Non-equilibrium fluctuations at the QCD phase transition

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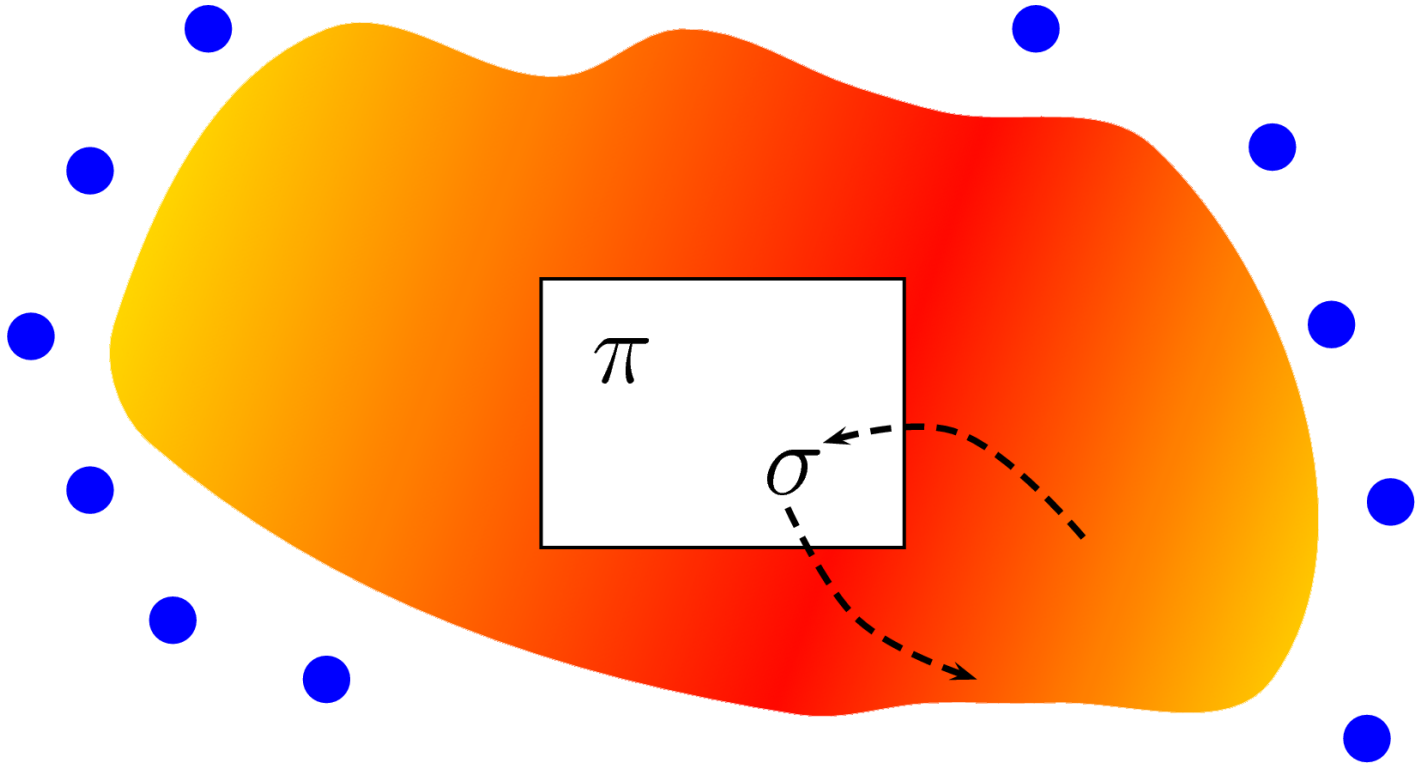
with Marcus Bleicher, Carsten Greiner, Stefan Leupold (Uppsala), Igor Mishustin

Hadronization workshop, 2010

Chiral fluid dynamics

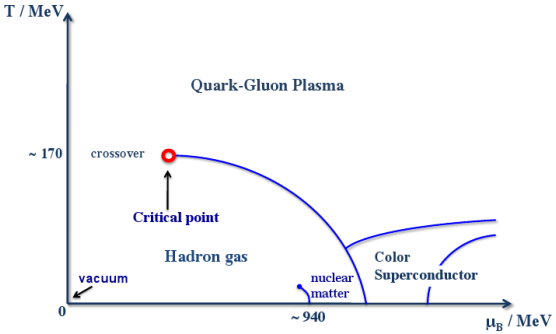


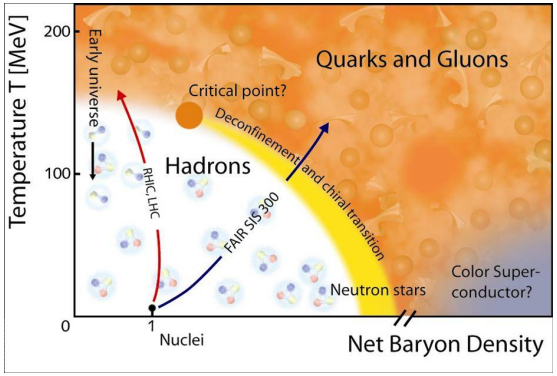
Fluid dynamics + phase transition model + dissipation and noise

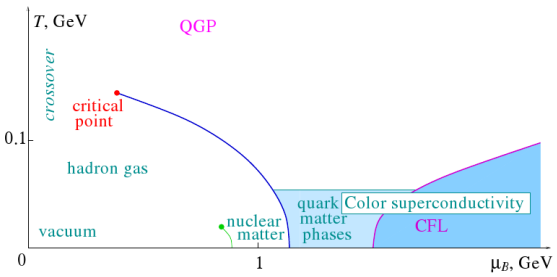
(I. N. Mishustin and O. Scavenius, PRL **83** (1999)

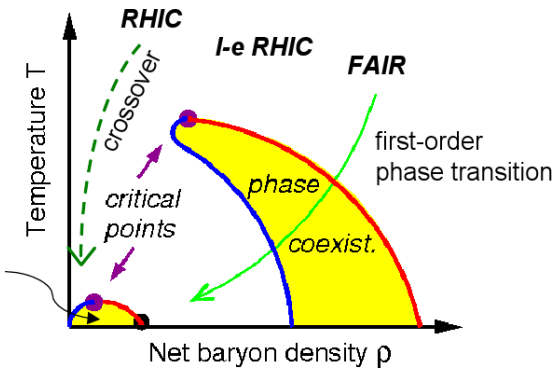
K. Paech, H. Stoecker and A. Dumitru, PRC **68** (2003))

