

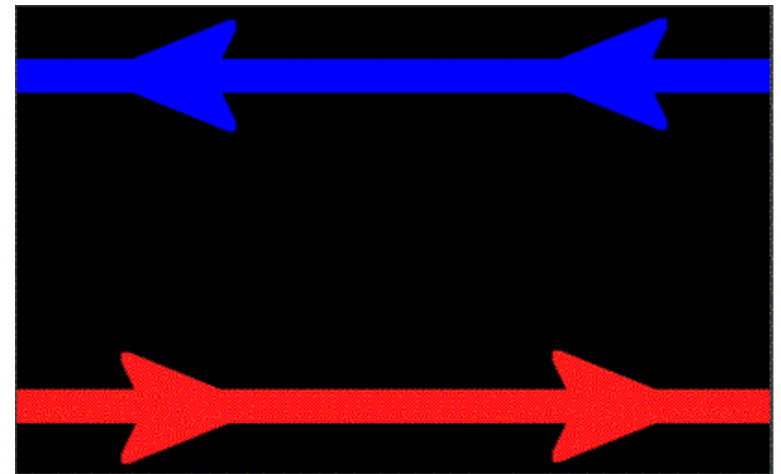
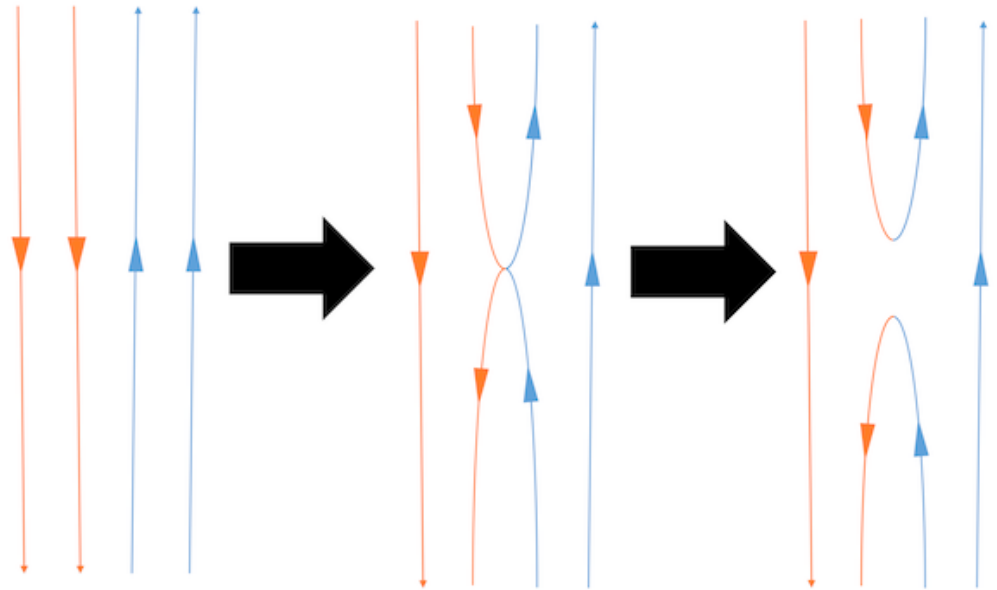
# Plasma Astrophysics

## Chapter 10: Magnetic Reconnection

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# Magnetic reconnection

- **Ideal MHD** gives frozen in magnetic fields.
- **Resistive MHD** allows diffusion of fields.
- Magnetic reconnection occurs through diffusion.
- *Magnetic reconnection* is the process of a rapid rearrangement of magnetic field topology.



# Magnetic reconnection (cont.)

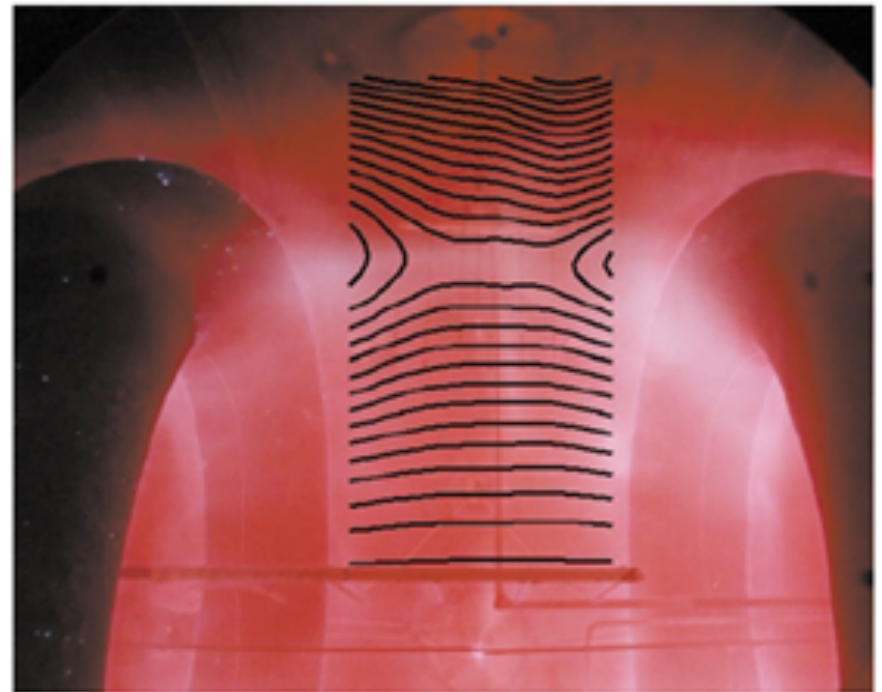
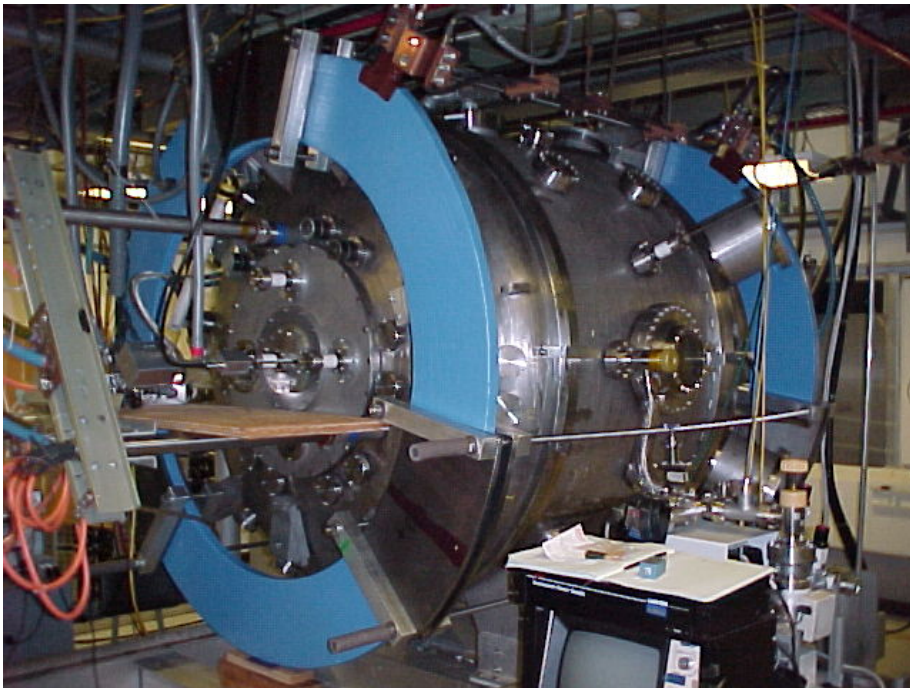
- Magnetic reconnection is **transient phenomena** (flaring event).
- This process leads rapid and violent release of stored magnetic energy (to thermal and kinetic energies, particle acceleration).
  
- In the universe,
  - interplanetary magnetic field => aurora in planet (earth etc.)
  - Sun (flare, CME)
  - Pulsar wind, Pulsar wind nebula and Pulsar jets (flare)
  - Magnetar flare
  - Relativistic jets (flare, time variability)
  - Gamma-ray bursts (time variability)
  
- Magnetic reconnection is observed in laboratory plasma experiment. Therefore, magnetic reconnection is **real physical phenomena**.

