Plasma Astrophysics
Chapter 7-1: Instabilities I

Yosuke Mizuno
Institute of Astronomy
National Tsing-Hua University

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- What is the instability? (How to analyze the instability)
- Rayleigh-Taylor instability
- Kelvin-Helmholtz instability

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- Parker instability
- Magneto-rotational instability
- Jeans instability
- Current-driven instability

Instability

- Questions of stability and instability are important for many astrophysical phenomena.
  - How a structure can remain *stable* for a long period

- Consider a particle motion in one-dimension
- Particle has a mass *m* and moves along x-axis under the action of conservative force

$$F(x)\hat{x} = -\frac{dW}{dx}\hat{x}$$

- Where $W(x)$ is the potential energy. It is in equilibrium position at x=0 and its equation of motion is

$$m\ddot{x} = F(x) \equiv -\left(\frac{dW}{dx}\right)$$

Instability (cont.)

- For small displacements, this reduces to the linear form

$$m\ddot{x} = F_1(x) \equiv -x\left(\frac{d^2W}{dx^2}\right)_0$$

- Where $F_1(x)$ is the first-order approximation to $F(x)$.
- One approach is to seek *normal-mode solutions* of the form $x = x_0 e^{i\omega t}$
- The equation of motion gives

$$\omega^2 = \frac{1}{m}\left(\frac{d^2W}{dx^2}\right)_0$$

- If $W(x)$ has a minimum at the origin, $(d^2W/dx^2)_0 > 0 \Rightarrow \omega^2 > 0$, so the particle oscillates about $x = 0$
- The force tends to restore its equilibrium (*stable*)
- If $W(x)$ has a maximum, $(d^2W/dx^2)_0 < 0 \Rightarrow \omega^2 < 0$, the displacement increases from the equilibrium position (*unstable*)
- When $(d^2W/dx^2)_0 = 0$, it said to *naturally stable*

Instability (cont.)

- The stability of a MHD system is studied in a following way (in general)

- First: linearizes the equations

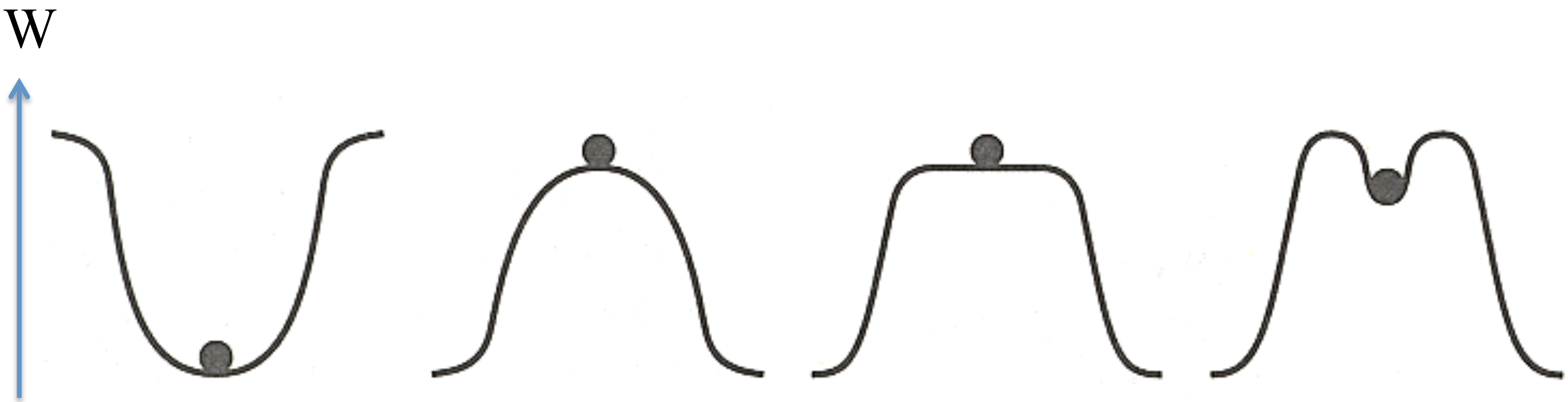
- Second: looks for *normal modes*
- Normal mode: can find a *dispersion relation* linking the frequency ω to the wave number k of the disturbance

- Another way: consider *the variation of energy* (*Variation method*)
- It may be applied more complex equilibrium states

Instability (cont.)

