# Recent results on QCD thermodynamics:

## Lattice QCD

versus

## Hadron Resonance Gas model

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Based on: S. Borsanyi, Z. Fodor, C. Hölbling, S. Katz, S. Krieg, C. R. and K. Szabó, 1005.3508

S. Borsanyi, G. Endrodi, Z. Fodor, A. Jakovac, S. Katz, S. Krieg, C. R. and K. Szabó 1007.2580

# Motivation

- Bielefeld-Brookhaven-Riken-Columbia (+ MILC = hotQCD) Collaboration:

  M. Cheng *et al.* PRD 74 (2006)

  ➡ Chiral susceptibility and Polyakov loop both give $T_c = 192(7)(4)$ MeV

- Wuppertal-Budapest (WB) Collaboration:

  Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, PLB 643 (2006)

  ➡ Chiral susceptibility gives $T_c = 151(3)(3)$ MeV

  ➡ Polyakov loop and strange quark number susceptibilities give $T_c = 175(2)(4)$ MeV

- ‘chiral $T_c$’: $\sim 40$ MeV difference

- ‘deconfinement $T_c$’: $\sim 15$ MeV difference

# Results from the two collaborations

## Wuppertal-Budapest collaboration

- 2006-2009

  Y. Aoki *et al.*, PLB (2006), JHEP (2009)

  - stout action
  - $N_t = 8, 10, 12$
  - $m_s/m_{u,d} = 28.15$

    $\Downarrow$

    $m_\pi = 135$ MeV

- 2010

  S. Borsanyi *et al.*, 1005.3508

  - $N_t = 16 \Rightarrow$ continuum

## hotQCD collaboration

- 2006-2009

  M.Cheng *et al.* PRD(2006),A.Bazavov *et al.* PRD(2009)

  - asqtad, p4 actions
  - $N_t = 6, 8$
  - $m_s/m_{u,d}=10 \Rightarrow m_\pi = 220$ MeV

- 2009-2010

  M. Cheng *et al.* PRD (2010)

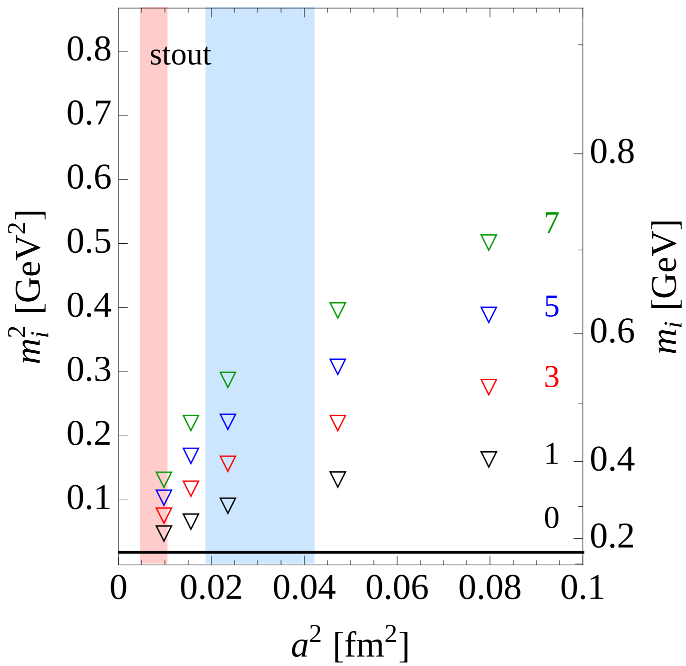
  - p4 action
  - $N_t = 8$
  - $m_s/m_{u,d}=20 \Rightarrow m_\pi = 160$ MeV

