Simulation of the Expansion and the Phase Transition of a Quark Plasma with the Nambu-Jona-Lasinio model

Rudy Marty

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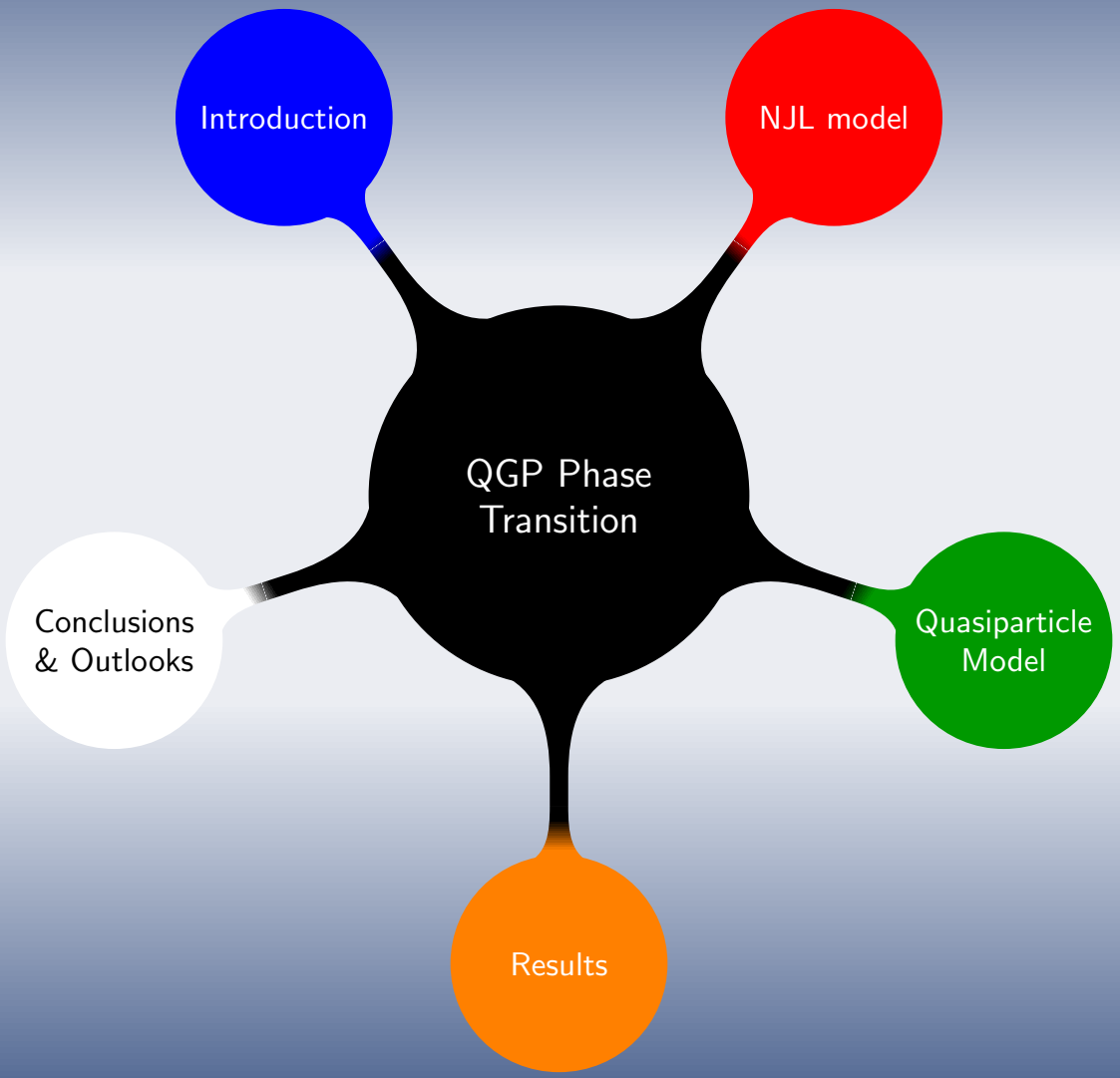
in collaboration with : J. Aichelin

Outline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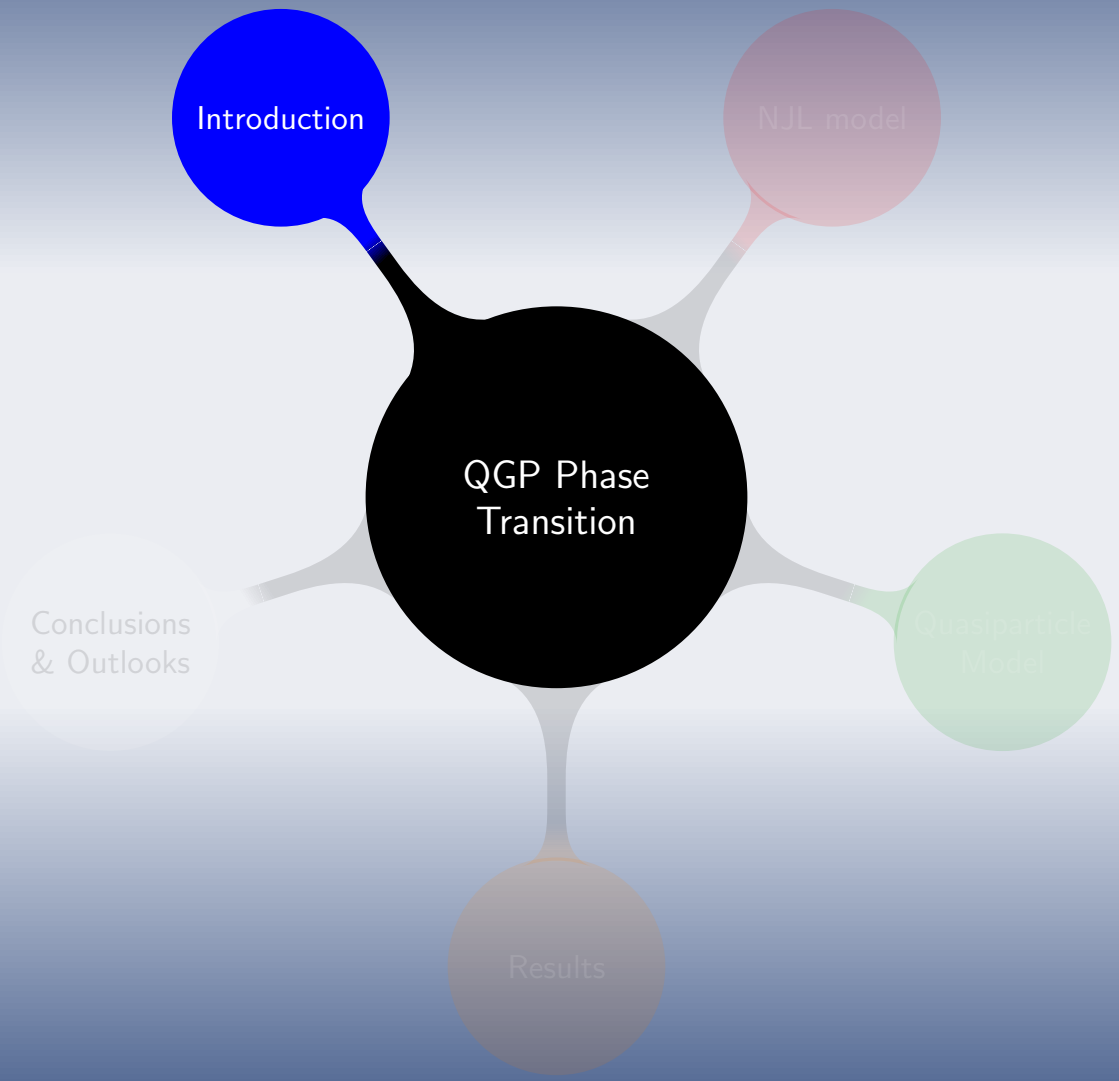
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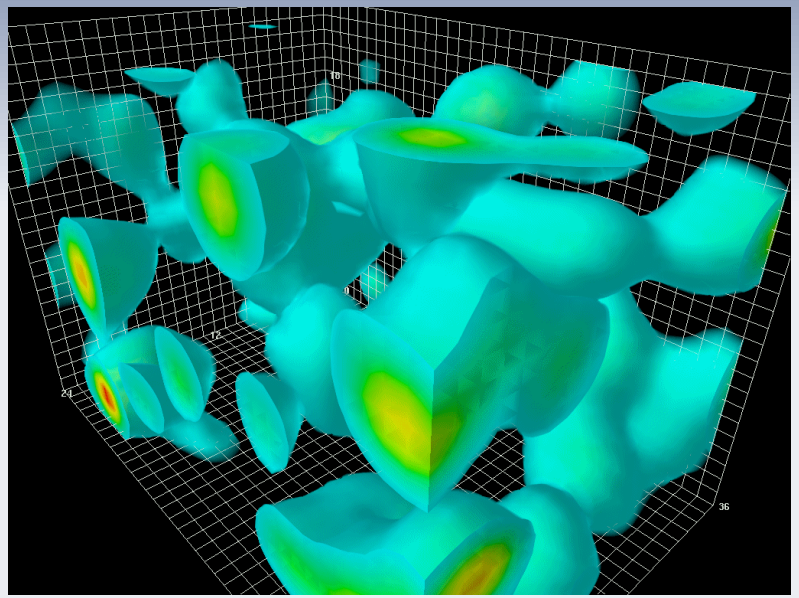
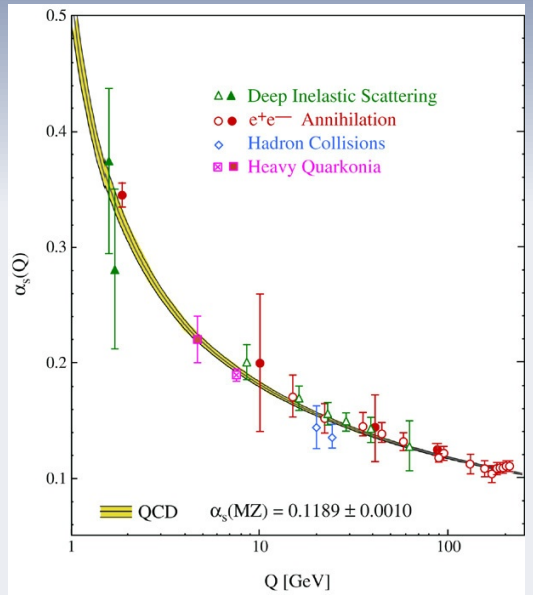
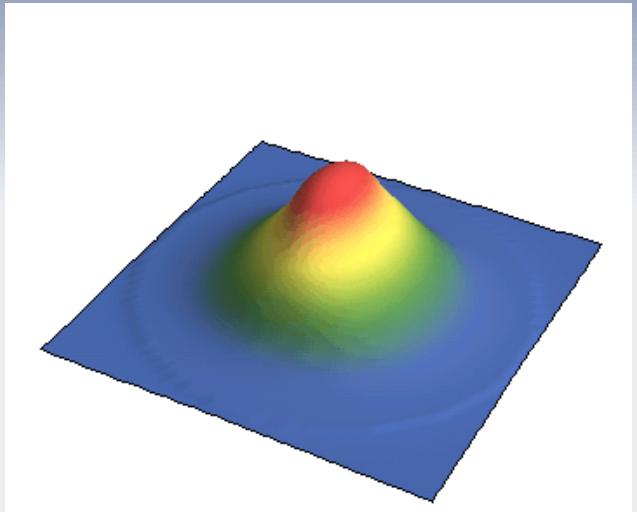
Lattice QCD framework





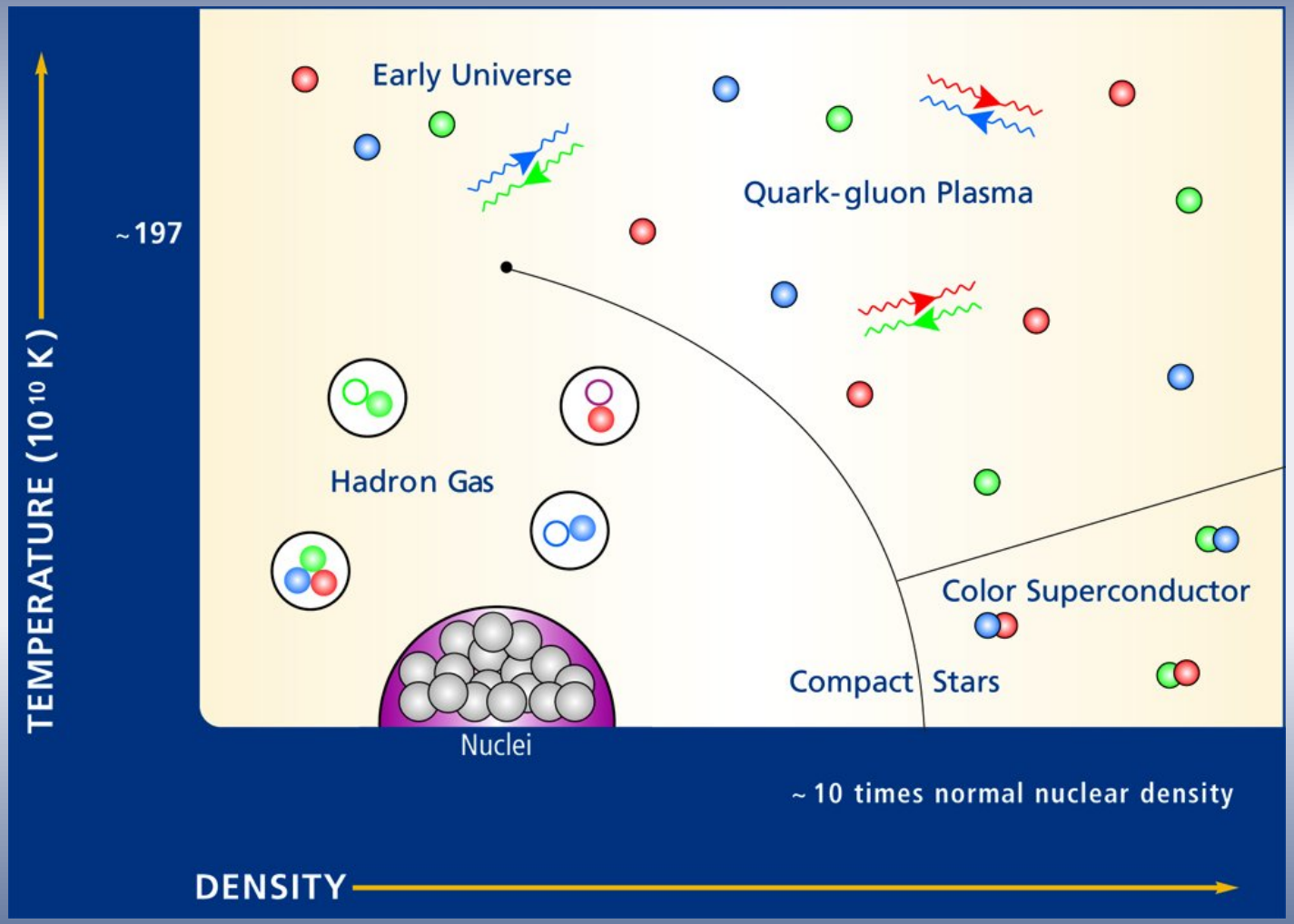
LQCD gives real QCD results for a microscopic system, but it is not a dynamical evolution at finite (T, μ). It describes bound states, but not an hadronization.