- Here we refer to *linear stability*
- But if considering deviations are not small, it is possible to investigate the *nonlinear stability* of a system
  - Linearly stable but nonlinearly unstable (*explosive*)
  - Linearly unstable but nonlinearly stable
- *Metastability*: naturally stable ($d^2W/dx^2=0$) to small-amplitude (linear) perturbation but unstable to large (finite-amplitude) one
  - $d^3W/dx^3=0$ and $d^4W/dx^4<0$
- Transfer from stability to instability occurs via a state of *marginal* (or normal) *stability*
  - ω^2 is real and decreases through zero, monotonic growth in perturbation. The marginal state is stationary ($\omega=0$)
  - Frequency (ω) is complex and its Imaginary part decreases from + to -, a state of growing oscillations appears (*overstability*). Marginal state is oscillatory motion.

Instability (cont.)



Linear stability

Linear instability

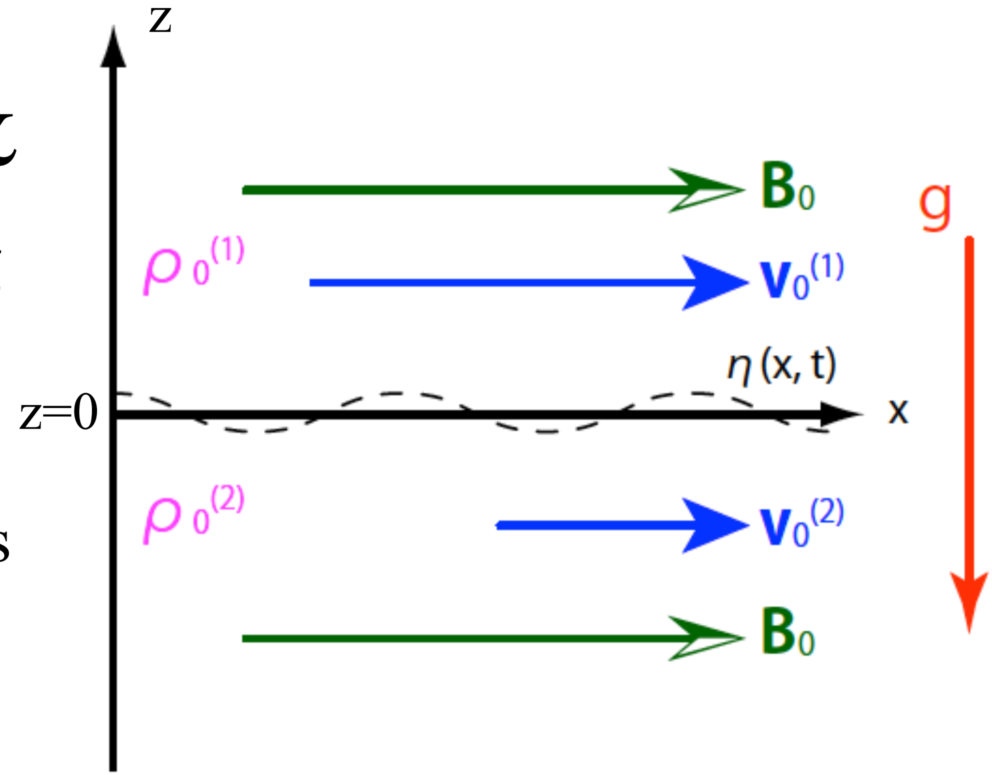
metastability

Non-linear instability

Normal mode method

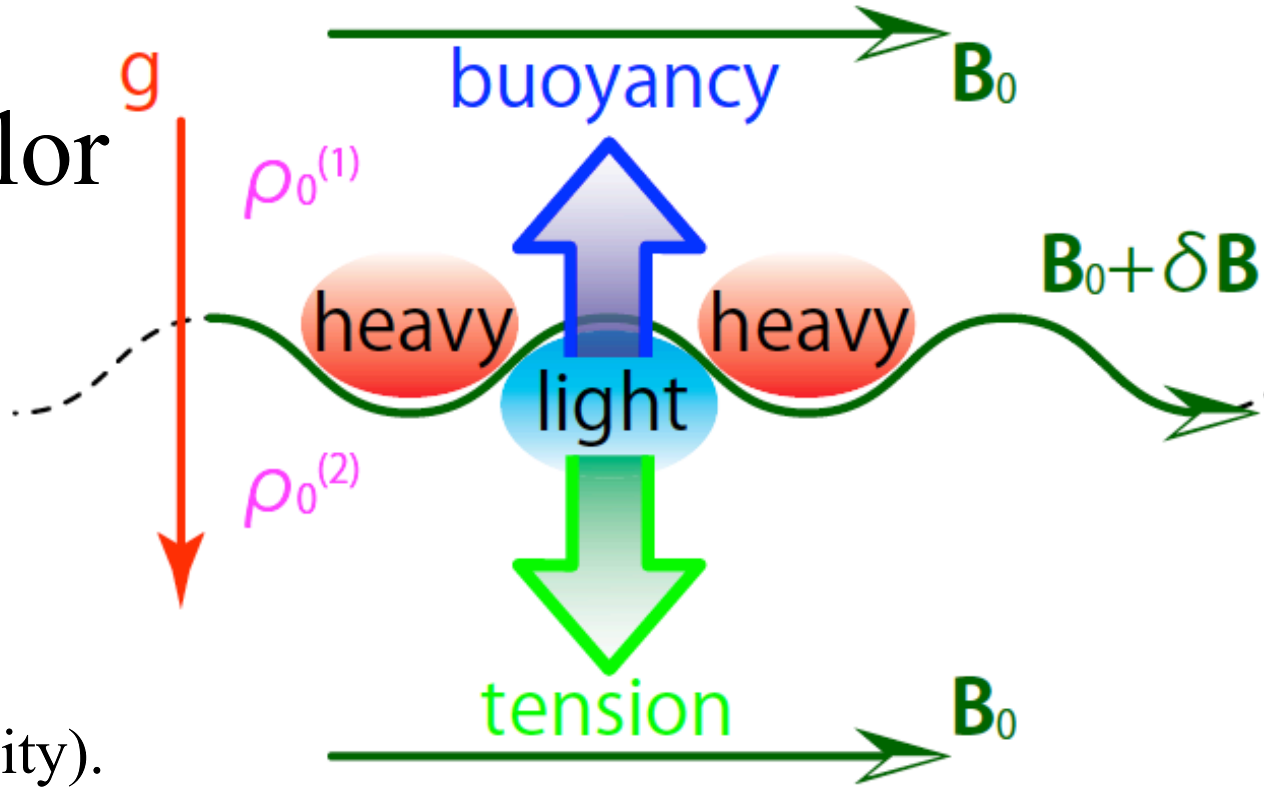
- Prescribe the equilibrium configuration and boundary condition
- The perturbed variables (ρ_1, $\boldsymbol{v}_1$, p_1, $\boldsymbol{j}_1$, and $\boldsymbol{B}_1$) are determined by the set of linearized equations
- Each variables may be decomposed into a spectrum of Fourier components which behave like $e^{i\omega t}$
- The resulting "*normal mode*" equations may be solved to determine which values of ω are allowed by the boundary condition
- If all the normal modes have real frequencies ($\omega^2 > 0$), the system just oscillates in the equilibrium configuration = *stable*
- If at least one of frequencies is imaginary ($\omega^2 < 0$), the system is *unstable*, the corresponding perturbations grow exponentially (*growth rate* is $\gamma = Im(\omega)$)