# Magnetic reconnection in Laboratory

Laboratory Plasma Experiment

Magnetic Reconnection Experiment (MRX)

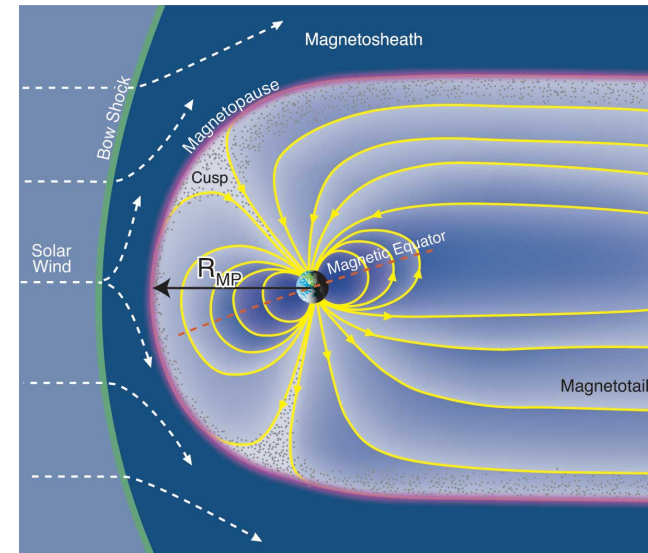
Magnetic reconnection seen in MRX



# Magnetic reconnection in Planetary magnetosphere

Planetary (Earth) magnetosphere, aurora (substorm)

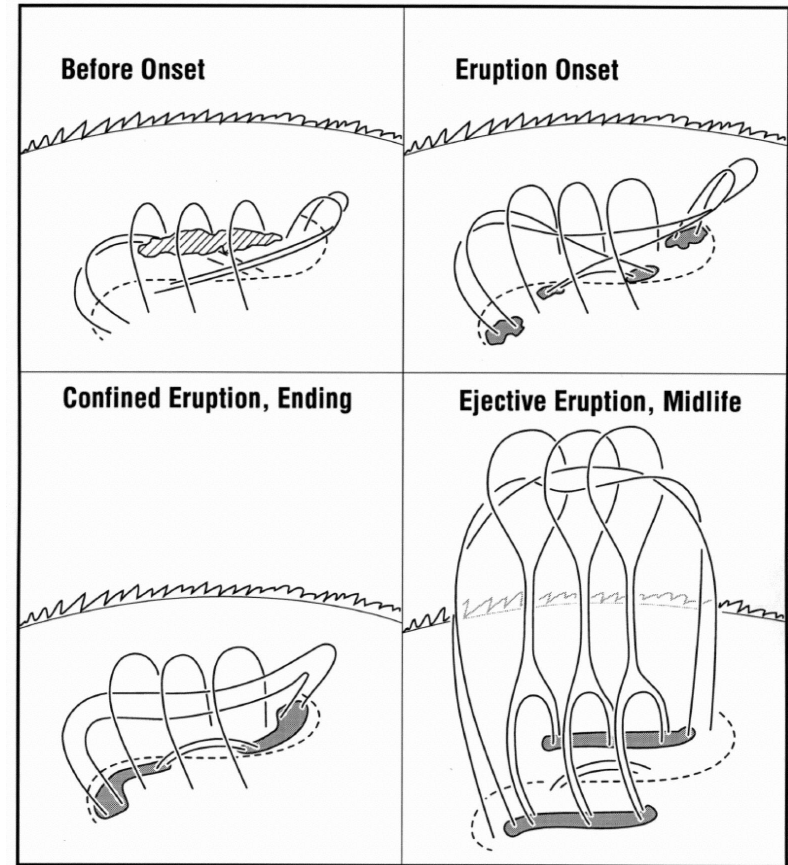
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# Magnetic reconnection in the Sun

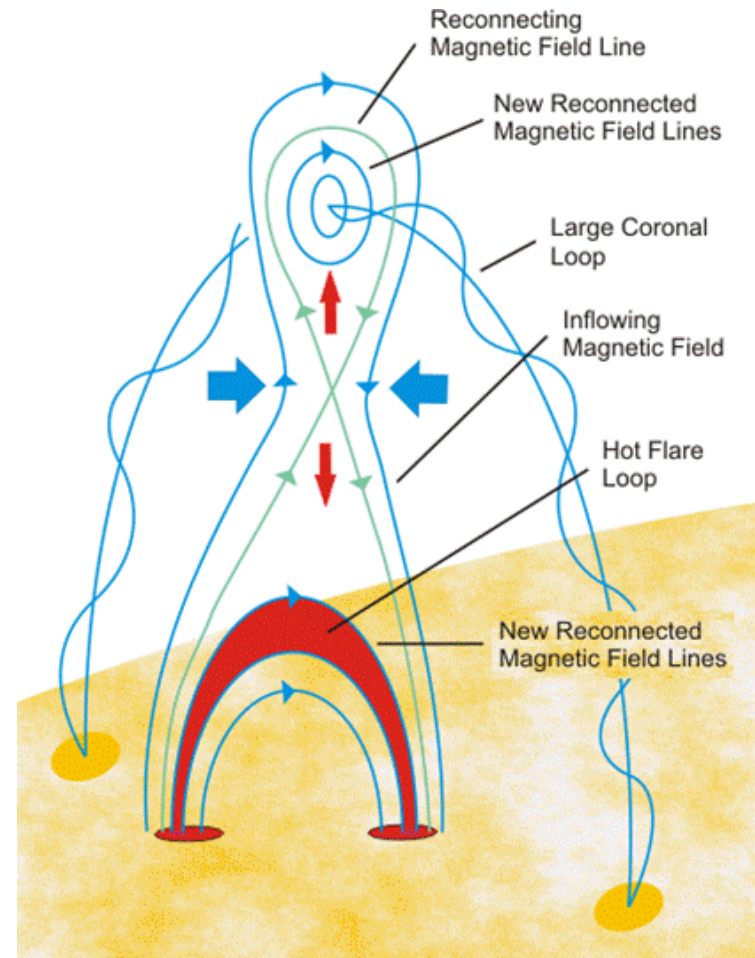
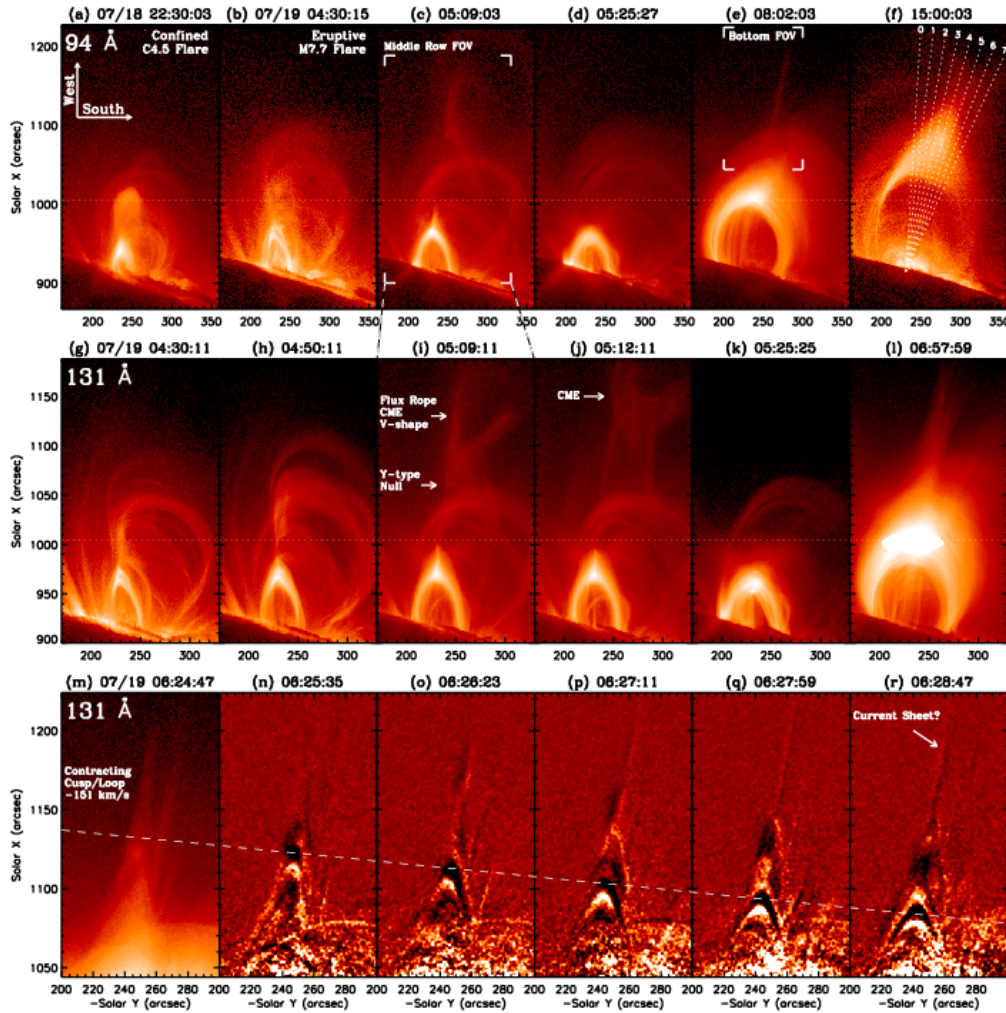
Two ribbon flare (top of view of solar flare)

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# Magnetic reconnection in the Sun (cont.)