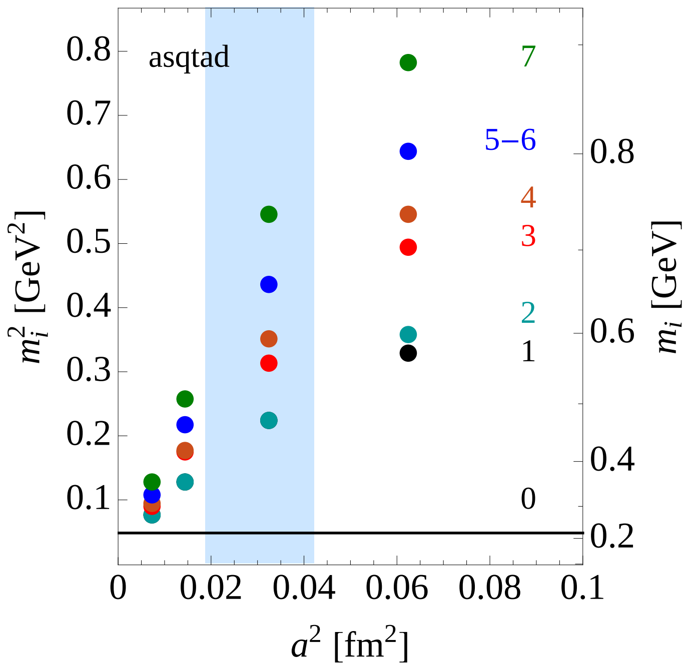
  A. Bazavov, P. Petreczky, 1005.1131 (2010)

  - asqtad, $N_t = 12$, hisq, $N_t = 6, 8$

# Pseudo-scalar mesons in staggered formulation

- Staggered formulation: four degenerate quark flavors ('tastes') in the continuum limit
- Rooting procedure: replace fermion determinant in the partition function by its fourth root
- At finite lattice spacing the four tastes are not degenerate
  - each pion is split into 16
  - the sixteen pseudo-scalar mesons have unequal masses
  - only one of them has vanishing mass in the chiral limit



# Purpose of this analysis

Use the Hadron Resonance Model

in order to identify

the origin of the discrepancy

In particular:

$\Rightarrow$ **Discretization effects**

$\Rightarrow$ **Effects due to heavy pions**

# Partition function of HRG model

- The pressure can be written as

$$p^{HRG}/T^4 = \frac{1}{VT^3}\sum_{i\in\ mesons} \ln \mathcal{Z}^M_{m_i}(T,V,\mu_{X^a}) + \frac{1}{VT^3}\sum_{i\in\ baryons} \ln \mathcal{Z}^B_{m_i}(T,V,\mu_{X^a}) \ ,$$

where

$$\ln \mathcal{Z}^{M/B}_{m_i} = \mp \frac{V d_i}{2\pi^2}\int_0^\infty dk k^2 \ln(1 \mp z_i e^{-\varepsilon_i/T}) \quad ,$$