Hydrodynamical model



Hydrodynamical model is the opposite of LQCD.
It is a model for dynamical evolution, but it is
not a microscopic description of partons and hadrons.
The Cooper-Frye formula is used for hadronization.

Mixed Phase


Early Universe
Quark-gluon Plasma
~197
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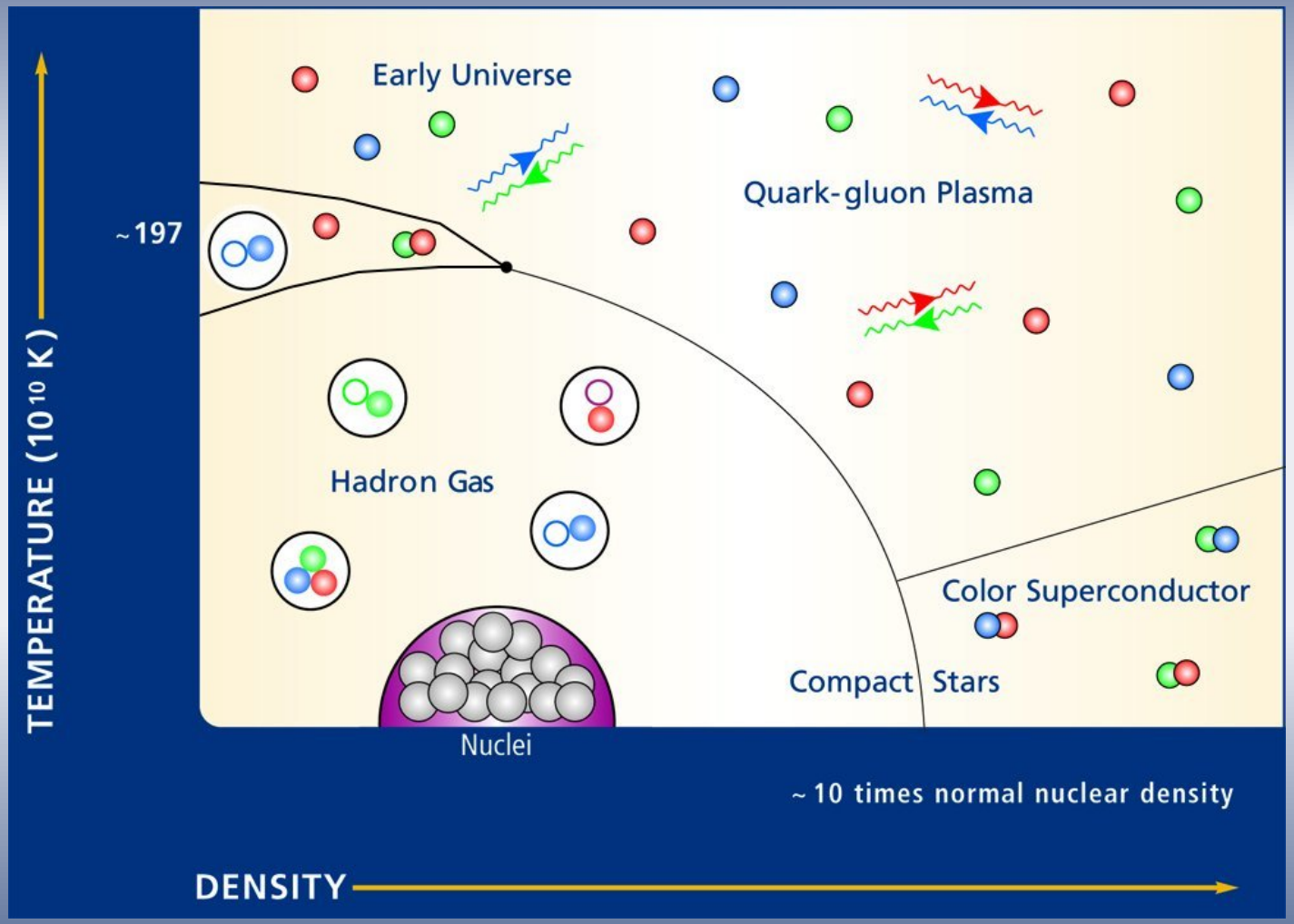
Nuclei
~10 times normal nuclear density
TEMPERATURE (10^{10} K)
DENSITY

Mixed Phase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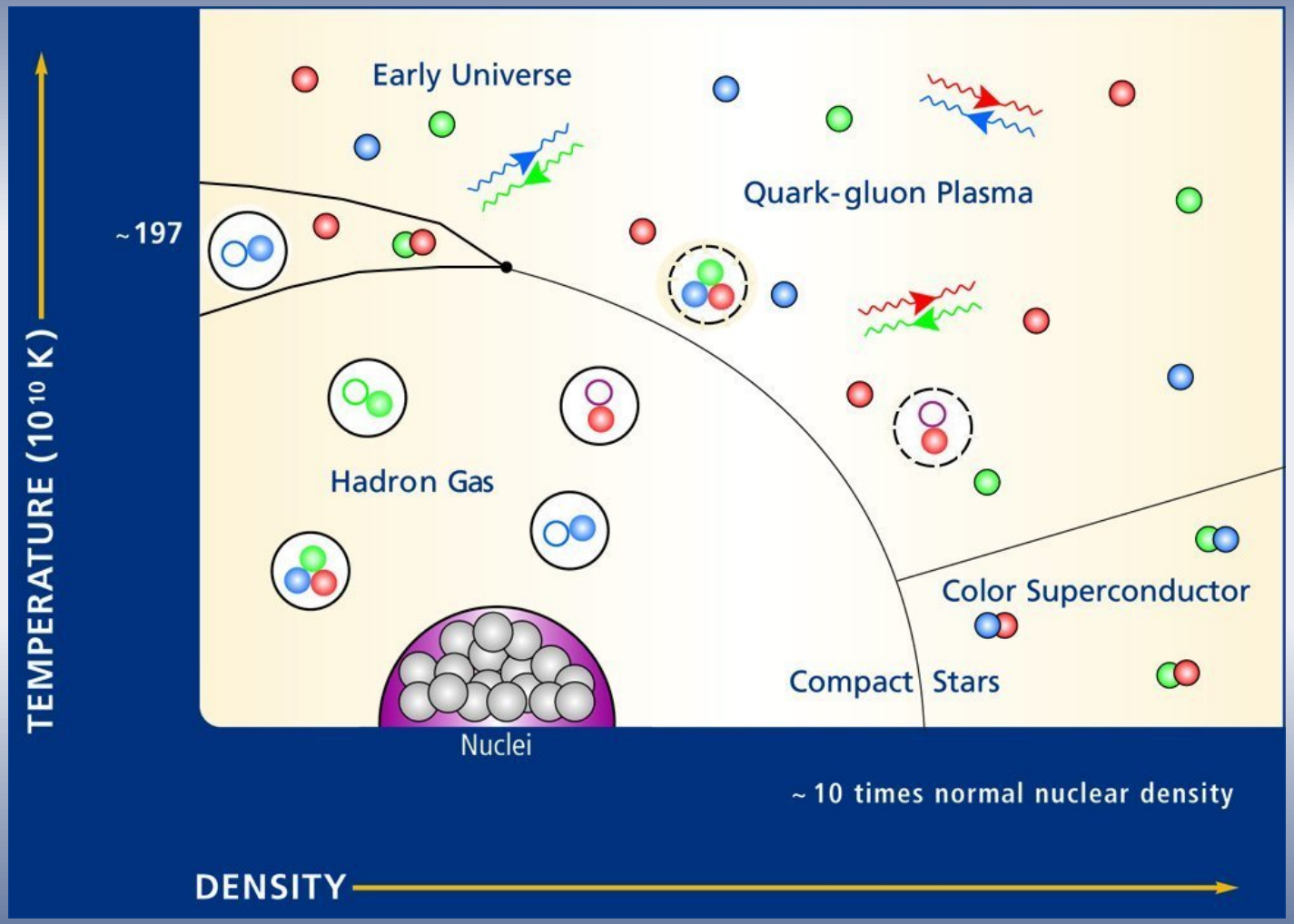
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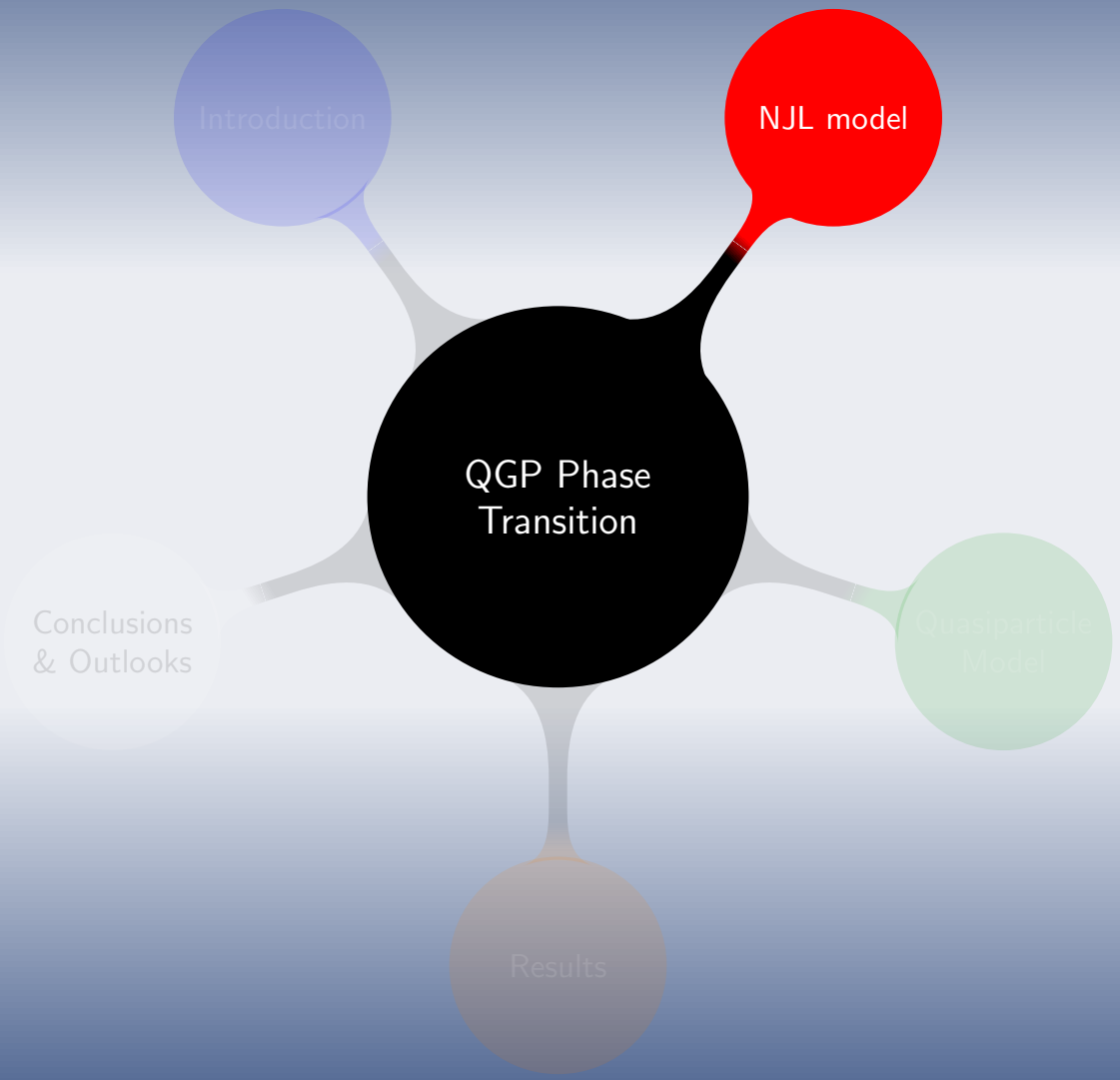
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Basis of model I

Nambu–Jona-Lasinio : model for nuclear matter (1961).
Now, a model for partons with respect of QCD symmetries.

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Lagrangian for $SU_R(3) \times SU_L(3) \times U_V(1)$

$$\mathscr{L} = \sum_{f=u,d,s} \underbrace{\bar{q}_f \left(i\partial\!\!\!/ - m_{0f}\right) q_f}_{\text{kinetic term}} + G_S \sum_{a=0}^{8} \left[\left(\bar{q}_f \lambda^a q_f\right)^2 + \left(\bar{q}_f i\gamma_5 \lambda^a q_f\right)^2 \right]$$
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Pay attention that there are not gluons ! Model for low energy : around T_c, gluons are massive and confined.

Basis of model II

NJL model does not have any free parameters.

We just start with the real mass of pion,
coupling constants and bare masses of quarks.

These two last parameters have to be chosen to fit
LQCD predictions about vacuum energy density.

But the NJL model is not renormalizable,
so parameters give us a cut-off (effective model).

Some improvements

Nevertheless this model does not respect all details of QCD :

- Adding a Polyakov Loop gives better inclusion of gluons properties (PNJL, H. Hansen) which allow to fit some LQCD predictions and improve hadronization speed (through high mass derivative),

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These improvements will not be discussed here because they just shift the masses and cross sections (same maximum values).

Quantum Field Theory

The NJL lagrangian gives us a self consistent mass using Hartree approximation (see S. P. Klevansky papers).

Mass of the quark u in $SU(3)$

$$m_u = m_{u0} - \frac{GN_c}{\pi^2} m_u A(m_u, \mu_u) + \frac{KN_c^2}{\pi^4} m_d m_s A(m_d, \mu_d) A(m_s, \mu_s)$$

where $A(m^*, T, \mu)$ is the one fermion loop integral at finite (T, μ). So the bare mass of quark is dressed with an effective mass (gluon mean field).

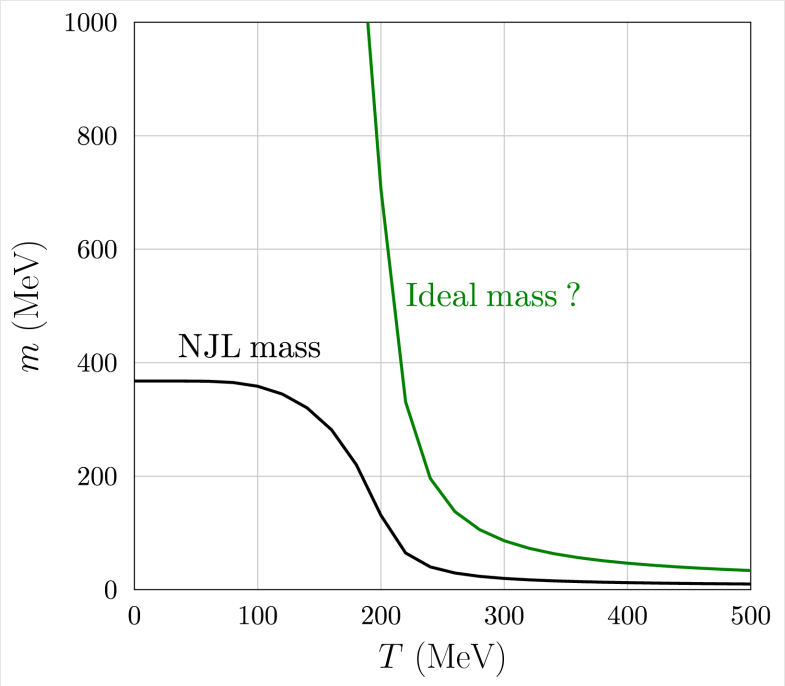
NJL quark mass

$$\underbrace{m_q}_{\text{dressed mass}} = \underbrace{m_0}_{\text{bare mass}} + \underbrace{G\langle q\bar{q}\rangle}_{\text{effective mass}}$$

Confinement

Recall that NJL does not include gluons but a dressed mass. The confinement is not perfect due to finite mass at $T = 0$. The ideal mass could be a divergent mass close to T_c (and so an infinite derivative $\rightarrow$ absolute confinement).