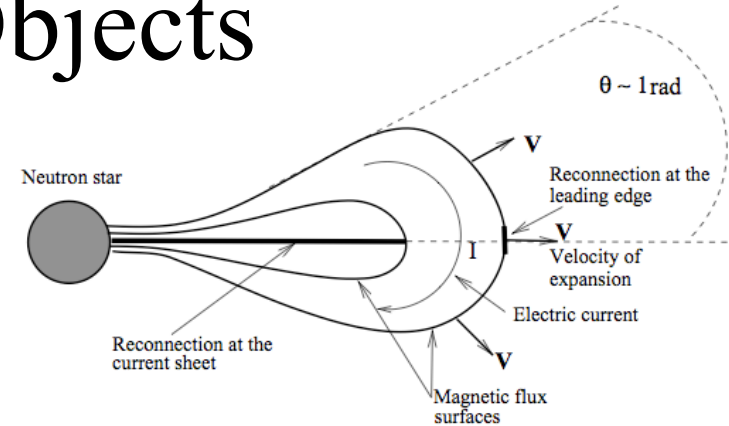
## Cusp flare (side view of Solar Flare)



# Magnetic Reconnection in Relativistic Astrophysical Objects

## Pulsar Magnetosphere & Striped pulsar wind

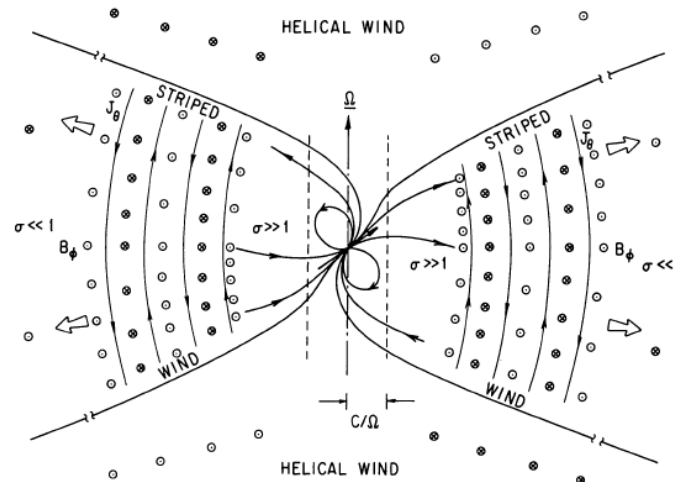
- obliquely rotating magnetosphere forms stripes of opposite magnetic polarity in equatorial belt
- magnetic dissipation via magnetic reconnection would be main energy conversion mechanism



## Magnetar Flares

- May be triggered by magnetic reconnection at equatorial current sheet

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# Magnetic diffusion

- Consider the simplest case in which magnetic field (or current) dissipates only by **resistivity** (= **diffusivity**)
- In this case, **diffusion time** for solar flare case becomes

$$\tau_D = L^2/\eta \simeq 10^{14} (L/10^9 \text{ cm})^2 (T/10^6 \text{ K})^{3/2} \text{ sec}$$

- Where  $L$  is typical size of solar flare,  $\eta$  is magnetic diffusivity (=  $c^2/4\pi\sigma$ ) and becomes

$$\eta \simeq 10^{13} T^{-3/2} \simeq 10^4 (T/10^6 \text{ K})^{-3/2} \text{ cm}^2 \text{ s}^{-1}$$

for coulomb collision. This is called *classical resistivity* or *Spitzer resistivity*.

- The diffusion time becomes  $10^{14}$  sec  $\sim$  3 million year. It is difficult to explain solar flare time scale (10-100 sec  $\sim$  10-100  $\tau_A$ ).

# Spitzer resistivity

- Classical model of electrical resistivity based on electron ion collisions
- Collision term:  $P_{ei} = \nu_{ei} n_e m_e (v_e - v_i)$  (see ch 3.5)
- Collisions are Coulomb collisions,  $P_{ei}$  is proportional to Coulomb force ( $e^2$ ) and densities, so we can write

$$P_{ei} = \eta e^2 n^2 (v_e - v_i) \quad (\text{see ch 4, here } q = e)$$

- So collision frequency is written as  $\nu_{ei} = (ne^2/m_e)\eta$
- Collision cross section:  $\sigma = \pi r_c^2 = e^4 / 16\pi\epsilon_0^2 m_e^2 v_e^4$  ( see ch 1, here  $mv^2 \sim 3/2 k_B T$ )
- Collision frequency:  $\nu_{ei} = n\sigma v_e = ne^4 / 16\pi\epsilon_0^2 m_e^2 v_e^3$
- Resistivity is ( $mv^2 \sim 3/2 k_B T$ )

$$\eta \approx \frac{\pi e^2 m_e^{1/2}}{(4\pi\epsilon_0)^2 (k_B T)^{3/2}}$$

# Spitzer resistivity (cont.)

- This resistivity is considered large-angle collision alone. Therefore we also need to consider small-angle collision
- Spitzer is shown as

$$\eta \approx \frac{\pi e^2 m_e^{1/2}}{(4\pi\epsilon_0)^2 (k_B T)^{3/2}} \ln \Lambda$$

*Spitzer resistivity*

- $\Lambda$  is plasma parameter (see ch.1)

# Magnetic diffusion (cont.)

- Dynamical time scale is defined as **the Alfvén transit time**,  $\tau_A$

$$\tau_A = L/v_A \simeq 1(L/10^9 \text{ cm})(B/100 \text{ G})^{-1}(n/10^9 \text{ cm}^{-3})^{1/2} \text{ sec}$$

$$v_A = B/(\mu_0 \rho)^{1/2} \simeq 10^4 (B/100 \text{ G})(n/10^9 \text{ cm}^{-3})^{-1/2} \text{ km/s}$$

- The magnetic Reynolds number (Lundquist number)  $R_m$  ( $S$ ) is defined by

$$R_m \equiv \mu_0 v L / \eta \quad (\text{see ch 4}) \quad S \equiv \mu_0 v_A L / \eta \quad (\text{Lundquist number})$$

$$S \simeq \tau_D / \tau_A = v_A L / \eta$$

$$\simeq 10^{14} (L/10^9 \text{ cm})(T/10^6 \text{ K})^{3/2} (B/100 \text{ G})(n/10^9 \text{ cm}^{-3})^{-1/2} \quad (10.1)$$

- Explosive magnetic energy release occurs even though  $S$  ( $\sim R_m$ )  $\gg 1$  in every kind of hot plasma. This puzzle is considered to be **one of the most difficult and fundamental problems in nature.**

# Magnetic Reynolds / Lundquist number

- The magnetic Reynolds number:  $R_m \equiv \mu_0 v L / \eta$
- Lundquist number:  $S \equiv \mu_0 v_A L / \eta$
- If the typical velocity of motion is Alfven speed,  $v=v_A$ , magnetic Reynolds number = Lundquist number
- The magnetic reconnection is treated the dynamics of magnetic field. So the typical velocity of motion is Alfven speed.
- Therefore in the study of magnetic reconnection

$$R_m = S = \mu_0 v_A L / \eta$$

# Magnetic diffusion (cont.)

	Magnetopause	Magnetotail	Solar corona (flares)
$n_0$ [cm <sup>-3</sup> ]	4	0.5	$6 \cdot 10^8$
$B_0$ [nT]	40	20	$3 \cdot 10^7$
$L_0$ [m]	$10^6$	$10^7$	$10^7$
$v_A$ [m/s]	$4.4 \cdot 10^5$	$6.2 \cdot 10^5$	$2.7 \cdot 10^7$
$\tau_A$ [s]	2.3	1.6	0.37
$T_i$ [K]	$10^6$	$5 \cdot 10^7$	$10^6$
$T_e$ [K]	$10^5$	$5 \cdot 10^6$	$10^6$
$\omega_{pe}$ [s <sup>-1</sup> ]	$1.1 \cdot 10^5$	$4.0 \cdot 10^4$	$1.4 \cdot 10^9$
$\Lambda$	$1.3 \cdot 10^{11}$	$1.3 \cdot 10^{13}$	$7 \cdot 10^7$
Collision frequency $\nu_c$ [s <sup>-1</sup> ]	$2 \cdot 10^{-7}$	$10^{-9}$	10
$\lambda_e$ [m]	$2.7 \cdot 10^3$	$8.0 \cdot 10^3$	0.2
$\tau_{diff}$ [s]	$7 \cdot 10^{11}$	$1.5 \cdot 10^{15}$	$2.5 \cdot 10^{14}$
$R$	$3 \cdot 10^{11}$	$10^{15}$	$10^{15}$