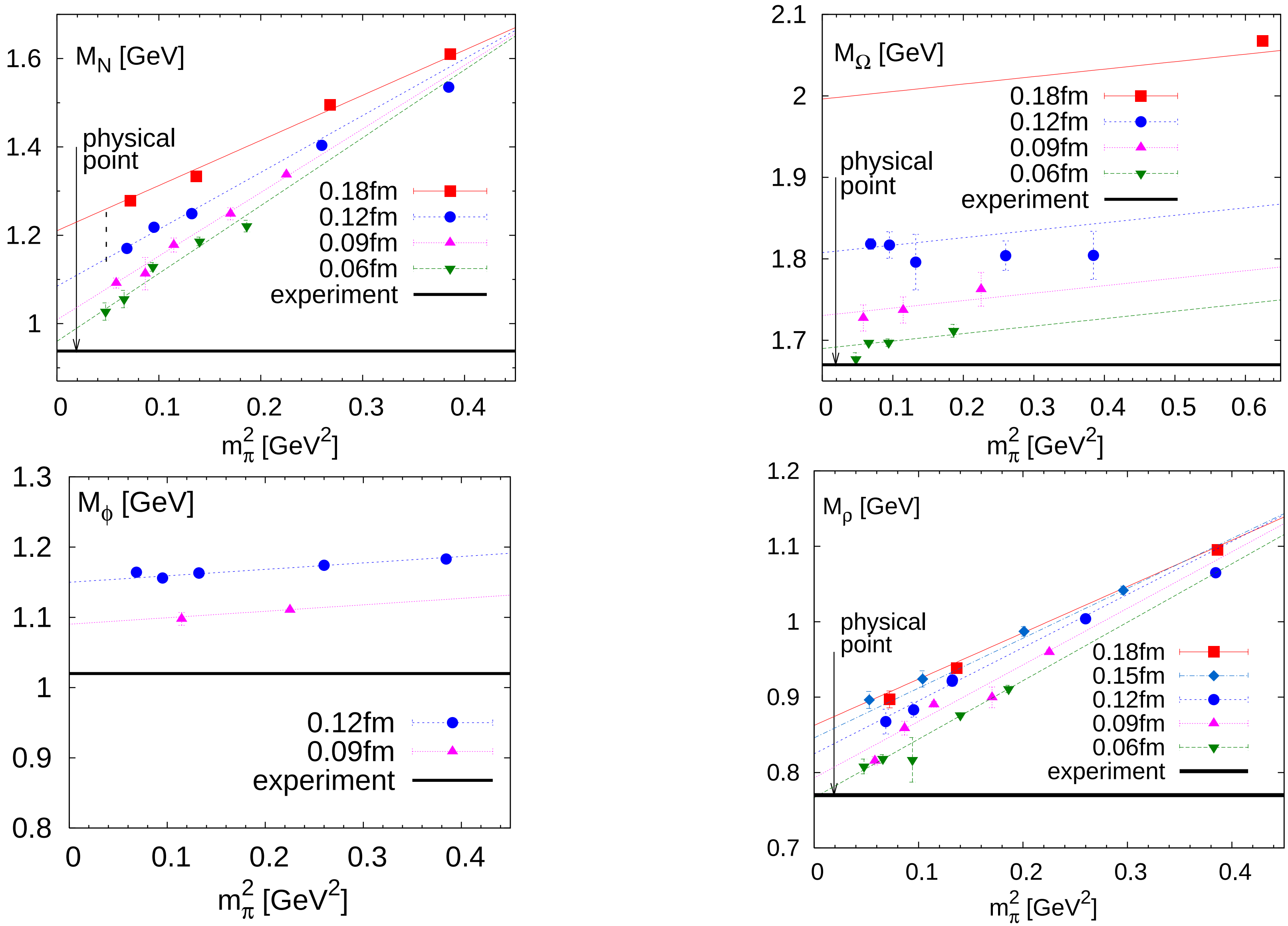
with energies $\varepsilon_i = \sqrt{k^2+m_i^2}$, degeneracy factors $d_i$ and fugacities

$$z_i = \exp\left( (\sum_a X^a_i \mu_{X^a})/T \right) \ .$$

$X^a$: all possible conserved charges, including the baryon number $B$, electric charge $Q$, strangeness $S$.

F. Karsch, A. Tawfik, K. Redlich; S. Ejiri, F. Karsch, K. Redlich

# Discretization effects




C. W. Bernard *et al.*, PRD (2001), C. Aubin *et al.*, PRD (2004), A. Bazavov *et al.*, 0903.3598.

# Hadron masses

- Non-strange baryons and mesons:

$$r_1 m = r_1 m_0 + \frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x}, \qquad x = (\frac{a}{r_1})^2$$