Masses NJL/Ideal



Hadronization process I

The hadronization comes from the fact that we can construct mesons from quarks interaction in the NJL model.

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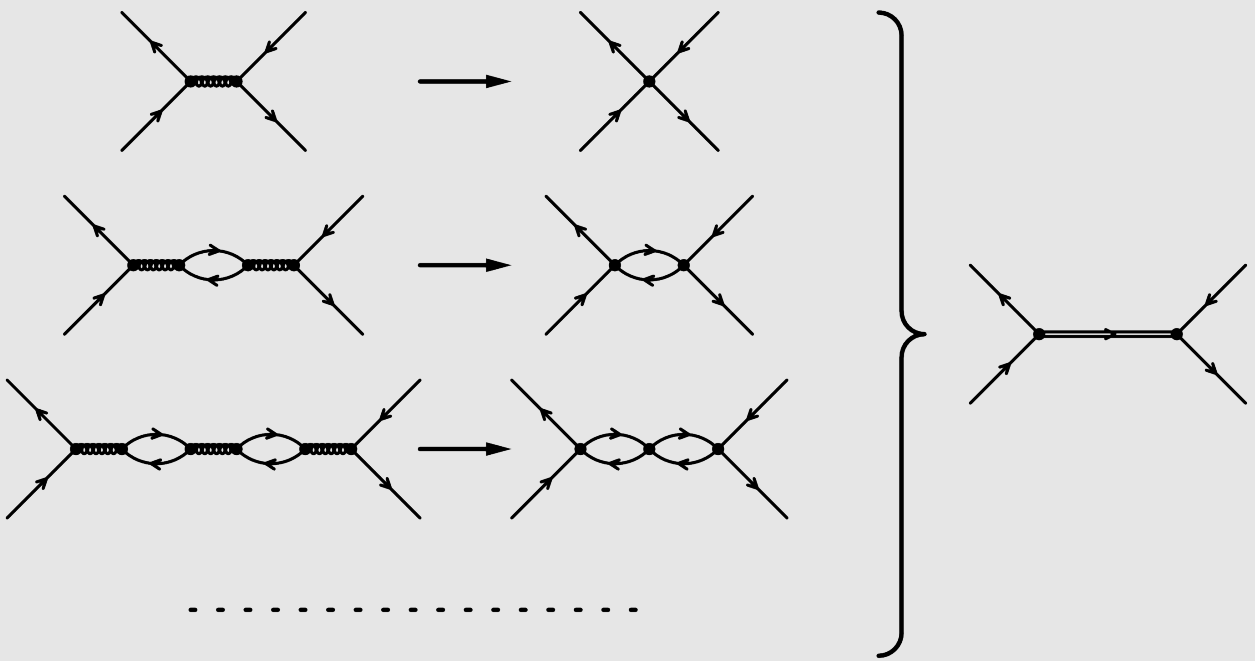
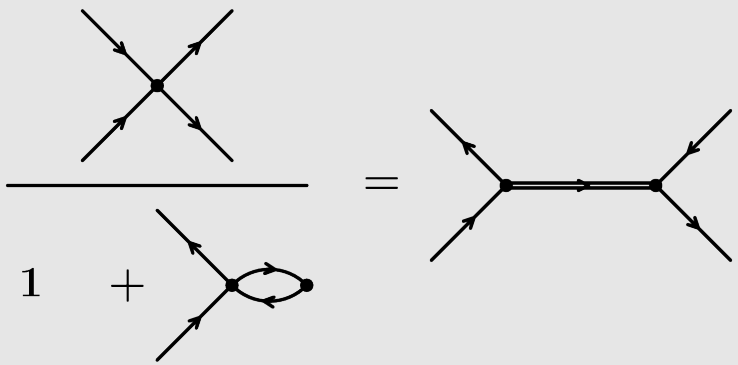
What is a mesons in NJL ?



Hadronization process II

The meson



mass of the meson π

$$M(k_0, \vec{k}) = \frac{2K_1^+}{1-4K_1^+\Pi^P_{q\bar{q}}(k_0,\vec{k})}$$

at the pole :

$$1 - 4K_1^+\Pi^P_{q\bar{q}}(m_\pi, \vec{0}) = 0$$

that gives

$$M(k_0, \vec{k}) = \frac{-g^2_{\pi q\bar{q}}}{k^2-m_\pi^2}$$

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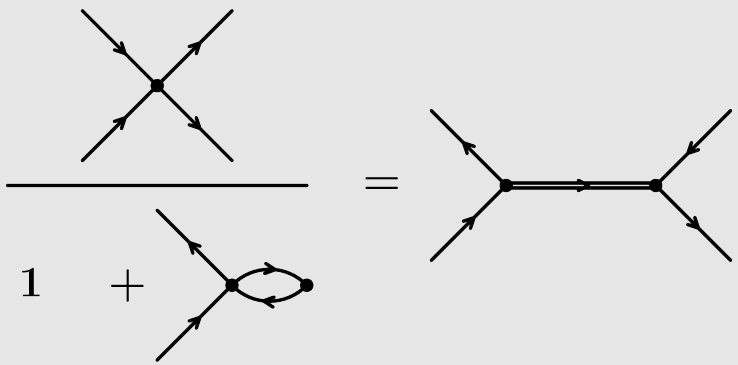

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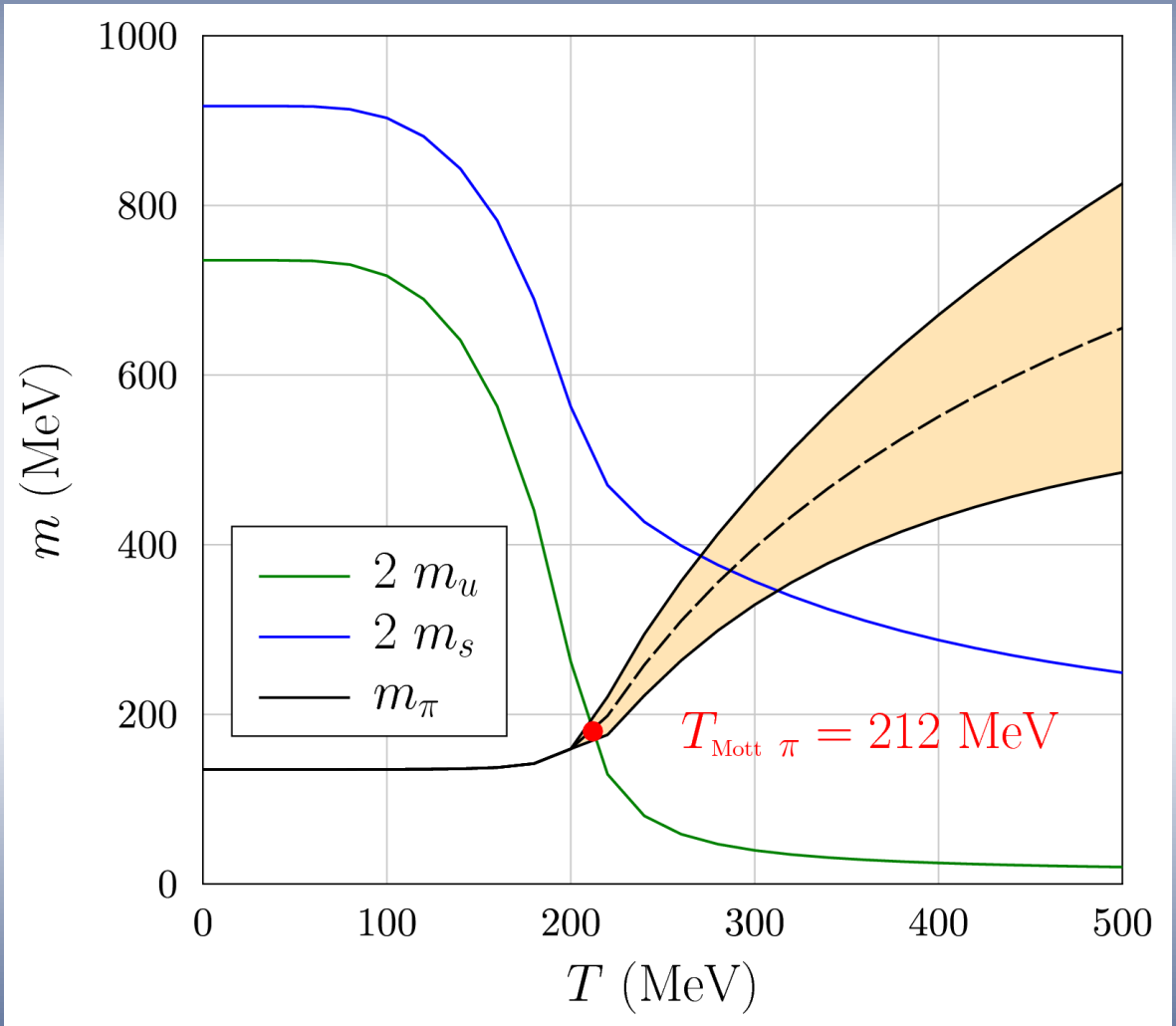
We note that mesons have an imaginary part in the mass : they are unstable above T_c. And the simplest hadronization process is $q\ \bar{q} \rightarrow M\ M$ so there is no entropy problem here.

Masses plots



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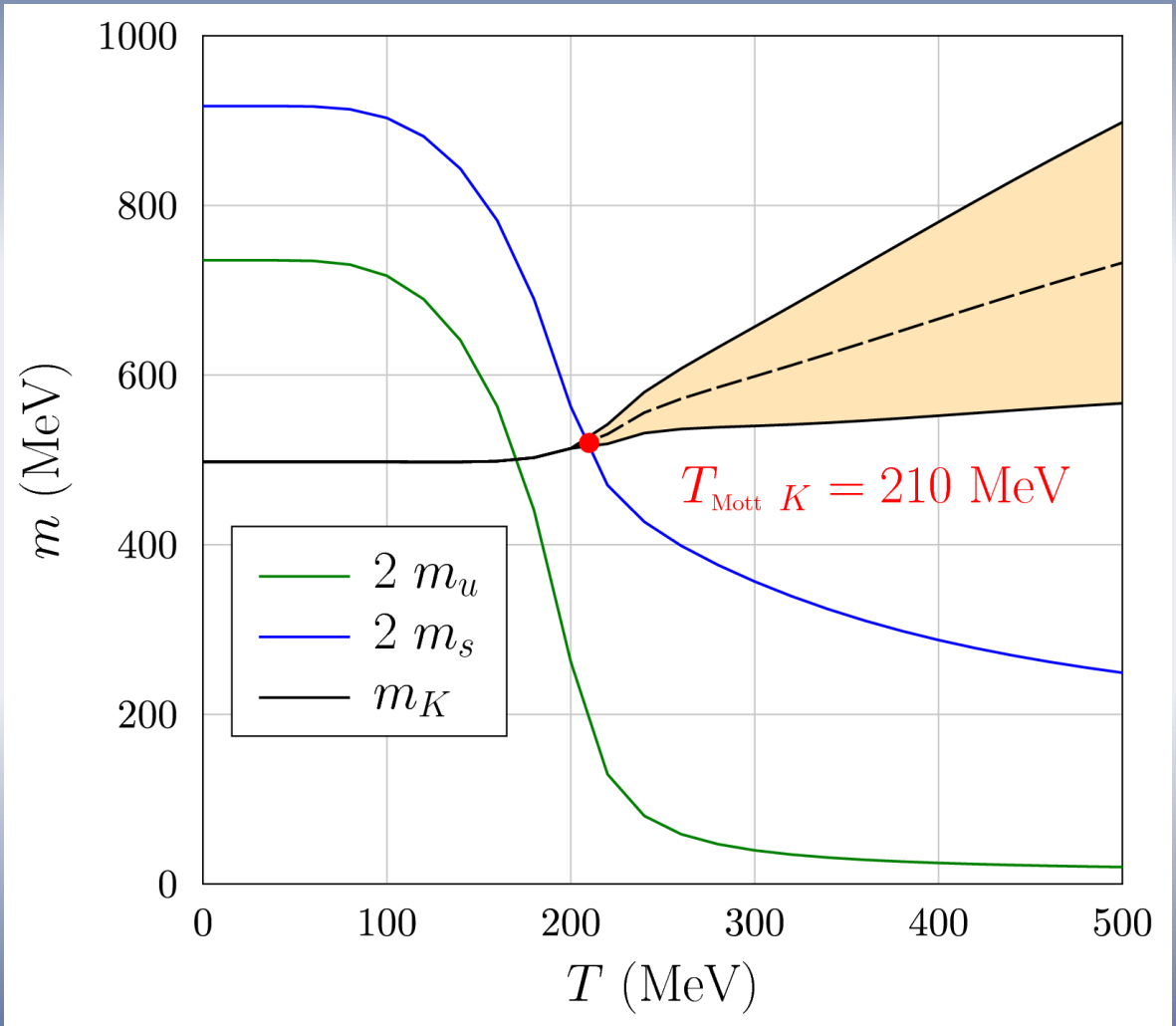