Collision  
frequency

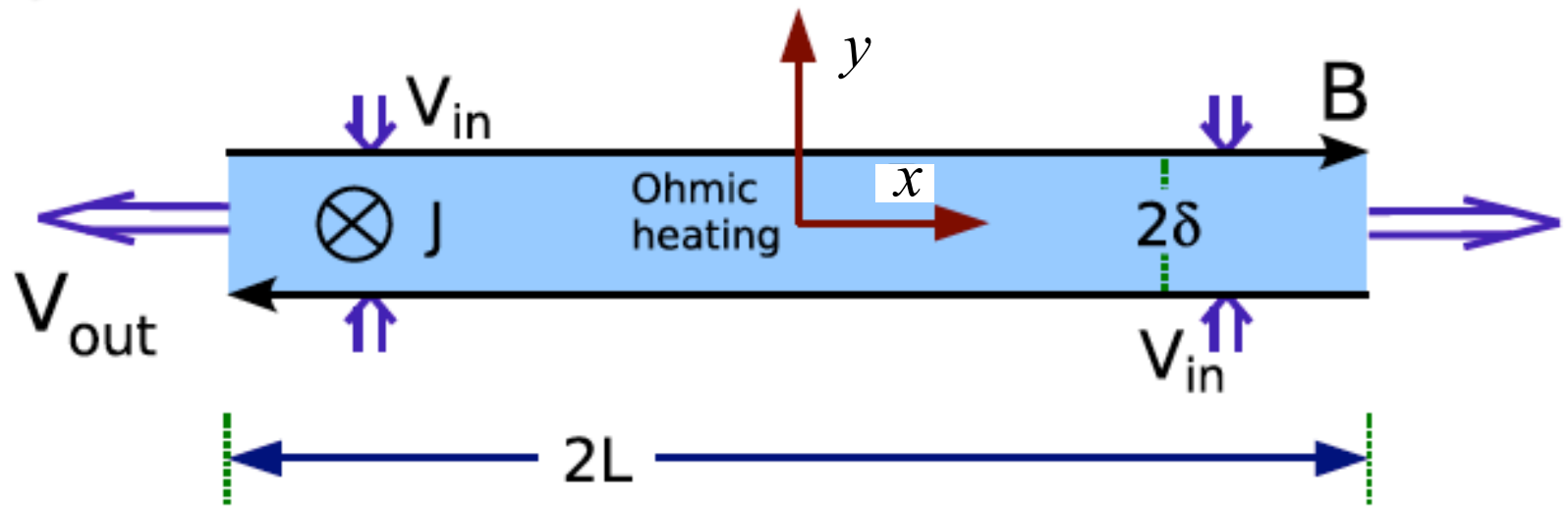
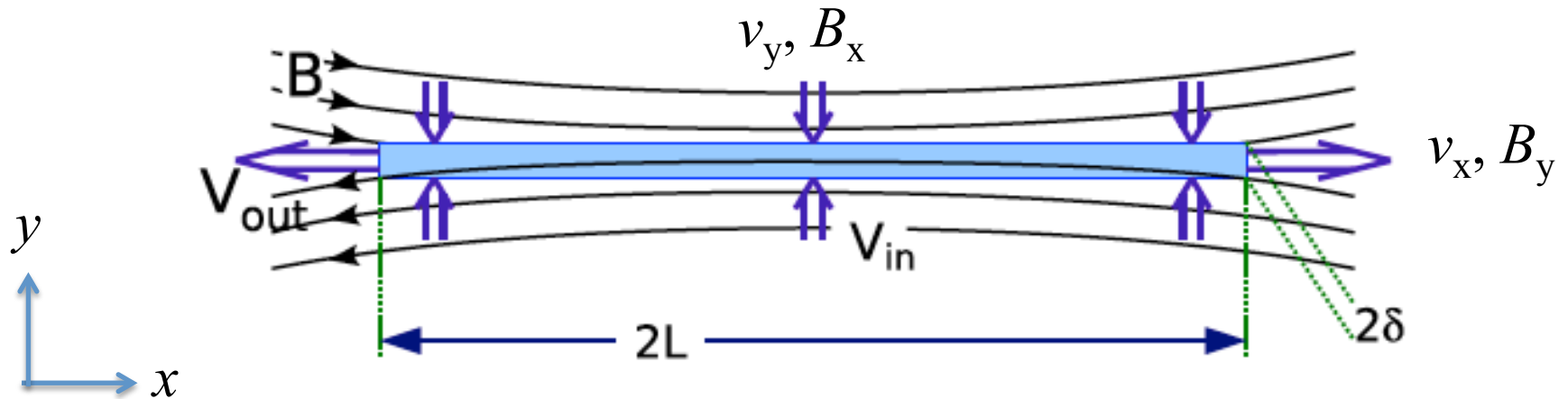
Lundquist number



# Sweet-Parker Model

- Sweet (1958) and Parker (1957) have developed **a simple theory of steadily driven reconnection**, taking into account the effect of plasma flow (2D model).
- Consider current sheet (or diffusion region) has a total length of  $2L$  and total thickness of  $2\delta$  (aspect ratio of the diffusion region is fixed).
- At the inflow region, a magnetic field  $B_0$  that is strongly tangential to the inflow boundary. The inflow velocity is approximately constant with  $v_{\text{in}}$ .
- At the outflow boundary the magnetic field is  $B_{\text{out}}$  and the outflow velocity is  $v_{\text{out}}$ .
- The the region inside the diffusion region is assumed to be **dominated by diffusion** and the outside region is assumed to be ideal.

# Sweet-Parker Model (cont.)



Diffusion region (current sheet)

$$v_{in} = v_y, B_{in} = B_x \quad v_{out} = v_x, B_{out} = B_y$$



# Sweet-Parker Model (cont.)

- First we estimate the **inflow velocity** to diffusion region.
- From the assumption, there is **no current** in inflow region.

Therefore, from Ohm's law,

$$E = -v_{in} B_{in}$$

$$\mathbf{E} = -(\mathbf{v} \times \mathbf{B}) + \eta \mathbf{j}$$

- At the center (diffusion region), there is **no magnetic field**. Hence from induction equation

$$E = \eta j$$

- From Ampere's law, The current at the center is roughly estimated as

$$j = \frac{B_{in}}{\mu_0 \delta}$$

$$\mathbf{j} = \nabla \times \mathbf{B} / \mu_0$$

- From them, the inflow velocity is

$$v_{in} = \frac{\eta}{\mu_0 \delta} \quad (10.2)$$

# Sweet-Parker Model (cont.)

- From mass conservation, inflowing mass and outflowing mass is conserved:

$$Lv_{in}\rho = \delta v_{out}\rho$$

$$v_{in} = v_y, \quad B_{in} = B_x$$

- This gives **velocity ratio**:

$$v_{out} = v_x, \quad B_{out} = B_y$$

$$\frac{\delta}{L} = \frac{v_{in}}{v_{out}} \quad (10.3)$$

- The same relation is hold for magnetic field

$$\frac{B_{in}}{B_{out}} = \frac{v_{in}}{v_{out}} = \frac{\delta}{L} \quad (10.4)$$

# Sweet-Parker Model (cont.)

- We calculate the **outflow velocity**. Consider the equation of motion (assuming steady and ignore pressure) in x-direction (parallel to outflow),

$$\rho(\mathbf{v} \cdot \nabla)v_x = (\mathbf{j} \times \mathbf{B})_x$$

$$\rho v_x \frac{\partial v_x}{\partial x} = \frac{1}{\mu_0} \left( -\frac{\partial}{\partial x} \frac{B^2}{2} + (\mathbf{B} \cdot \nabla)B_x \right)$$

- Along center of diffusion region,  $\partial B_x / \partial x = 0$ ,  $\partial B_y / \partial x \approx 0$
- Therefore the equation of motion is

$$\rho v_x \frac{\partial v_x}{\partial x} \approx \frac{B_y}{\mu_0} \left( \frac{\partial B_x}{\partial y} \right)$$

$$v_{in} = v_y, \quad B_{in} = B_x$$

$$v_{out} = v_x, \quad B_{out} = B_y$$

- This gives the result:

$$\rho \frac{v_{out}^2}{L} \approx \frac{B_{out} B_{in}}{\mu_0 \delta} \quad (10.5)$$

# Sweet-Parker Model (cont.)

- We put the relation  $B_{out}/\delta = B_{in}/L$  (10.4) into this equation,

$$\rho \frac{v_{out}^2}{L} \approx \frac{B_{out} B_{in}}{\mu_0 \delta}$$

- This gives

$$v_{out}^2 \approx \frac{B_{in}^2}{\mu_0 \rho} = v_{A,in}^2 \quad (10.6)$$

- That is, the outflow velocity (**reconnection jet(outflow)**) is of the order of the **Alfven speed** in inflow region.