- Strange baryons and mesons:

$$r_1 \cdot m_\Lambda(a, m_\pi) = r_1 m_\Lambda^{phys} + \frac{2}{3}\frac{a_1 (r_1 m_\pi)^2}{1+a_2 x} + \frac{b_1 x}{1+b_2 x} + \frac{r_1 \cdot (m_\Lambda^{phys} - m_p^{phys})}{1+a_2 x}\left(\frac{m_s}{m_s^{phys}}\right),$$

$$r_1 \cdot m_\Sigma(a, m_\pi) = r_1 m_\Sigma^{phys} + \frac{1}{3}\frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Sigma^{phys} - m_p^{phys})}{1 + a_2 x}\left(\frac{m_s}{m_s^{phys}}\right),$$

$$r_1 \cdot m_\Xi(a, m_\pi) = m_\Xi^{phys} + \frac{1}{3}\frac{a_1 (r_1 m_\pi)^2}{1 + a_2 x} + \frac{b_1 x}{1 + b_2 x} + \frac{r_1 \cdot (m_\Xi^{phys} - m_p^{phys})}{1 + a_2 x}\left(\frac{m_s}{m_s^{phys}}\right)$$

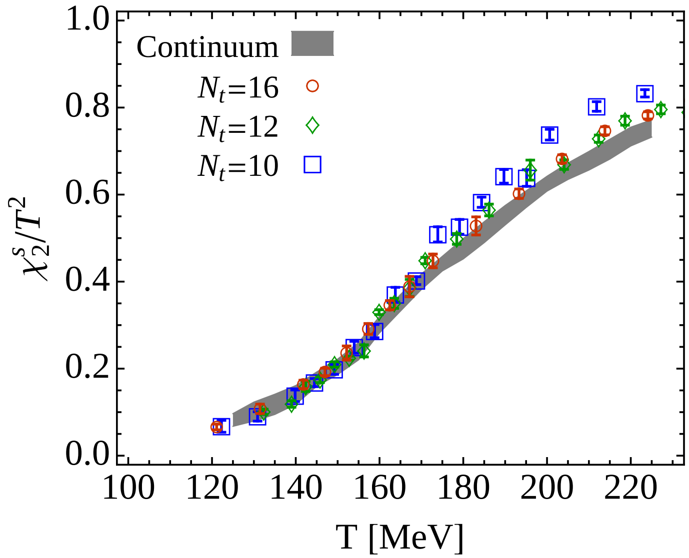
$$r_1 m_\Omega(a, m_\pi) = r_1 m_\Omega^{phys} + a_1 (r_1 m_\pi)^2 - a_1 (r_1 m_\pi^{phys})^2 + b_1 x + (m_\Omega^{phys} - m_\Delta^{phys}) \cdot 1.02 x$$

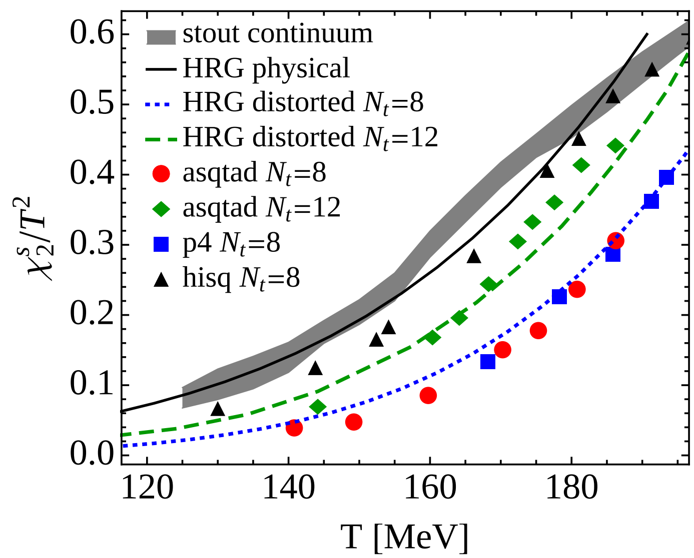
- Distorted spectrum implemented in the HRG model
- Assumption: all resonances behave as their fundamental states