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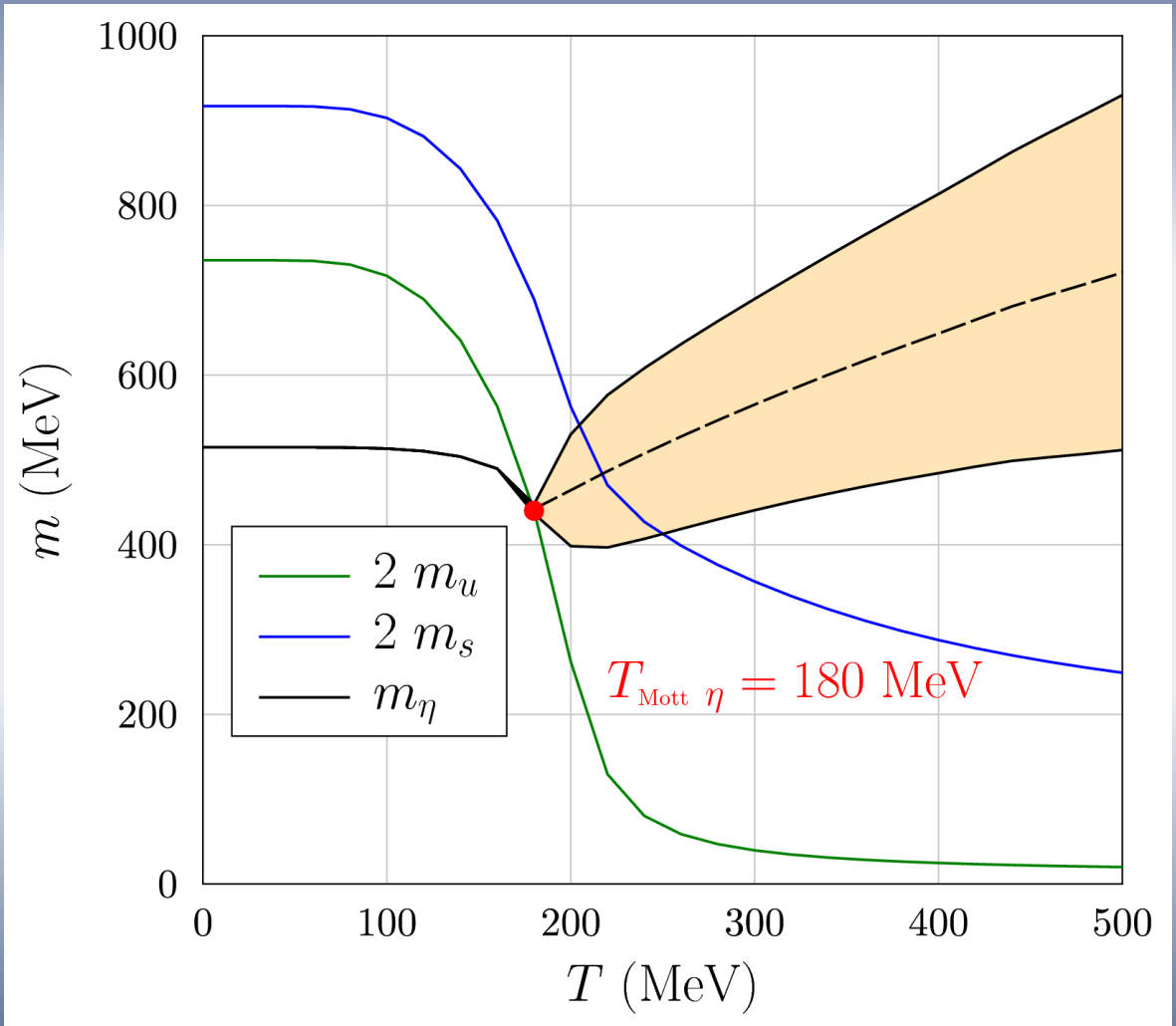
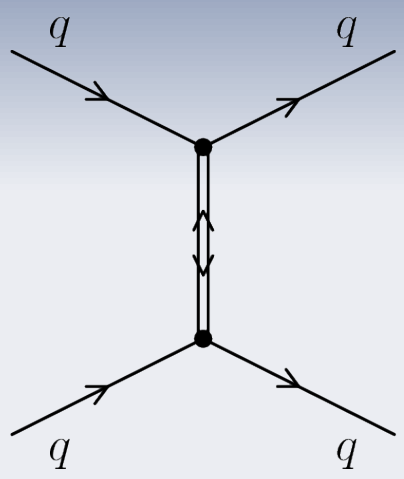
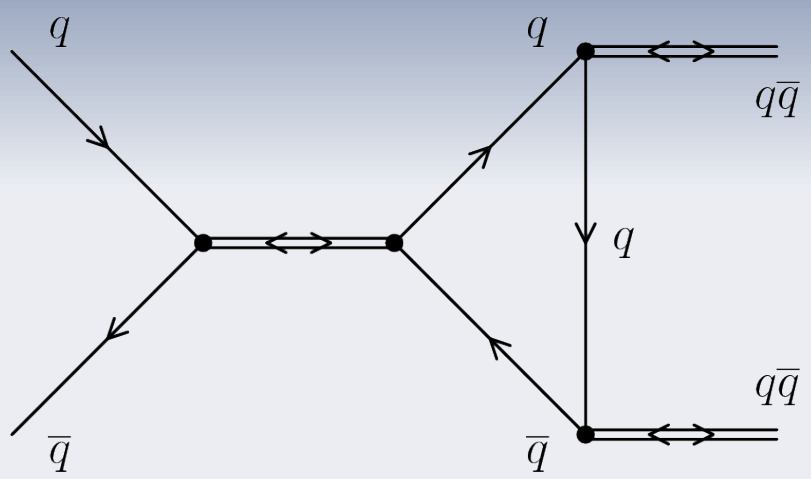


Cross Sections

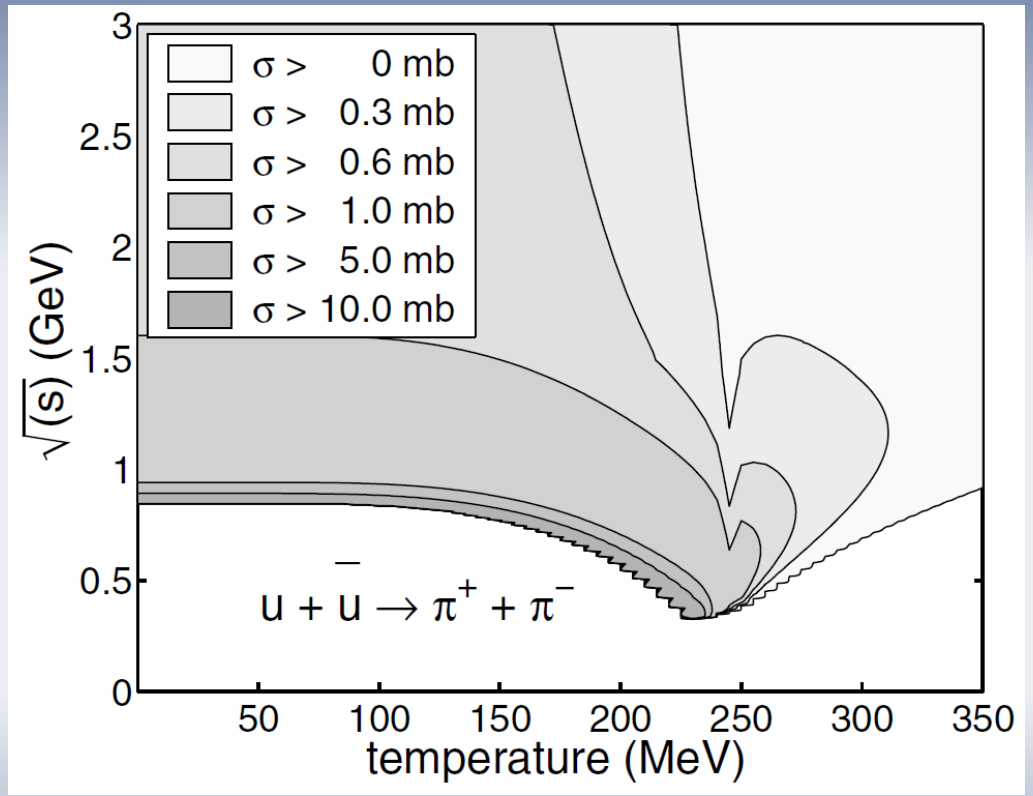




For the moment it miss some processes I do not have calculated :

$$q\ q \rightarrow D\ D,$$
$$M\ M \rightarrow M\ M,$$
$$M\ D \rightarrow B\ q, \ldots$$

Cross Section plot



An example of NJL cross section : $u\ \bar{u} \rightarrow \pi^+\ \pi^-$. We can see the explosion close to the phase transition.

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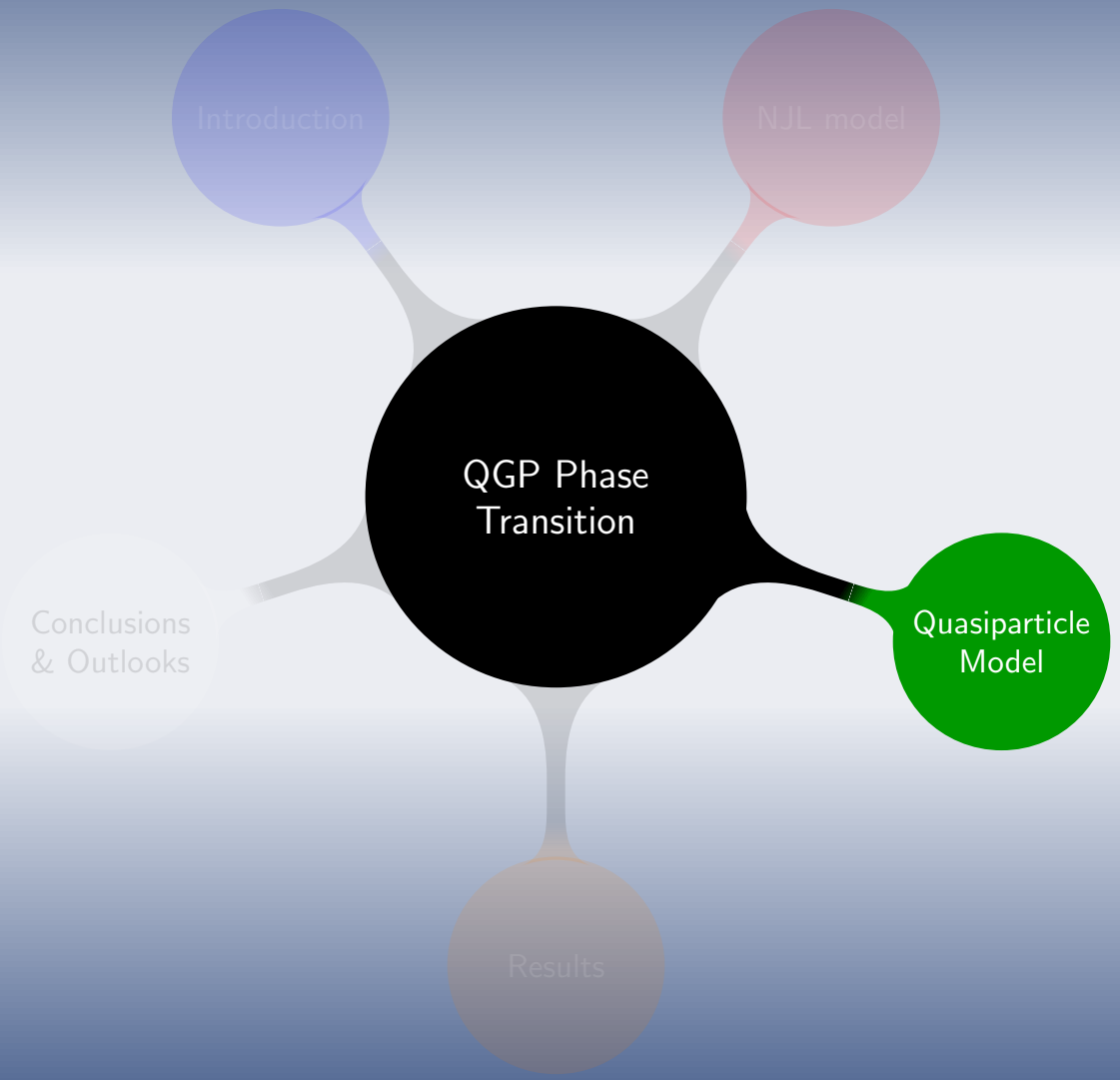
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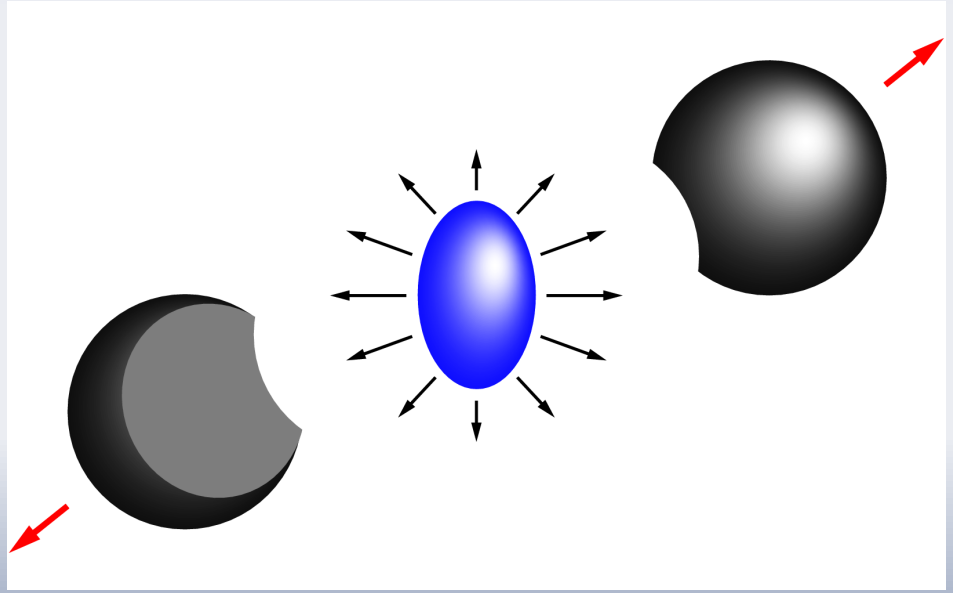
Initial Conditions

Our framework is the heavy ions collision. We start after the thermal equilibrium. Our parameters are :$(A, b, \sqrt{s})$, (T, μ), η (pseudo-rapidity), t (simulation time).



At the beginning we set $\mu(r) = \mu_0$ and $T(r) = T_0\ (1 - (r/R)^3)^{1/3}$ in the almond (distance from center : R).

Dynamics of particles

Dynamics = motion + collisions

Motion

$$(q_\mu, p_\mu) \longrightarrow (\vec{q}, \vec{p})$$

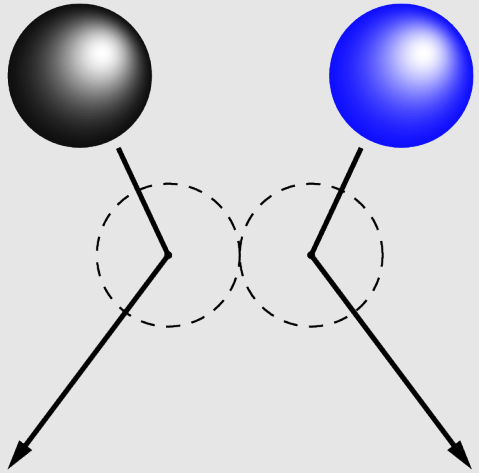
Relativistic constrained dynamics to get the equations of motion :

$$\mathrm{d}q/\mathrm{d}\tau = \{q, \mathcal{H}\}$$
$$\mathrm{d}p/\mathrm{d}\tau = \{p, \mathcal{H}\}$$

(τ = Relative time)

Collisions

"Billard ball" collisions (using geometrical cross sections)



$$d = \sqrt{\sigma/\pi}$$

Dynamical temperature

NJL quantities depend on the temperature.
During the expansion, we suppose a local equilibrium.
So we use a local temperature which comes from :

Fermionic density → temperature

$$\rho_i = \alpha^{-1}\ T_i^3 = \sum_{j\neq i} R_{ij} \quad \rightarrow \quad T_i = \alpha(\sum_{j\neq i} R_{ij})^{1/3}$$

with a typical distance for the interaction :

Weighting function

$$R_{ij} = \exp\left(-\Delta x^2/L^2\right) \quad \text{and} \quad L = \hbar c/T_c \approx 1\text{fm}$$

Decay of unstable hadron

Hadrons are unstable inside the plasma ($T > T_c$).
We make a random test at each time step :

Decay test

$$rand() < \Gamma \ \Delta t / \hbar c$$

Due to the local aspect of the temperature, and to avoid the Pauli blocking for fermion : created quarks from the decay are spaced by

Decay security

$$\Delta x = \hbar c / T_{\mathrm{decay}}$$

Final aim

Why the study of the expansion is so important ?

- We start with quark phase and we must finish with 100% of hadrons : this is the total hadronization,
- Understand observables : study of initial states and fluctuations to show differences and similarities.

Of course we keep in mind that we have to be carefull :
this is a relativistic quantum system.
So we have to do a lot of simulations to be consistent.