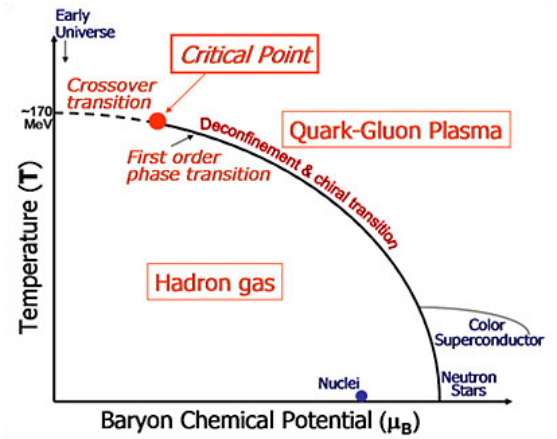
QCD phase diagrams

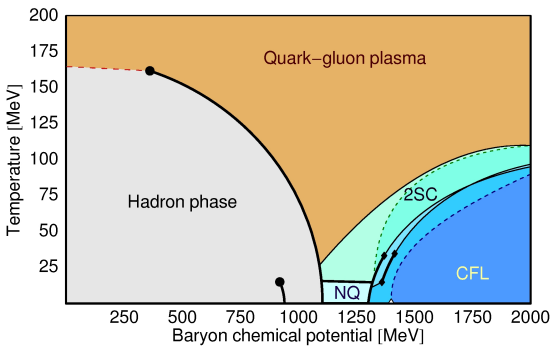


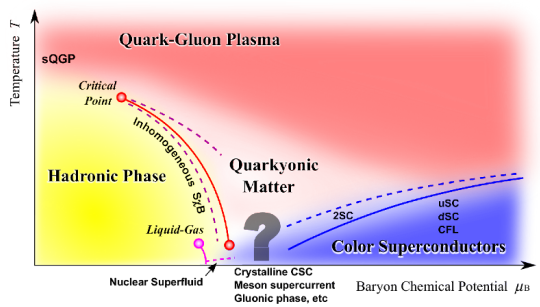


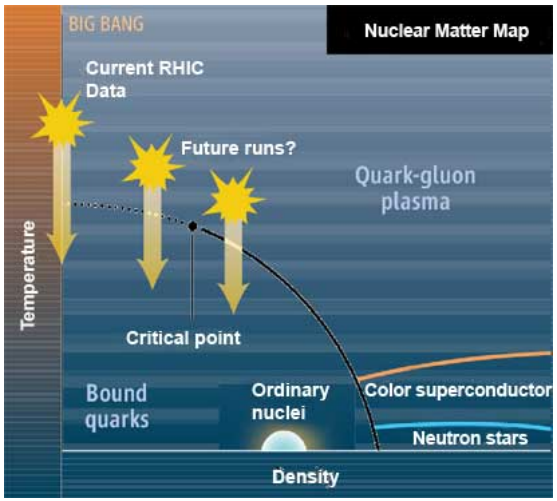


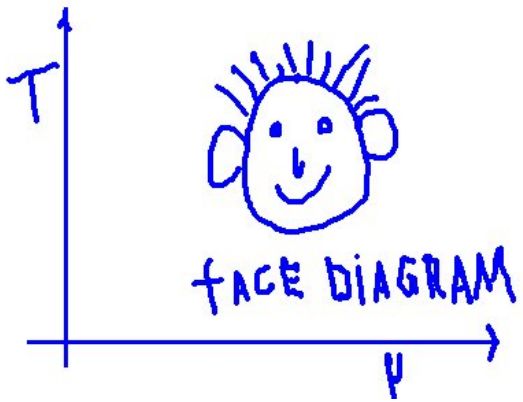






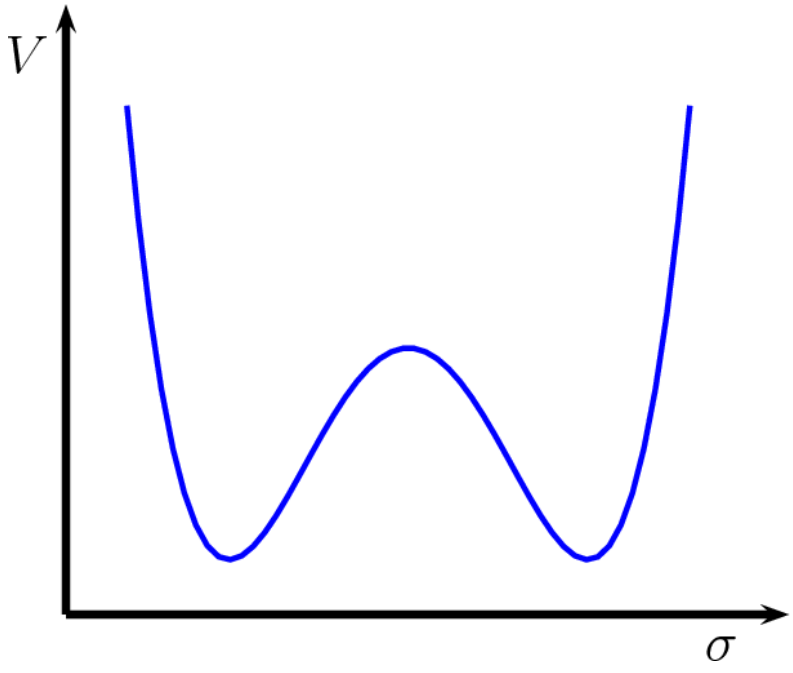


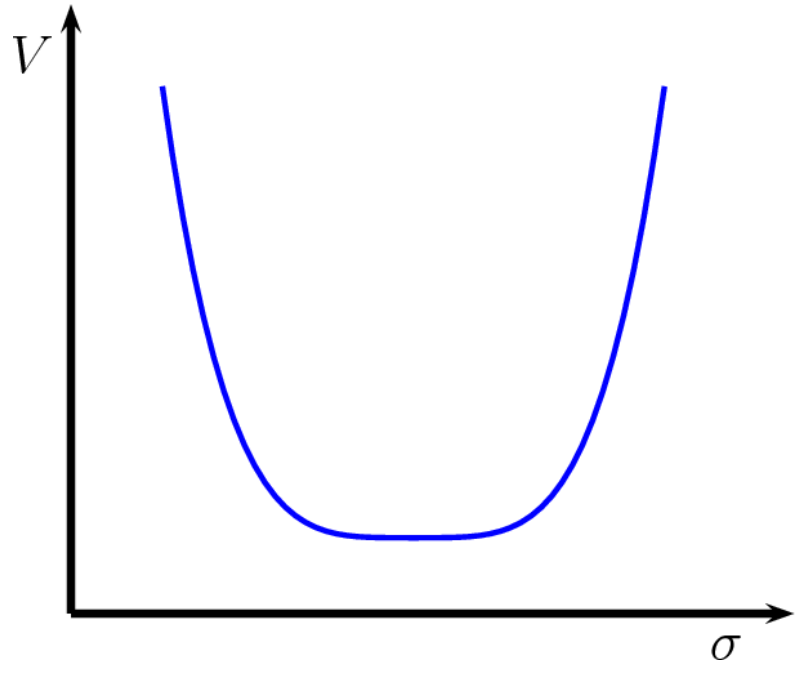




first order phase transition

critical point





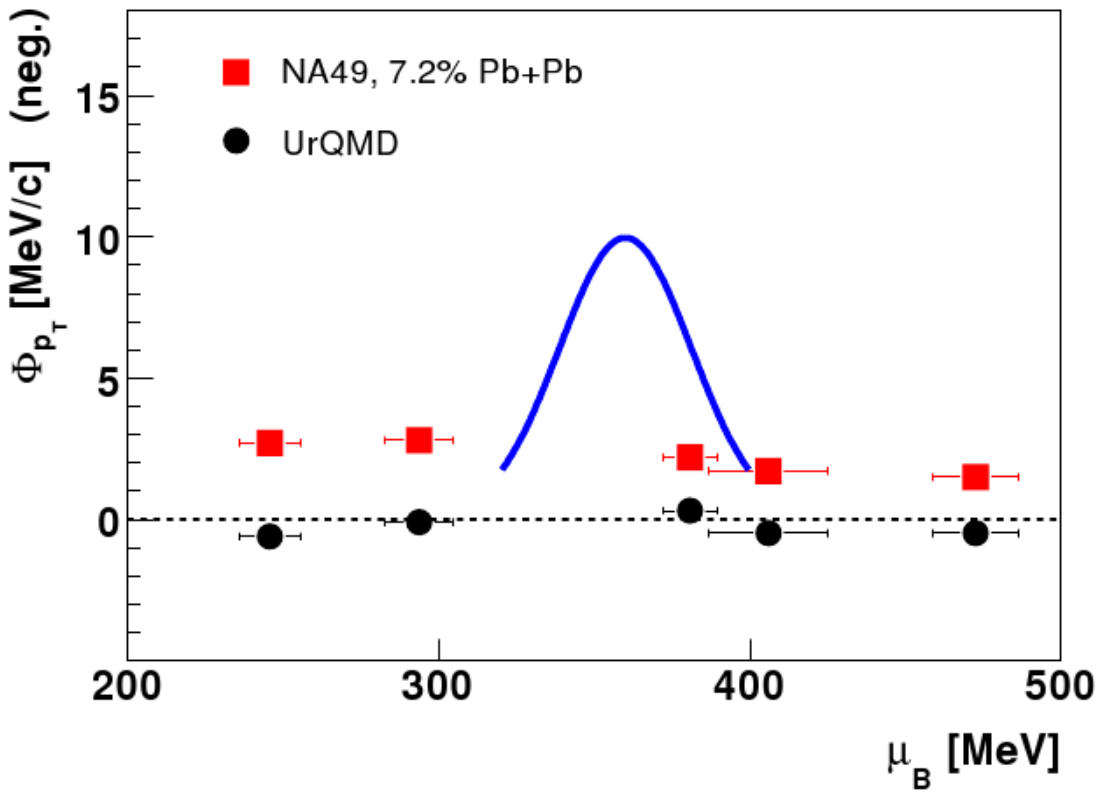
spinodal decomposition

critical phenomena

(I. N. Mishustin, PRL **82** (1999),
J. Randrup, PRL **92** (2004))

Critical phenomena

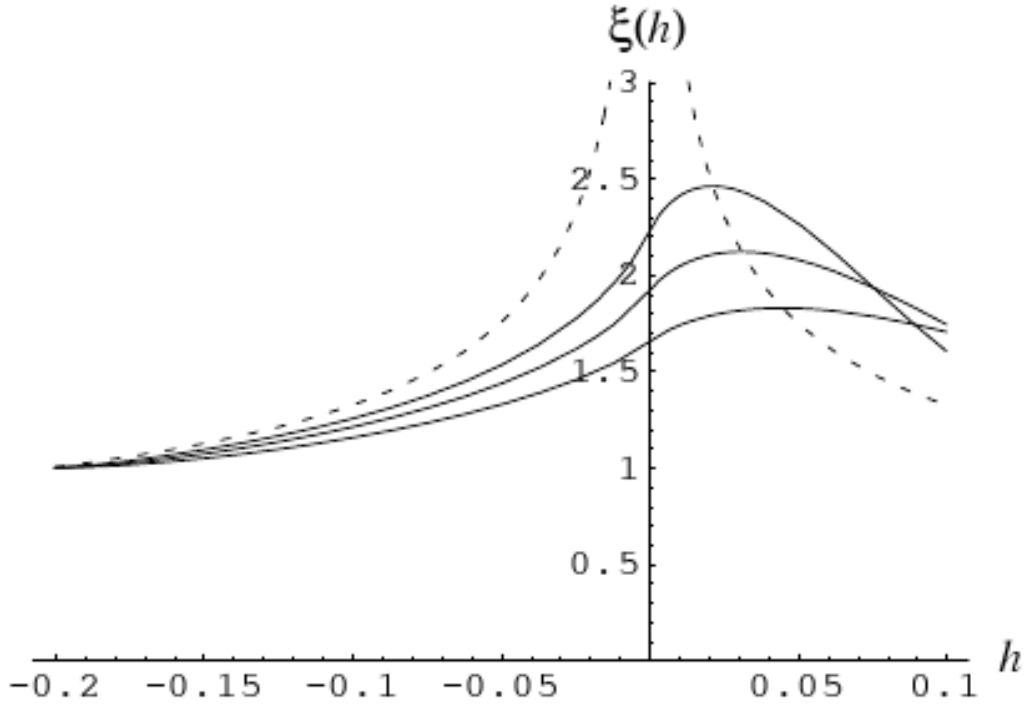
$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$



(M. A. Stephanov, K. Rajagopal and E. V. Shuryak, PRD **60** (1999), NA49 collaboration J. Phys. G **35** (2008))

The critical point in dynamic systems

ξ : correlation length, $\quad h = \frac{T_C - T}{T_C}$



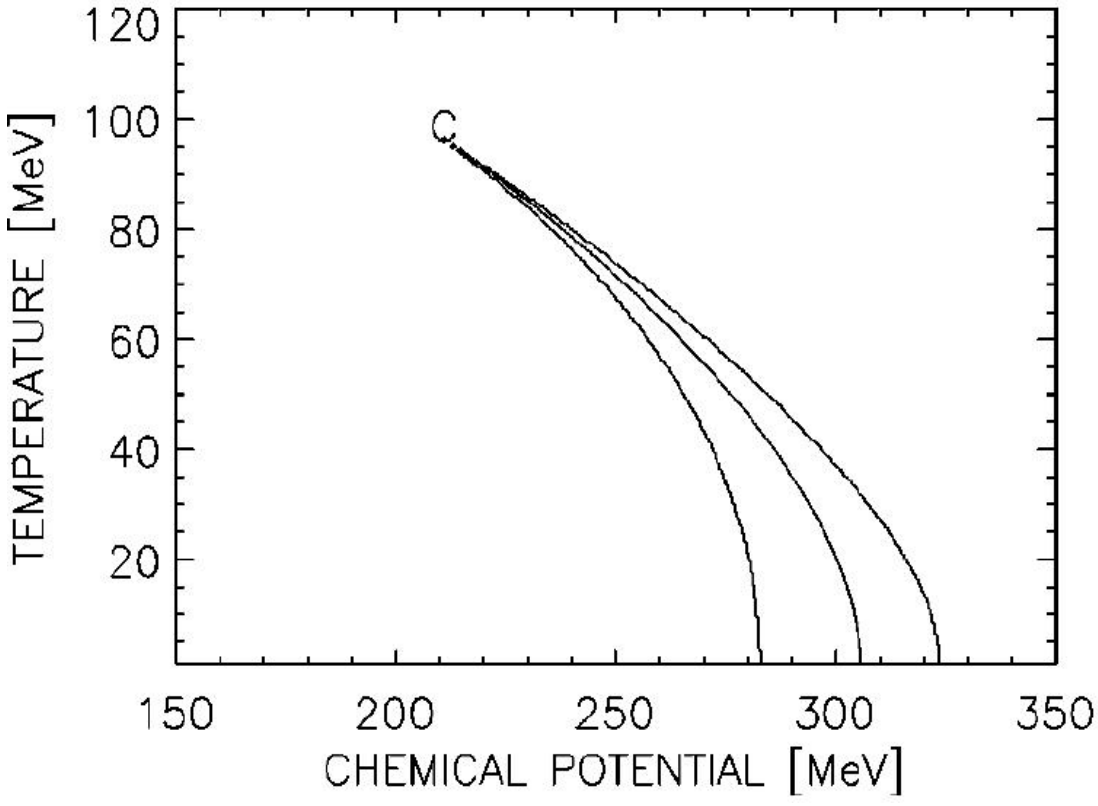
(B. Berdnikov and K. Rajagopal, PRD **61** (2000))