# Sweet-Parker Model (cont.)

- We calculate the reconnection rate. Using eq(10.2) and (10.3), the inflow velocity is written as

$$v_{in} = v_{A,in} \frac{\delta}{L} = \frac{v_{A,in}}{\sqrt{S}} \quad (10.7) \quad \begin{aligned} v_{in}^2 &= \eta v_{A,in} / \mu_0 L \\ S &= \mu_0 v_A L / \eta \end{aligned}$$

- The resulting *reconnection rate* for the Sweet-Parker model is

$$E = \eta j = \frac{\eta B_{in}}{\mu_0 \delta} = \frac{B_{in} v_{A,in}}{\sqrt{S}}$$

- Thus the *reconnection rate* normalized to the typical electric field  $E = B_{in} v_{A,in}$  is

$$r = \frac{v_{in}}{v_{A,in}} = \frac{\delta}{L} = \frac{1}{\sqrt{S}}$$

*Normalized reconnection rate*

# Sweet-Parker Model (cont.)

- Since Lundquist (magnetic Reynolds) number is typically **very large number**, the resulting reconnection rate is **very small**.
- This implies also a **very large aspect ratio**  $L/\delta$  (see normalized reconnection rate).

- In solar flare case, **the time scale of Sweet-Parker reconnection** is

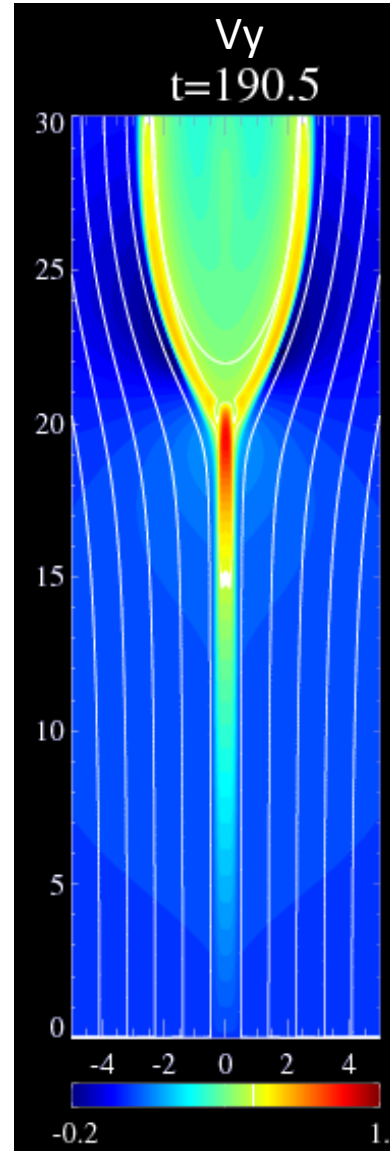
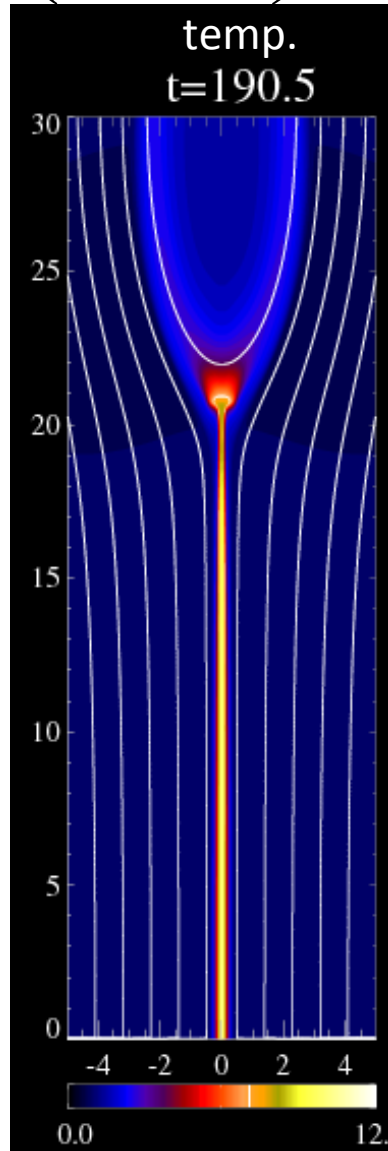
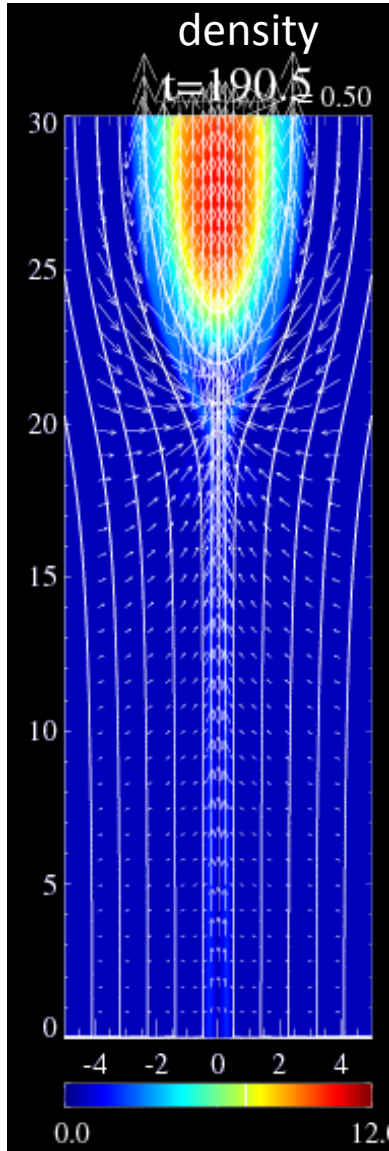
$$\tau_{SP} = L/v_{in} = L\sqrt{S}/v_{A,in} \approx 10^7 \text{ sec}$$

- This is **not fast** enough to explain the solar flare (10-100 sec).
- Sweet-Parker reconnection is *slow reconnection*
- Using classical resistivity, the thickness of diffusion region for solar flare is  
$$\delta \simeq L/\sqrt{S} \sim 1 \text{ m}$$

# Sweet-Parker Model

density

(cont.)



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# Petschek Model

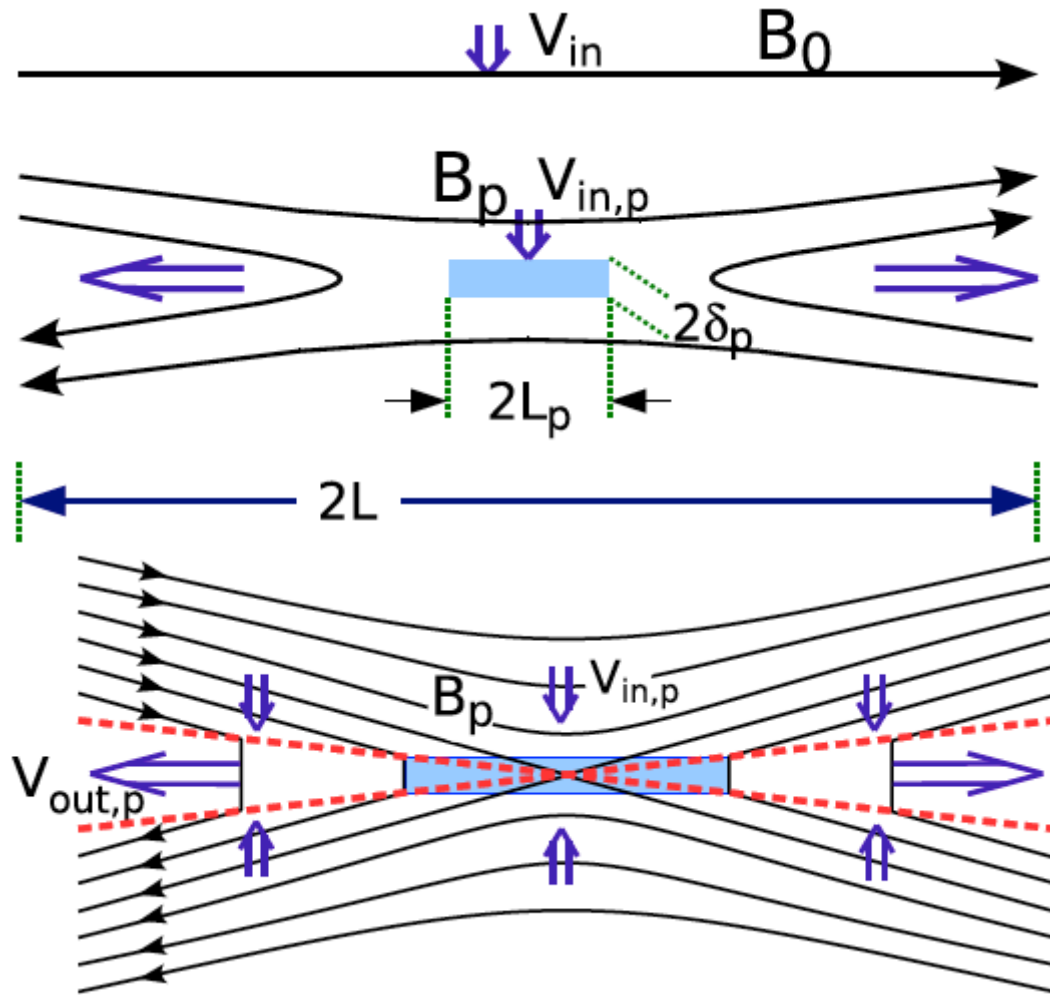
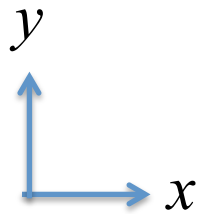
- The **large aspect ratio** limits the possible reconnection rate in the Sweet-Parker model
- Petschek (1964) realized that a much larger reconnection rate would be possible if **the diffusion region were much shorter**.
- The main point in Petschek's model is that the length of the diffusion region should be much **shorter**  $L_p \ll L$  in order to realize a higher reconnection rate.
- A much shorter diffusion region also implies a **thinner diffusion region**, i.e.,  $\delta_p \ll \delta$ , because faster reconnection implies a larger electric field and a higher current density at the x-line.
- The fast transport out of this diffusion region over the scale from  $L_p$  to  $L$  occurs through a flow layer which is bounded by **slow shocks**.



