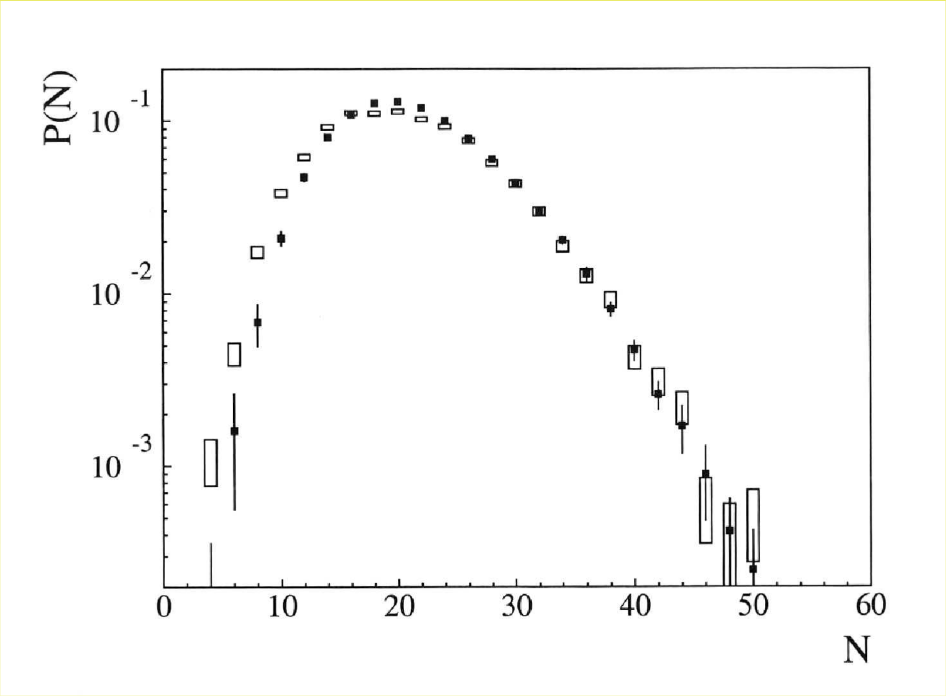
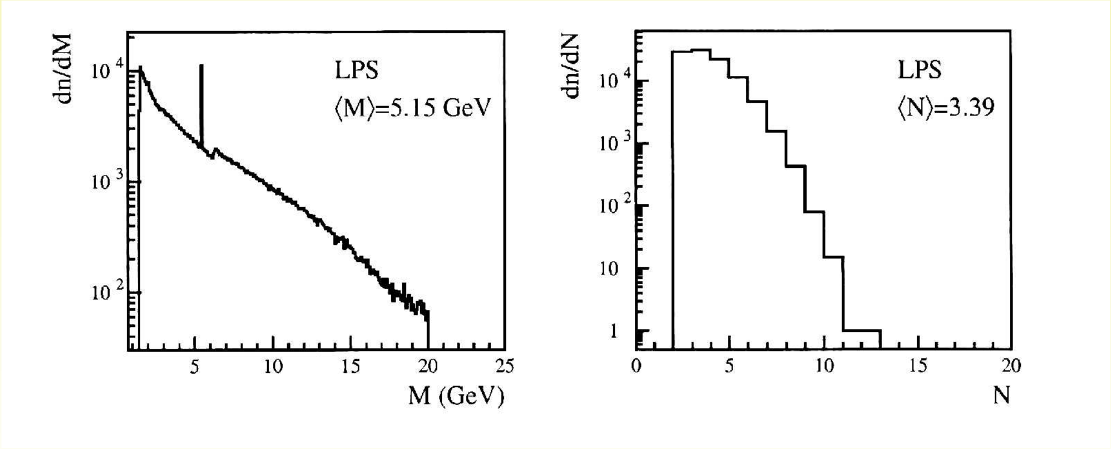
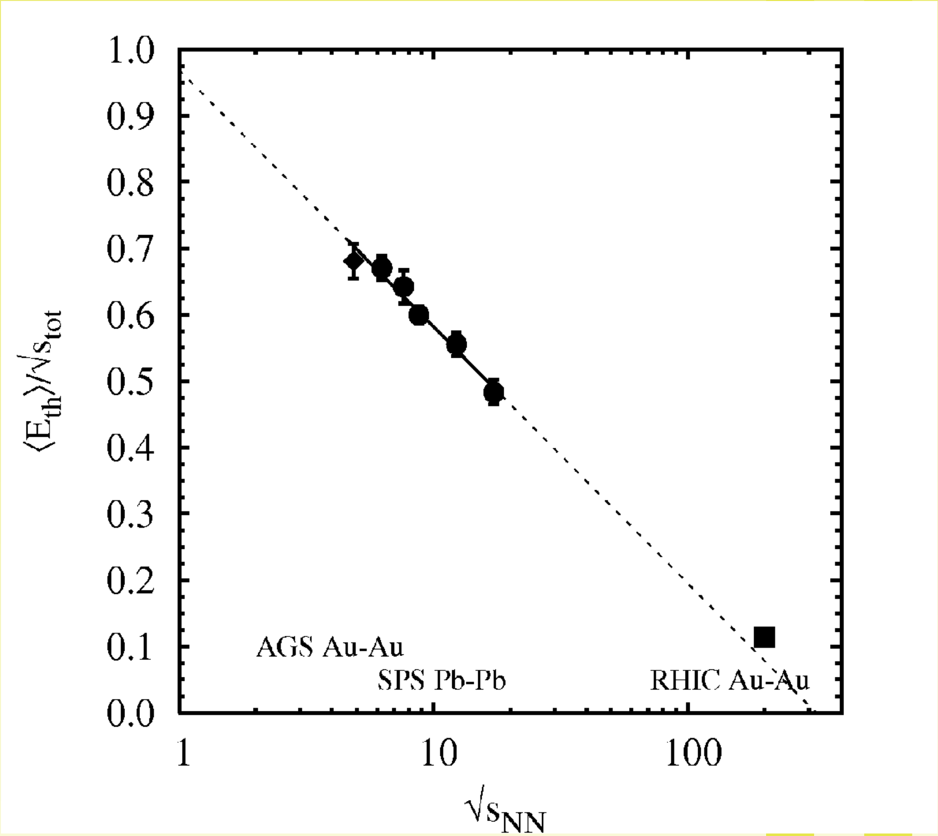
e^+e^- 91.2 GeV (Jetset +Statistical model)





Thermal energy ~17.5 GeV

HIC (Equivalent Global Cluster)



F. Becattini, J. Manninen, M. Gazdzicki, Phys.Rev.C73:044905,2006.

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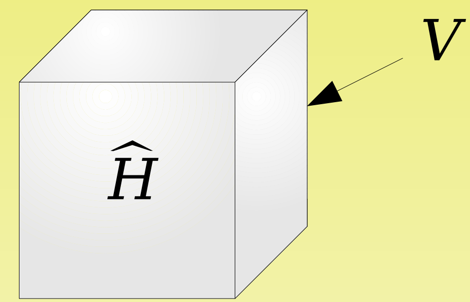
Definition of the microcanonical partition function

$$\widehat{H}' = \widehat{H} + \text{Infinite potential walls}$$

$$\Omega = \sum_{states} \delta(E - E_{state}) \xrightarrow{\text{quantum system}} \Omega = Tr\, \delta(E - \widehat{H}')$$

$$\Omega = \sum_{\mathbf{k}} \delta\left(E - \frac{k^2}{2m}\right)$$



in general, a set of integers k_x, k_y, k_z fulfilling this constraint dos not exist!

