Plasma Astrophysics
Chapter 1: Basic Concepts of Plasma

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What is a Plasma?

- A plasma is a quasi-neutral gas consisting of positive and negative charged particles (usually ions & electrons)
- Liquid is heated, atoms vaporize => gas
- Gas is heated, atoms collide each other and knock their electrons => decompose into ions & electrons (*plasma*)
- Plasma state: fourth state of matter

What is a Plasma? (cont.)

- Ions & electrons interact
  - via short-range atomic forces (during collision)
  - via long-range electro-magnetic forces due to currents and charge
- Long range nature of electromagnetic forces means that plasma can show collective behavior (oscillations, instabilities)
- Plasmas can also contain some neutral particles
  - Which interact with charged particles via collisions or ionizations
  - Ex. interstellar medium, molecular clouds etc.
- Simplest Plasma: equal numbers of electrons and protons (formed by ionization of atomic hydrogen)

Examples of Plasmas

- Liquid => gas
  - thermal energy > Van del Waasl force (10^{-2} eV)
- Ionize to neutral atoms
  - Need 1~ 30 eV (10^4 ~ $10^{5.5}$ K in temperature)
- To make a fully ionized gas, we must give large energies on the matter
- Most of matter are not in plasma state on the earth

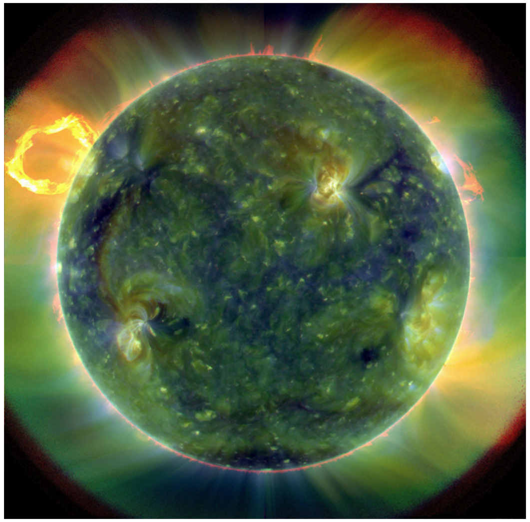
Plasmas in the Universe

- Most of (visible) universe is in form of plasma
- Plasma form wherever temperatures are high enough or radiation is strong enough to ionize atoms

- For examples
  - Sun's and star's atmosphere and winds
  - Interstellar medium
  - Astrophysical jet, outflows
  - Pulsars and their magnetosphere
  - Accretion disk around stars and compact objects etc.
- Plasma exist wide range of number densities and temperatures

Plasma in Nature and Technology

Sun & stars

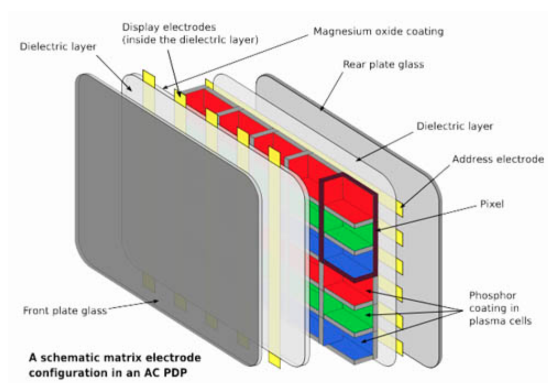


Aurorae

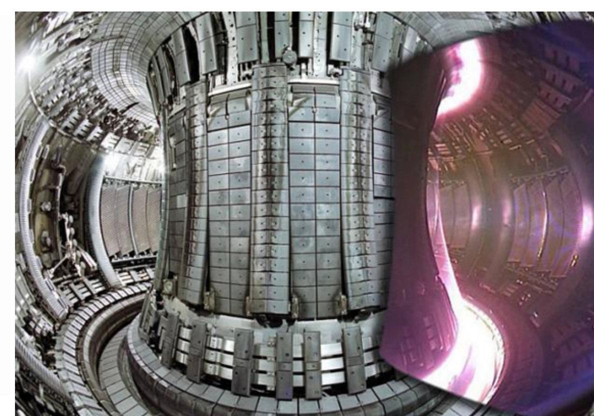
Molecular cloud

Laser produced plasma

plasma TV



Tokamaks



Basic parameters

- For ensemble of N particles of mass m and velocity v, average energy per particle is

$$\langle E \rangle = \frac{1}{2N} \sum_{i=1}^{N} m_i v_i^2$$

- In thermal equilibrium, particles have Maxwell-Boltzmann distribution of speeds.