Linear sigma model with constituent quarks

$$\begin{aligned}\mathcal{L} =& \bar{q}(i\gamma^{\mu}\partial_{\mu} - g(\sigma + i\gamma_5\tau\vec{\pi}))q \\ &+ \frac{1}{2}(\partial_{\mu}\sigma)^2 + \frac{1}{2}(\partial_{\mu}\vec{\pi})^2 \\ &- U(\sigma, \vec{\pi})\end{aligned}$$

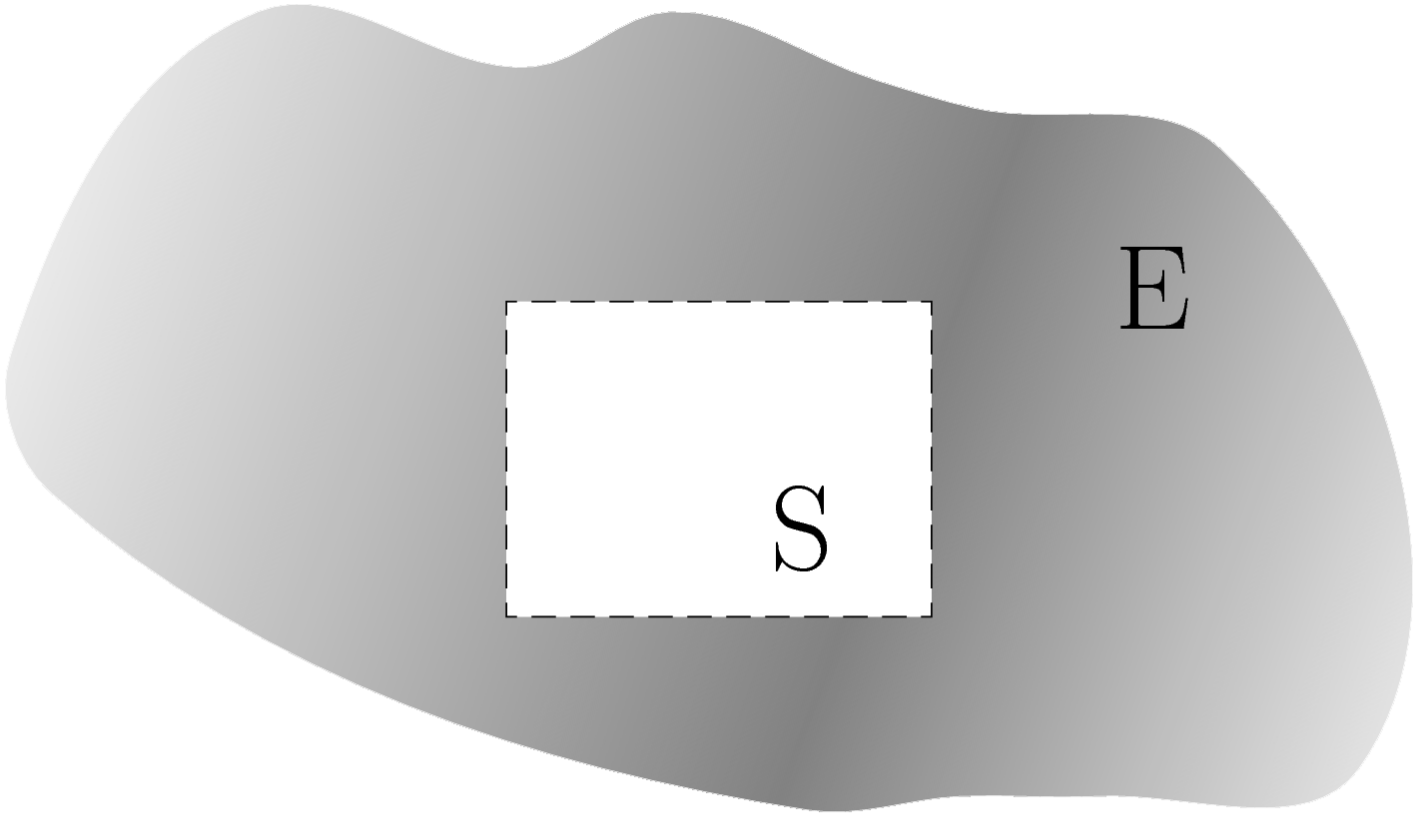
$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \vec{\pi}^2 - \nu^2)^2 - h_q\sigma - U_0$$

Phase diagram of the linear sigma model



(O. Scavenius, A. Mocsy, I. N. Mishustin and D. H. Rischke, PRC **64** (2001))

Influence functional method



$$S[x,q] = S_{\mathrm{S}}[x] + S_{\mathrm{E}}[q] + S_{\mathrm{int}}[x,q]$$

Classical equation of motion for the sigma field

fluctuations

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g \langle \bar{q} q \rangle_\sigma = \xi$$

interaction between the field and the heat bath

Chiral condensate - lowest order

$$\langle \bar{q}q \rangle_\sigma^{(0)} = 2 d_q m_q \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{n_\mathrm{F}(p)}{E}$$

$$E = \sqrt{p^2 + g^2\sigma^2} \quad , \quad m_q = g\sigma$$

equation of motion for the sigma field:

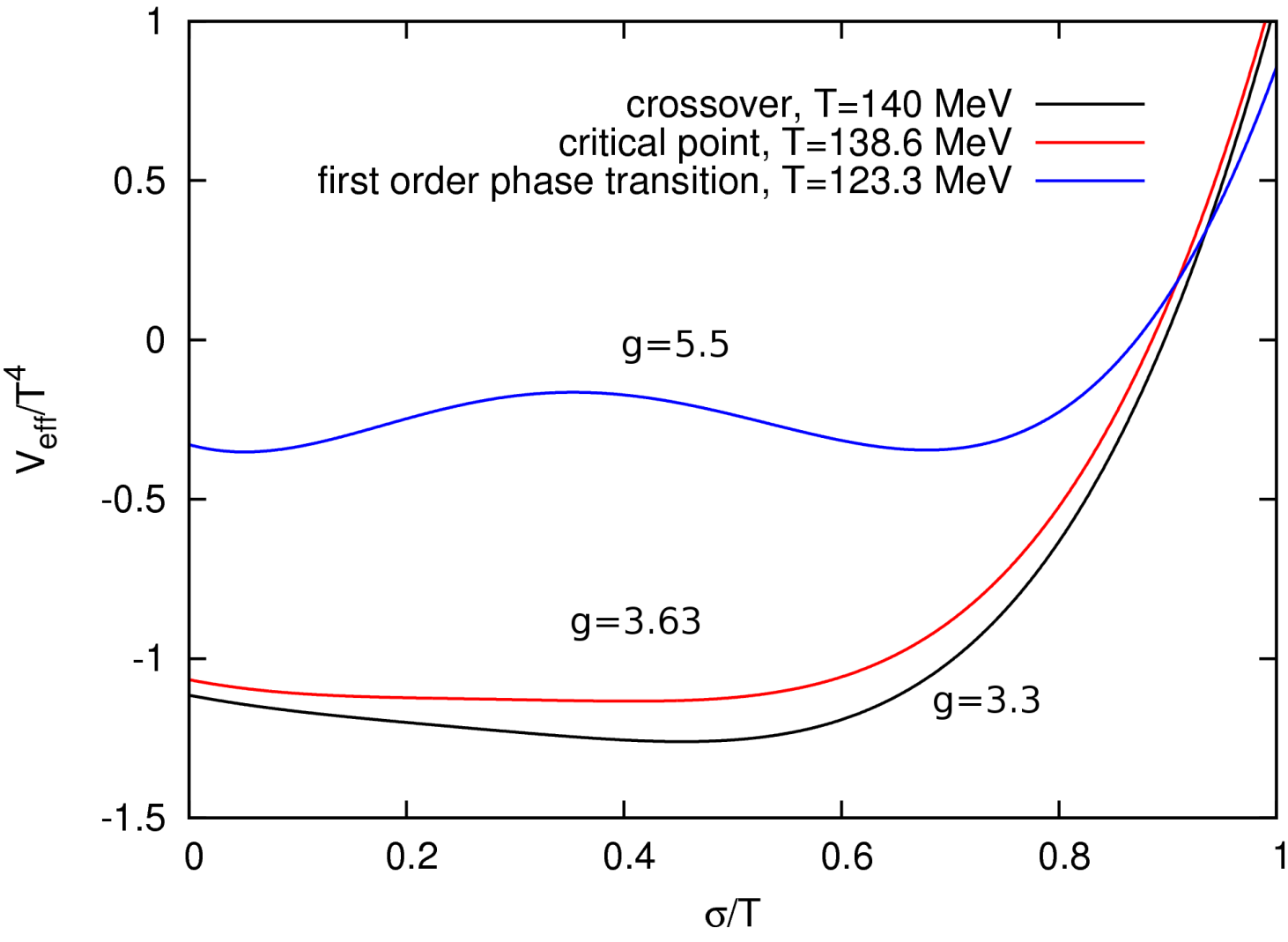
$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g \langle \bar{q}q \rangle_0 = 0$$

Effective potential - lowest order

$$\mathcal{Z} = \int \mathcal{D}\overline{q}\mathcal{D}q\mathcal{D}\sigma\mathcal{D}\vec{\pi}\,\exp\left[\int_0^{1/T} \mathrm{d(it)} \int_V \mathrm{d}^3x\mathcal{L}\right]$$