Rayleigh-Taylor & Kelvin-Helmholtz instabilities



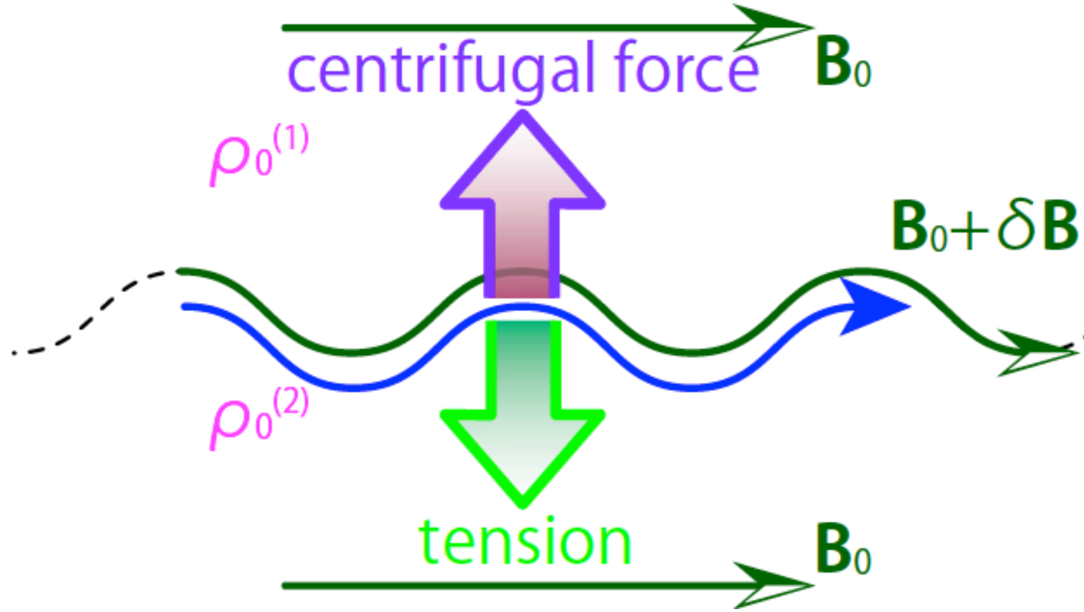
- Very famous (usual) instabilities in the universe.
- Important for entrainment (mixing) materials in interstellar medium, supernova remnant, astrophysical jets etc.
- Consider a boundary separating two perfectly conducting plasmas, superscript (1) and (2)
- The boundary is situated at z=0, gravitational force acts normal to it (-z direction), undisturbed magnetic field is parallel to the boundary (x-direction)

Rayleigh-Taylor instability



- Consider denser (heavier) plasma rests on the top of lighter one (no velocity).
- If displacement is occurred at the boundary, the lighter plasma which pushed up by the displacement is lighter than surrounding plasma so that it has a buoyancy then continue to move up.
- Denser plasma which pushed down is heavier than surrounding plasma so that it has stronger gravity force then continue to down.
- Therefore, both dense and lighter plasmas are mixing each other.
- This is so-called *Rayleigh-Taylor instability*

Kelvin-Helmholtz instability



- Consider the velocity shear between the boundary ($v_0^{(0)} \neq v_0^{(1)}$) with no gravity force
- If displacement is occurred at the boundary, the plasma flowing parallel to the boundary is bended and feel a centrifugal force so that this bending is growing up.
- Then two plasmas are mixing each other.
- This is so-called *kelvin-Helmholtz instability*

Linearized equations for RTI &KHI

- Assume: incompressible fluid, $\nabla \cdot \boldsymbol{v} = 0$ (7.1)
- Mass conservation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0 \qquad (7.2)$$

- Momentum conservation (including gravity):

$$\frac{\partial}{\partial t}(\rho \boldsymbol{v}) + \nabla \cdot \left[\rho \boldsymbol{v}\boldsymbol{v} + \left(p + \frac{B^2}{2\mu_0} \right) \boldsymbol{I} - \frac{1}{\mu_0} \boldsymbol{B}\boldsymbol{B} \right] = -\rho g \hat{z} \qquad (7.3)$$

- Induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) \qquad (7.4)$$

- Gauss's law: $\nabla \cdot \boldsymbol{B} = 0$ (7.5)

Linearized equations for RTI &KHI (cont.)