$$f(v) = N\sqrt{\frac{m}{2\pi k_B T}} \exp\left(\frac{-1/2mv^2}{k_B T}\right)$$

- Average kinetic energy can be calculated using

$$\langle E \rangle = \frac{\int_{-\infty}^{+\infty} 1/2mv^2 f(v) dv}{\int_{-\infty}^{+\infty} f(v) dv}$$

- Integrating numerator by parts, and using $\int_{-\infty}^{+\infty} e^{-a^2x^2} dx = \sqrt{\pi}/a$
- we have $\langle E \rangle = 1/2\, k_B T$ or in 3D,

$$\langle E \rangle = 3/2 k_B T$$

Condition for a ionized gas

- The gas particles are freely moving, the averaged kinetic energy is much greater than interaction energy among them
- For hydrogen gas (atomic number Z=1)

$$\frac{3}{2}k_B T \gg \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} \quad (1.1)$$

k_B: Bolzmann constant, T: Temperature, ε_0: vacuum permittivity, e: charge on the electron, r: average distance between two particles

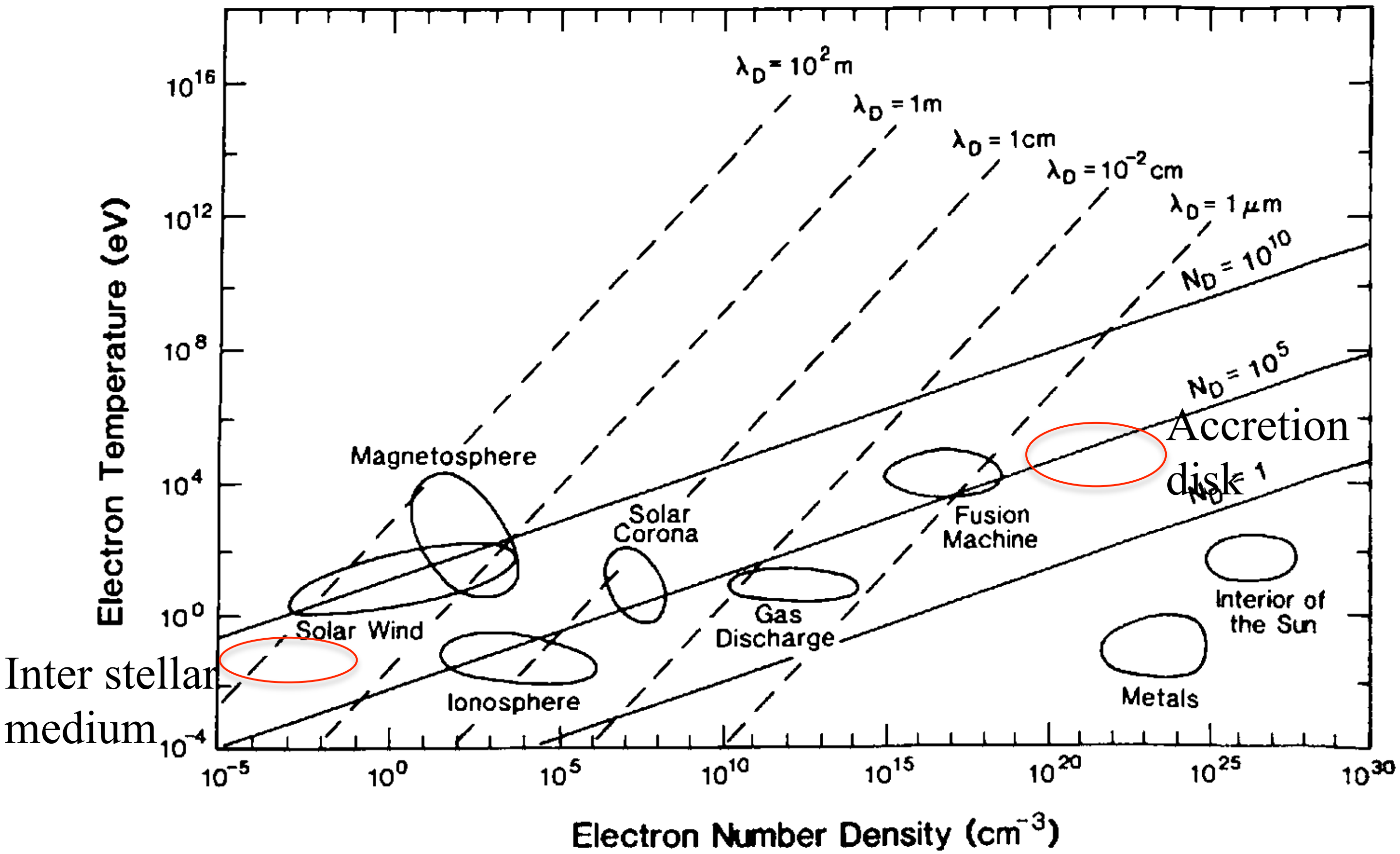
- Introducing particle number density $n=3/(4\pi r^3)$