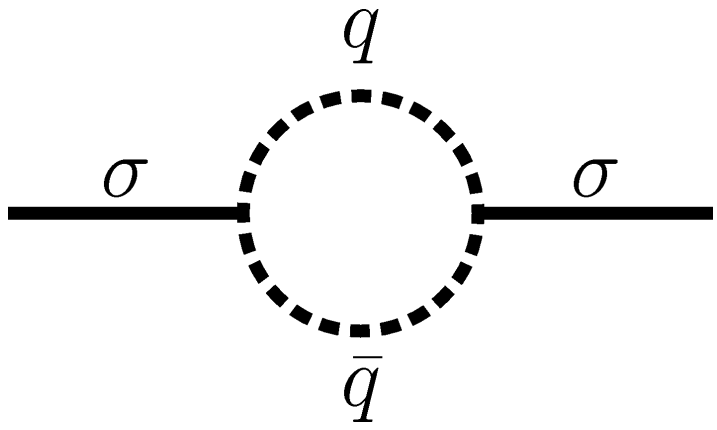
$$V_{\mathrm{eff}} = -\frac{T}{V}\log\mathcal{Z} = -2d_qT\int\frac{\mathrm{d}^3p}{(2\pi)^3}\log(1+\mathrm{e}^{-E/T}) + U(\sigma,\vec{\pi})$$

Effective potential - lowest order



Chiral condensate - first order

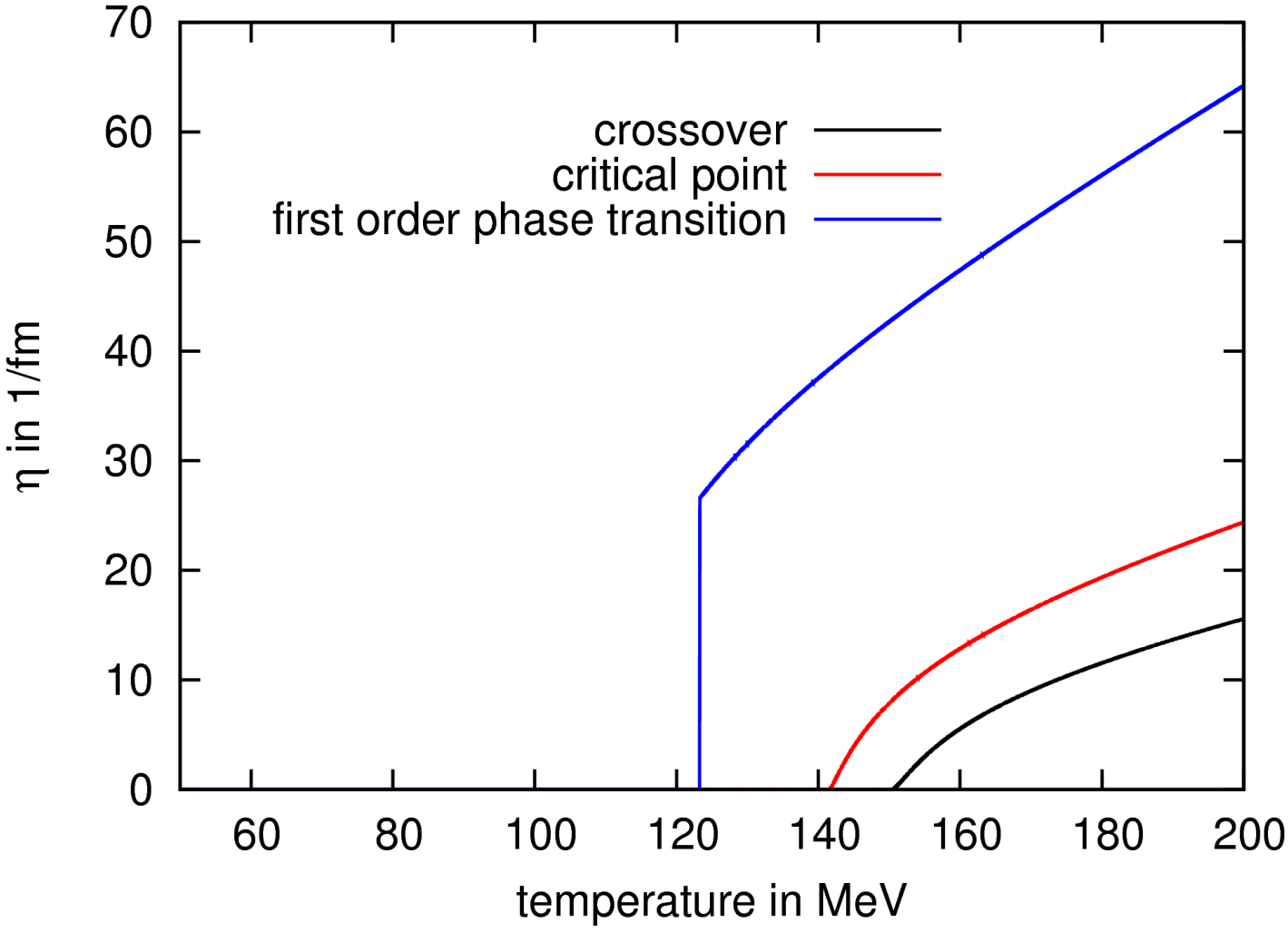
damping term η and noise ξ for $\mathbf{k} = 0$



$$\eta = g^2 \frac{d_q}{\pi} \left(1 - 2 n_{\mathrm{F}}\left(\frac{m_\sigma}{2}\right)\right) \frac{\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{\frac{3}{2}}}{m_\sigma^2}$$

$$\langle \xi(t) \xi(t') \rangle_\xi = \frac{1}{V} \delta(t - t') m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right)$$

Damping term η



large compared to $\eta = 2.2/\mathrm{fm}$

(T. S. Biro and C. Greiner, PRL **79** (1997))

Equation of motion for the sigma field

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g \langle \bar{q} q \rangle_\sigma^{(0)} + \eta \partial_t \sigma = \xi$$

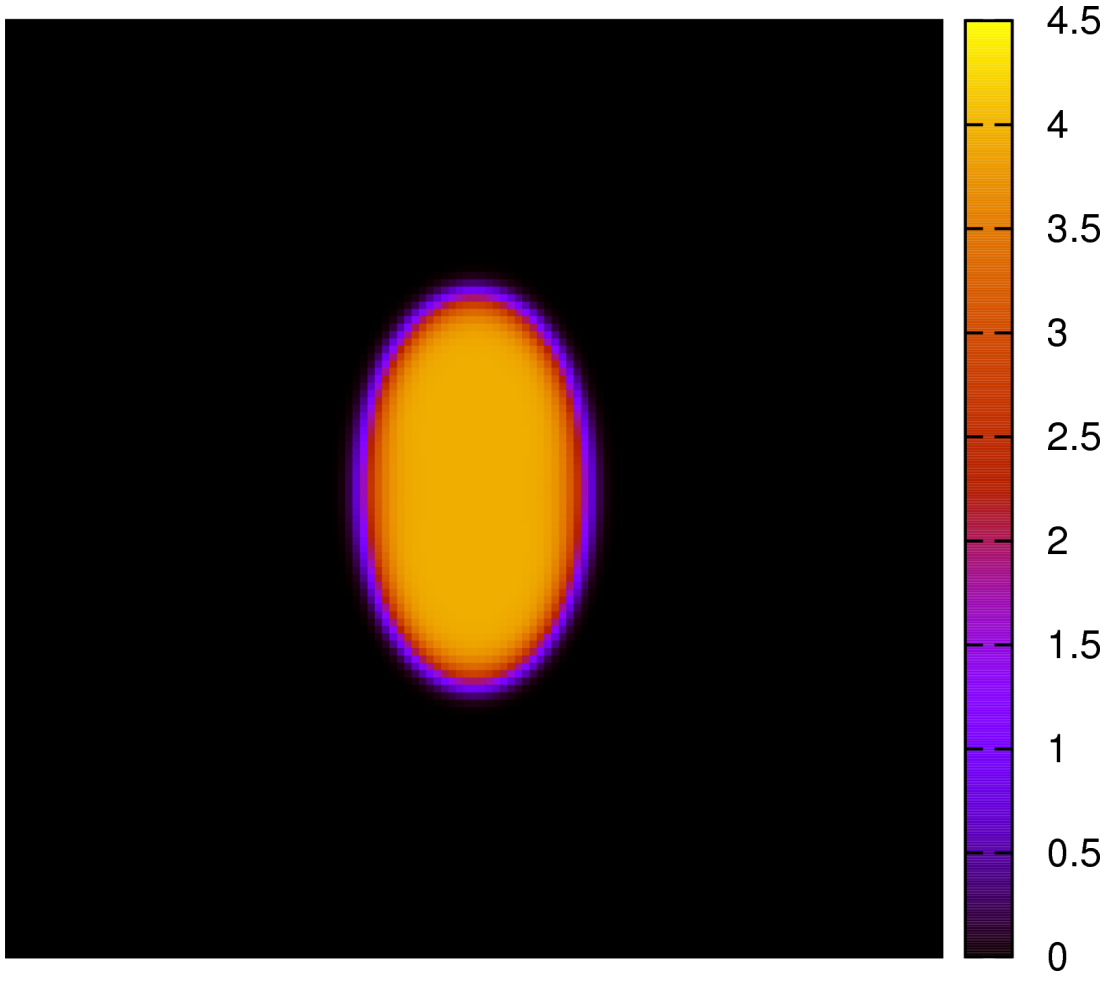
$$\begin{aligned} \eta &= 20/\mathrm{fm} \\ \langle \xi(t) \rangle &= 0 \\ \langle \xi(t) \xi(t') \rangle &= \frac{2T}{V} \eta \delta(t - t') \end{aligned}$$

Stochastic source term

$$\partial_\mu T^{\mu\nu} = S_\nu$$

$$\begin{aligned} S^\nu = -\partial_\mu T^{\mu\nu}_{\text{field}} &= -\left(\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma}\right)\partial^\nu \sigma \\ &= -\left(-g\langle \bar{q} q \rangle_\sigma^{(0)} - \eta \partial_t \sigma + \xi\right)\partial^\nu \sigma \end{aligned}$$