It is only meaningful in the infinite-volume limit where:

$$\sum_{cells} \rightarrow \frac{V}{(2\pi)^3} \int d^3 p$$

We use a better definition:

$$\Omega = Tr_V\, \delta(E - \widehat{H}) \equiv \sum_{h_V} \langle h_V | \delta(E - \widehat{H}) | h_V \rangle$$

Now Ω is a continuous function of E and has the right thermodynamical limit

The microcanonical partition function

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$$\Omega=\sum_{h_V}\langle h_V|\delta(E-\hat{H})|h_V\rangle \rightarrow \Omega=\sum_{h_V}\langle h_V|\delta(P-\hat{P})|h_V\rangle$$

Generalization

$$\Omega=\sum_{h_V}\langle h_V|P_i|h_V\rangle$$

All conserved quantities

$$\Omega=\sum_{h_V}\langle h_V|P_i|h_V\rangle=\sum_{f}\langle f|P_i P_V|f\rangle=Tr\,P_i P_V$$

$$P_V\equiv\sum_{h_V}|h_V\rangle\langle h_V|$$

we will calculate the partition function according to the decomposition:

$$\Omega=\sum_{f}\Omega_f$$

where

$$\Omega_f=\langle f|P_i P_V|f\rangle$$

$$\rho_f=\frac{\Omega_f}{\Omega}$$

(Probability of the final channel f)

Enforcing conservation laws

Energy-momentum conservation

F. Becattini, LF, Eur.Phys.J.C38:225-246,2004

$$\Omega_{\{N_j\}} = \left[\prod_j \sum_{\{h_{n_j}\}} (\mp 1)^{N_j+H_j} \frac{1}{\prod_{n_j=1}^{N_j} n_j^{4h_{n_j}} h_{n_j}!} \left[\prod_{l_j=1}^{H_j} \frac{V(2J_j+1)}{(2\pi)^3} \int \mathrm{d}^3 \mathrm{p}'_{l_j} \right] \right] \delta^4 (P - \sum_{j,l_j=1}^{H_j} p'_{l_j})$$

Energy-momentum, Angular momentum, Isospin, Parity, C-parity

$$\Omega_{\{N_j\}} = \sum_{\boldsymbol{\rho}} \left[\prod_{j=1}^{K} \chi(\rho_j)^{b_j} \right] \frac{1}{8\pi} \int_0^{4\pi} \mathrm{d}\psi \left[\prod_{j=1}^{k} \frac{1}{N_j!} \prod_{n_j=1}^{N_j} \int \mathrm{d}^3 \mathrm{p}_{n_j} \right]$$
$$\times \delta^4 \left(P - \sum_{n=1}^{N} p_n \right) \sin\frac{\psi}{2} \sin\left[\left(J + \frac{1}{2} \right) \psi \right] \prod_{j=1}^{K} \left[\prod_{l_j=1}^{L_j} \left[\frac{\sin[(S_j+\frac{1}{2}) l_j \psi]}{\sin(\frac{l_j \psi}{2})} \right]^{h_{l_j}(\rho_j)} \right]$$
$$\times \left(\prod_{j=1}^{K} \prod_{l_j=1}^{L_j} F_V^{(s)}(\mathbf{p}_{\rho_j(l_j)} - \mathsf{R}_3^{-1}(\psi)\mathbf{p}_{l_j}) + \Pi \Pi_f \prod_{j=1}^{K} \prod_{l_j=1}^{L_j} F_V^{(s)}(\mathbf{p}_{\rho_j(l_j)} + \mathsf{R}_3^{-1}(\psi)\mathbf{p}_{l_j}) \right)$$
$$\times \left(\mathcal{I}_{\boldsymbol{\rho}}^{\{N_j\}}(I, I_3) \prod_{j=1}^{K} \prod_{l_j=1}^{L_j} \delta_{\alpha_{\rho_j(l_j)}\alpha_{l_j}} + \chi_C^0 \chi_C \overline{\mathcal{I}}_{\boldsymbol{\rho}}^{\{N_j\}}(I, I_3) \prod_{j=1}^{K} \prod_{l_j=1}^{L_j} \delta_{-\alpha_{\rho_j(l_j)}\alpha_{l_j}} \right)$$

F. Becattini, LF, Eur.Phys.J.C52:597-615,2007.

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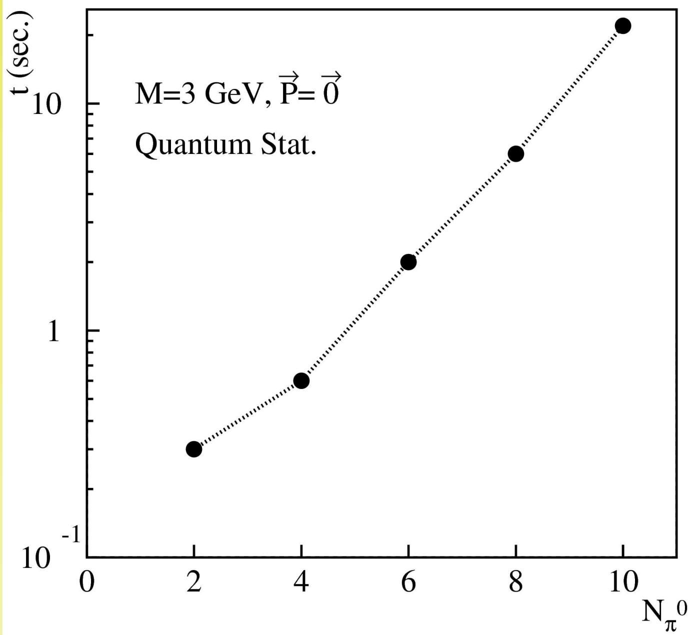
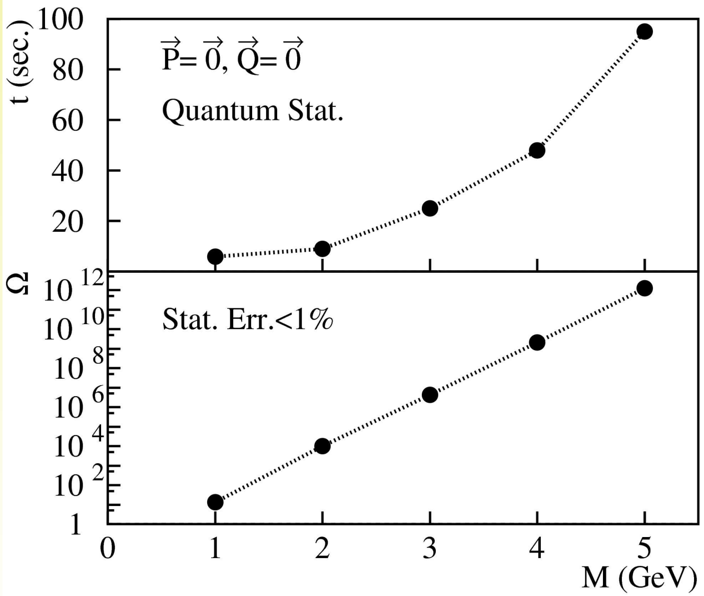
Estimate of the calculation time

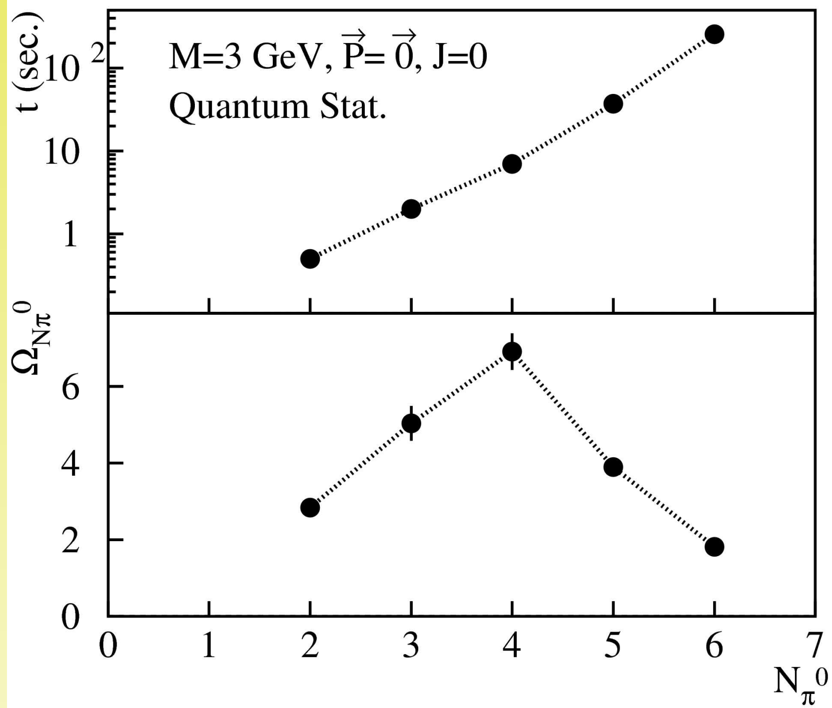
CPU ~ 2GHz

Energy-momentum cons.





Energy-momentum, ang.momentum cons.



The calculation above, includes finite-volume Fourier integrals that account for HBT correlations.