$$\frac{T}{\mathrm{eV}} \gg 1.54 \times 10^{-9}\left(\frac{n}{\mathrm{m}^{-3}}\right)^{1/3} \quad (1.2)$$

- In Plasma physics, energy units are used for temperature. Because Joules are very large, electron volts (eV) are usually used
- $1\ \mathrm{eV} = 1.16 \times 10^4\ \mathrm{K} = 1.602 \times 10^{-19}$ Joules (1.3)

Condition for a ionized gas (cont.)

- The condition (1.2) is satisfied under various situations in the Universe (density scale ~ 30 order, temperature scale ~ 10 order)



Quasi-neutrality

- Plasma tends to be electrically neutral at each point due to large charge-to-mass ratio ($e/m_e = 1.8 \times 10^{11}$ C /kg) of electrons

- If charge neutrality breaks down at some point

⇒ Electric field is exerted around it

⇒ Electrons are accelerated towards positive charge region

⇒ Recovering a charge neutrality in a very short time

Quasi-neutrality (cont.)

- Examples: laboratory plasma
  - contains 10^{15}m^{-3} ions and neutral atoms
  - Small spherical region (r~10^{-2}m), 1% deviation from change neutrality
  - Electric field arises (from Gauss's theorem)

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = 6.0 \times 10^2 \text{ V m}^{-1} \quad (1.4)$$

  - This electric field accelerate an electron at the rate

$$\frac{eE}{m_e} = 10^{14} \text{ m s}^{-2} \quad (1.5)$$

  - 10^{13} times greater than the gravitational acceleration
- Even small deviation occurs from charge neutrality, electrons immediately migrate to recover the charge neutrality

Plasma Oscillation

- A dynamic aspect of the plasma's tendency toward neutralization shows as plasma oscillation

- If some region is (slightly) charged positively,

⇒ Incurred electric field attract (accelerate) electrons toward the region (for cancelling the charge inhomogeneity)