- Initial state (unperturbed state) is steady and each physical quantities are

$$\rho = \rho_0,\ \boldsymbol{v} = (v_0, 0, 0),\ \boldsymbol{B} = (B_0, 0.0),\ p = p_0$$

- In the current situation, ρ_0, p_0, v_0 are only the function of z
- Assume: Magnetic field is uniform in x-direction
- From eq (7.1) – (7.5),

$$\nabla \cdot \boldsymbol{v}_0 = 0 \qquad (7.6)$$

$$\nabla \cdot (\rho_0 \boldsymbol{v}_0) = 0 \qquad (7.7)$$

$$\nabla \cdot \left[\rho_0 \boldsymbol{v}_0 \boldsymbol{v}_0 + \left(p_0 + \frac{B_0^2}{2\mu_0}\right)\boldsymbol{I} - \frac{1}{\mu_0}\boldsymbol{B}_0\boldsymbol{B}_0\right] = -\rho_0 g \hat{z} \qquad (7.8)$$

$$\nabla \times (\boldsymbol{v}_0 \times \boldsymbol{B}_0) = 0 \qquad (7.9)$$

$$\nabla \cdot \boldsymbol{B}_0 = 0 \qquad (7.10)$$

Linearized equations for RTI &KHI (cont.)

- Added small perturbation in initial state

$$\rho = \rho_0 + \delta\rho, \boldsymbol{v} = \boldsymbol{v}_0 + \delta\boldsymbol{v} = (v_0 + \delta v_x, 0, \delta v_z),$$

$$\boldsymbol{B} = \boldsymbol{B}_0 + \delta\boldsymbol{B} = (B_0 + \delta B, 0, \delta B_z),\ \ p = p_0 + \delta p$$

- These put into eq (7.1) – (7.5) and taken First-order term,

$$(7.1) \rightarrow \nabla \cdot \delta\boldsymbol{v} = 0 \qquad (7.11)$$

$$(7.2) \rightarrow \frac{\partial}{\partial t}\delta\rho + (\boldsymbol{v}_0 \cdot \nabla)\delta\rho + (\delta\boldsymbol{v} \cdot \nabla)\rho_0 = 0 \qquad (7.12)$$

$$(7.3) \rightarrow \frac{\partial}{\partial t}(\rho_0\delta\boldsymbol{v} + \delta\rho\boldsymbol{v}_0) + \nabla \cdot \Big[\delta\rho\boldsymbol{v}_0\boldsymbol{v}_0 + \rho_0\boldsymbol{v}_0\delta\boldsymbol{v} + \rho_0\delta\boldsymbol{v}\boldsymbol{v}_0 + \left(\delta p + \frac{1}{\mu_0}\delta\boldsymbol{B} \cdot \boldsymbol{B}_0\right)\boldsymbol{I} - \frac{1}{\mu_0}(\boldsymbol{B}_0\delta\boldsymbol{B} + \delta\boldsymbol{B}\boldsymbol{B}_0)\Big] = \delta\rho g\hat{z} \qquad (7.13)$$

$$(7.4) \rightarrow \frac{\partial}{\partial t}\delta\boldsymbol{B} - \nabla \times (\boldsymbol{v}_0 \times \delta\boldsymbol{B} + \delta\boldsymbol{v} \times \boldsymbol{B}_0) = 0 \qquad (7.14)$$

$$(7.5) \rightarrow \nabla \cdot \delta\boldsymbol{B} = 0 \qquad (7.15)$$

Linearized equations for RTI &KHI (cont.)

- Divide into each components, linearized equations are given as

$$(7.11) \rightarrow \frac{\partial}{\partial x}\delta v_x + \frac{\partial}{\partial z}\delta v_z = 0 \qquad (7.16)$$

$$(7.12) \rightarrow \frac{\partial}{\partial t}\delta\rho + v_0\frac{\partial}{\partial x}\delta\rho + \delta v_z\frac{\partial}{\partial z}\rho_0 = 0 \qquad (7.17)$$

$$(7.15) \rightarrow \frac{\partial}{\partial x}\delta B_x + \frac{\partial}{\partial z}\delta B_z = 0 \qquad (7.18)$$

$$(7.13)_x \rightarrow \rho_0\frac{\partial}{\partial t}\delta v_x + \rho_0 v_0\frac{\partial}{\partial x}\delta v_x + \frac{\partial}{\partial x}\delta p + p_0\delta v_z\frac{\partial}{\partial z}v_0 = 0 \qquad (7.19)$$