P. Huovinen and P. Petreczky (2009).

# Results: strangeness susceptibilities

$$\chi_n^S = T^n \frac{\partial^n p(T,\mu_B,\mu_S,\mu_I)}{\partial \mu_S^n}\Big|_{\mu_X=0}$$



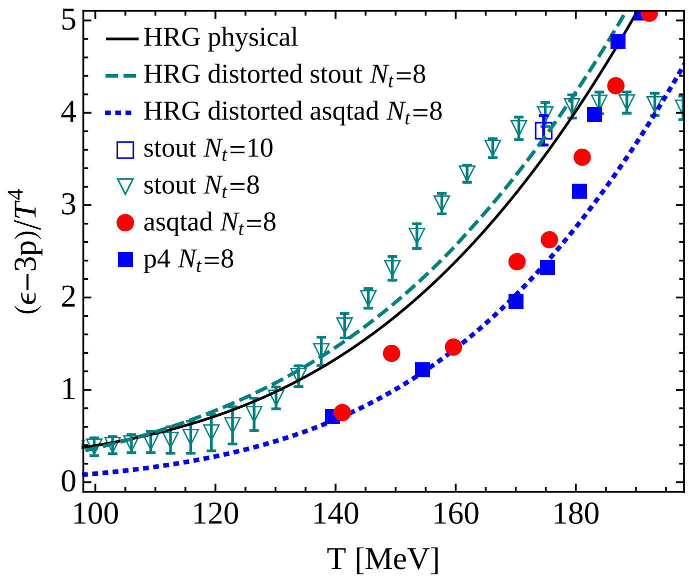



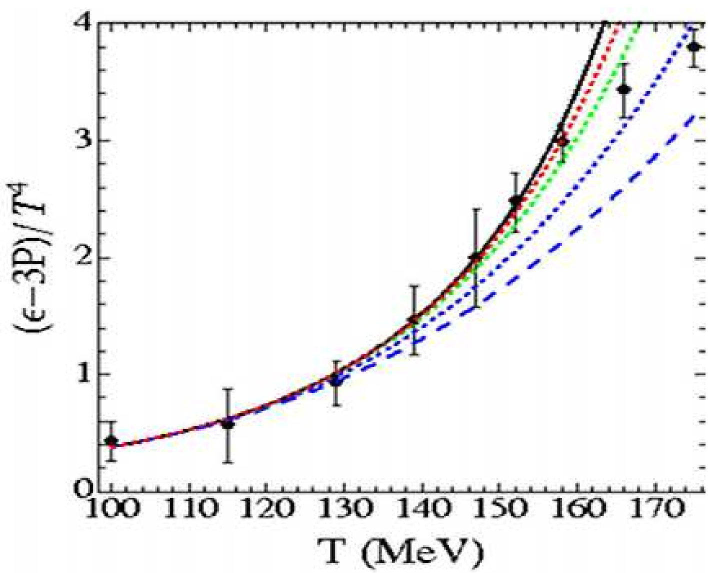

- HRG results in good agreement with stout action
- asqtad and p4 results show similar shape but shift in temperature
  - ➡ HRG results with corresponding distorted spectrum reproduce asqtad and p4 results

S. Borsanyi *et al.*, 1005.3508

# Results: trace anomaly

$$\frac{\theta(T)}{T^4} = \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4)$$




- For large masses few states are known experimentally
- Inclusion of exponentially growing hadron mass spectrum