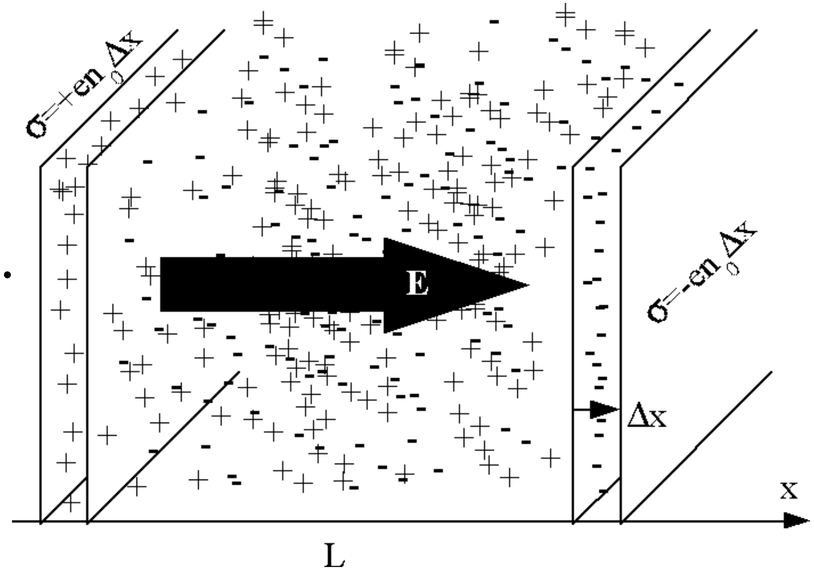
⇒ However, electron's motion overshoots

⇒ Electrons are pulled back (oscillations)

- Note: the electrons move much faster than the ions because of their masses
- We consider such a high-frequency variation of electrons' position in a static ion system

Plasma Oscillation (cont.)

- Consider plasma of equal number of positive and negative charges. Overall, plasma is neutral, so $n_e = n_i = n$
- Now displace group of electrons by Δx.
- Charge separation gives E, which accelerates electrons towards initial position. Electrons overshoot equilibrium position.



- Using Newton's Law $m_e \frac{d^2 \Delta x}{dt^2} = eE$ (1.6)
- Displacement sets up E across distance L, similar to parallel plate capacitor.
- Charge per unit area of slab is $\sigma = -ne\Delta x \Rightarrow E = \sigma/\varepsilon_0 = -ne\Delta x / \varepsilon_0$

Plasma Oscillation (cont.)

- Therefore, Eq. (1.16) can be written $\frac{d^2\Delta x}{dt^2} = -\frac{ne^2}{m_e\epsilon_0}\Delta x = -\omega_p^2\Delta x$
- Where $\omega_p = \sqrt{\frac{ne^2}{m_e\epsilon_0}}$ *Plasma frequency*
- Plasma oscillations are result of plasma trying to maintain charge neutrality.
- Plasma frequency commonly written $f_p = \omega_p/2\pi = 9000\sqrt{n_e}$ Hz where n_e is in cm^{-3}
- In Solar System, f_p ranges from hundreds of MHz (in solar corona) to <1 kHz (near outer planets).

Plasma criteria

- In a partially ionized gas where collisions are important, plasma oscillations can only develop if the collision time (τ_c) is longer than the oscillation period ($\tau_p = 1/\omega_p$).
- That is, $\tau_c >> \tau_p$ or $\boxed{\tau_c/\tau_p >> 1}$ *Plasma criteria #1*
- Above is a *criterion* for an ionized gas to be considered a plasma.

- Plasma oscillations can be driven by natural thermal motions of electrons ($E = 1/2 k_B T_e$). Work by displacement of electron by Δx is (using $E = -ne\Delta x / \varepsilon_0$)

$$
\begin{aligned}
W &= \int F dx \\
&= \int_0^{\Delta x} eE(x) dx \\
&= \frac{e^2 n \Delta x^2}{2\epsilon_0}
\end{aligned}
$$

Plasma criteria (cont.)