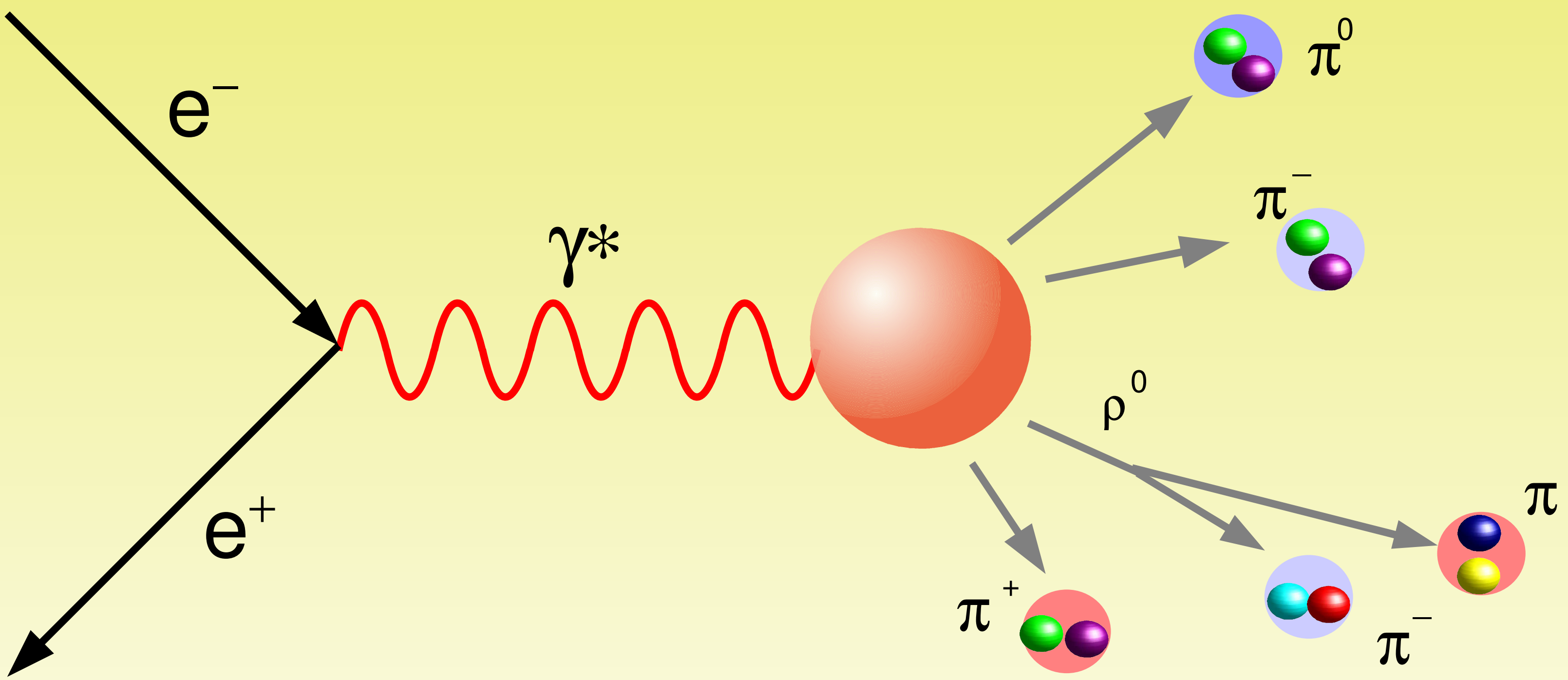
Not optimized for channels with many particles. There is room for improvements.

e^+e^- low-energy collisions

Model assumptions:

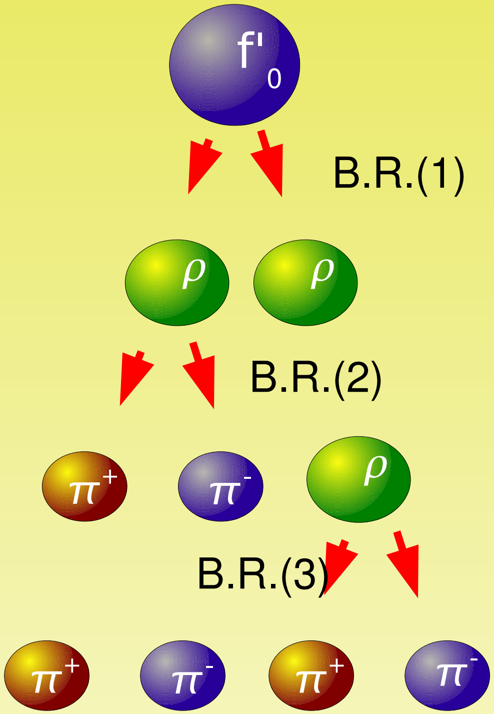
Minimize χ^2

$$\Omega^{final}(channel) = A \cdot \sigma(channel)$$



- $SU(3) \rightarrow SU(2) \otimes U(1)$
- Isospin Mixing $\quad I_0 |0\rangle\langle 0| + (1-I_0)|1\rangle\langle 1|$

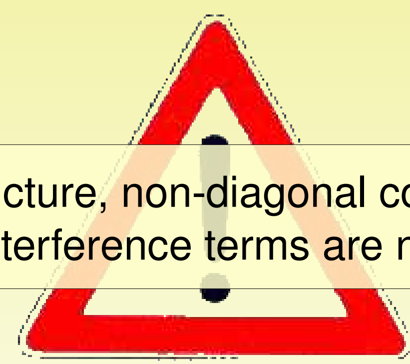
Hadron resonance gas model



from primary to final

$$\Omega^{\text{final}}(\pi^+\pi^-\pi^+\pi^-) = \Omega(\pi^+\pi^-\pi^+\pi^-) + \text{B.R.}(3)\,\Omega(\pi^+\pi^-\rho)$$

$$+\text{B.R.}(2)\,\Omega(\rho\rho) + \text{...all possible decay trees}$$



In this picture, non-diagonal contributions and interference terms are neglected!

Safe energy range: $2\ \text{GeV} \leq \sqrt{s} \leq 3\ \text{GeV}$

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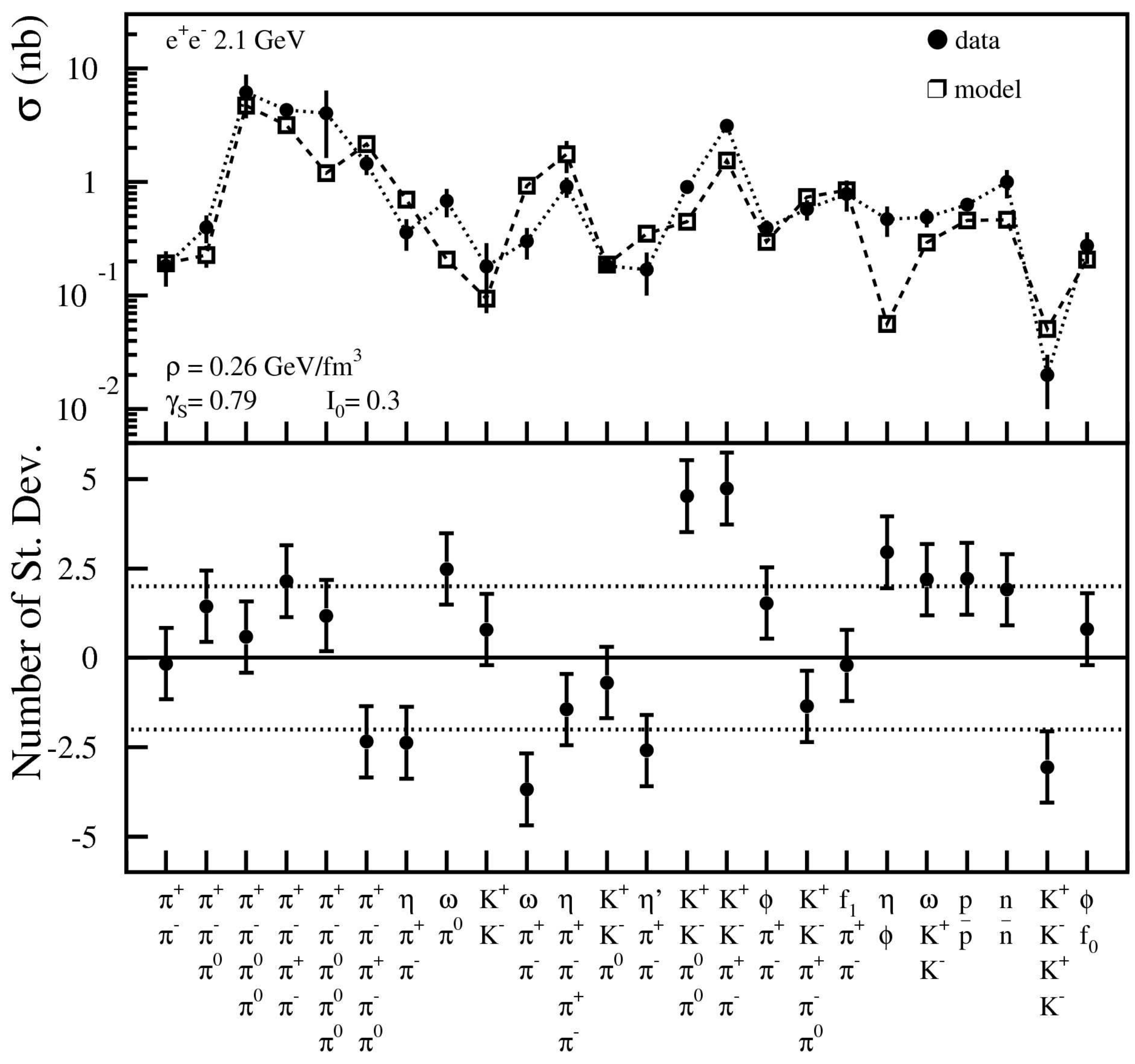
e+e- √s=2.1 GeV