# Petschek Model (cont.)

$$v_{in} = v_y, \quad B_{in} = B_x$$

$$v_{out} = v_x, \quad B_{out} = B_y$$



# Petschek Model (cont.)

- First look at the **scaling** in the vicinity of diffusion region.
- The magnetic field in the inflow of the Petschek region is denoted  $B_{in,p}$  which is smaller than  $B_{in}$

- However, from magnetic flux conservation

$$v_{in} B_{in} = v_{in,p} B_{in,p} \text{ (and assuming constant mass density)}$$

using  $v_{A,p} = B_{in,p} / \sqrt{\mu_0 \rho} = v_A B_{in,p} / B_{in}$

- the **inflow Alfvén Mach number** is ( $\mathcal{M}_{in} = v_{in} / v_A$ )

$$\frac{\mathcal{M}_{in}}{\mathcal{M}_{in,p}} = \frac{v_{in}}{v_{in,p}} \frac{v_{A,p}}{v_A} = \frac{B_{in,p}^2}{B_{in}^2} \quad (10.8)$$

# Petschek Model (cont.)

- Using  $S = \mu_0 L v_A / \eta$ ,  $S_p = \mu_0 L_p v_{A,p} / \eta$   
and the relation  $S_p = v_{A,p}^2 / v_{in,p}^2 = 1 / \mathcal{M}_{in,p}^2$   
for length scale we obtain

$$\frac{L_p}{L} = \frac{S_p}{S} \frac{v_A}{v_{A,p}} = \frac{1}{S} \frac{v_A}{v_{A,p}} \frac{v_{A,p}^2}{v_{in,p}^2} = \frac{1}{S} \frac{v_{A,p}^2}{v_{in,p}^2} \frac{B_{in}}{B_{in,p}} \quad (10.9)$$

- Using  $B_{in} / B_{in,p} = \mathcal{M}_{in,p}^{1/2} / \mathcal{M}_{in}^{1/2}$  (from eq(10.8)), we obtain

$$\frac{L_p}{L} = \frac{1}{S} \frac{1}{\mathcal{M}_{in,p}^{3/2}} \frac{1}{\mathcal{M}_{in}^{1/2}} \quad (10.10)$$

- or

$$\frac{\delta_p}{L} = \frac{L_p}{L} \frac{1}{\sqrt{S_p}} = \frac{1}{S} \frac{1}{\mathcal{M}_{in,p}^{1/2}} \frac{1}{\mathcal{M}_{in}^{1/2}} \quad (10.11)$$

# Petschek Model (cont.)

- Thus, once we have determined  $B_{in,p}/B_{in}$ , we can obtain the ratio of the inflow Mach numbers as well as the scale of the Petschek diffusion region.
- This is just a re-scaling of the diffusion region in size and is applicable to any smaller scale diffusion region.
- Note that similar to the Sweet-Parker model, Petschek reconnection model also **does not treat the diffusion region self-consistently**.
- The main insight from Petschek model is that the transport from  $L_p$  to  $L$  does not require the thin outflow layer.
- Petschek suggested that the outflow region is bounded by **two slow switch-off shocks**.
- This is strictly only true if **the shocks are horizontal**, i.e., aligned with the x-direction.
- But, since the shocks are only very slightly inclined, the error in the assumption of switch-off shocks is small

# Petschek Model (cont.)

- From the discussion of switch-off slow shocks, away from diffusion region, to satisfy **the switch-off shock condition**, the plasma in outflow region is jetting with at the speed of

$$v_{out,p} = v_{At,p} \approx B_{in,p} / \sqrt{\mu_0 \rho} \quad (10.12)$$

- Noted that the actual normal velocity in the downstream (outflow) region is 0.
- This means that the slow shock is moving with a velocity

$$v_{nu} = v_{An} = B_n / \sqrt{\mu_0 \rho}$$

toward the upstream (inflow) region.

# Petschek Model (cont.)

- The prior discussion assumes that the density is **constant**, which is **incorrect** for a slow shock.
- We can compute the outflow density from the switch-off shock properties.
- To do so, one has to determine the angle of the upstream magnetic field with the shock normal.
- The tangential and normal components are  $B_{in,p}$  and  $B_n$
- Magnetic flux conservation implies  $v_{nu} B_{in,p} = v_{At,p} B_n$
- The angle the upstream magnetic field is

$$\tan \theta = \frac{B_n}{B_{in,p}} = \frac{v_{nu}}{v_{At,p}} = r_p \quad (10.12)$$

- where  $r_p$  is the **reconnection rate for the Petschek diffusion region**

# Petschek Model (cont.)

$$\tan \theta = \frac{B_n}{B_{in,p}} = \frac{v_{nu}}{v_{At,p}} = r_p$$

- This indicates that  $\theta \approx r_p \ll 1$ .
- In this case, inflow velocity is  $v_{in} \approx \theta v_{A,p}$

# Petschek Model (cont.)

- Determine the magnetic field at the diffusion region  $B_{in,p}$
- This field is modified because the field in the vicinity of the diffusion region is curved and different from the field  $B_{in}$  at large distance from the diffusion region
- From long calculations, the magnetic field at the diffusion region is obtained as

$$B_{in,p} = B_{in} - \frac{4B_n}{\pi} \ln \frac{L}{L_p} = B_{in} \left( 1 - \frac{4\mathcal{M}_{in}}{\pi} \ln \frac{L}{L_p} \right) \quad (10.13)$$



# Petschek Model (cont.)

- For  $\frac{4\mathcal{M}_{in}}{\pi} \ln \frac{L}{L_p} \leq \frac{1}{2}$ , eq(10.13) implies roughly  $B_{in,p} \approx B_{in}$

such that  $\mathcal{M}_{in,p} \approx \mathcal{M}_{in}$  (modification is small)

- From eq (10.10), the relations for the diffusion region scales become

$$\frac{L_p}{L} = \frac{1}{S} \frac{1}{\mathcal{M}_{in}^2}, \quad \frac{\delta_p}{L} = \frac{1}{S} \frac{1}{\mathcal{M}_{in}}$$

- Petschek suggested that the process becomes **inefficient** if  $B_{in,p}$  becomes too **small**

# Petschek Model (cont.)

- Assuming that a reasonable value for the minimum  $B_{in,p}$  is  $B_{in,p} \sim B_{in}/2$  yields for the approximate maximum inflow Mach number (or reconnection rate)

$$\mathcal{M}_{in} = r_p \approx \frac{\pi}{8 \ln(L/L_p)} = \frac{\pi}{8 \ln(\mathcal{M}_{in}^2 S)} \sim \frac{\pi}{8 \ln S}$$

- This reconnection rate is much **larger** than the Sweet-Parker rate (*fast reconnection*).
- For instance, the case of  $R_m = 10^8$ ,
  - the Petschek reconnection rate is  $r_p \sim 2 \times 10^{-2}$
  - The Sweet-Parker reconnection rate is  $r_{sp} \sim 10^{-4}$

# Petschek Model (cont.)

- The reason is that the aspect ratio  $\delta_p/L_p$  is much **larger** than  $\delta/L$  for the Sweet-Parker diffusion region.
- This is accomplished by a much **smaller** length of the diffusion region

$$L_p \approx 64L \ln^2 S / (\pi^2 S)$$

- However, the thickness of diffusion region is also much **smaller**

$$\delta_p \approx 8\delta \ln S / (\pi \sqrt{S})$$

- In solar corona case, using classical resistivity ( $S \sim 10^{14}$ ), the diffusion length of Petschek reconnection becomes **smaller** than ion gyro-radius ( $\sim 10$  cm) (*MHD is acceptable?*).
- Although Petschek model is a very attractive idea, the question arises whether the single Petschek reconnection controls the entire flare process or not (large scale difference).