Simulation

In practice, it is a hard program to do (now I have 6000 lines of C++ code). And a very time consumming program ! ($t_{\mathrm{simu}} \propto O(N^3)$ due to matrix inversion in relativistic dynamic). The results I will present come only from the following simulation :

- $(A, b, \sqrt{s}) = (208, 13.4\ \mathrm{fm}, 2\ \mathrm{GeV})$ (small plasma $\sim$ 50 particles),
- $(T, \mu) = (350\ \mathrm{MeV}, 0\ \mathrm{MeV})$,
- $\eta = 0$ (isotrope),
- $L = 1\ \mathrm{fm}$ (interaction length $\approx$ thermal length),
- $t = 15\ \mathrm{fm/c}$.

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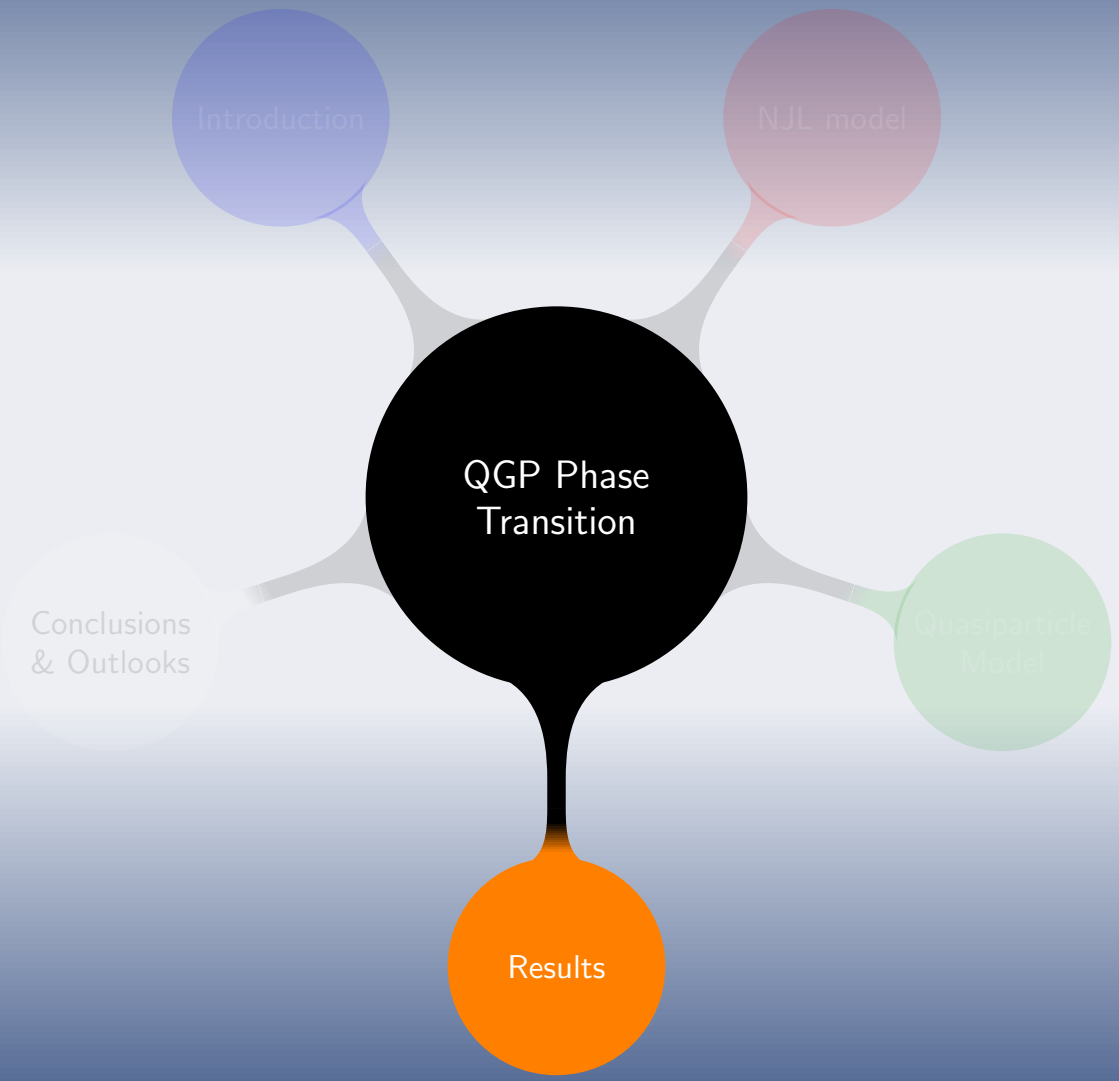
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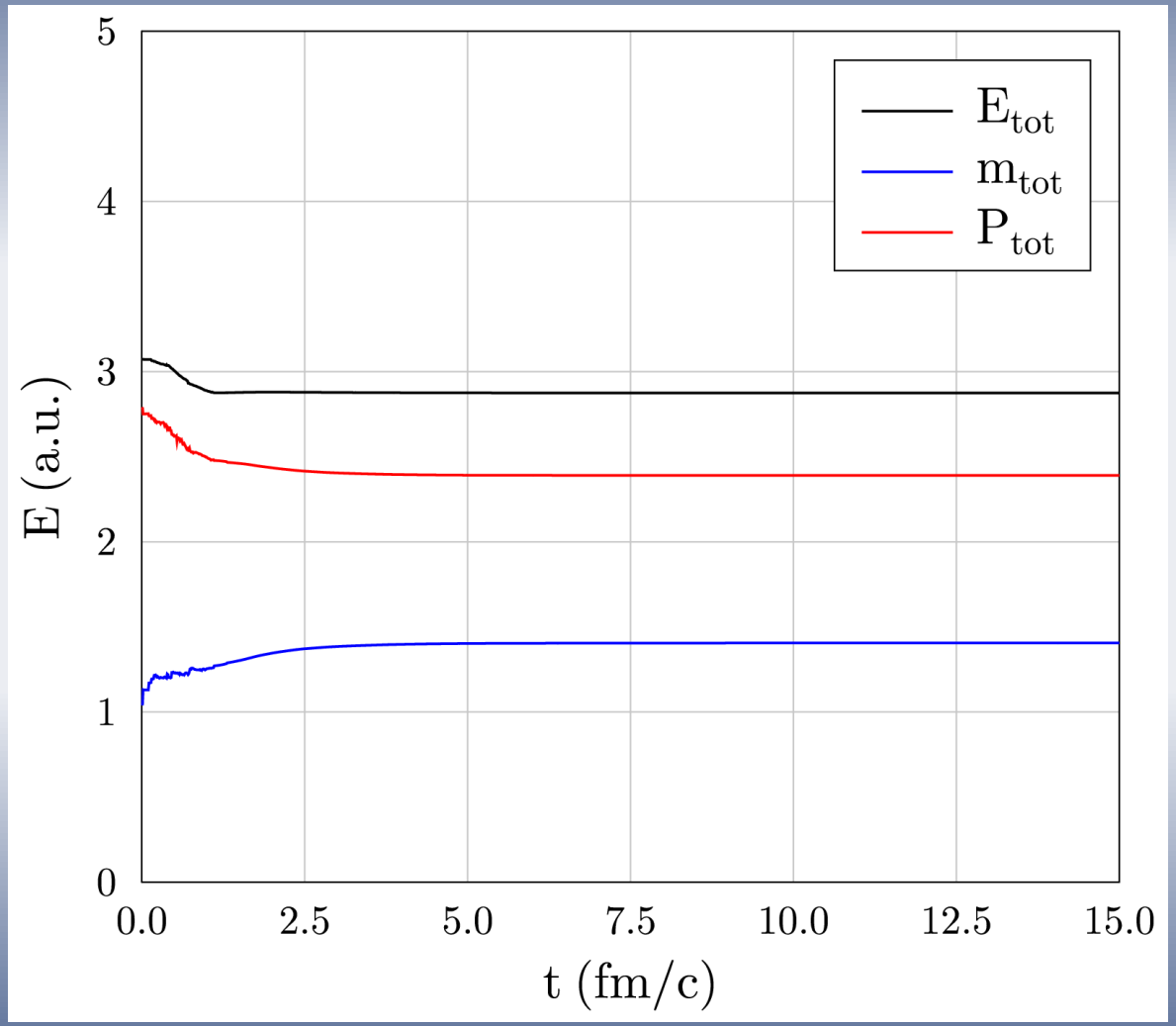
Energy conservation



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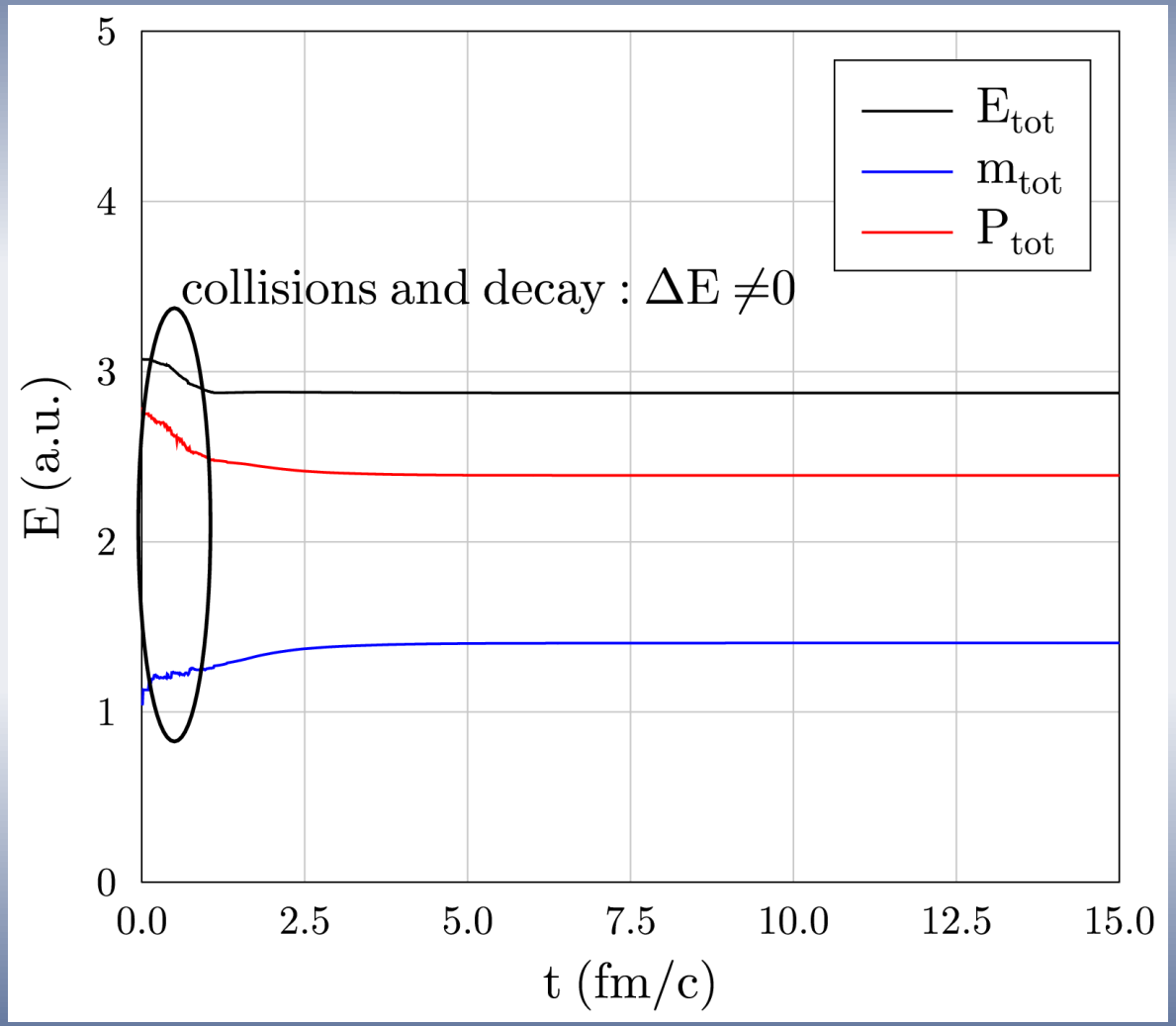