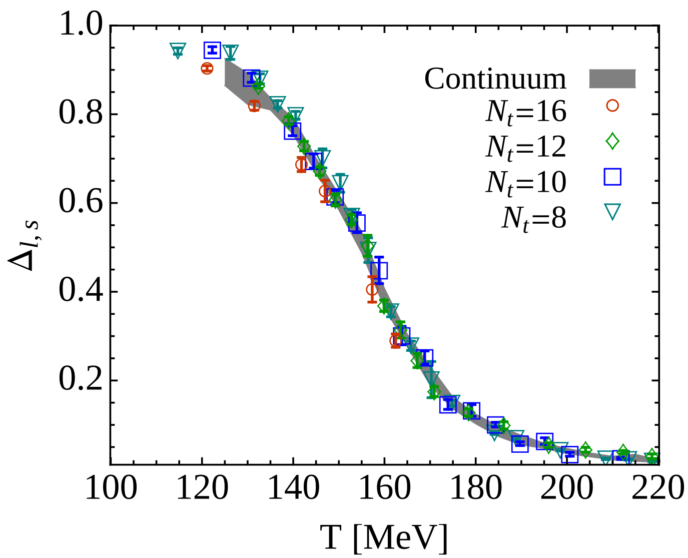
J. Noronha-Hostler, C. Greiner, I. Shovkovy (2008); J. Noronha-Hostler, M. Beitel, C. Greiner, I. Shovkovy (2010)

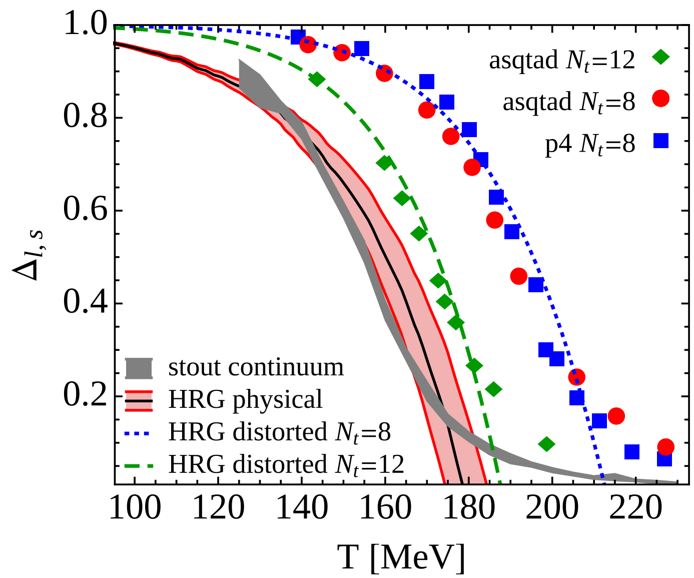
- Agreement between lattice and HRG improved up to $T \sim 155$ MeV

(A. Majumder, B. Müller: 1008.1747)

# Results: subtracted chiral condensate

$$\Delta_{l,s} = \frac{\langle\bar\psi\psi\rangle_{l,T} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,T}}{\langle\bar\psi\psi\rangle_{l,0} - \frac{m_l}{m_s}\langle\bar\psi\psi\rangle_{s,0}} \quad \text{with} \quad \langle\bar\psi\psi\rangle_i = \frac{T}{V}\frac{\partial \ln Z}{\partial m_i}$$




$$\langle\bar\psi\psi\rangle_l = \langle\bar\psi\psi\rangle_{l,0} + \langle\bar\psi\psi\rangle_\pi + \sum_{i\in mesons}\frac{\partial \ln Z^M_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_\pi^2}\frac{\partial m_\pi^2}{\partial m_l} + \sum_{i\in baryons}\frac{\partial \ln Z^B_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_\pi^2}\frac{\partial m_\pi^2}{\partial m_l}.$$