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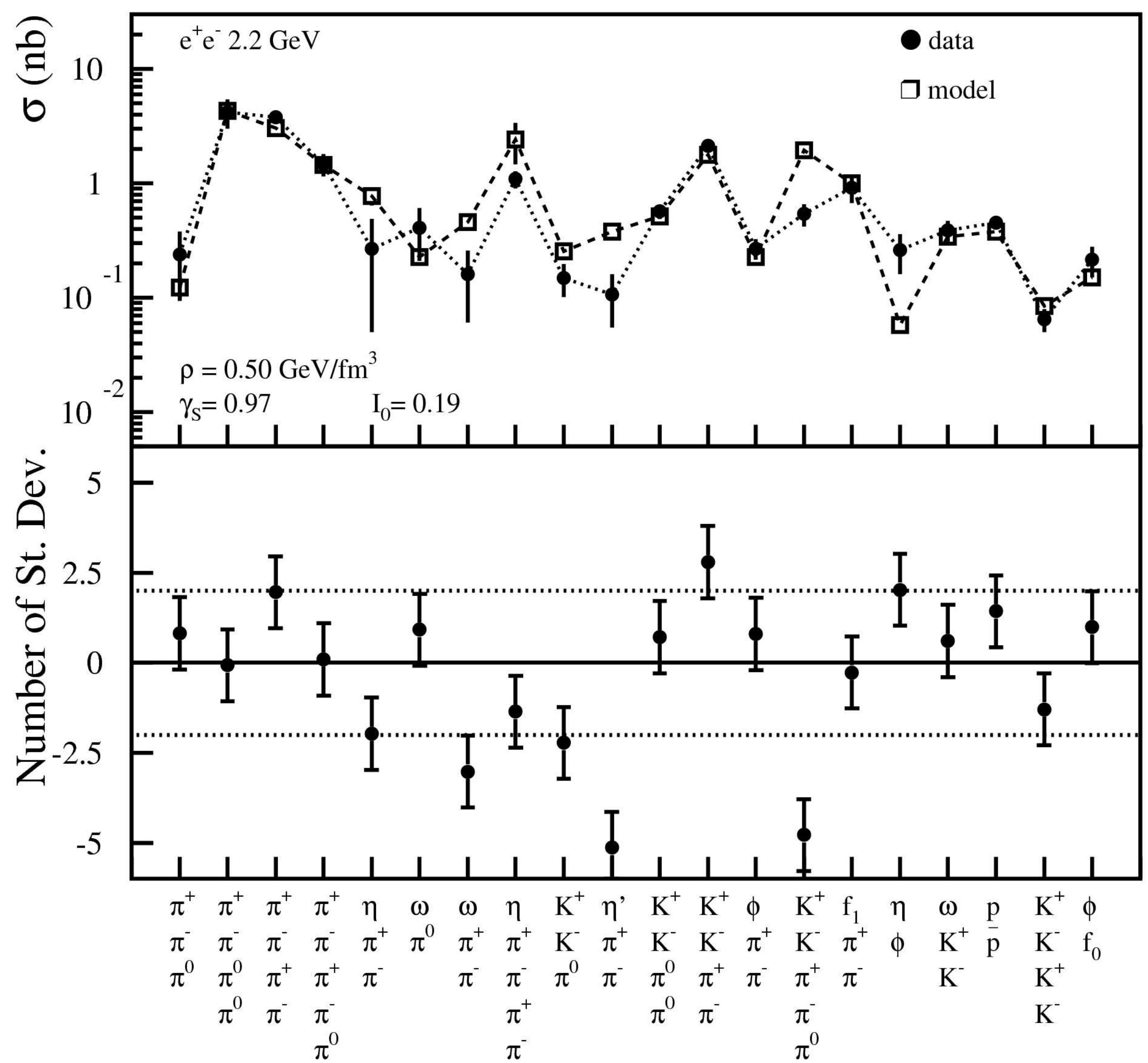
e+e- √s=2.2 GeV



e+e- √s=2.4 GeV

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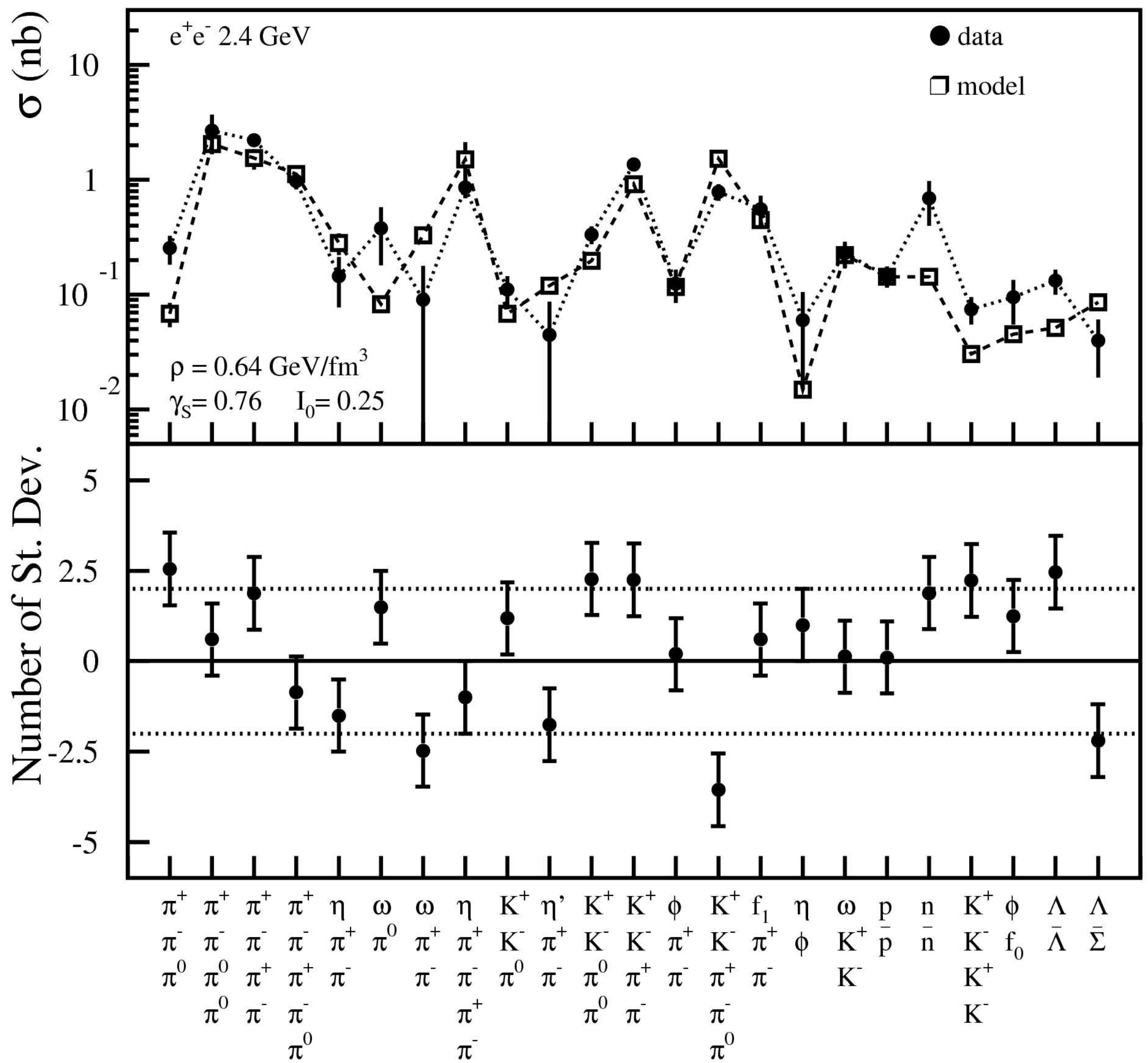