Initial conditions



energy density in units of e_0

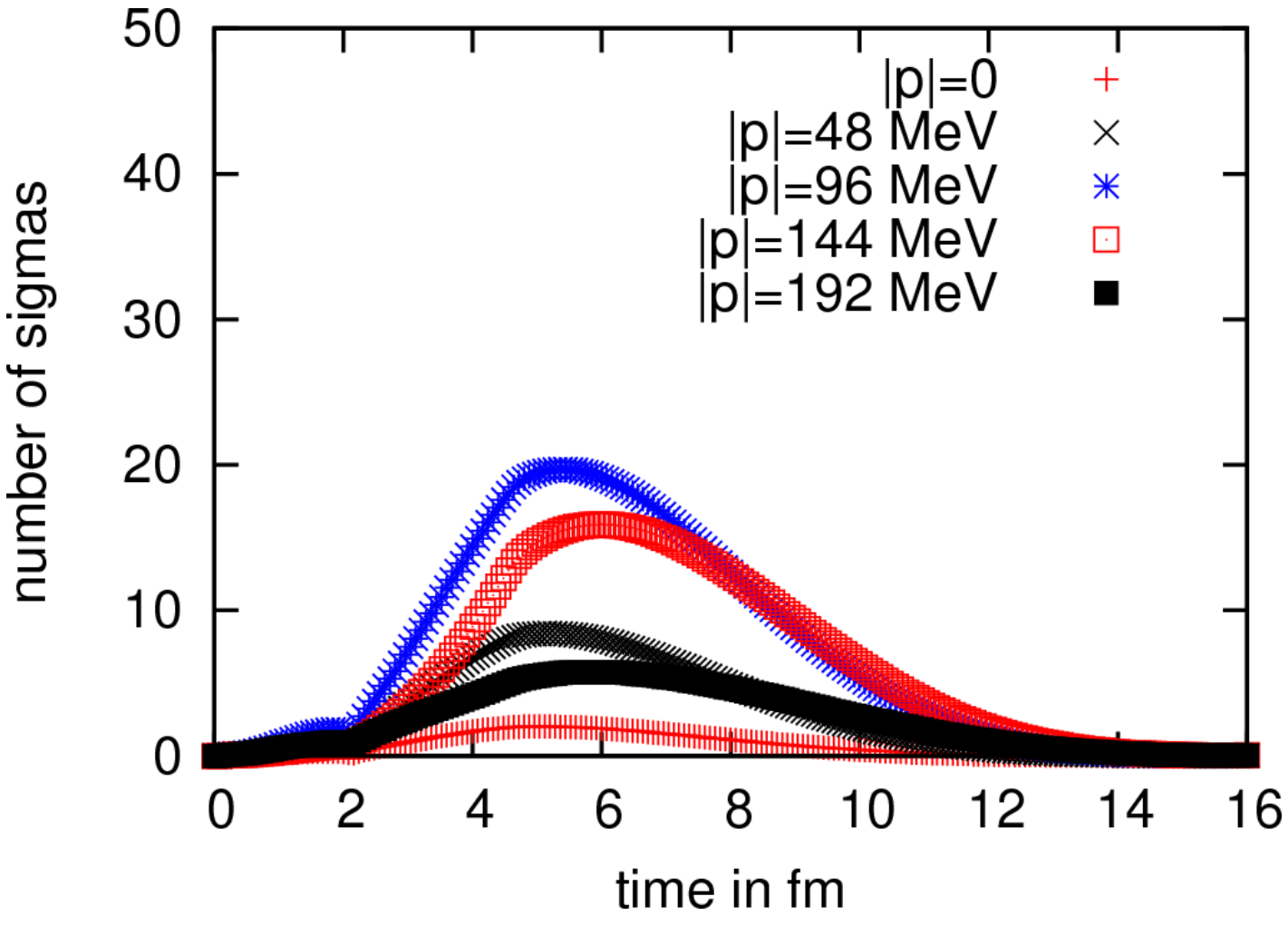
Intensity of sigma fluctuations

$$\frac{\mathrm{d}N_\sigma}{\mathrm{d}^3k} = \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \frac{1}{(2\pi)^3 2\omega_k}\left(\omega_k^2|\sigma_k|^2 + |\partial_t\sigma_k|^2\right)$$

$$\omega_k = \sqrt{|k|^2 + m_\sigma^2}$$

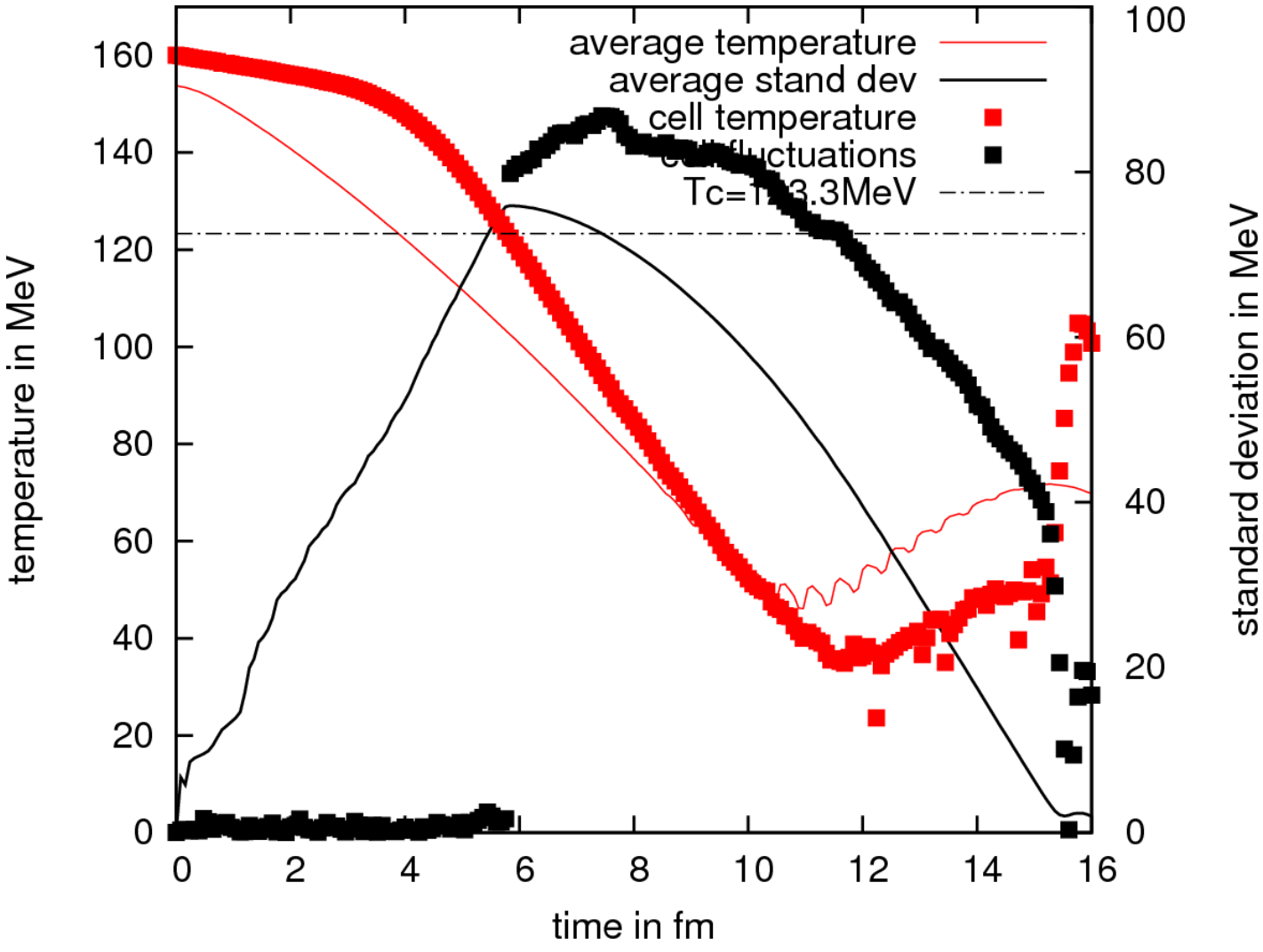
$$m_\sigma = \sqrt{\frac{\partial^2 V_{\mathrm{eff}}}{\partial\sigma^2}\Big|_{\sigma=\sigma_{\mathrm{eq}}}}$$