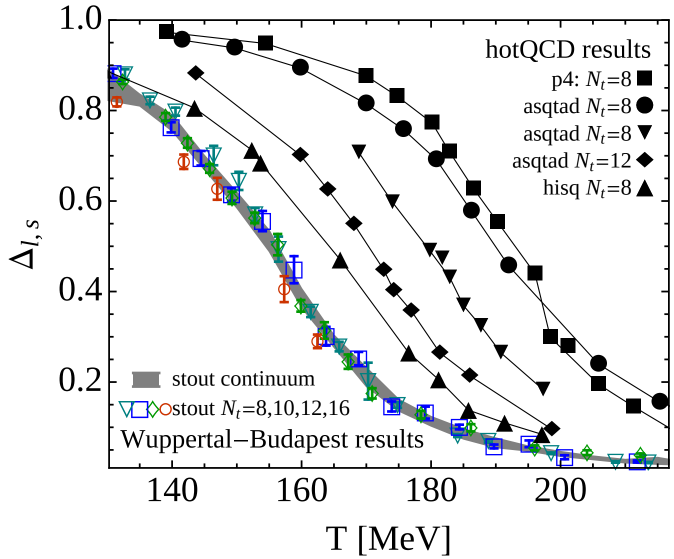
$$\langle\bar\psi\psi\rangle_s = \langle\bar\psi\psi\rangle_{s,0} + \langle\bar\psi\psi\rangle_K + \sum_{i\in mesons}\frac{\partial \ln Z^M_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_s} + \sum_{i\in baryons}\frac{\partial \ln Z^B_{m_i}}{\partial m_i}\frac{\partial m_i}{\partial m_s}.$$

- $\frac{\partial m_i}{\partial m_\pi^2}$ and $\frac{\partial m_i}{\partial m_s}$ from fit to lattice data Camalich, Geng and Vacas (2010)

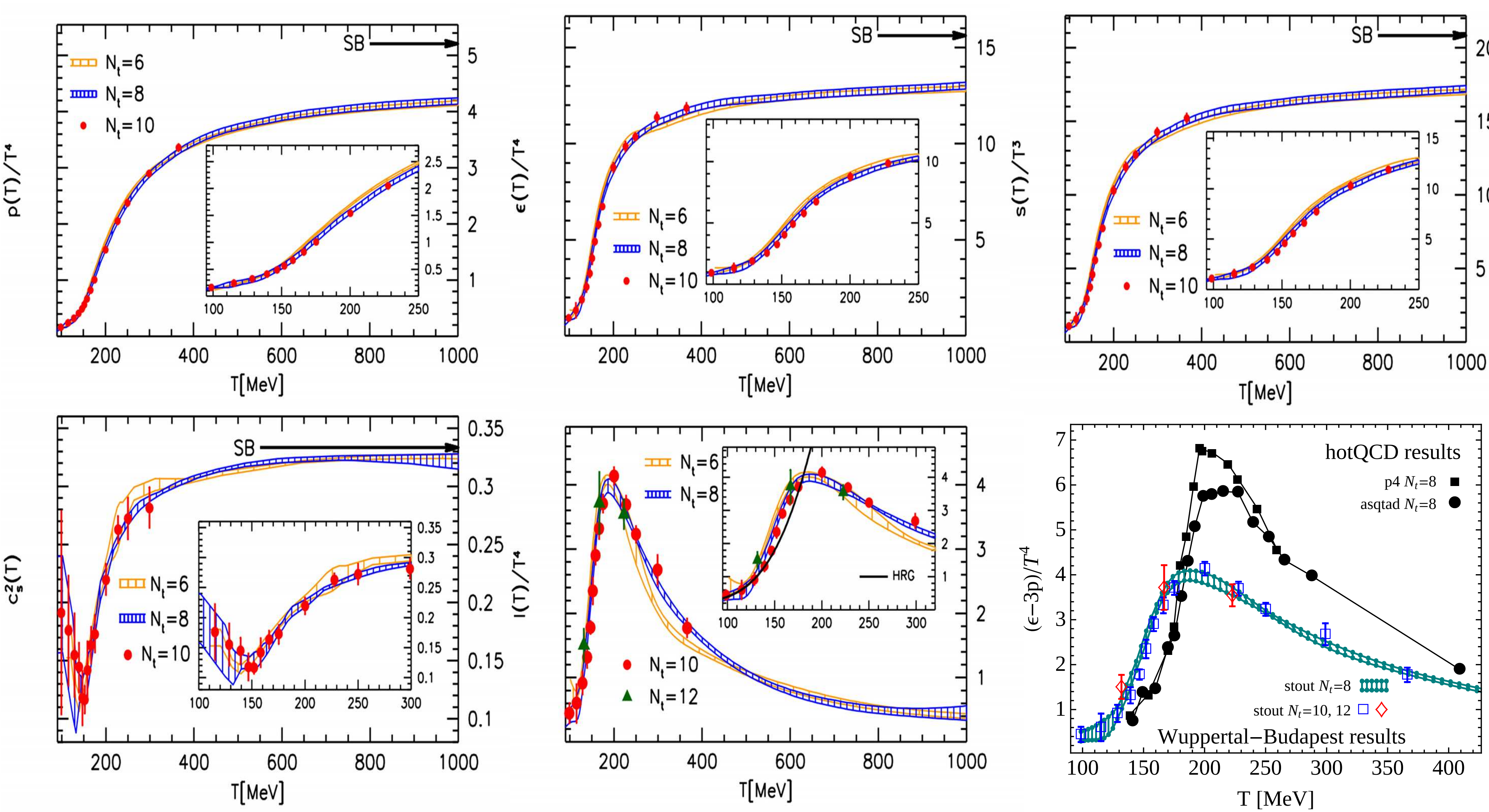
# $T_c$ summary from Wuppertal-Budapest collaboration