- Equating work done by displacement with average energy in thermal agitation

$$\frac{e^2 n \Delta x^2}{2\epsilon_0} \simeq \frac{1}{2} k_B T_e$$

- The maximum distance an electron can travel is $\Delta x_{max} = \lambda_D$, where

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 n}}$$ *Debye length*

- Gas is considered a plasma if length scale of system is larger than Debye Length:

$$\lambda_D \ll \lambda$$ *Plasma criteria #2*

- Debye Length is spatial scale over which charge neutrality is violated by spontaneous fluctuations.
- Debye Number is defined as $N_D = 4\pi n \lambda_D^3 / 3$

Debye shielding

- Even though a plasma is electrically neutral in an average sense, charge density deviates from zero if we look at a very small region

- Electro-static potential around an ion
  - In the vicinity of ion, electrons are moving around by its thermal motion
  - Forming a kind of “cloud”
  - Screens the positive charge of ion

- Investigate this screening effect quantitatively

Debye shielding (cont.)

- Suppose immerse test particle $+Q$ within a plasma with $n_\mathrm{i} = n_\mathrm{e} = n$
- At $t = 0$, electric potential is $\Phi(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$
- As time progresses, electrons are attracted, while ions are repelled. As $m_\mathrm{i} >> m_\mathrm{e}$, we neglect motion of ions.
- At $t >> 0$, $n_\mathrm{e} > n_i$ and a new potential is set up, with charge density

Debye shielding (cont.)

- New potential evaluated using Poisson's equation:

$$\nabla^2\Phi(r) = -\frac{\rho}{\epsilon_0} = -\frac{e(n_e - n_i)}{\epsilon_0}$$

- In presence of potential, electron number density is

$$n_e(r) = ne^{-e\Phi(r)/k_B T_e}$$

- Subbing this into Poisson's equation in spherical coordinates.

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) = -\frac{en}{\epsilon_0}\left[e^{-e\Phi/k_B T_e} - 1\right]$$

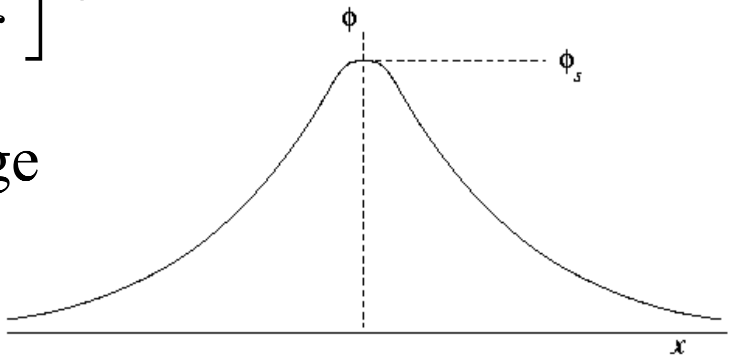
- For $|e\Phi| \ll k_B T_e$, it can be done Taylor expansion: $e^x \approx 1 + x \rightarrow$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Phi}{dr}\right) \approx \left[\frac{ne^2}{\epsilon_0 k_B T_e}\right]\Phi(r) = \frac{1}{\lambda_D^2}\Phi(r)$$

- Where λ_D is the *Debye shielding length*.

Debye shielding (cont.)

- Solution to previous is $\Phi(r) = \left[\frac{1}{4\pi\epsilon_0} \frac{Q}{r} \right] e^{-r/\lambda_D}$



- As $r \Rightarrow 0$, potential is that of a free charge in free space, but for $r >> \lambda_D$ potential
- falls exponentially.
- Coloumb force is long range in free space, but only extends to Debye length in plasma.
- For positive test charge, shielding cloud contains excess of electrons.
- Recall $\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 n}}$

$\Rightarrow$ size of shielding cloud increases as electron temperature becomes high which electrons can overcome Coulomb attraction. Also, λ_D is smaller for denser plasma because more electrons available to populate shielding cloud.