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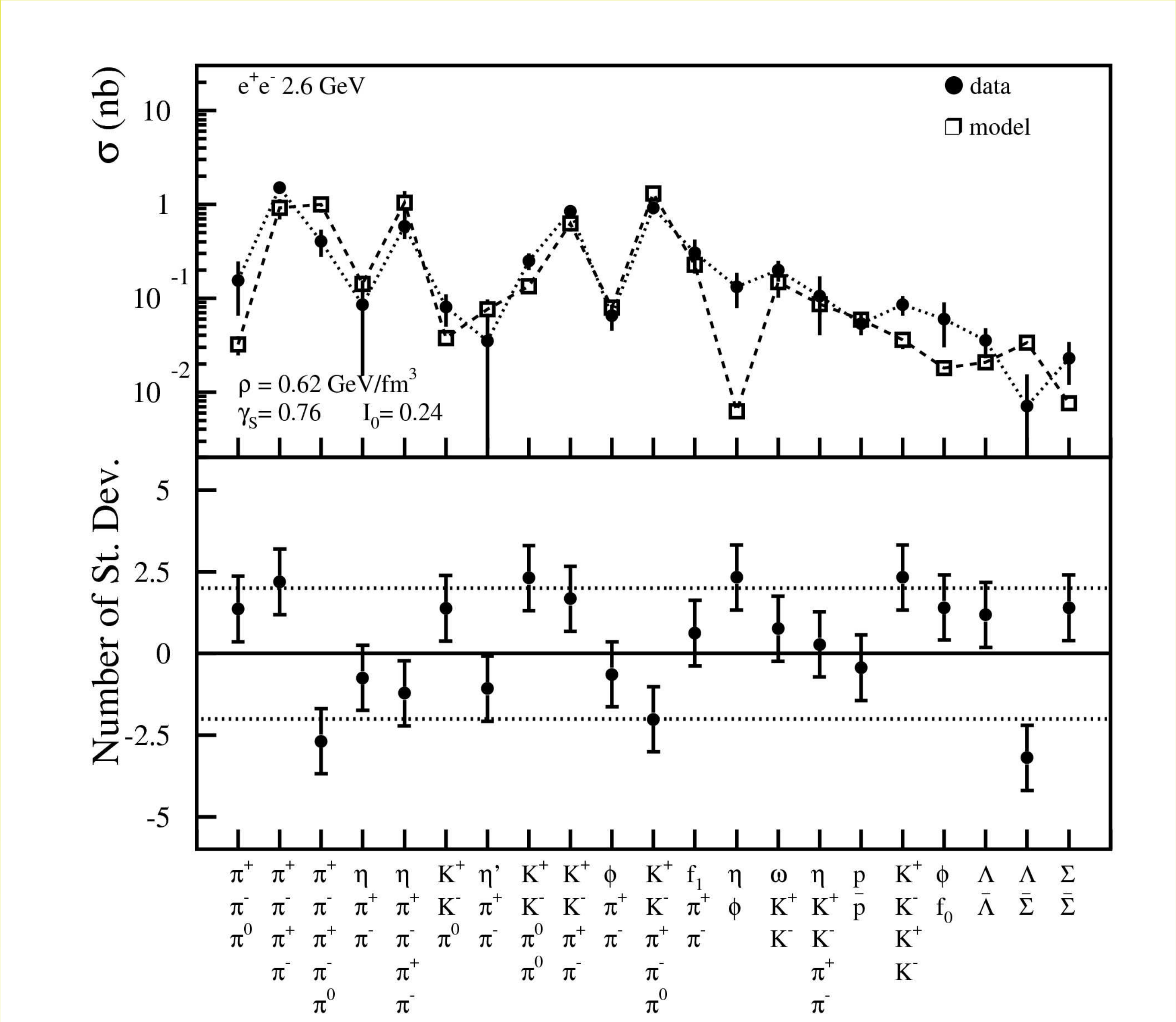
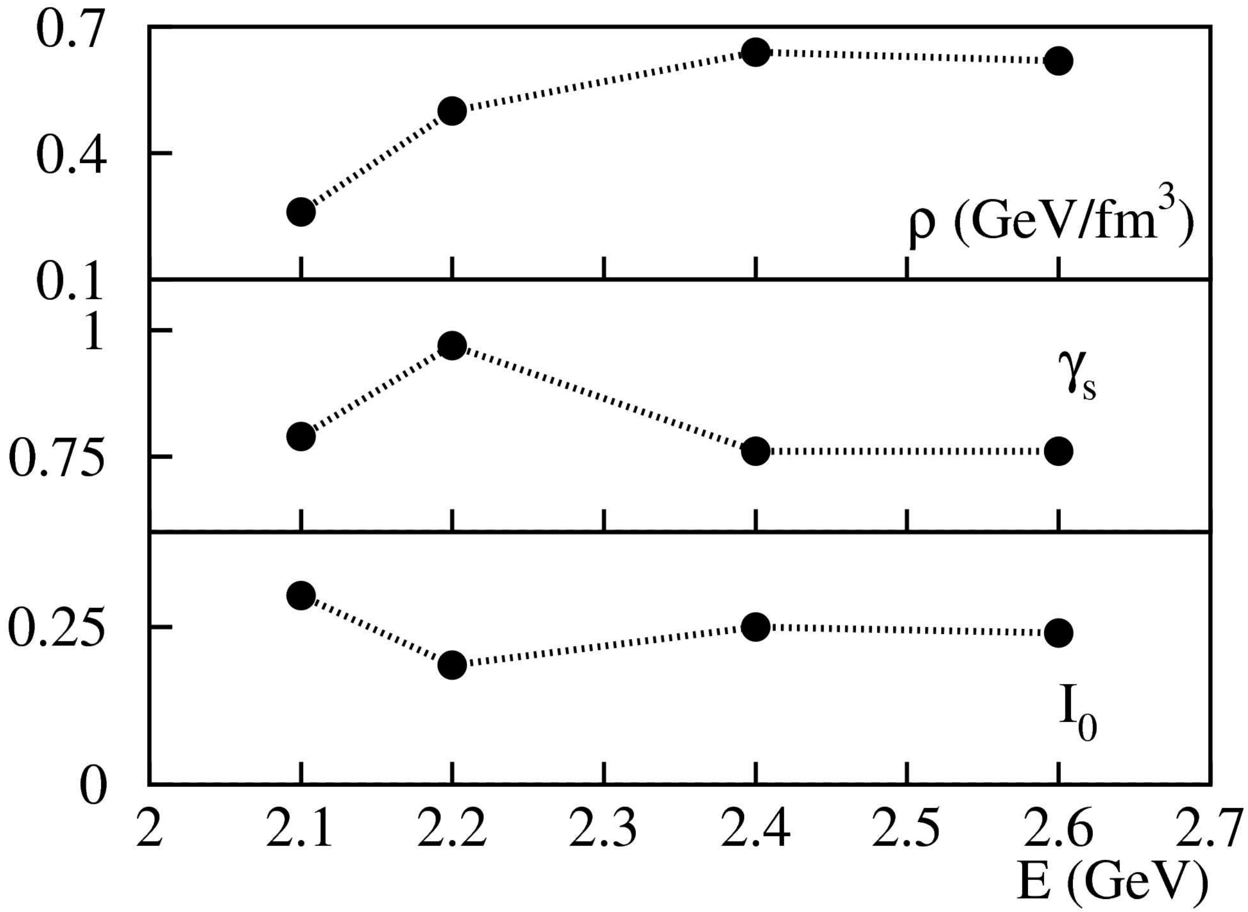
e+e- √s=2.6 GeV