$$(7.13)_z \rightarrow \rho_0\frac{\partial}{\partial t}\delta v_z + \rho_0 v_0\frac{\partial}{\partial x}\delta v_z - \frac{1}{\mu_0}B_0\frac{\partial}{\partial x}\delta B_z + \frac{\partial}{\partial z}\delta p + \frac{1}{\mu_0}\frac{\partial}{\partial z}\delta B_x = -\delta\rho g \qquad (7.20)$$

$$(7.14)_x \rightarrow \frac{\partial}{\partial t}\delta B_x - \delta B_z\frac{\partial}{\partial z}v_0 + v_0\frac{\partial}{\partial x}\delta B_x - B_0\frac{\partial}{\partial x}\delta v_x = 0 \qquad (7.21)$$

$$(7.14)_z \rightarrow \frac{\partial}{\partial t}\delta B_z + v_0\frac{\partial}{\partial x}\delta B_z - B_0\frac{\partial}{\partial x}\delta v_z = 0 \qquad (7.22)$$

Linear analysis for RTI &KHI

- We consider normal mode, $\delta\rho, \delta\boldsymbol{v}, \delta\boldsymbol{B}, \delta p \propto e^{i(kx-\omega t)}$

- It put into linearized equations

$$(7.16) \rightarrow \delta v_x = \frac{i}{k}\frac{\partial}{\partial z}\delta v_z \qquad (7.23)$$

$$(7.17) \rightarrow \delta\rho = \frac{i}{kv_0 - \omega}\delta v_z \frac{\partial}{\partial z}\rho_0 \qquad (7.24)$$

$$(7.19) \rightarrow \delta p = i\frac{\omega - kv_0}{k^2}\rho_0\frac{\partial}{\partial z}\delta v_z + i\frac{\rho_0}{k}\delta v_z\frac{\partial}{\partial z}v_0 \qquad (7.25)$$

Here using (7.23)

Linear analysis for RTI &KHI (cont.)

$$(7.20) \rightarrow \quad -i\rho_0(\omega - kv_0)\delta v_z - \frac{ik}{\mu_0}B_0\delta B_z$$

Here using (7.24) & (7.25)

$$+\frac{\partial}{\partial z}\left[i\frac{\omega - kv_0}{k^2}\rho_0\frac{\partial}{\partial z}\delta v_z + i\frac{\rho_0}{k}\delta v_z\frac{\partial}{\partial z}v_0\right] + \frac{1}{\mu_0}B_0\frac{\partial}{\partial z}\delta B_x$$

$$= \frac{ig}{\omega - kv_0}\delta v_z\frac{\partial}{\partial z}\rho_0 \quad (7.26)$$

$$(7.22) \rightarrow \delta B_z = -\frac{k}{\omega - kv_0}B_0\delta v_z \quad (7.27)$$

$$(7.18) \rightarrow \delta B_x = -i\frac{\partial}{\partial z}\left(\frac{B_0}{\omega k v_0}\delta v_z\right) \quad (7.28)$$

Here using (7.27)

Linear analysis for RTI &KHI (cont.)

- From eq (7.26), (7.27), (7.28) x (k^2/i)

$$\frac{\partial}{\partial z}\left[(\omega - kv_0)\rho_0\frac{\partial}{\partial z}\delta v_z + \rho_0 k\delta v_z\frac{\partial}{\partial z}v_0\right] - \rho_0 k^2(\omega - kv_0)\delta v_z$$
$$= \frac{k^2B_0^2}{\mu_0}\left(\frac{\partial^2}{\partial z^2} - k^2\right)\left(\frac{\delta v_z}{\omega - kv_0}\right) + \frac{gk^2}{\omega - kv_0}\delta v_z\frac{\partial}{\partial z}\rho_0 \tag{7.29}$$

- Equation of δv_z only.
- Next we look for the profile of δv_z and the boundary condition

Linear analysis for RTI &KHI (cont.)

