Dr. J. Reinhardt Summer semester 2009

Nonlinear Dynamics and Complex Systems

Exercise No. 1

Population Dynamics

1) A Predator-Prey-Model (Rabbits vs. Foxes)

Two species of animals with population numbers N_1 and N_2 coexist in a common habitat. The predators (N_2, "foxes") feed by eating the prey (N_1, "rabbits"). The prey feeds on plants and is assumed to multiply unboundedly if left alone. A simple ansatz for the evolution of the populations (continuous in time) is given by

$$\frac{dN_1}{dt} = N_1(a - bN_2) \ , \qquad (1)$$

$$\frac{dN_2}{dt} = N_2(cN_1 - d) \qquad (2)$$

with positive constants a, b, c, d.

a) Interpret these equations and the meaning of the constants.

b) By rescaling the constants and the time coordinate show that (1), (2) can be transformed into the dimensionless form (the *Lotka-Volterra equation*)

$$\frac{dx}{d\tau} = x(1 - y) \ , \qquad (3)$$

$$\frac{dy}{d\tau} = \alpha y(x - 1) \ . \qquad (4)$$

c) Investigate the fixed points of (3), (4) and their stability.

d) By integrating dy/dx resulting from (3), (4) show that the phase space trajectories satisfy the equation

$$\alpha x + y - \ln(x^\alpha y) = C$$

where C is a conserved quantity ($C \geq 1 + \alpha$).

e) Using MAPLE: Draw typical trajetories $x(t)$, $y(t)$ and phase space plots. Interpret the results

2) A Competition Model (Rabbits vs. Sheep)

Two species of animals (N_1, "rabbits" and N_2, "sheep") live in the same habitat and compete for the same limited food source. An ansatz for the time evolution of the populations is given by

$$\frac{dN_1}{dt} = r_1 N_1 \Big(1 - \frac{N_1}{K_1} - b_1 N_2\Big) \ , \tag{5}$$

$$\frac{dN_2}{dt} = r_2 N_2 \Big(1 - \frac{N_2}{K_2} - b_2 N_1\Big) \tag{6}$$

with positive constants r_1, r_2, K_1, K_2, b_1, b_2.

a) Interpret these equations and the meaning of the constants.

b) By rescaling the constants and the time show that (5), (6) can be transformed into the dimensionless form

$$\frac{dx}{d\tau} = x(1 - x - ay) \equiv f(x, y) \ , \tag{7}$$

$$\frac{dy}{d\tau} = \rho y(1 - y - bx) \equiv g(x, y) \ . \tag{8}$$

c) By investigating the "nullclines" (i.e., the curves with $f(x, y) = 0$ or $g(x, y) = 0$) find the fixed points of the system.

d) As an example, study the case $a = \frac{4}{3}$, $b = \frac{3}{2}$, $\rho = \frac{2}{3}$. Investigate the stability of the fixed points and draw a phase space diagram. Can the two species coexist peacefully? Could this outcome change for other values of the parameters?