Intensity of sigma fluctuations - critical point

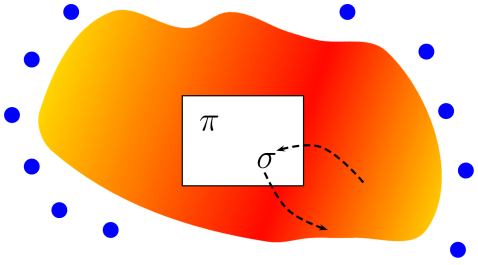


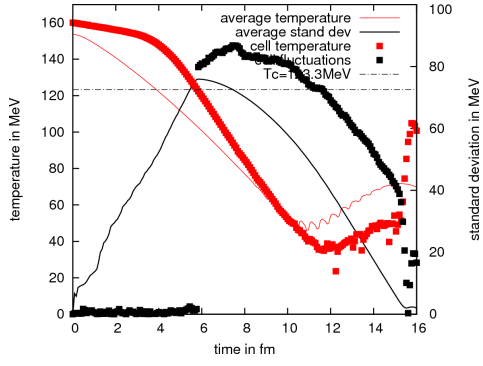
Fluctuations - critical point

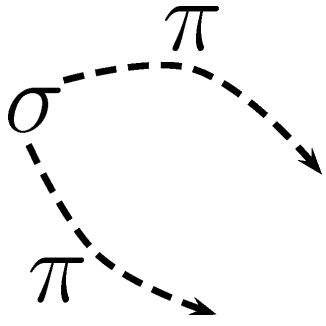
Fluctuations - first order PT

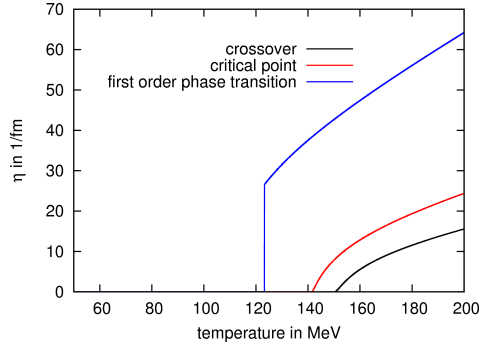


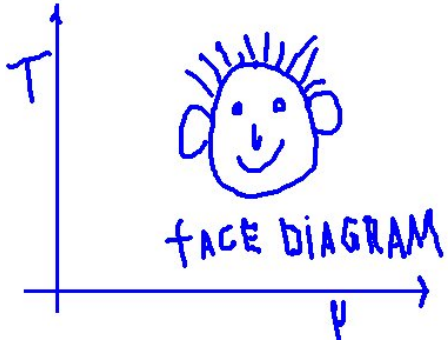
Summary & outlook





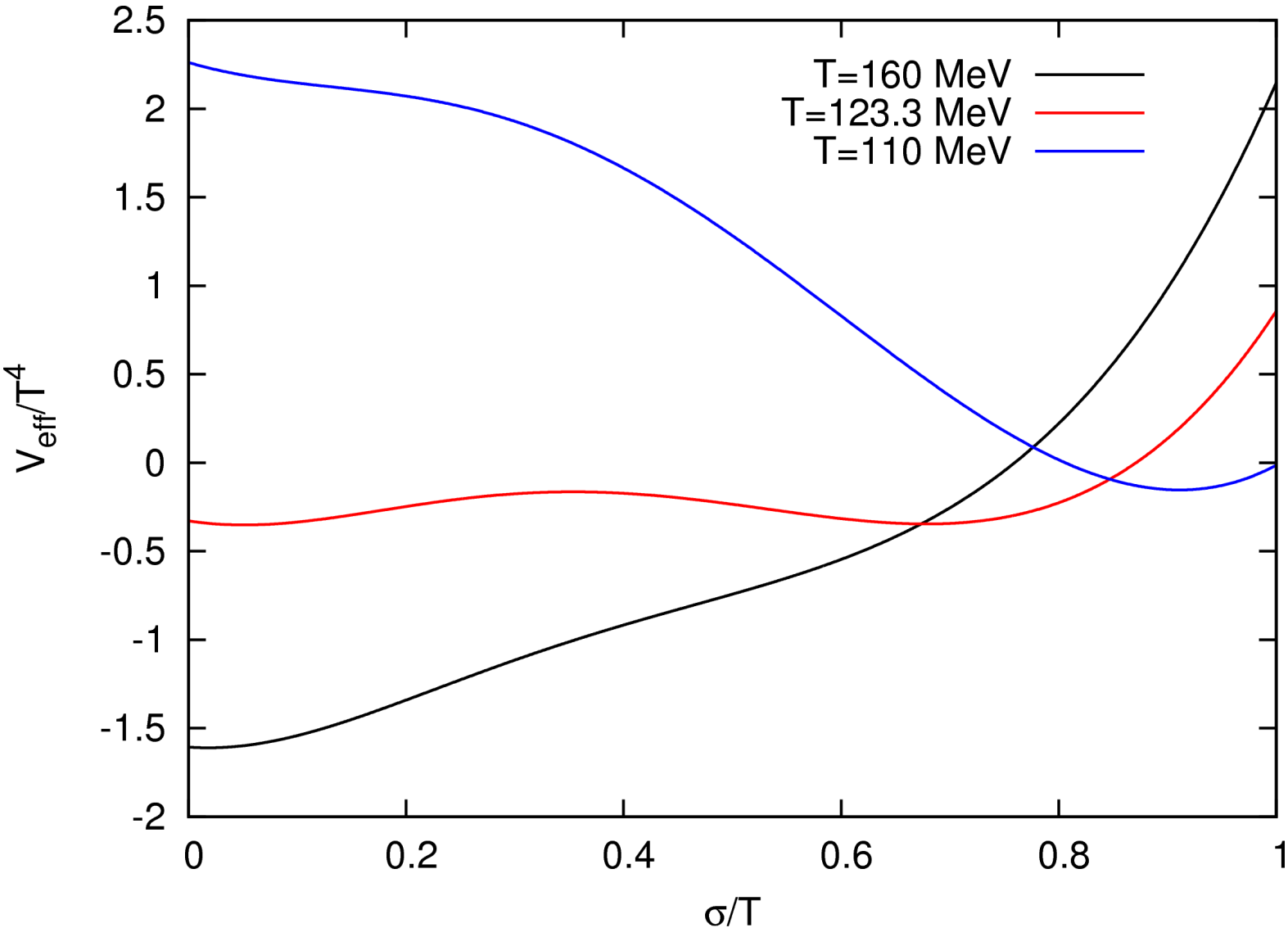