- At $z \neq 0,\ \partial\rho_0/\partial z = \partial v_0/\partial z = 0$ (uniform density and velocity)

$$(7.29) \rightarrow \left[(\omega - kv_0)^2\rho_0 - \frac{k^2B_0^2}{\mu_0}\right]\left(\frac{\partial^2}{\partial z^2} - k^2\right)\delta v_z = 0 \quad (7.30)$$

- Moreover, we expect that the perturbation becomes small in the region far from the boundary (z=0)
- So from eq (7.30),

$$\delta v_z = Ae^{-k|z|} \quad (7.31)$$

Linear analysis for RTI &KHI (cont.)

- Next, we consider the boundary which interact two plasma (fluid) (z=0)
- The boundary is deformed by the perturbation (displacement).
- The deformed form can be expected $Y = \eta(x,t)$
- The boundary moves with fluid motion
- Therefore

$$\delta v_z = \frac{D\eta}{Dt} = \left[\frac{\partial}{\partial t} + (v_0 + \delta v_x)\frac{\partial}{\partial x}\right]\eta$$

- The perturbation of the boundary can be written as $\eta \propto e^{i(kx-\omega t)}$
- And we assume the amplitude is small then it can be linearized

$$\delta v_z = [-i\omega + (v_0 + \delta v_x)ik]\eta \simeq -i(\omega - kv_0)\eta \qquad (7.32)$$

Linear analysis for RTI &KHI (cont.)

- From eq (7.32), at z=0 we take a ratio between upper (1) and lower (2) fluids

$$\frac{\delta v_z^{(1)}}{\delta v_z^{(2)}} = \frac{\omega - k v_0^{(1)}}{\omega - k v_0^{(2)}} \qquad (7.33)$$

- Such condition is satisfied at the boundary
- From eq (7.31) & (7.33), at $z \neq 0$

$$\begin{cases} \delta v_z^{(1)} = (\omega - k v_0^{(1)}) e^{-kz} \\ \delta v_z^{(2)} = (\omega - k v_0^{(2)}) e^{kz} \end{cases} \qquad (7.34)$$

Linear analysis for RTI &KHI (cont.)

- Next derive the condition at boundary (z=0) of eq (7.29)
- Define the integral of the small region between the boundary [-ε : ε] as

$$\Delta_s(f) \equiv \lim_{\epsilon \to 0} \int_{0-\epsilon}^{0+\epsilon} \frac{\partial f}{\partial z} dz = \lim_{\epsilon \to 0} [f(\epsilon) - f(-\epsilon)]$$

- Consider the integral of eq (7.29) in the small region between the boundary [-ε : ε] with respect to z-direction

$$\underline{\frac{\partial}{\partial z}\left[(\omega - kv_0)\rho_0 \frac{\partial}{\partial z}\delta v_z + \rho_0 k \delta v_z \frac{\partial}{\partial z} v_0\right]} \; \underline{- \rho_0 k^2 (\omega - kv_0)\delta v_z}$$
$$= \frac{k^2 B_0^2}{\mu_0} \underline{\left(\frac{\partial^2}{\partial z^2} - k^2\right)\left(\frac{\delta v_z}{\omega - kv_0}\right)} + \underline{\frac{gk^2}{\omega - kv_0}\delta v_z \frac{\partial}{\partial z}\rho_0} \qquad (7.29)$$

Linear analysis for RTI &KHI (cont.)

$$\lim_{\epsilon\to 0}\int_{0-\epsilon}^{0+\epsilon}\frac{\partial}{\partial z}\left[(\omega-kv_0)\rho_0\frac{\partial}{\partial z}\delta v_z+\rho_0 k\delta v_z\frac{\partial}{\partial z}v_0\right]dz$$

$$=\Delta_s\left[(\omega-kv_0)\rho_0\frac{\partial}{\partial z}\delta v_z\right]+\Delta_s\left[\rho_0 k\delta v_z\frac{\partial}{\partial z}v_0\right] \quad (7.35)$$

$\partial v_0/\partial z=0\ (\text{at } y\neq 0)$

$$\lim_{\epsilon\to 0}\int_{0-\epsilon}^{0+\epsilon}\rho_0 k^2(\omega-kv_0)\delta v_z dz\to 0 \quad (7.36)$$

$$\lim_{\epsilon\to 0}\int_{0-\epsilon}^{0+\epsilon}\left(\frac{\partial^2}{\partial z^2}-k^2\right)\left(\frac{\delta v_z}{\omega-kv_0}\right)dz=\Delta_s\left(\frac{\frac{\partial}{\partial z}\delta v_z}{\omega-kv_0}\right)+\Delta_s\left(\frac{k\frac{\partial v_0}{\partial z}}{(\omega-kv_0)^2}\right) \quad (7.37)$$