| | $\chi_{\bar{\psi}\psi}/T^4$ | $\Delta_{l,s}$ | $\langle\bar{\psi}\psi\rangle_R$ | $\chi_2^s/T^2$ | $\epsilon/T^4$ | $(\epsilon-3p)/T^4$ |
|---|---|---|---|---|---|---|
| WB'10 | 147(2)(3) | 157(3)(3) | 155(3)(3) | 165(5)(3) | 157(4)(3) | 154(4)(3) |
| WB'09 | 146(2)(3) | 155(2)(3) | - | 169(3)(3) | - | - |
| WB'06 | 151(3)(3) | - | - | 175(2)(4) | - | - |

- Different variables give different $T_c$ values: the transition is broad S. Borsanyi *et al.*, 1005.3508
- progress in T dependence of chiral condensate



# Equation of state




S. Borsanyi *et al.*, 1007.2580

# Conclusions

- The present analysis concludes the WB investigation of $T_c$ with stout action
- Results from 2006 and 2009 are improved:
  - physical quark masses used in simulations also at $T = 0$
  - smaller lattice spacings $N_t = 16$
  - continuum limit provided for all observables
- The new results are in perfect agreement with those from 2006 and 2009
- The QCD transition is a broad analytic crossover
- Good agreement between HRG model predictions and WB continuum results
- hotQCD results can be reproduced in HRG model with distorted spectrum
- New results for EoS with physical masses and fine lattices

# Backup slides

# What happens below $T_c$?

- At low $T$ and $\mu = 0$, QCD thermodynamics is dominated by pions
- The interaction between pions is suppressed
  - chiral perturbation theory: pion contribution to the thermodynamic potential
  - the energy density of pions from 3-loop ChPT differs only less than 15% from the ideal gas value

    P. Gerber and H. Leutwyler (1989)
- as $T$ increases, heavier hadrons start to contribute
- for $T \geq 120$ MeV heavy states dominate the energy density
- their mutual interactions are proportional to $n_i n_k \sim \exp[-(M_i + M_k)/T]$: they are suppressed
  - the virial expansion can be used to calculate the effect of the interaction

# Why HRG?

- In the virial expansion, the partition function can be split into a non-interacting piece and a piece which includes all interactions Dashen, Ma and Bernstein (1969)
- virial expansion and experimental information on scattering phase shift
  Prakash and Venugopalan (1992)
  - interplay between attractive and repulsive interaction

Interacting hadronic matter

can be well approximated by

a non-interacting gas of resonances