Energy dependence of the parameters
(preliminary study)



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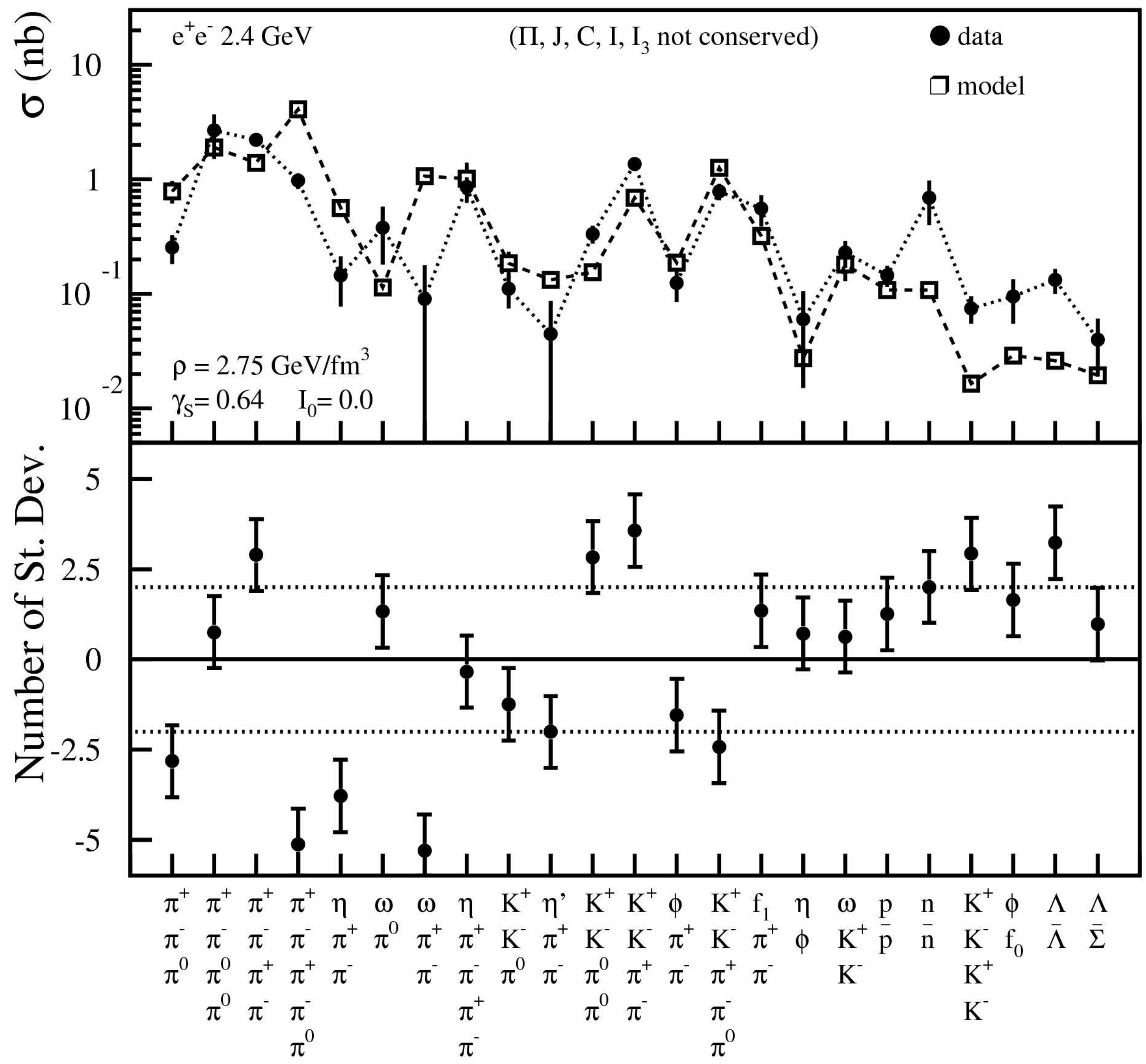
Importance of conservation laws



Q,E,P

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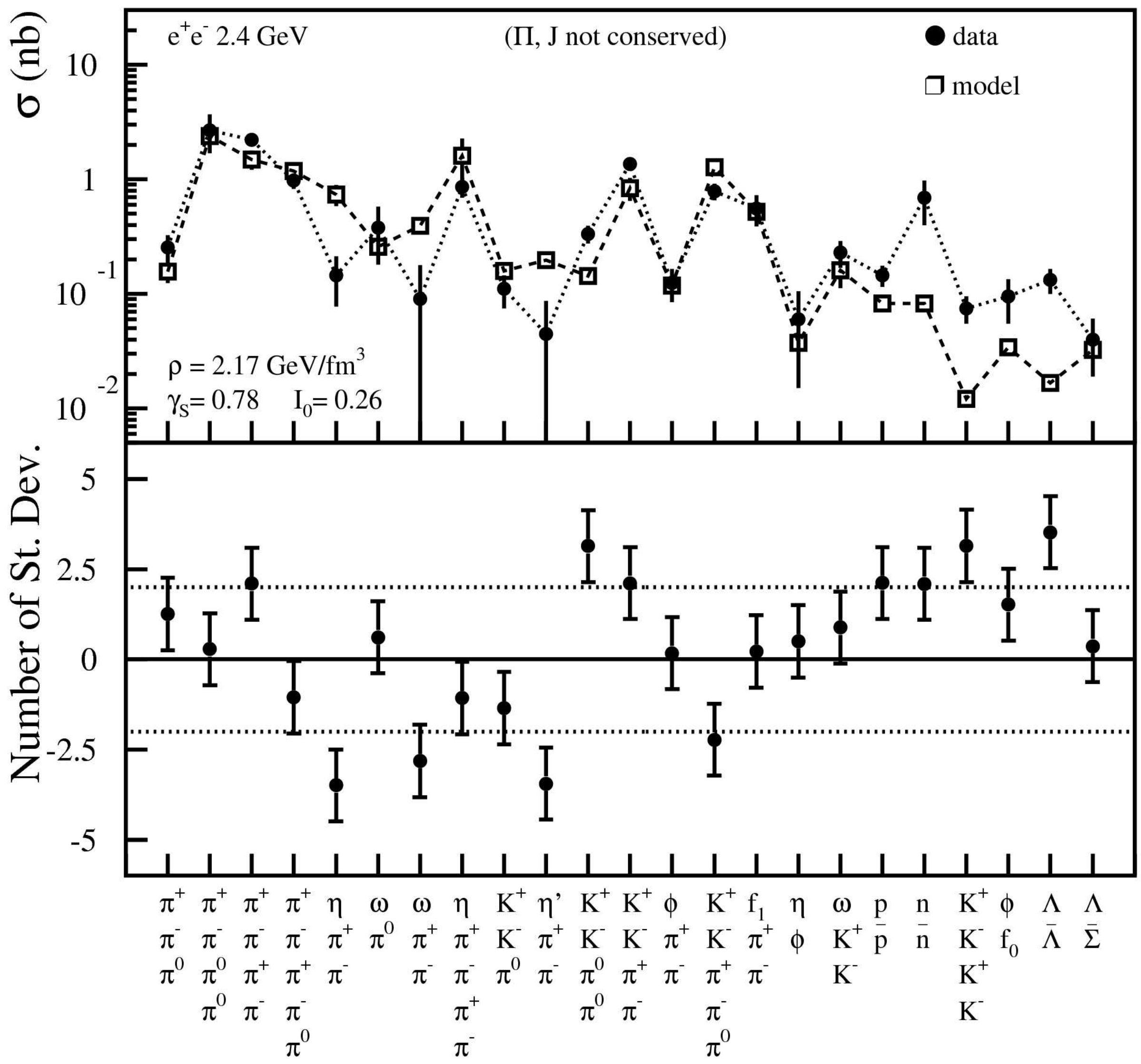
Importance of conservation laws