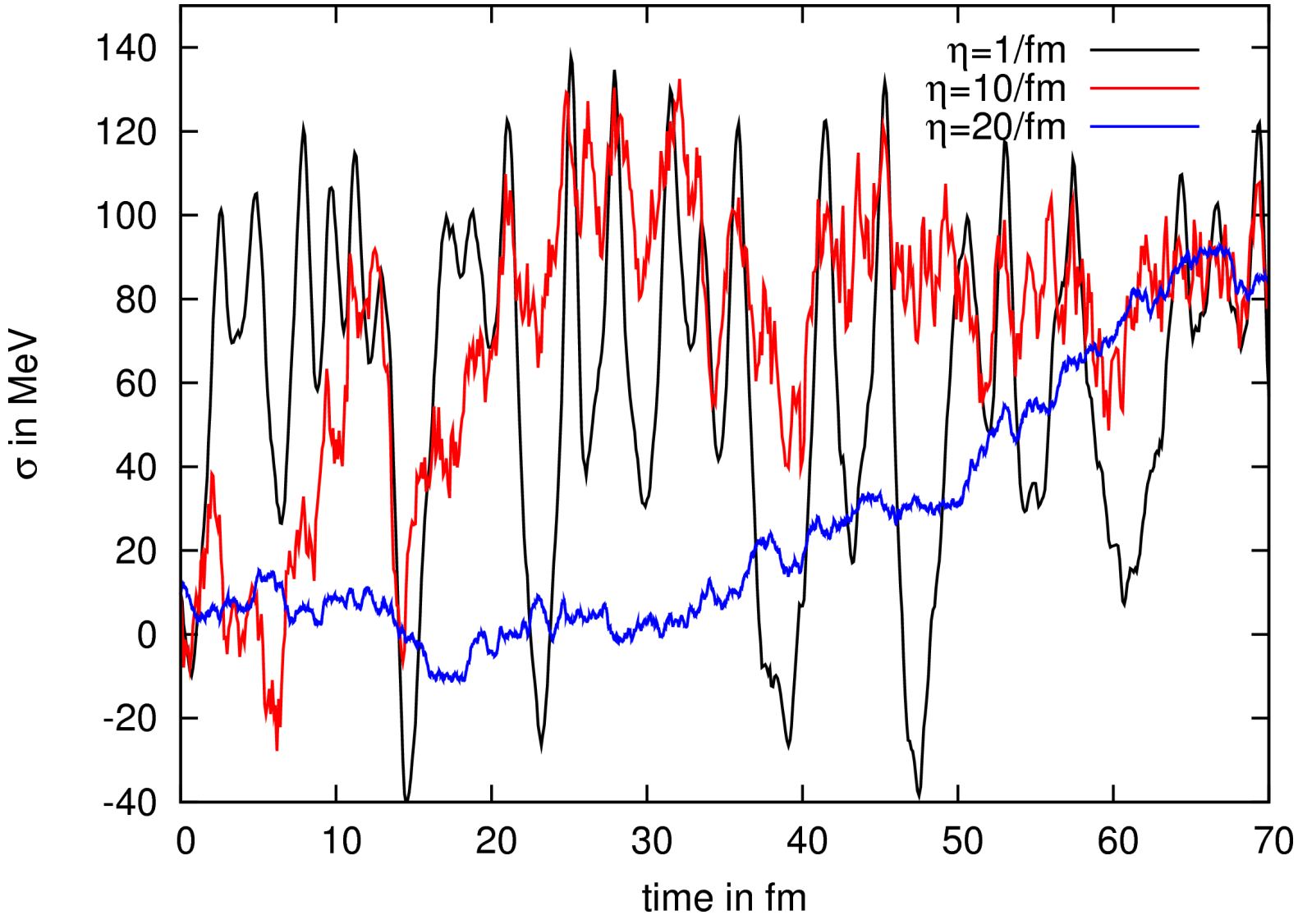


backup

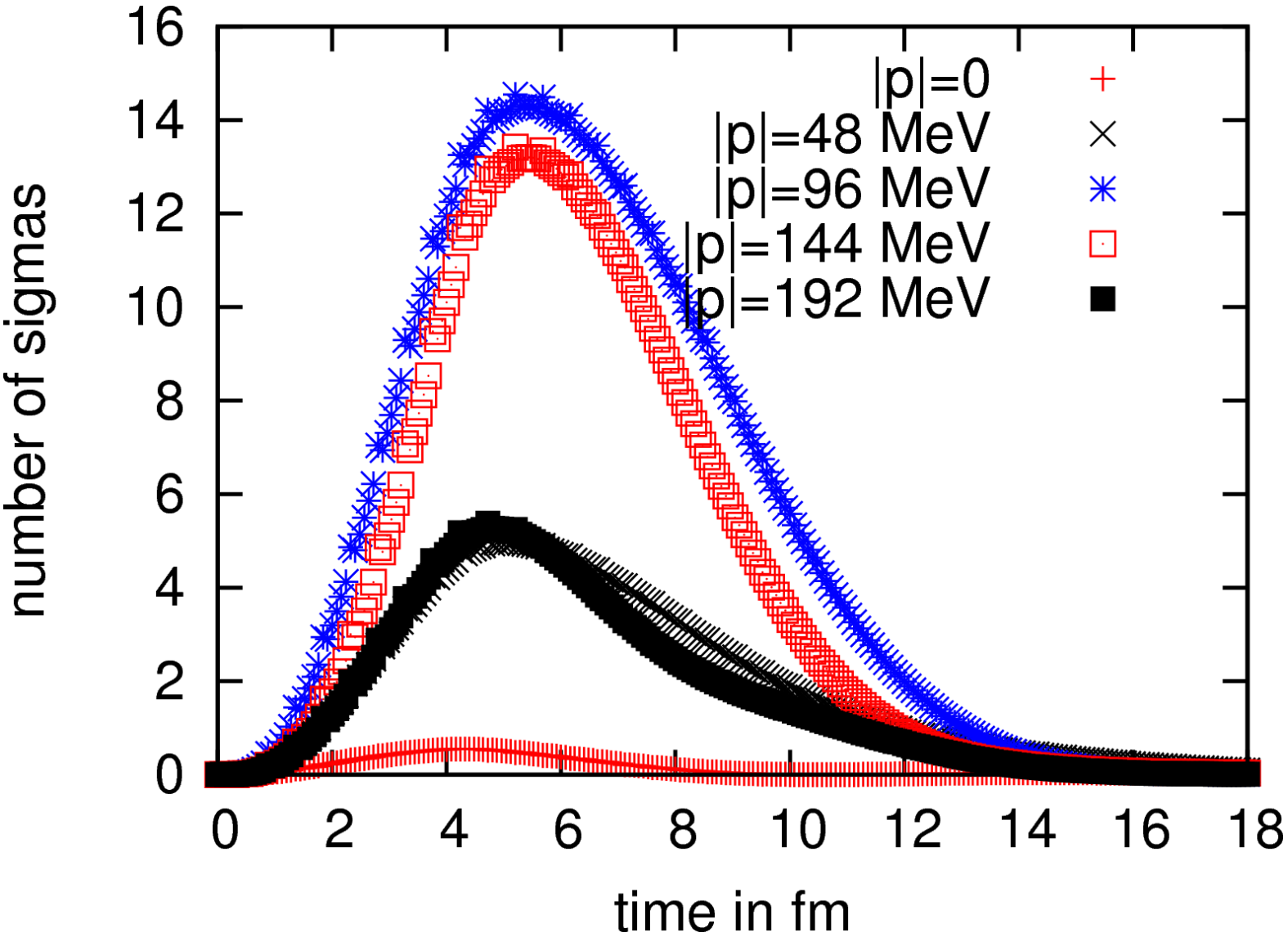
Effective potential



Relaxation of the sigma field

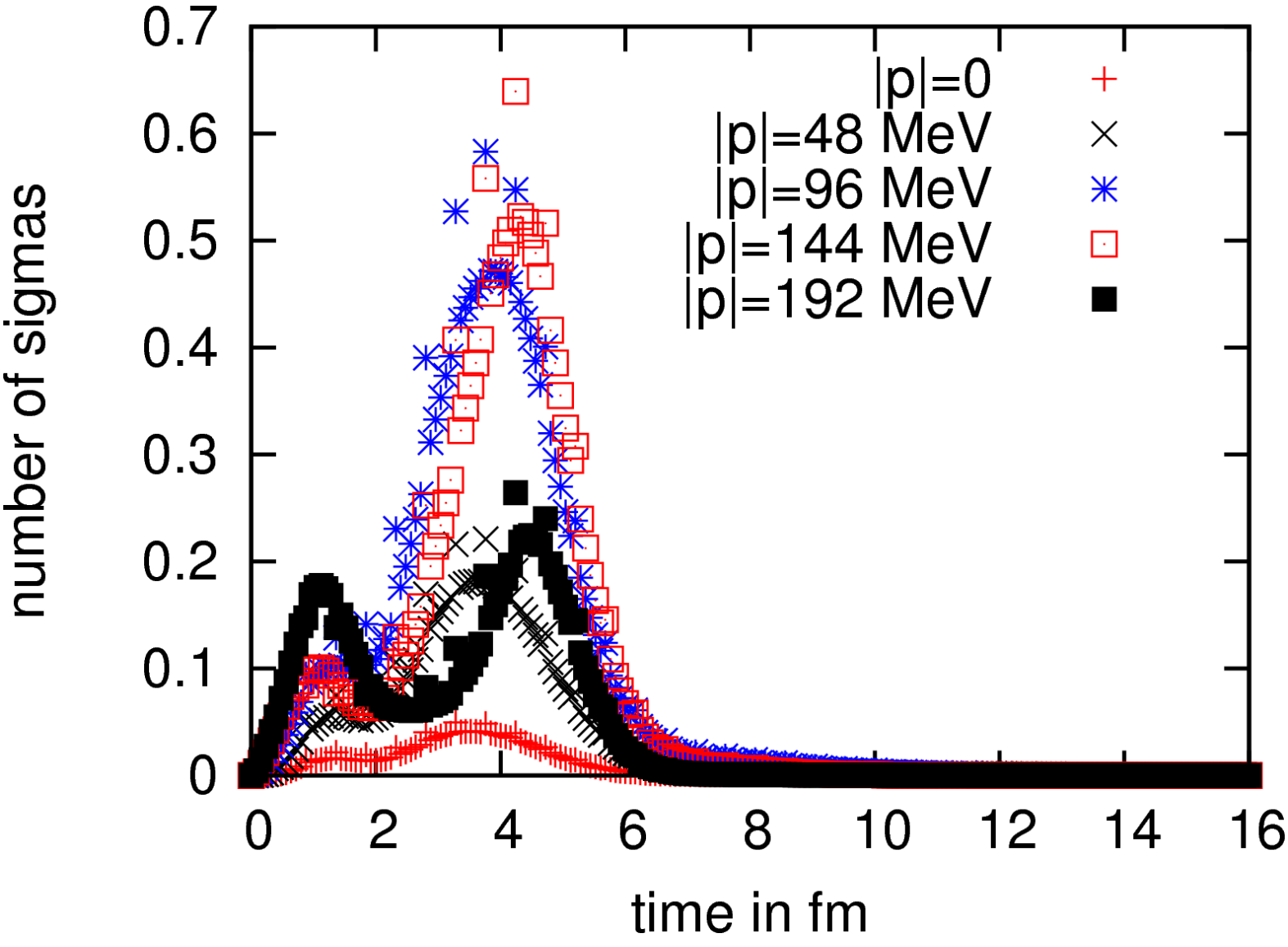


Intensity of sigma fluctuations - crossover

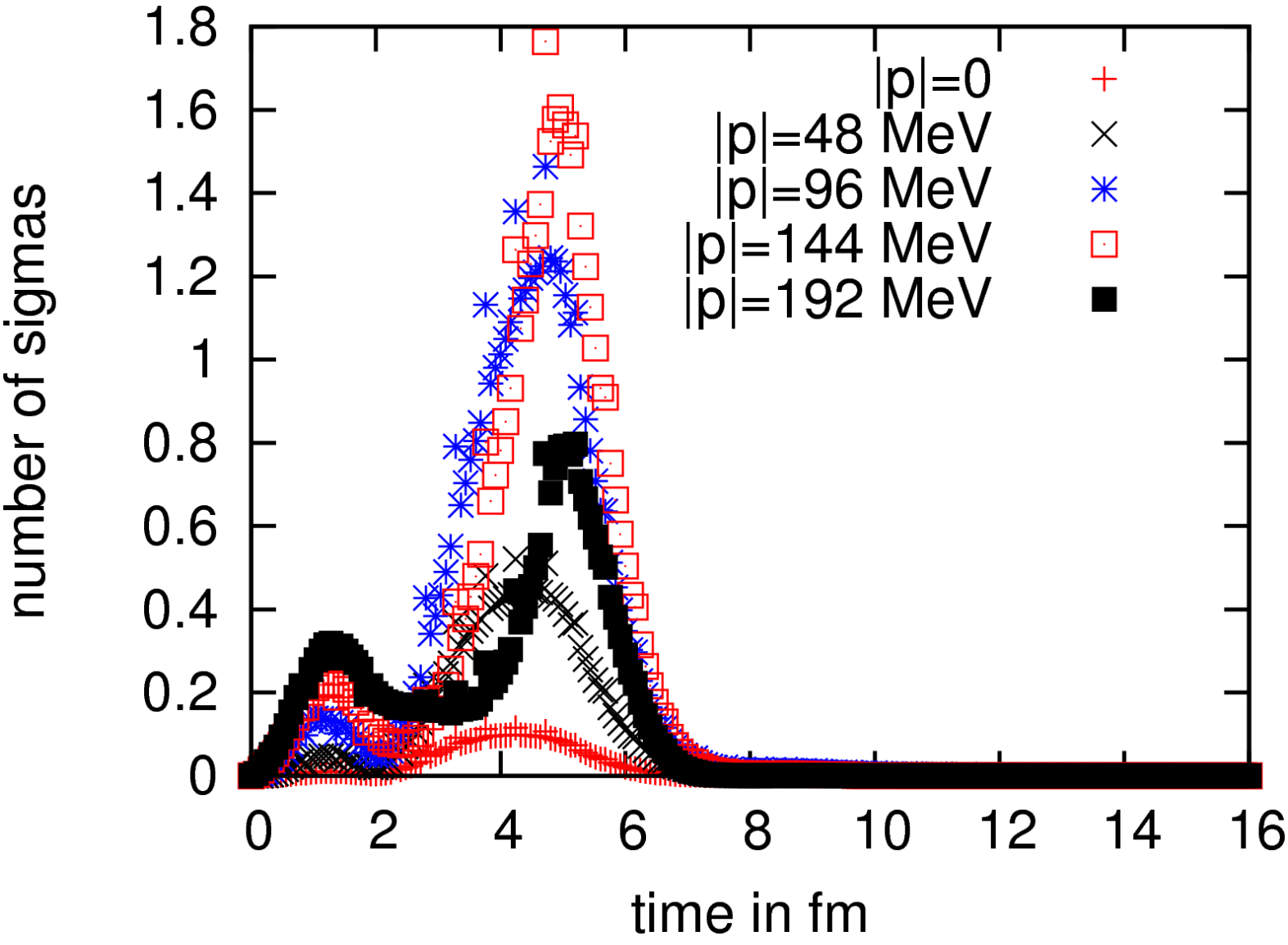


$$\eta_1 = 2.2\,/\,\mathrm{fm}$$

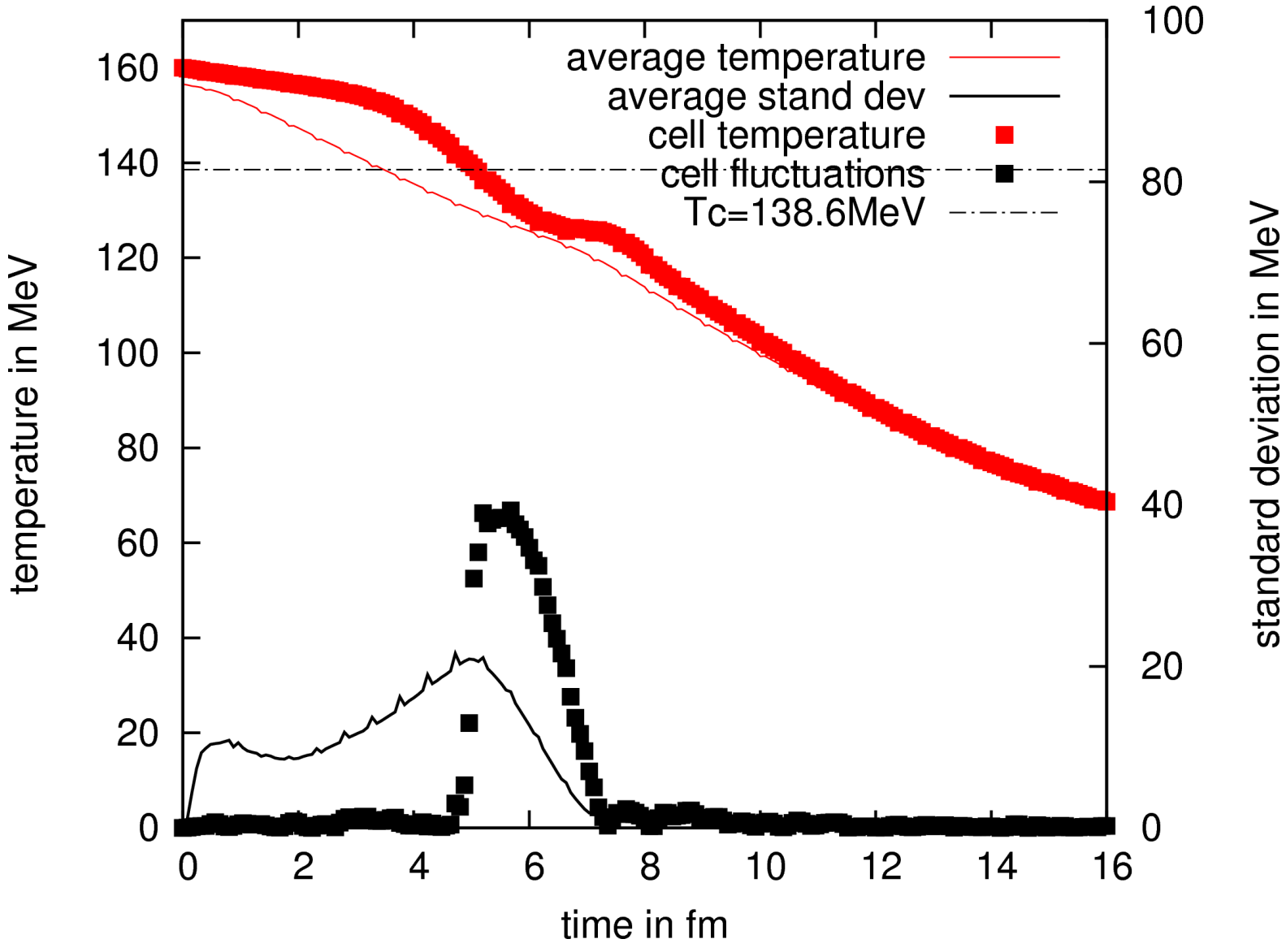
Intensity of sigma fluctuations - crossover