The plasma parameter

- The typical number of particles in a Debye sphere is given by *the plasma parameter*:

$$\Lambda = 4\pi n \lambda_D^3 = \frac{1.38 \times 10^6 T_e^{3/2}}{n^{1/2}}$$

- If $\Lambda \ll 1$, the Debye sphere is sparsely populated, corresponding to a strongly coupled plasma.
- Strongly coupled plasmas tend to be cold and dense, whereas weakly coupled plasmas tend to be diffuse and hot.
- Strongly coupled plasma: White dwarf, Neutron star atmosphere
- Weakly coupled plasma: space plasma, Magnetic fusion

Collisions

- Neutral particles have quite small collision cross sections. As Coulomb force is long range, charged particles are more frequent.
- A collisional plasma is $\lambda_{\mathrm{mfp}} << L$,
  where L is the observational length scale and $\lambda_{\mathrm{mfp}} = 1/\sigma n$ is the mean free path.
- The effective Coulomb cross-section is $\sigma = \pi r_c^2$
- An electron will be affected by a neighboring ion if the Coulomb potential is of the order of the electron thermal energy

$$\frac{e^2}{4\pi\epsilon_0 r_c} \approx \frac{3}{2} k_B T$$

$$\rightarrow \sigma = \pi \left(\frac{e^2}{6\pi k_B \epsilon_0} \right)^2 \frac{1}{T^2}$$

- At $T = 10^6$ K, $\sigma \sim 10^{-22}$ m^2, which is much larger than the geometric nuclear cross section of 10^{-33} m^2.

Collisions (cont.)

- In terms of the plasma parameter, the collision frequency ($\nu = nev$) is
$$\nu \approx \frac{\omega_p}{64\pi}\frac{\ln \Lambda}{\Lambda}$$
- Where ln(Λ) is the Coulomb logarithm. Used as Λ is large, but 10 < ln(Λ) < 30.
- In a weakly coupled plasma, $\nu \ll \omega_p$ => collisions do not effect plasma oscillations
- More rigorously, it can be shown that

$$\nu \approx \frac{\sqrt{2}\omega_p^4}{64\pi n_e}\left(\frac{k_B T}{m_e}\right)^{-3/2} \ln \Lambda$$

(Plasma frequency: $\omega_p \sim n^{1/2}$)

- Thus, diffuse, high temperature plasmas tend to be collisionless.

Mean free path

- Using Debye number, mean free path is

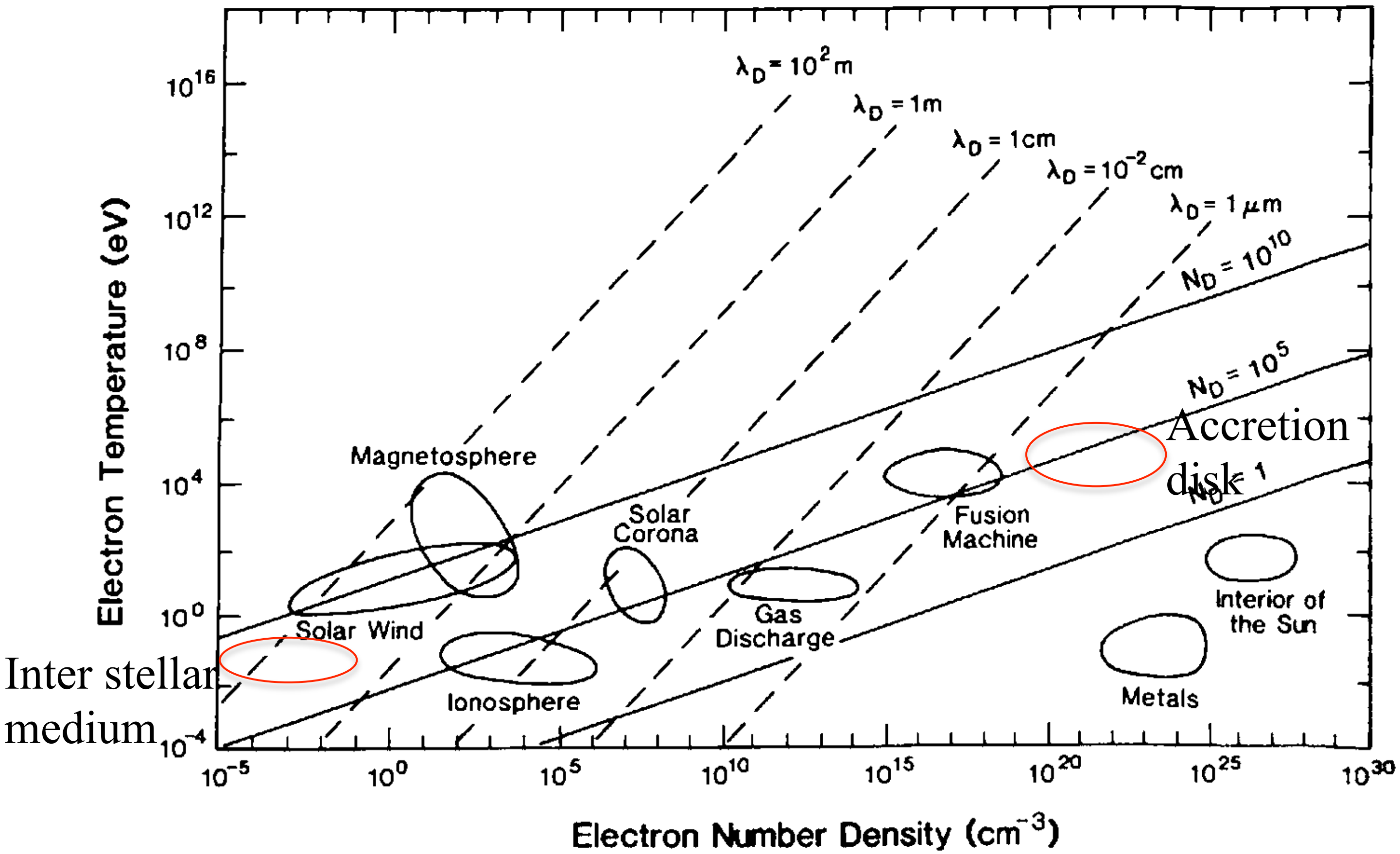
$$\lambda_{mfp} \approx \frac{36\pi}{n}\left(\frac{\epsilon_0 k_B T}{e^2}\right)^2$$
$$\approx 36\pi n \lambda_D^4 \sim \lambda_D N_D$$