C-parity

Isospin

Q,E,P

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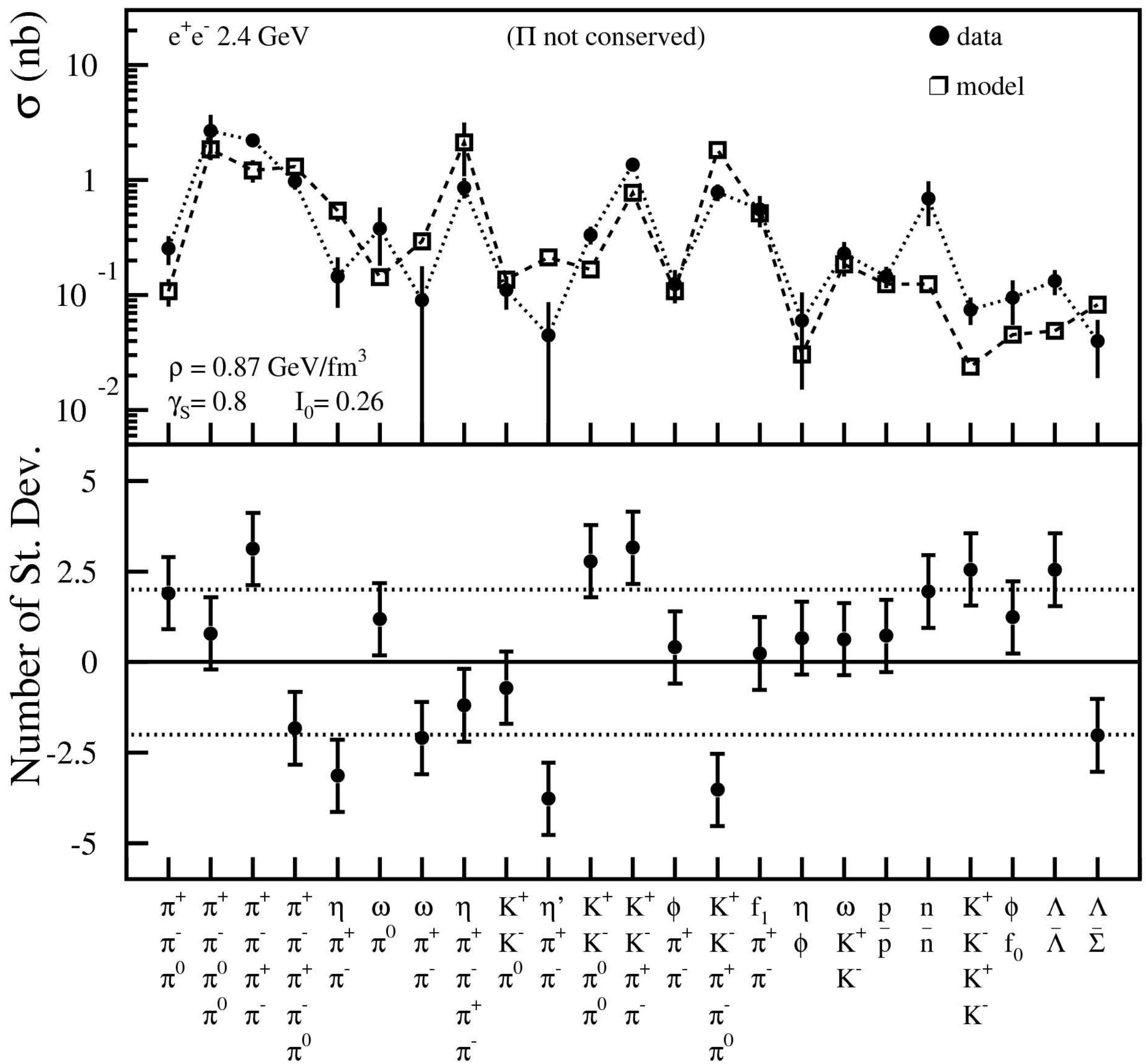
Importance of conservation laws



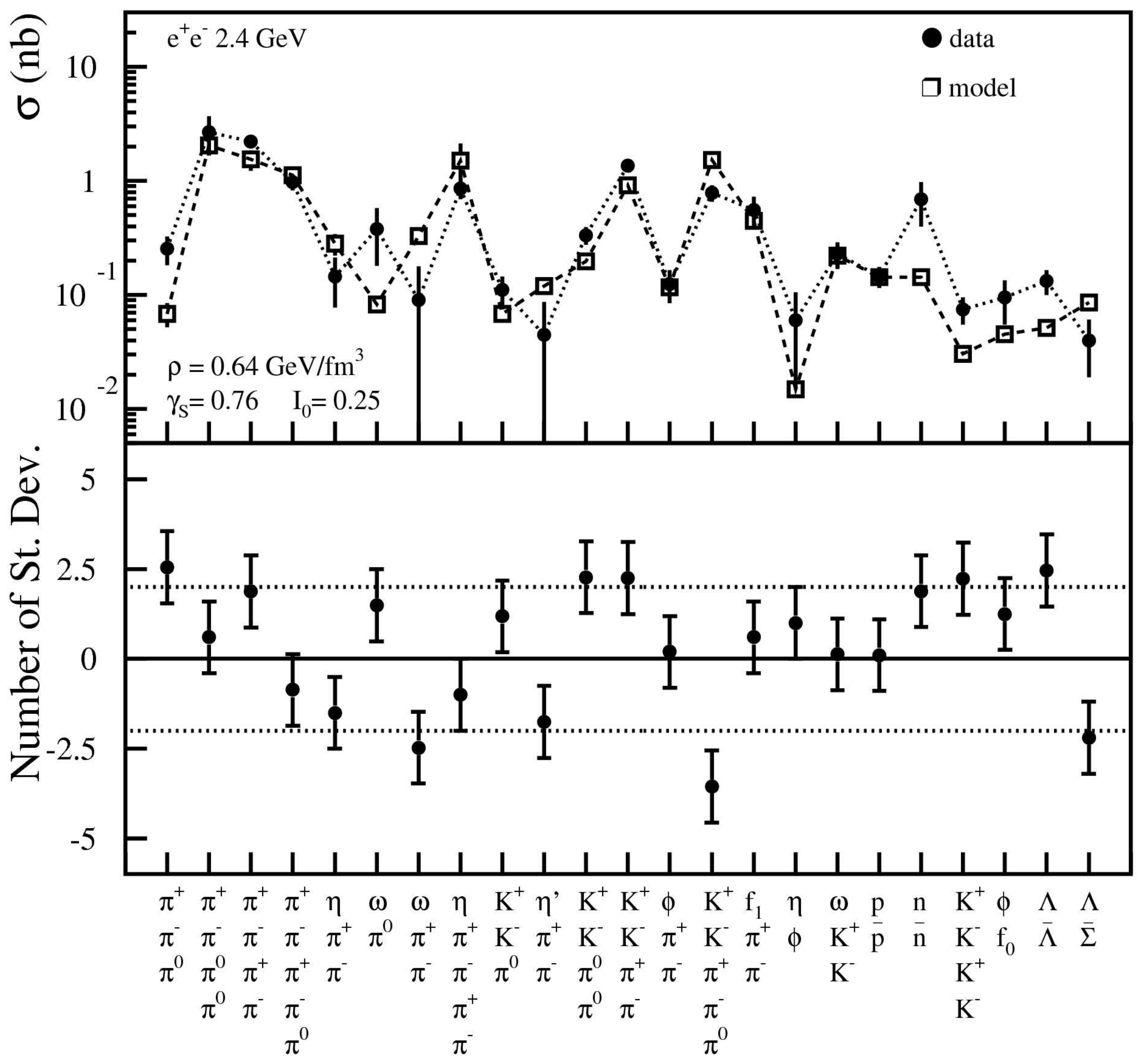
J

C-parity

Isospin

Q,E,P

Importance of conservation laws



Parity

J

C-parity

Isospin

Q,E,P

Summary

- The statistical hadronization model (SHM) can in principle be matched to transport, hydro codes or QCD-inspired models as a model for the adronization (see also the talk from C. Bignamini). The problem resides in finding a good assumption to identify the degrees of freedom of the early evolution of the collision (partons or fluid cells etc.) with colorless clusters.
- Interactions between already formed hadrons might also be described with the SHM, but:
  - The SHM cannot predict cross sections, but only relative probability of exclusive channels.
  - Detailed balance?
- The exact conservation of energy momentum is still important for clusters with mass 12 GeV also for mean multiplicities.
- For production rates of exclusive channels, the full set of conservation laws is required.