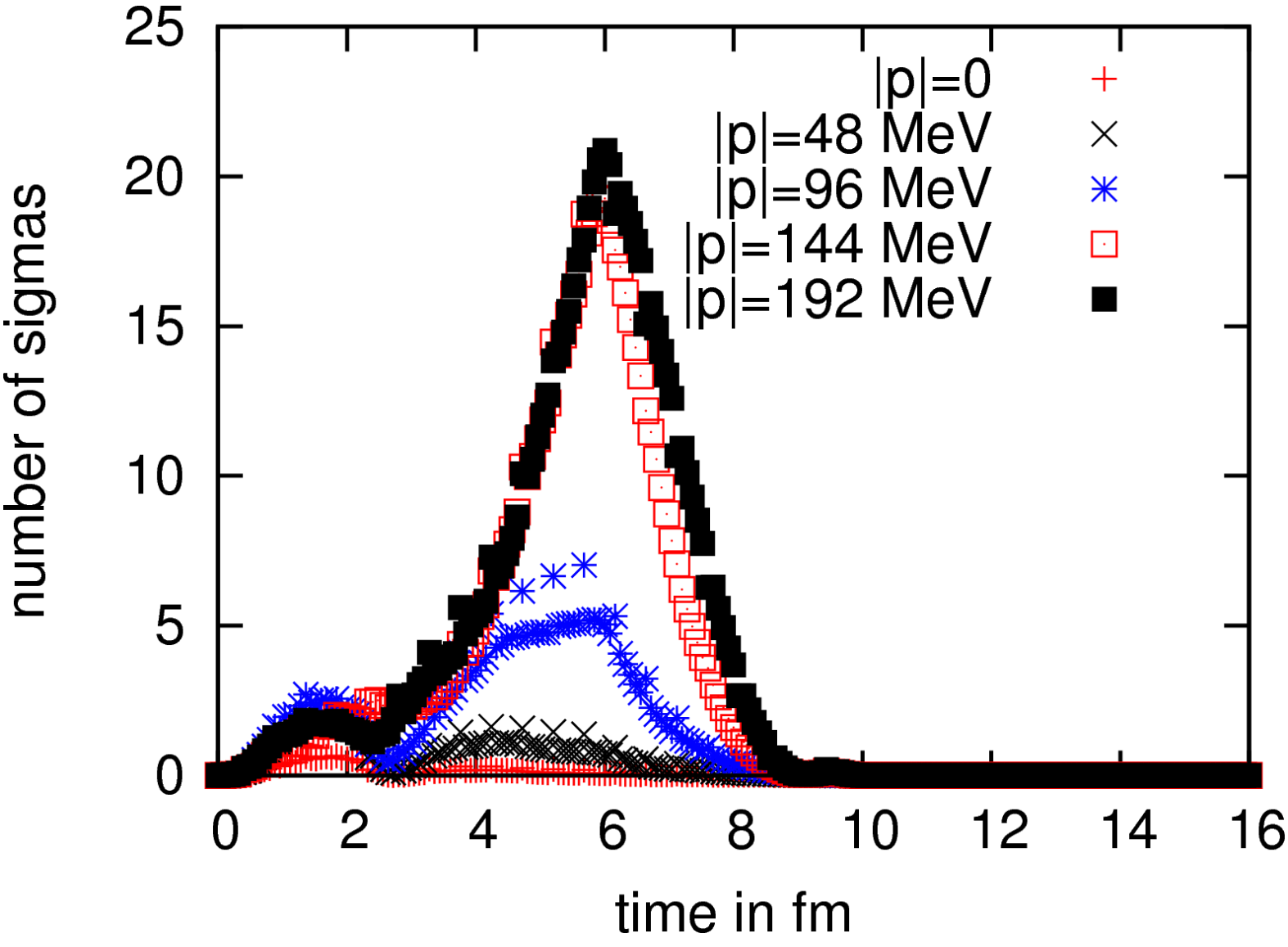
Intensity of sigma fluctuations - critical point



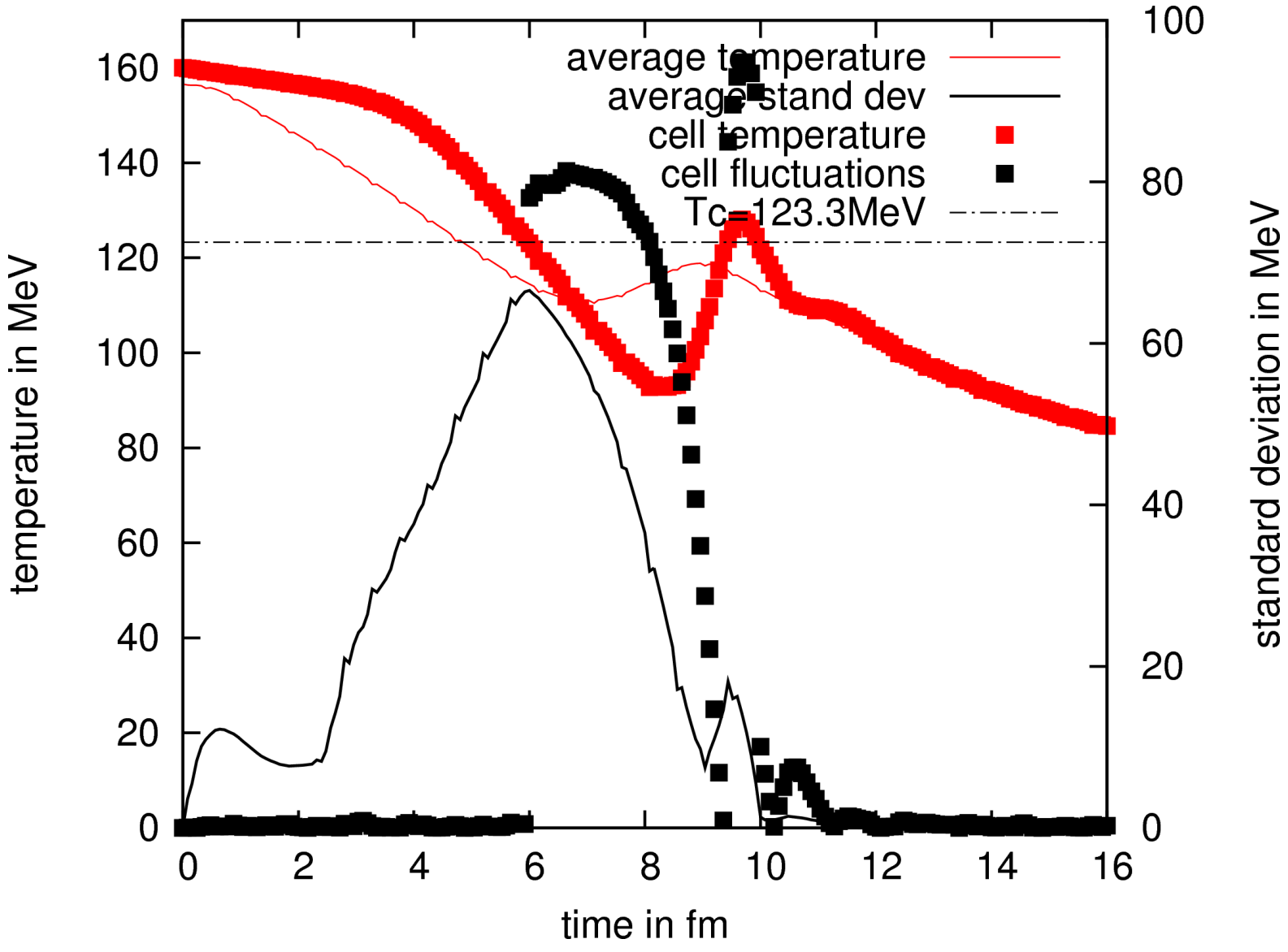
Fluctuations - critical point



Intensity of sigma fluctuations - first order PT



Fluctuations - first order PT



The equation of state

$$e(\sigma, T) = T\frac{\partial p(\sigma, T)}{\partial T} - p(\sigma, T)$$

$$p(\sigma, T) = -V_{\text{eff}}(\sigma, T) + U(\sigma)$$