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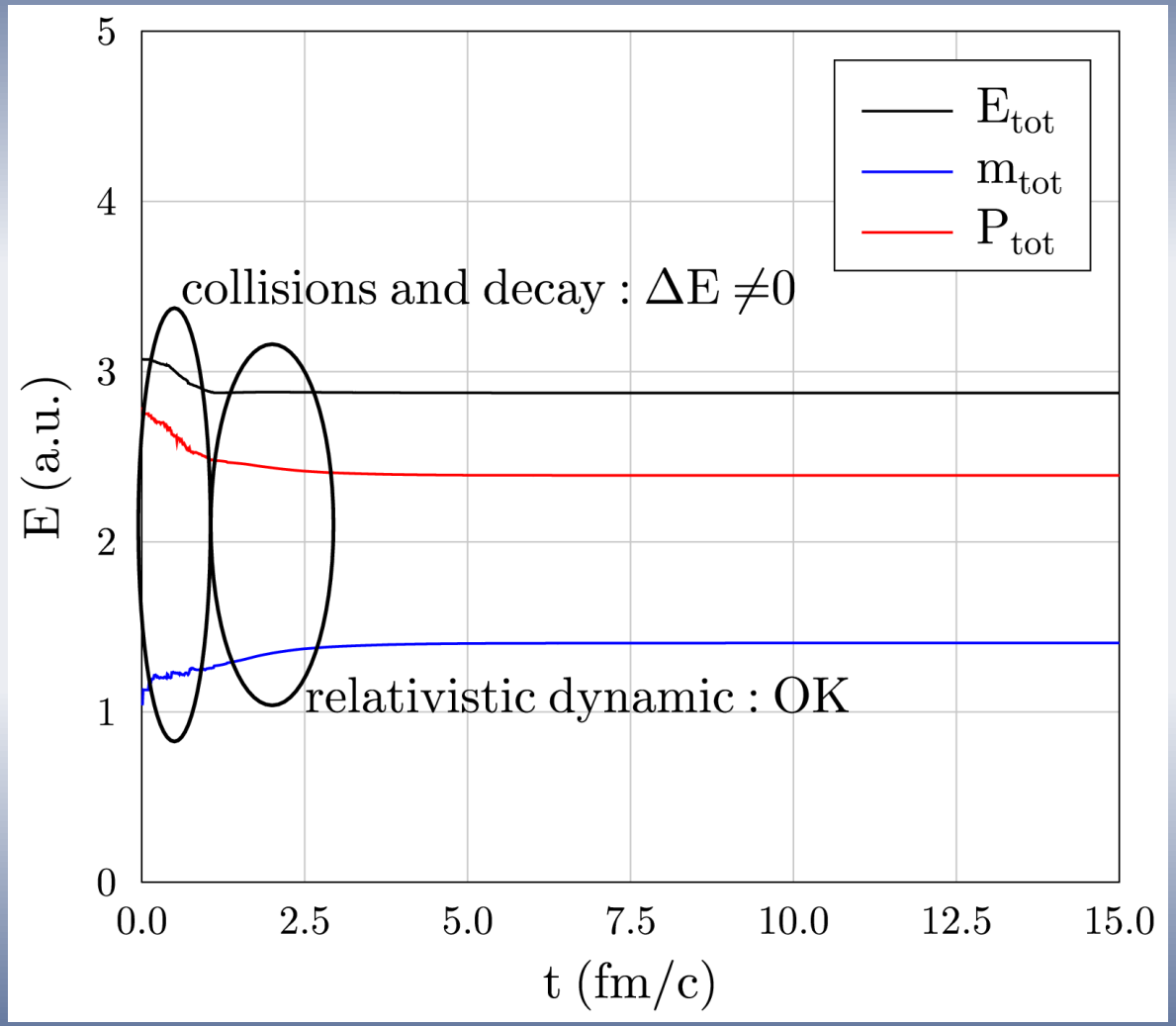
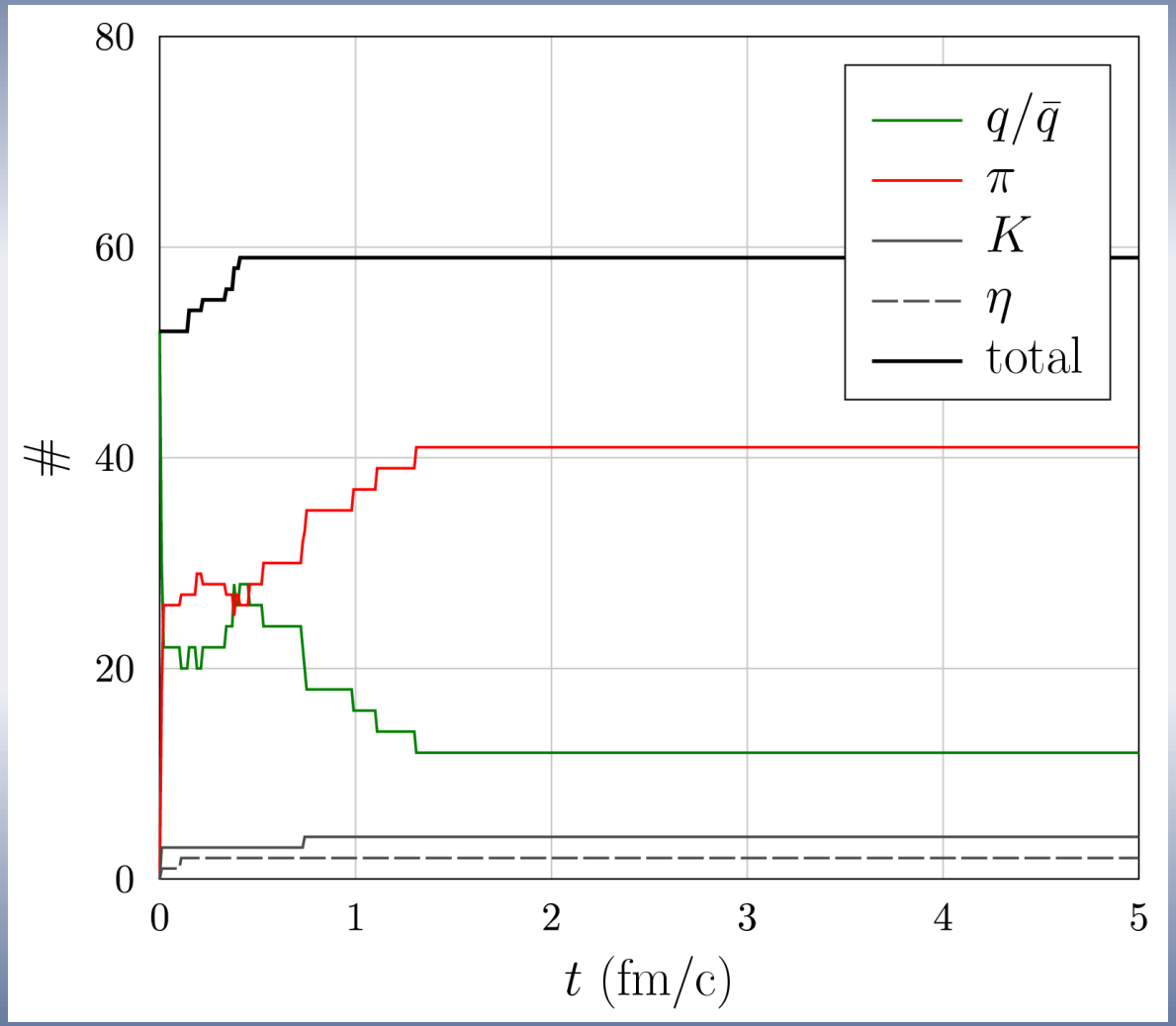


Multiplicity



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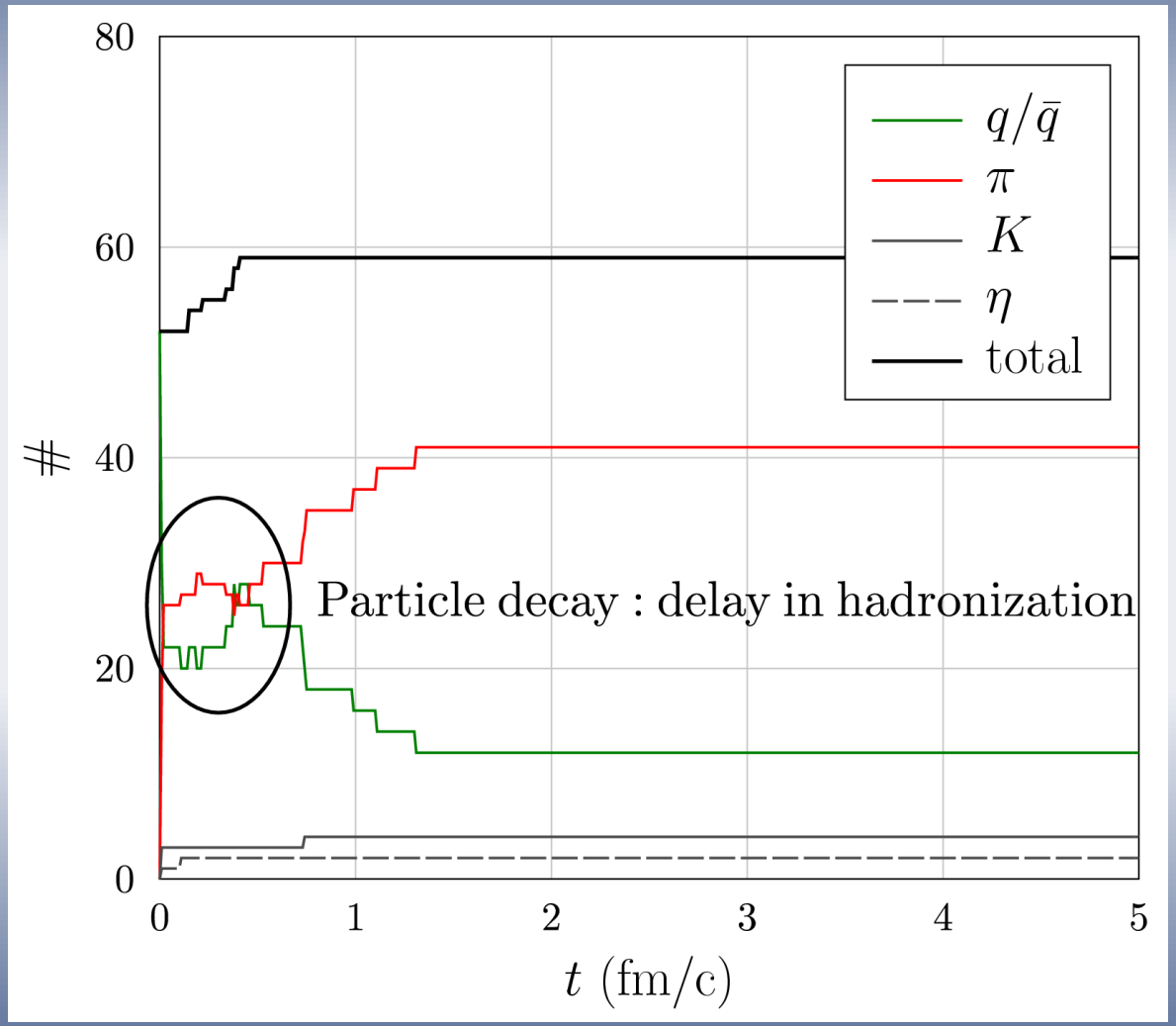
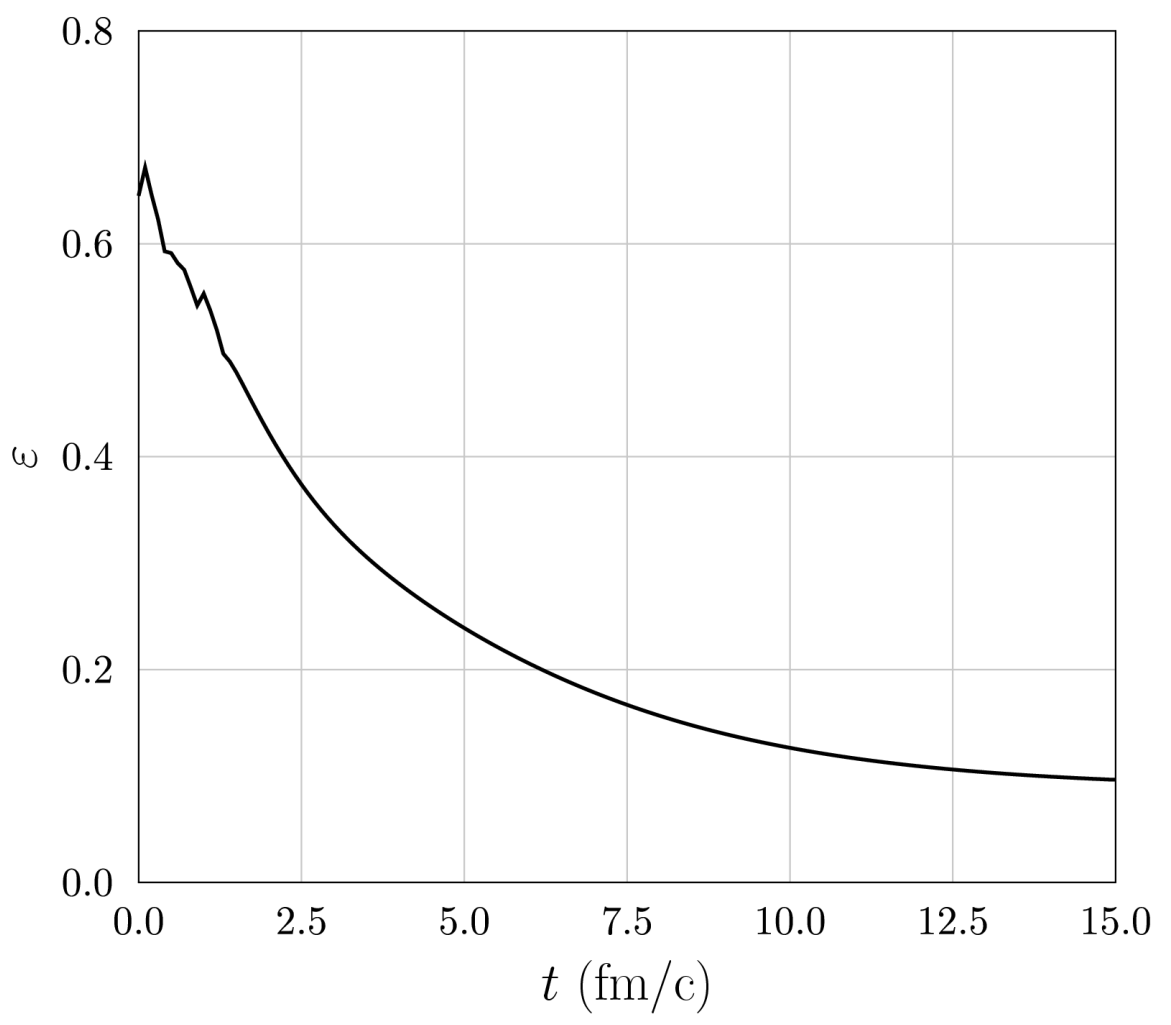
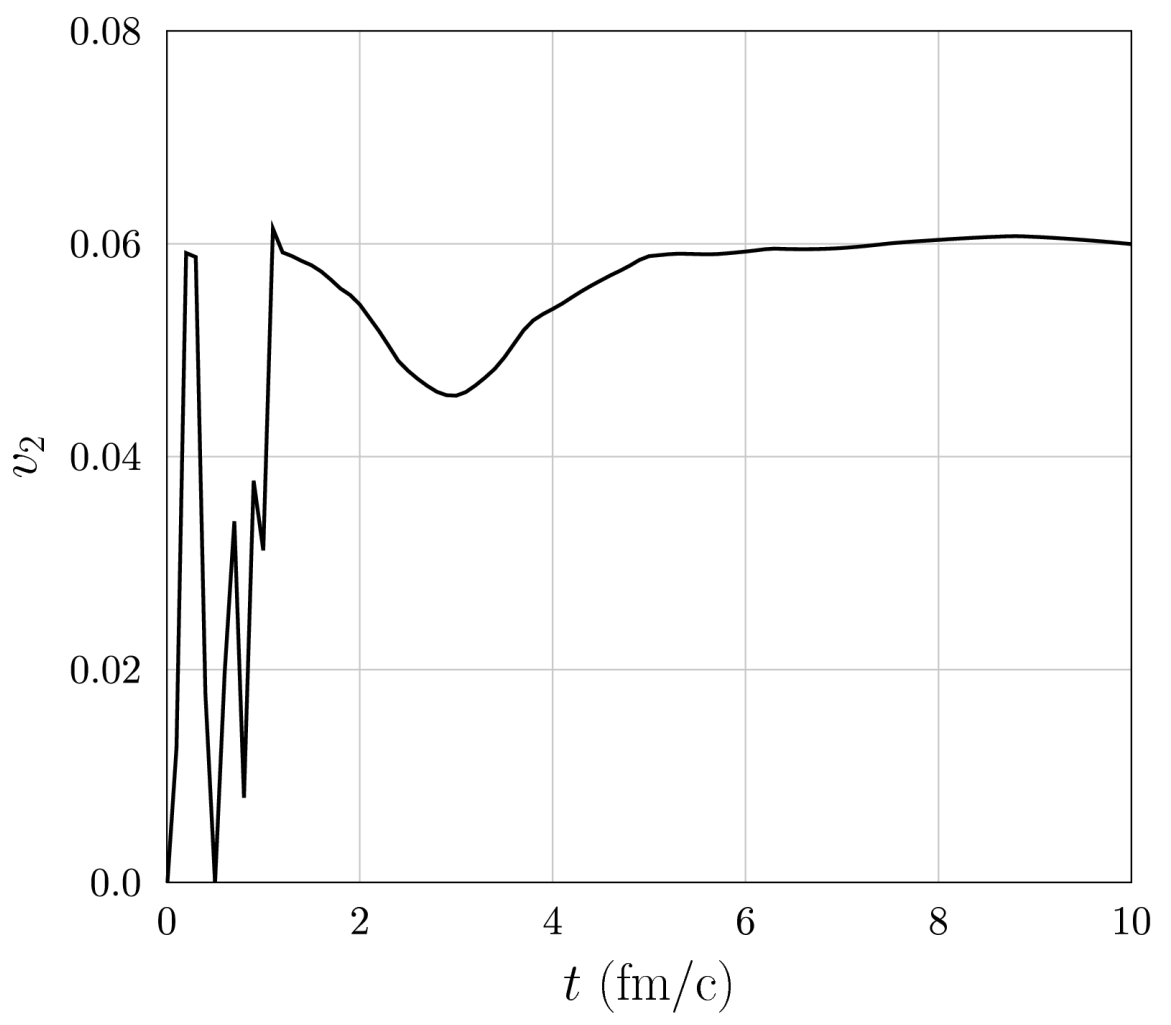