- The condition $N_D >> 1$ is equivalent to $\lambda_{mfp} >> \lambda_D$
- If $N_D >>1$, electrons are moving around almost free from the collision (important for long-range collective effects)
- A single electron cannot effectively screen the ion's potential

= collision is not very effective when an electron approaches the ion at the distance λ_D.

- $N_D >> 1$ is generally satisfied in many astrophysical objects.
- If $N_D \sim 1$, the individual particles cannot be treated as a smooth continuum (short range correlation also important).

Plasma state



Example of plasma key parameter

	$n(m^{-3})$	$T(eV)$	$\omega_p(sec^{-1})$	$\lambda_D(m)$	Λ
Interstellar	10^6	10^{-2}	6×10^4	0.7	4×10^6
Solar Chromosphere	10^{18}	2	6×10^{10}	5×10^{-6}	2×10^3
Solar Wind (1AU)	10^7	10	2×10^5	7	5×10^{10}
Ionosphere	10^{12}	0.1	6×10^7	2×10^{-3}	1×10^5
Arc discharge	10^{20}	1	6×10^{11}	7×10^{-7}	5×10^2
Tokamak	10^{20}	10^4	6×10^{11}	7×10^{-5}	4×10^8
Inertial Confinement	10^{28}	10^4	6×10^{15}	7×10^{-9}	5×10^4

Table 1.1: *Key parameters for some typical weakly coupled plasmas.*

Summery

- A plasma is a quasi-neural ionized gas consisting of positive and negative charged particles
- Plasma oscillations are result of plasma trying to maintain charge neutrality.
- Plasma criteria #1: the collision time is longer than the oscillation period.
- The charge neutrality holds only for the average over the scale that is greater than Debye (shielding) length
- Plasma criteria #2: length scale of system is larger than Debye Length
- The condition Debye number $N_D >> 1$ is equivalent to mean-free path $\lambda_{mfp} >> \lambda_D$
- In many astrophysical plasma, collective effects dominate over collisions