Statistical hadronization of small systems

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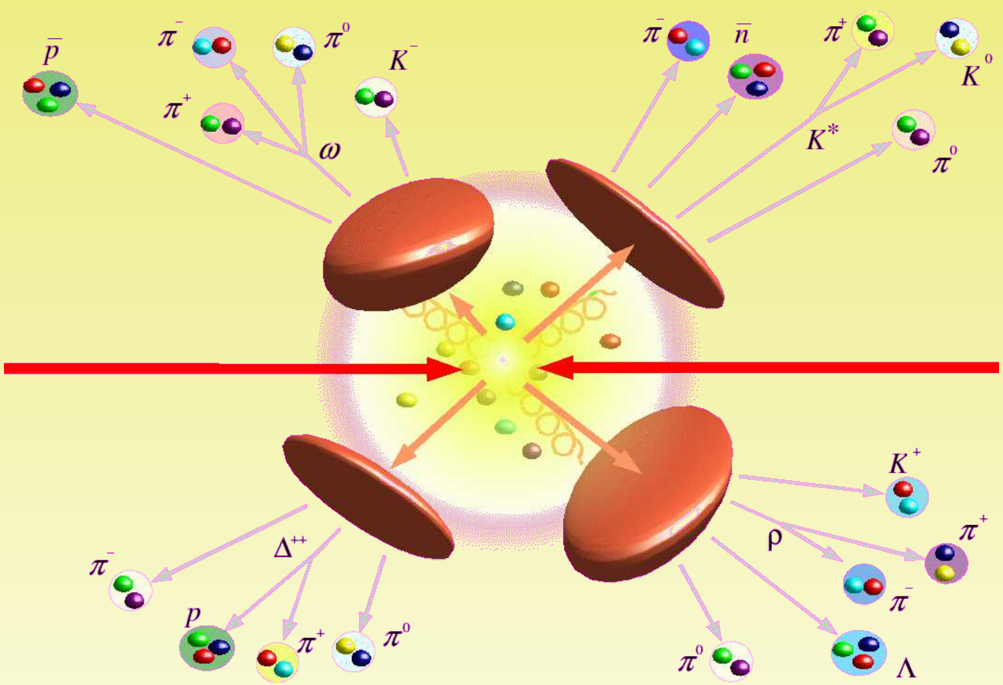
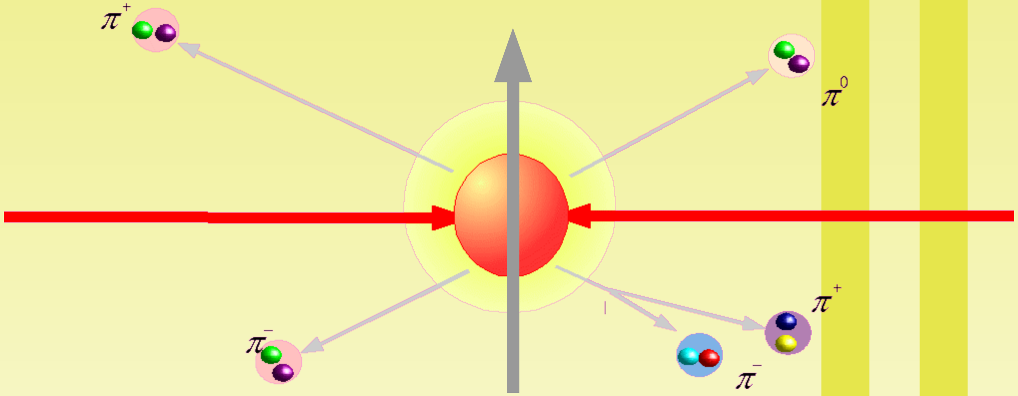
high E

(Multi-cluster scenario)



low E<~ 3 GeV

(Single cluster at rest)



P, J, I, I_3, Π , C

Angular momentum conservation

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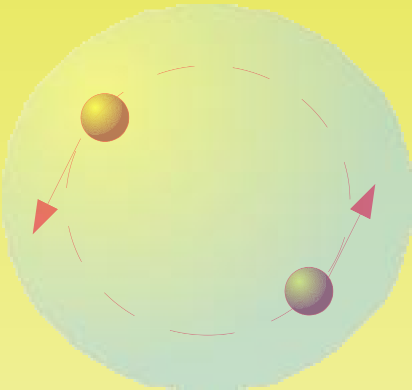
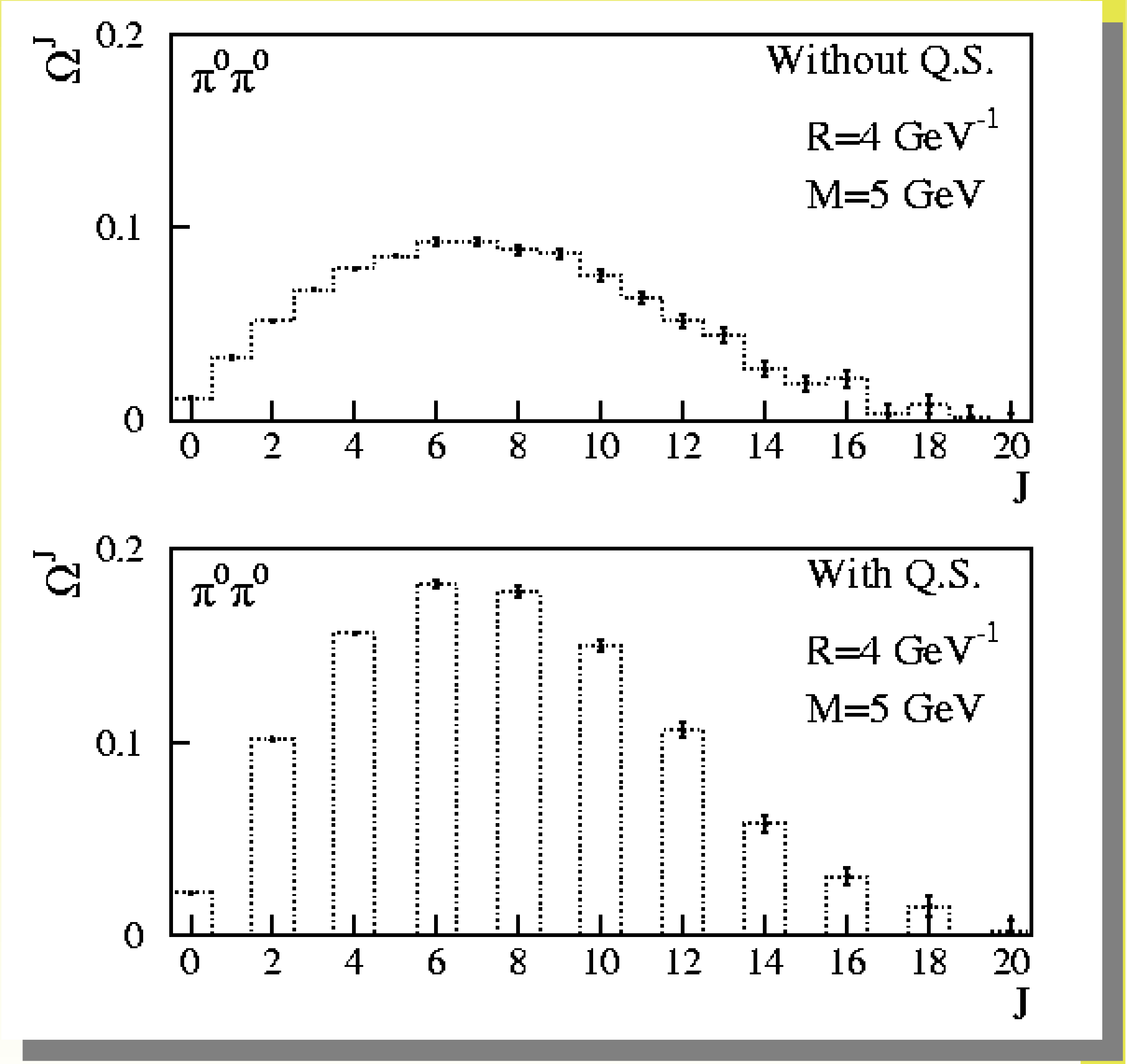


Small V & low p= suppression of high angular momentum



Microcanonical weight of the channel $\pi^0\pi^0$ as a function of the total spin of the system with Boltzmann statistics (upper panel) and quantum statistics (lower panel).