# Sweet-Parker vs Petschek

	Magnetopause	Magnetotail	Solar corona (flares)
$n_0$ [cm <sup>-3</sup> ]	4	0.5	$6 \cdot 10^8$
$B_0$ [nT]	40	20	$3 \cdot 10^7$
$L$ [m]	$10^6$	$10^7$	$10^7$
$v_A$ [m/s]	$4.4 \cdot 10^5$	$6.2 \cdot 10^5$	$2.7 \cdot 10^7$
$E_0 = v_A B_0$ [V]	$1.8 \cdot 10^{-2}$	$1.3 \cdot 10^{-2}$	$8 \cdot 10^5$
$\tau_A$ [s]	2.3	1.6	0.37
$v_{thi}$ [K]	$10^5$	$7 \cdot 10^5$	$10^5$
$v_{the}$ [K]	$1.3 \cdot 10^6$	$10^7$	$4.3 \cdot 10^6$
$\lambda_e$ [m]	$2.7 \cdot 10^3$	$8.0 \cdot 10^3$	0.2
$\tau_{diff}$ [s]	$7 \cdot 10^{11}$	$1.5 \cdot 10^{15}$	$2.5 \cdot 10^{14}$
Lundquist number $\rightarrow R$	$3 \cdot 10^{11}$	$10^{15}$	$10^{15}$
Reconnection rate $r_{sp}$	$1.8 \cdot 10^{-6}$	$3 \cdot 10^{-8}$	$3 \cdot 10^{-8}$
$r_p$	$1.5 \cdot 10^{-2}$	$1.1 \cdot 10^{-2}$	$1.1 \cdot 10^{-2}$
$r_{obs}$	$3 \cdot 10^{-2}$	$3 \cdot 10^{-2}$	$10^{-4}$ to $10^{-2}$
Thickness of diffusion region $\delta_{sp}$ [m]	1.8	0.3	0.3
$\delta_p$ [m]	$2 \cdot 10^{-4}$	$10^{-6}$	$10^{-6}$
$L_p$ [m]	$1.4 \cdot 10^{-2}$	$10^{-4}$	$10^{-4}$

# Sweet-Parker vs Petschek (cont.)

	Magnetopause	Magnetotail	Solar corona (flares)
$L$ [m]	$10^6$	$10^7$	$10^7$
$v_A$ [m/s]	$4.4 \cdot 10^5$	$6.2 \cdot 10^5$	$2.7 \cdot 10^7$
$\tau_A$ [s]	2.3	1.6	0.37
$R$	$3 \cdot 10^{11}$	$10^{15}$	$10^{15}$
$\tau_{diff}$ [s]	$7 \cdot 10^{11}$	$1.5 \cdot 10^{15}$	$2.5 \cdot 10^{14}$
Reconnection time $\tau_{sp}$ [s]	$10^6$	$5 \cdot 10^7$	$10^7$
$\tau_p$ [s]	150	150	35
$v_{diff}$ [m/s]	$1.5 \cdot 10^{-6}$	$6 \cdot 10^{-10}$	$2.7 \cdot 10^{-8}$
Outflow velocity $v_{sp}$ [m/s]	0.8	0.02	0.1
$v_p$ [m/s]	$3 \cdot 10^3$	$5.5 \cdot 10^3$	$2.5 \cdot 10^5$

# Petschek Model (cont.)

- The numerical simulations show that the Petschek model for fast reconnection does not arise under **uniform resistivity** in the limit of large  $S$ . (Biskamp 1986)
- In order to develop Petschek-type reconnection, we need to use non-uniform resistivity (spatially-localized resistivity), so-called *anomalous resistivity*
- What is the anomalous resistivity?
- Until now, it is still unknown. But it may be related microscopic (kinetic) process.

# Petschek Model (cont.)

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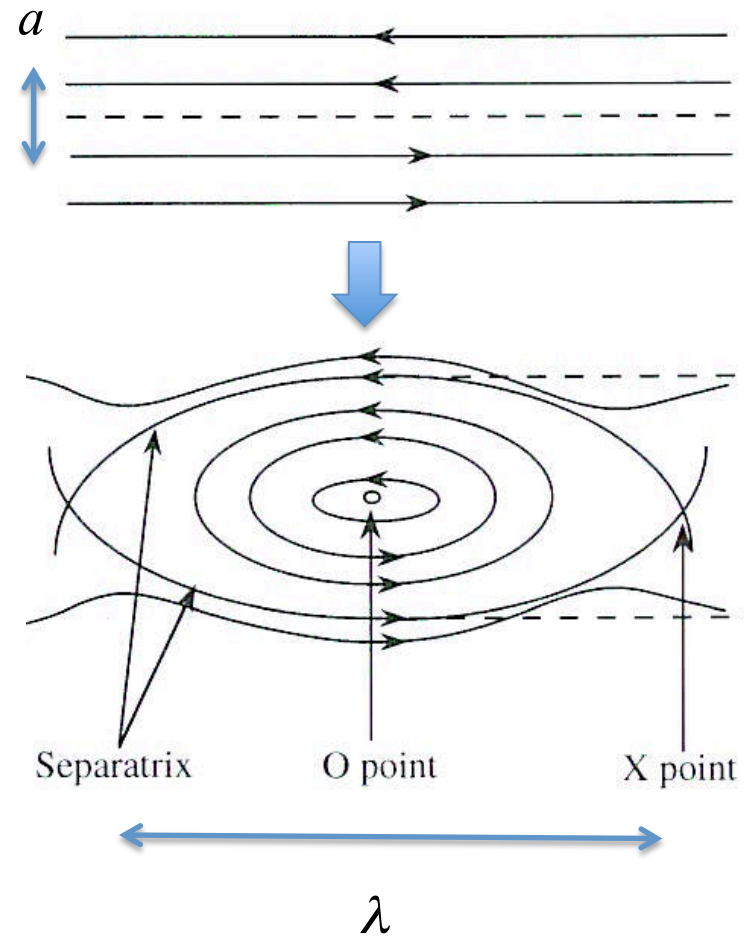
# Tearing instability

- Sweet-Parker model is **slow reconnection process**. It can not be explained solar flare.
- Petschek model is **fast reconnection process**. It may be explained solar flare time scale but physically not-perfect.
- Here we again focus on *Sweet-Parker model*.
- The problem for slow reconnection is the **large aspect ratio**.
- Question is can we make small aspect ratio?
- In the evolution of Sweet-Parker reconnection, the diffusion region (current-sheet) is expanded.
- Such **long current-sheet** is **unstable** against *tearing instability*



# Tearing instability

- Tearing mode is **resistive instability** that occurs in presence of current sheet
- The tearing mode forms magnetic islands
- The magnetic island grow unsteadily.
- We follow Furth, Killeen & Rosenbluth (1963) (FKR) study



# Tearing instability (cont.)

- According to their analysis, the growth rate of the tearing instability is of the order of

$$\gamma \sim \alpha^{-2/5} \tau_{D,*}^{-3/5} \tau_{A,*}^{-2/5} \sim \alpha^{-2/5} \tau_{A,*}^{-1} S_*^{-3/5}$$

- For **large**  $S_*$  and **long wavelength**  $\lambda > 2\pi a$  in parallel to current sheet, where

$$\tau_{D,*} = a^2 / \eta,$$

$$\tau_{A,*} = a / v_A,$$

$$S_* \simeq \tau_{D,*} / \tau_{A,*} = a v_A / \eta,$$

$$\alpha = k a = 2\pi a / \lambda$$

- $a$  is **the thickness of current sheet** and  $k$  is transverse wavenumber of the tearing mode

# Tearing instability (cont.)

- From FKR, the instability only exists for

$$S_*^{-1/4} < \alpha < 1$$

- Using the fastest growing mode, **the maximum growth rate** is

$$\gamma_{max} \sim \tau_{A,*}^{-1} S_*^{-1/2}$$

- From numerical analysis of the linear tearing instability without using the so-called constant  $\psi$  approximation (Steinolfson & van Hoven 1984), the maximum growth rate is

$$\begin{aligned} \gamma_{max} &\sim \tau_{A,*}^{-1} \alpha^{2/3} S_*^{-1/3} \\ \alpha_{max} &\sim S_*^{-1/4} \end{aligned}$$

- This growth rate is faster than that found by FKR.