Eccentricity



Elliptic flow



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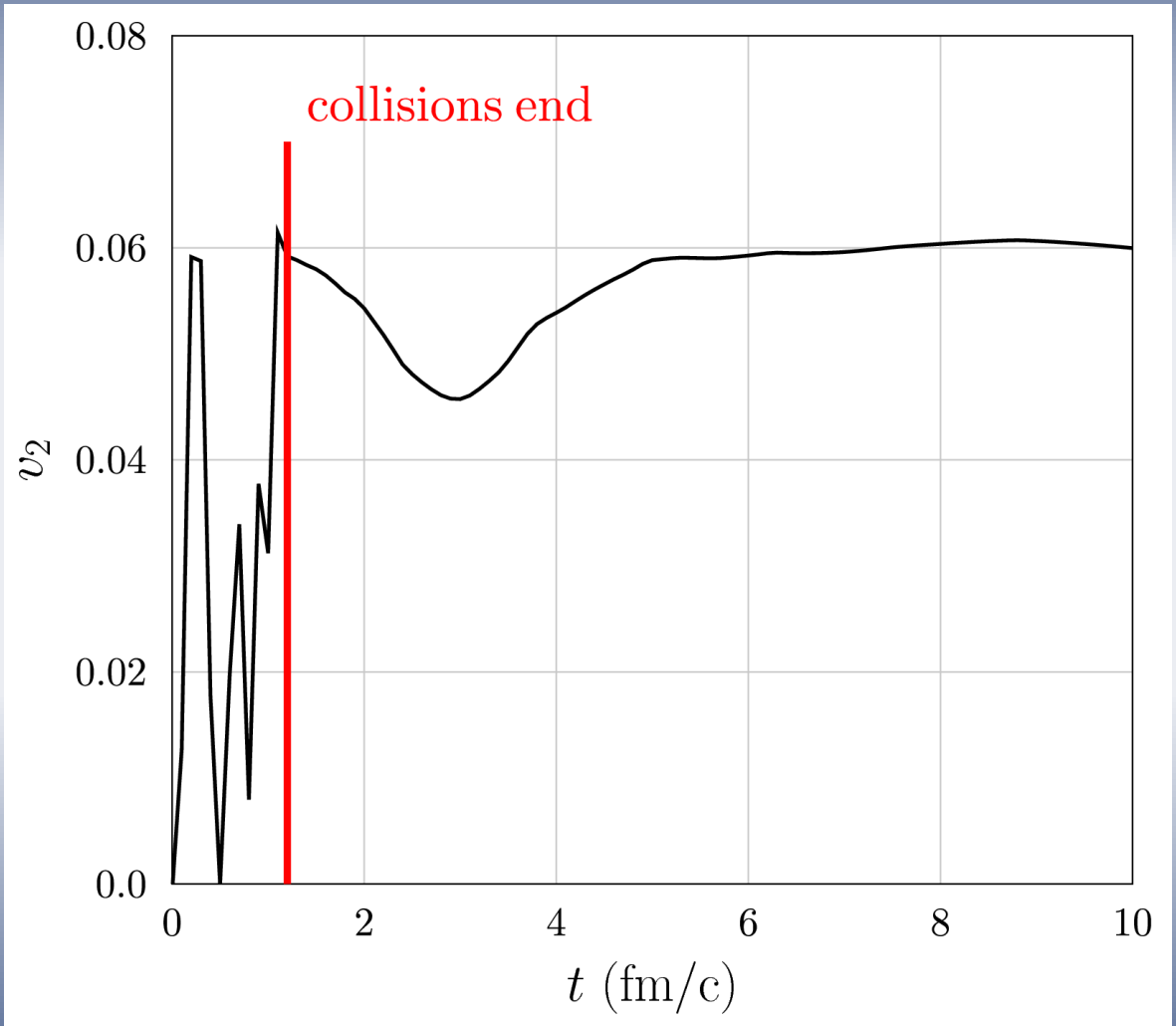

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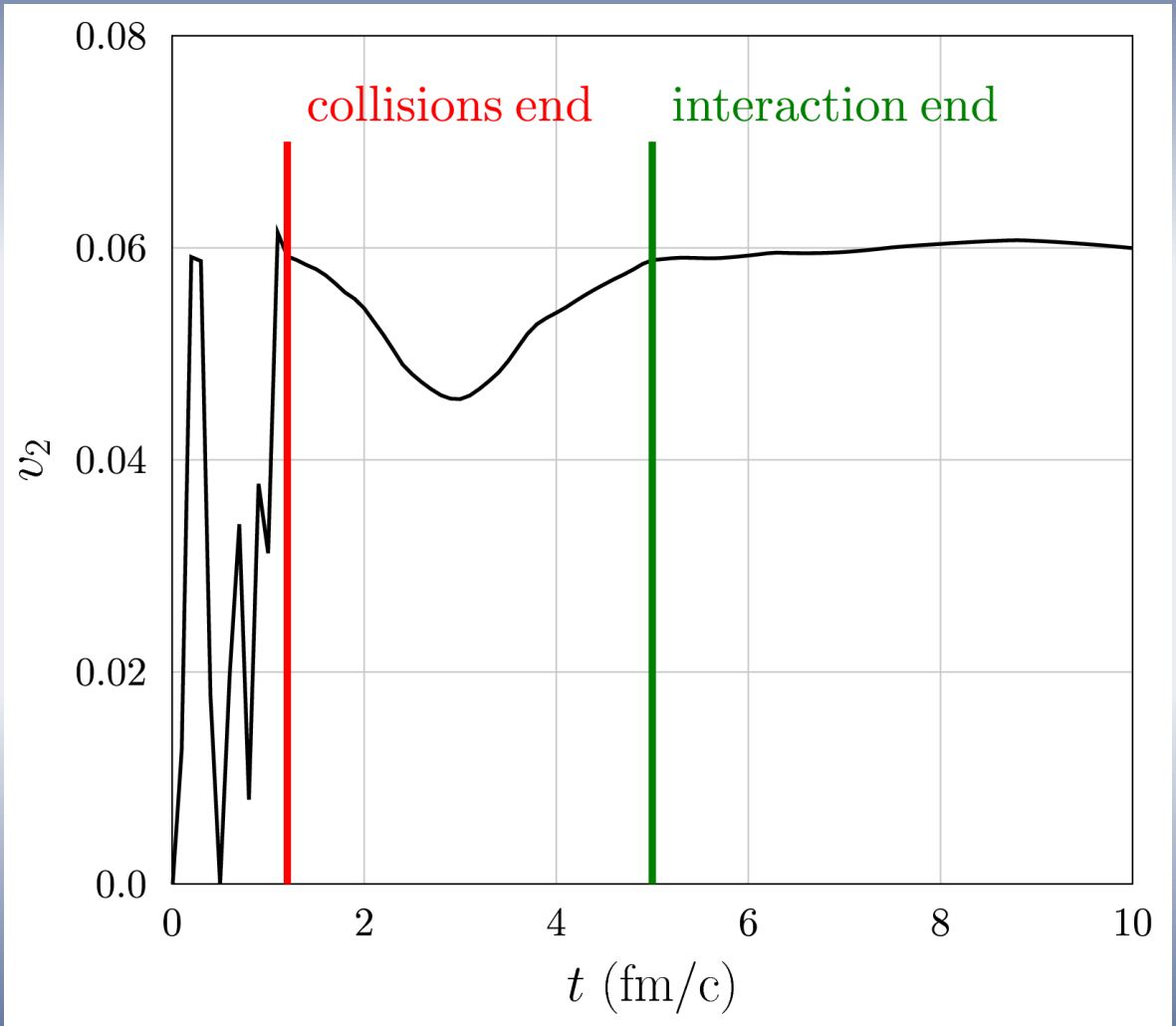
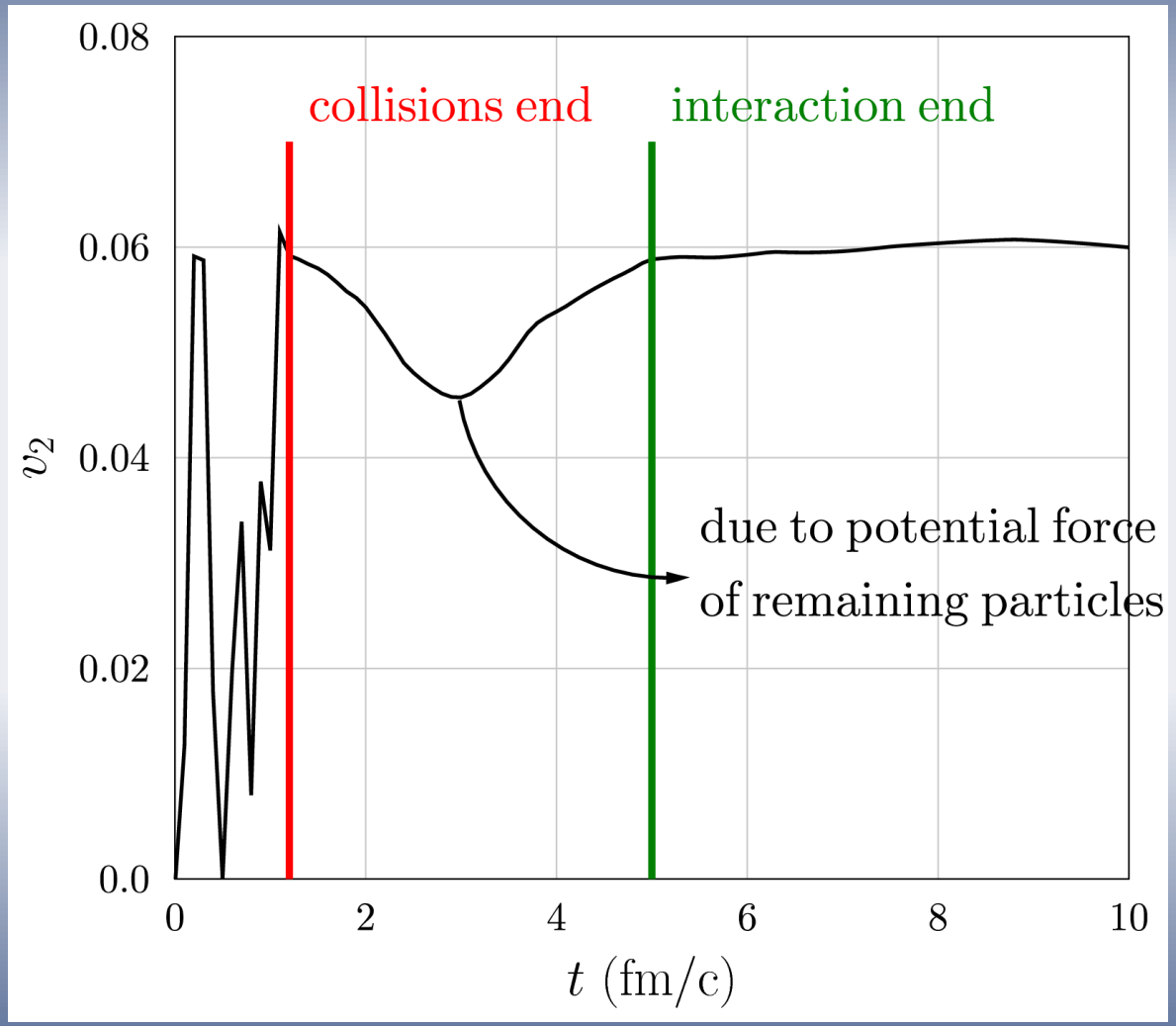


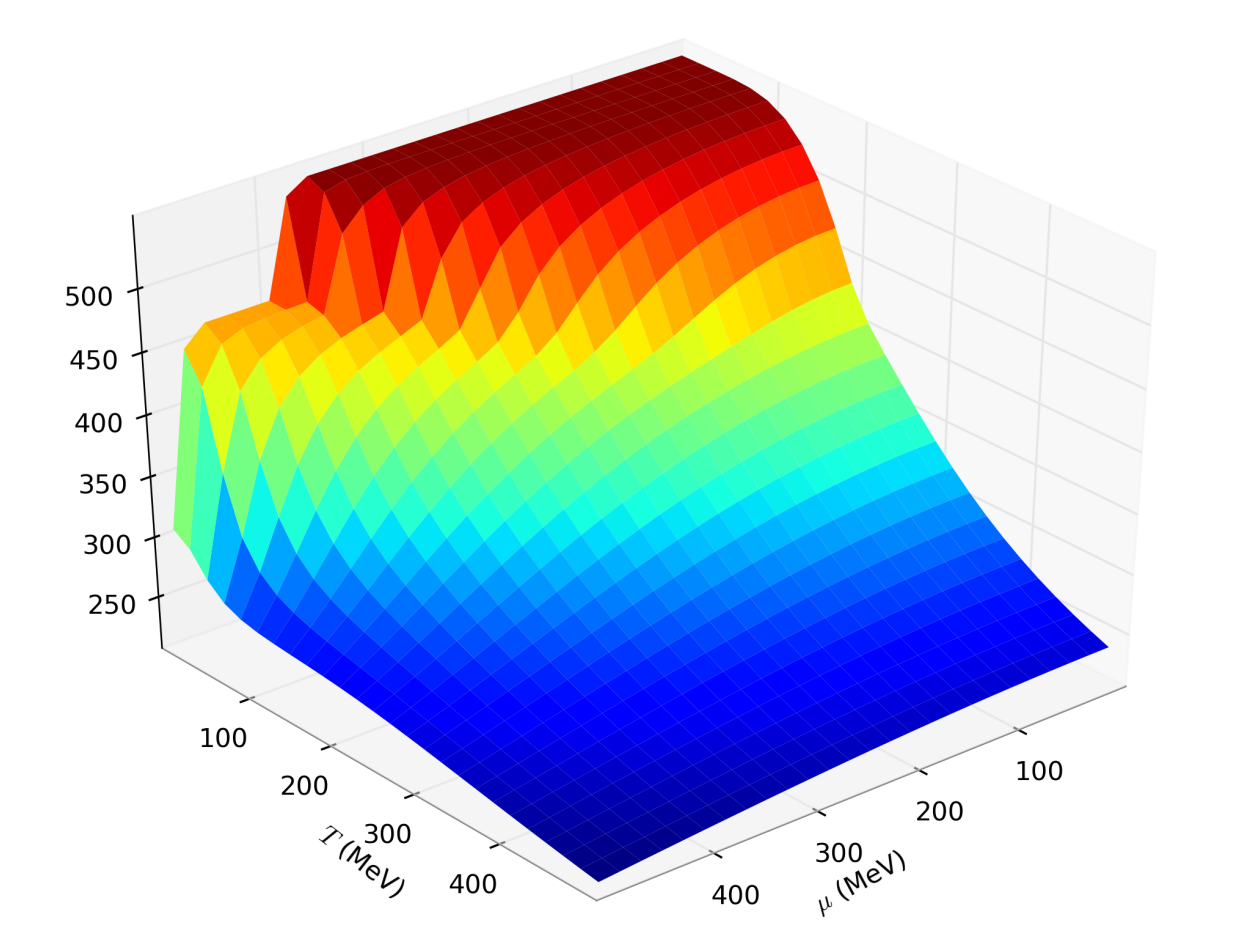
Elliptic flow



Mesons production

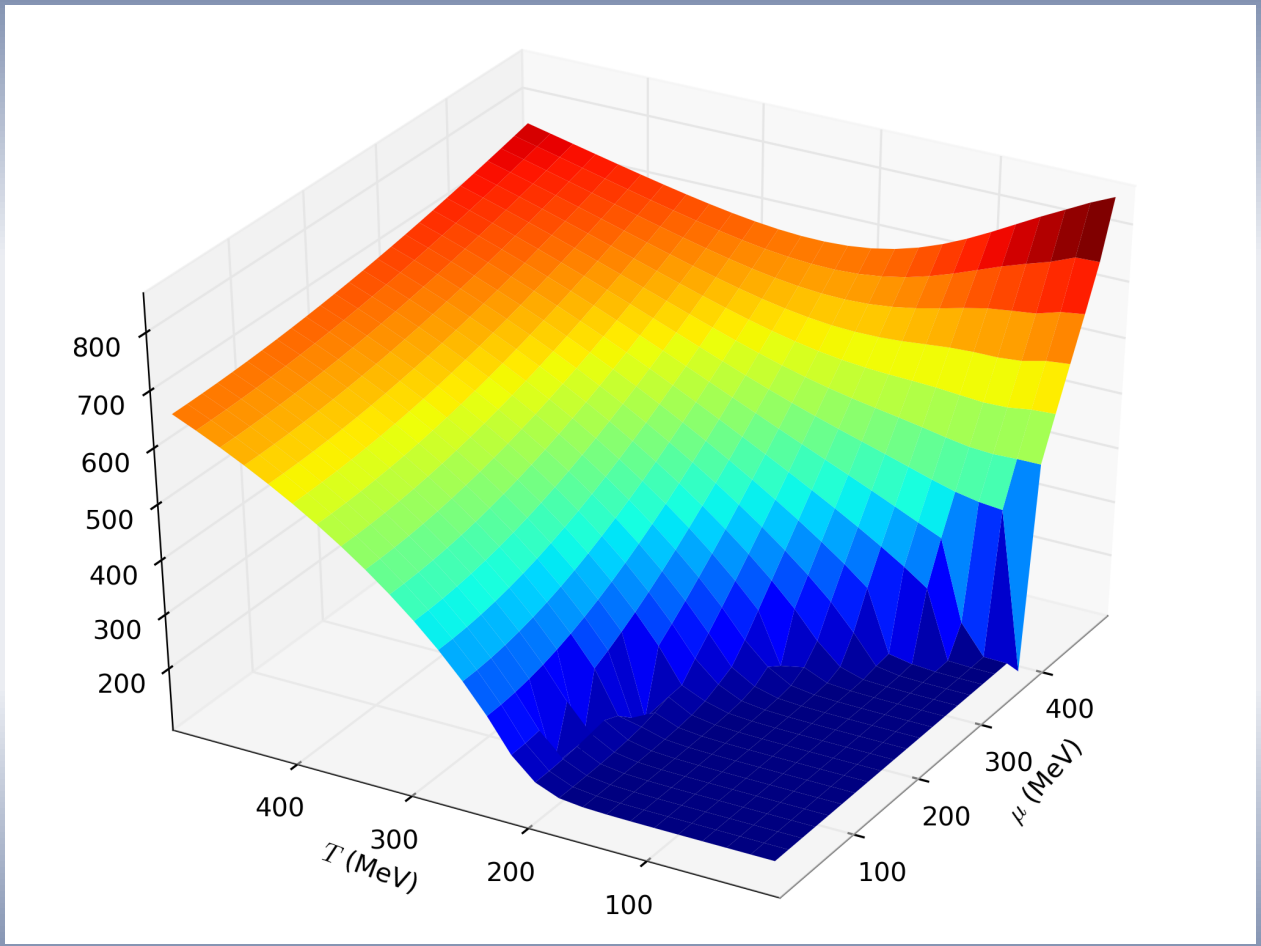
I do not have enough statistic to show the temperature creation of the pion but I can show this from an article (Blanquier, 2005) :

Masses at finite (T, μ)



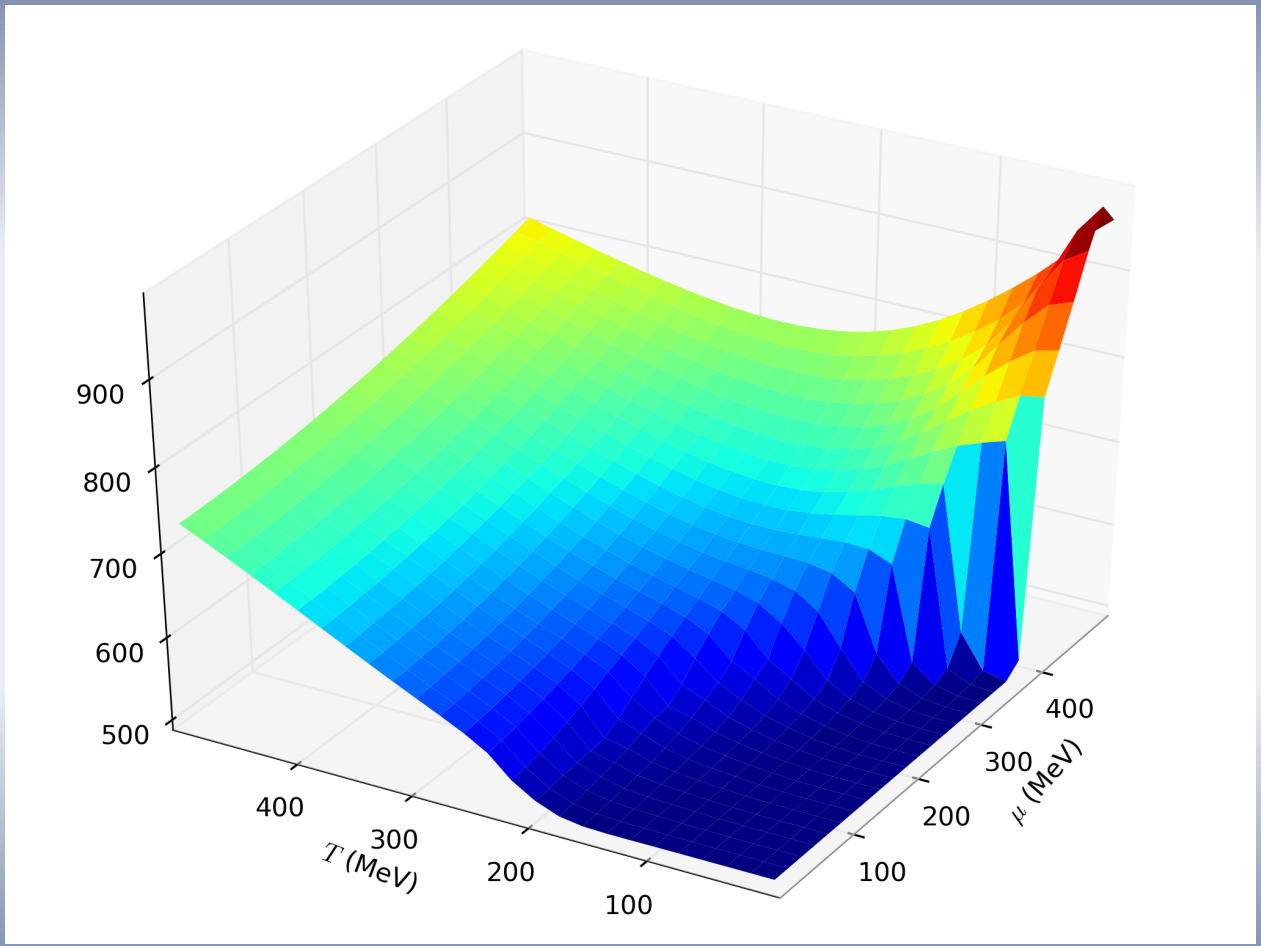
Mass s

Masses at finite (T, μ)



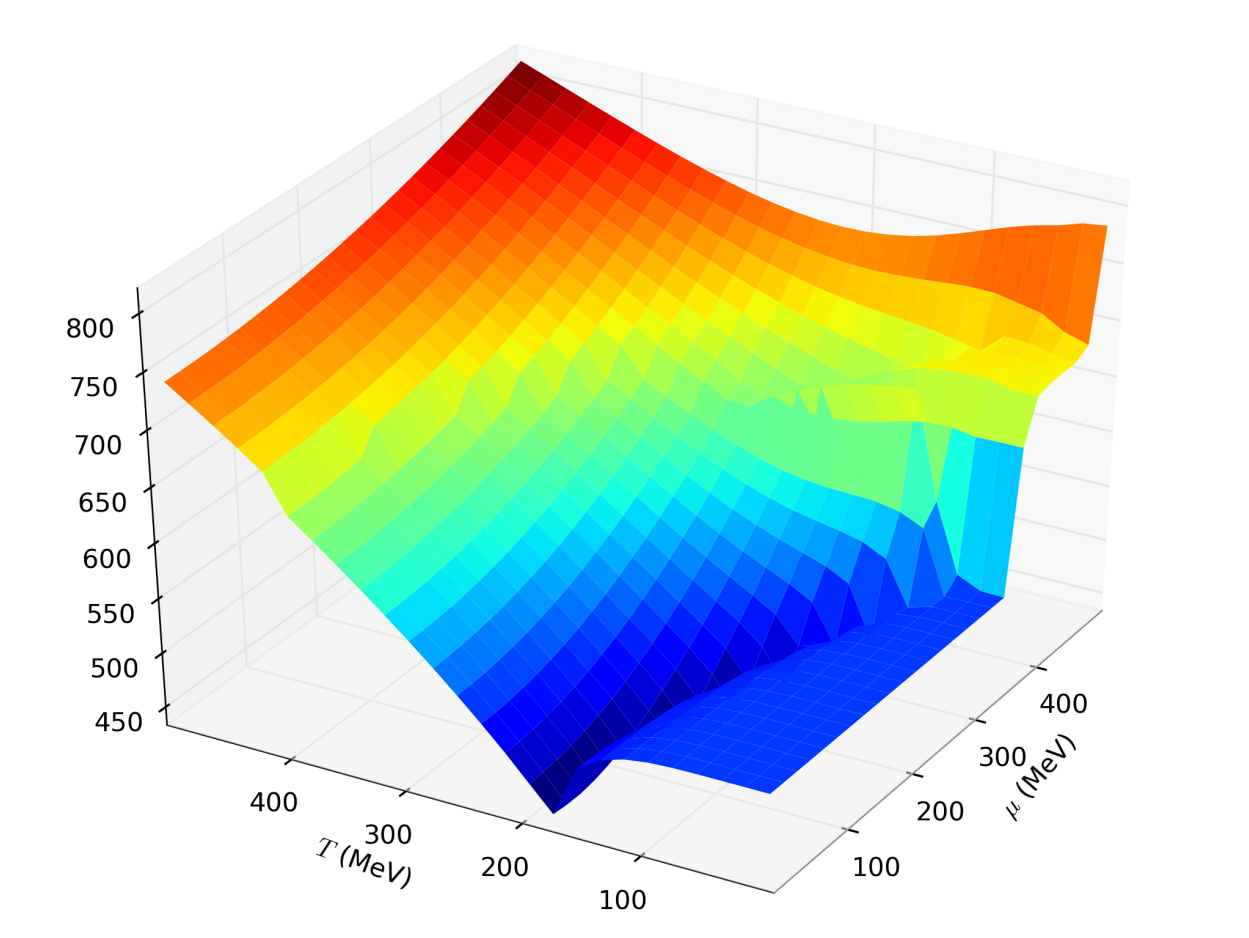
Mass π

Masses at finite (T, μ)



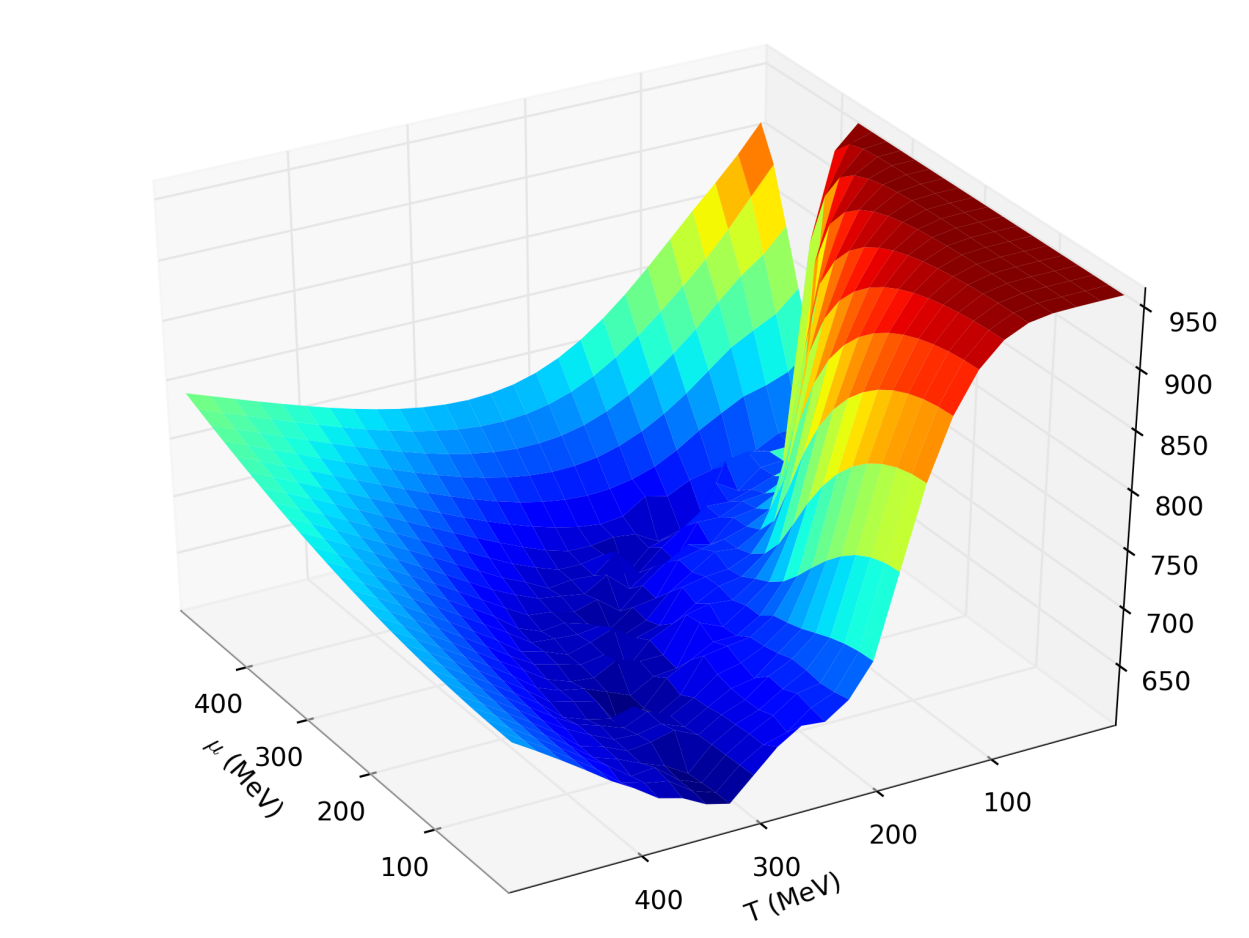
Mass K

Masses at finite (T, μ)



Mass η

Masses at finite (T, μ)



Mass η'