# Tearing instability (cont.)

- Let us now apply to solar corona, If we assume  $a = 10^4$  km (typical length of solar flare), the Lundquist (magnetic Reynolds) number becomes  $S_* \sim 10^{14}$ . Then we find

$$\alpha_{max} \sim 10^{-3.5}, \text{ i.e., } \lambda_{max} \sim 2 \times 10^4 a \sim 2 \times 10^8 \text{ km} > R_{\odot}$$

- Hence the (most unstable) tearing mode cannot be applied to the solar corona.
- But if we assume  $a = 1$  km ( $10^3$  times larger thickness estimated from SP model), we have  $S_* = 10^{10}$ , and

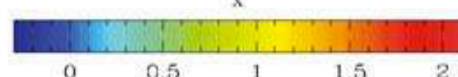
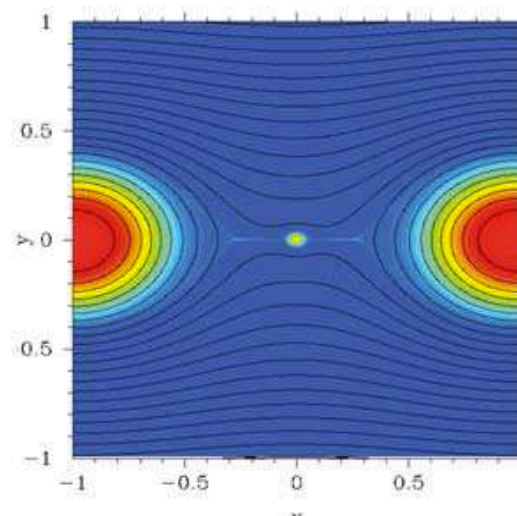
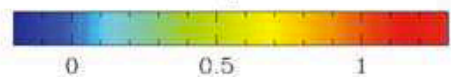
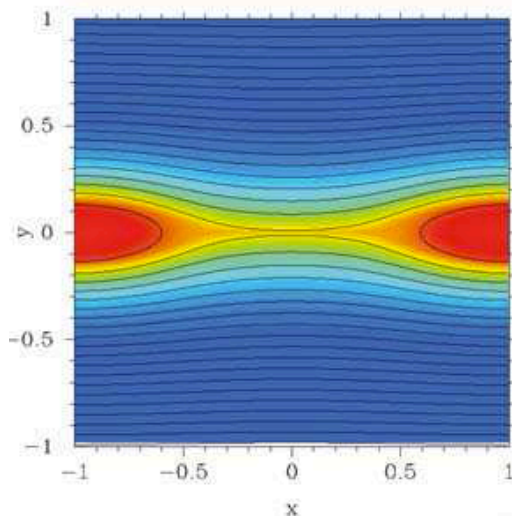
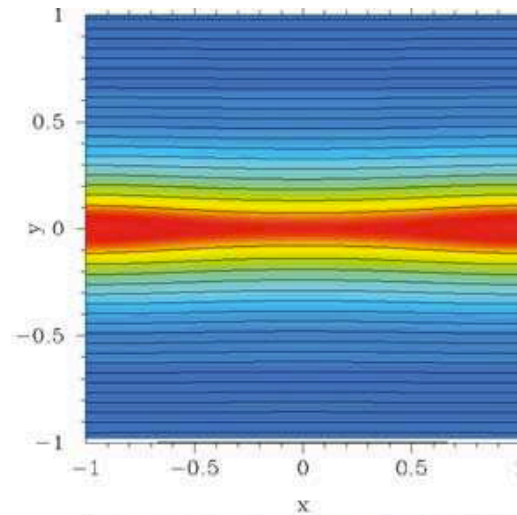
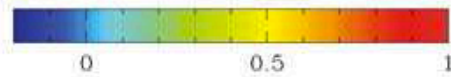
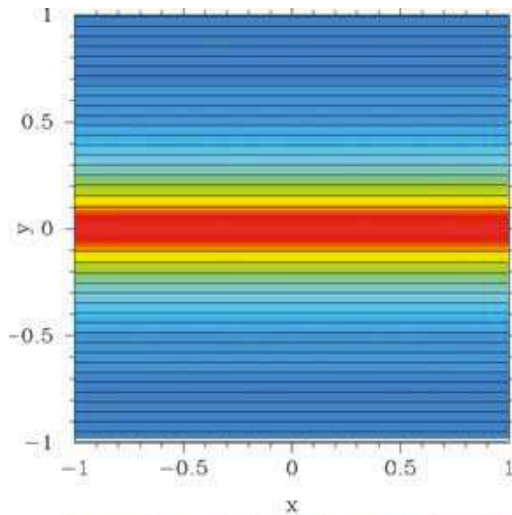
$$\alpha_{max} \sim 10^{-2.5}, \text{ i.e., } \lambda_{max} \sim 2 \times 10^3 a \sim 2 \times 10^3 \text{ km}$$

- And the **growth time** becomes  $\tau_{tearing} \sim 10$  sec
- So in this case, the tearing instability can occur in the solar corona and will form multiple magnetic islands with a size  $2 \times 10^3$  km in the coronal current sheet

# Tearing instability (cont.)

Current density

Force-free  
simulation



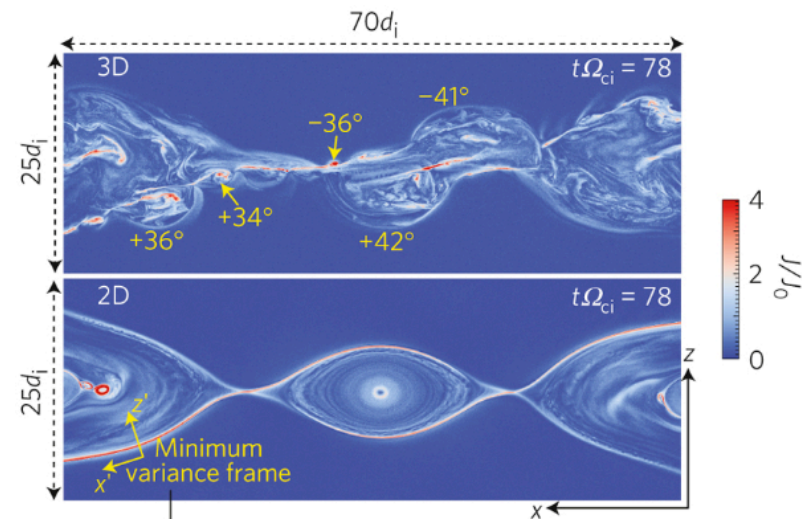
# Tearing instability (cont.)

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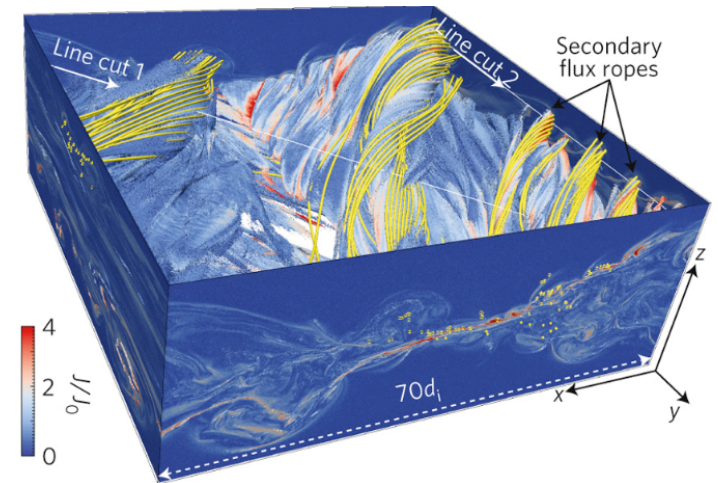
# Current trend of magnetic reconnection

- *Instability (turbulence) in collisional reconnection*
  - Trigger of fast reconnection from Sweet-Parker type condition
  - Time-dependent, non-stationary reconnection in very large systems (**Sweet-Parker type situation**) show **the development of multiple magnetic islands** via **tearing instability**
  - Growing instability leads to **turbulence** in reconnection.
  - Turbulence makes **small aspect ratio of diffusion region** and leads to **fast reconnection**.
  - Now we consider interaction between two fundamental plasma processes: reconnection and turbulence
- *Collisional reconnection to collision-less reconnection.*
  - Development of Physically correct Petschek type reconnection
  - **Spatially-localized anomalous resistivity** due to plasma micro-instabilities
  - Hall-term effect (two fluid effect) enables Petschek-like structure ( $v_{\text{rec}} < 0.1 v_A$ )

# 3D Magnetic reconnection



Movie here





# Summary

- Magnetic reconnection is the process of a rapid rearrangement of magnetic field topology which leads to rapidly and violent release of magnetic energy.
- Flaring event (rapid energy release) in the universe may be related magnetic reconnection process.
- In basic theory of magnetic reconnection, there are two physical models, Sweet-Parker model and Petscheck model.
- Both models have some advantage and disadvantage.
- Until now, we have not developed perfect magnetic reconnection theory
- Magnetic reconnection is one of the most important fundamental questions in physics.