$\partial v_0/\partial z=0\ (\text{at } y\neq 0)$

$$\lim_{\epsilon\to 0}\int_{0-\epsilon}^{0+\epsilon}\frac{\delta v_z}{\omega-kv_0}\frac{\partial\rho_0}{\partial z}dz=\frac{\delta v_z}{\omega-kv_0}\Delta_s(\rho_0) \quad (7.38)$$

Here using eq(7.34)

Linear analysis for RTI &KHI (cont.)

- Combine eq (7.35) – (7.38), the boundary condition is

$$\Delta_s \left[(\omega - kv_0)\rho_0 \frac{\partial}{\partial z} \delta v_z \right] = \frac{k^2 B_0^2}{\mu_0} \Delta_s \left(\frac{\frac{\partial}{\partial z}\delta v_z}{\omega - kv_0} \right) + k^2 g \frac{\delta v_z}{\omega - kv_0} \Delta_s(\rho_0) \quad (7.39)$$

- To obtain the dispersion relation, put eq (7.34) into this equations

$$\Delta_s \left[(\omega - kv_0)\rho_0 \frac{\partial}{\partial z} \delta v_z \right] \quad (7.40)$$

$$= (\omega - kv_0^{(1)})\rho_0^{(1)}(\omega - kv_0^{(1)})(-k)e^{-kz} - (\omega - kv_0^{(2)})\rho_0^{(2)}(\omega - kv_0^{(2)})ke^{kz}$$

$$\rightarrow -k[(\omega - kv_0^{(1)})^2 \rho_0^{(1)} + (\omega - kv_0^{(2)})^2 \rho_0^{(2)}]$$

$$\Delta_s \left(\frac{\frac{\partial}{\partial z}\delta v_z}{\omega - kv_0} \right) = (-k)e^{-kz} - ke^{kz} \rightarrow -2k \quad (7.41)$$

$$\Delta_s(\rho_0) = \rho_0^{(1)} - \rho_0^{(2)} \quad (7.42)$$

Dispersion Relation for for RTI &KHI

- Consider (7.40)- (7.42), eq (7.39) is written as

$$-k[(\omega - kv_0^{(1)})^2\rho_0^{(1)} + (\omega - kv_0^{(2)})^2\rho_0^{(2)}] = -\frac{2k^3B_0^2}{\mu_0} + k^2 g(\rho_0^{(1)} - \rho_0^{(2)})$$

- The (general) dispersion relation is

$$(\rho_0^{(1)} + \rho_0^{(2)})\omega^2 - 2k(\rho_0^{(1)}v_0^{(1)} + \rho_0^{(2)}v_0^{(2)})\omega + k^2(\rho_0^{(1)}v_0^{(1)2} + \rho_0^{(2)}v_0^{(2)2})$$
$$-\frac{2k^2B_0^2}{\mu_0} + kg(\rho_0^{(1)} - \rho_0^{(2)}) = 0 \quad (7.43)$$

RTI in uniform magnetic field

- If no velocity shear, i.e., $v_0^{(1)} = v_0^{(2)} = 0$
- The dispersion relation is

$$\omega^2 = -gk\frac{\rho_0^{(1)} - \rho_0^{(2)}}{\rho_0^{(1)} + \rho_0^{(2)}} + \frac{2B_0^2 k^2}{\mu_0(\rho_0^{(1)} + \rho_0^{(2)})}$$

- In more general, we consider perturbation $\propto e^{i(k_x x + k_y y - \omega t)}$
- The dispersion relation is written as

$$\omega^2 = -gk\frac{\rho_0^{(1)} - \rho_0^{(2)}}{\rho_0^{(1)} + \rho_0^{(2)}} + \frac{2B_0^2 k_x^2}{\mu_0(\rho_0^{(1)} + \rho_0^{(2)})}$$

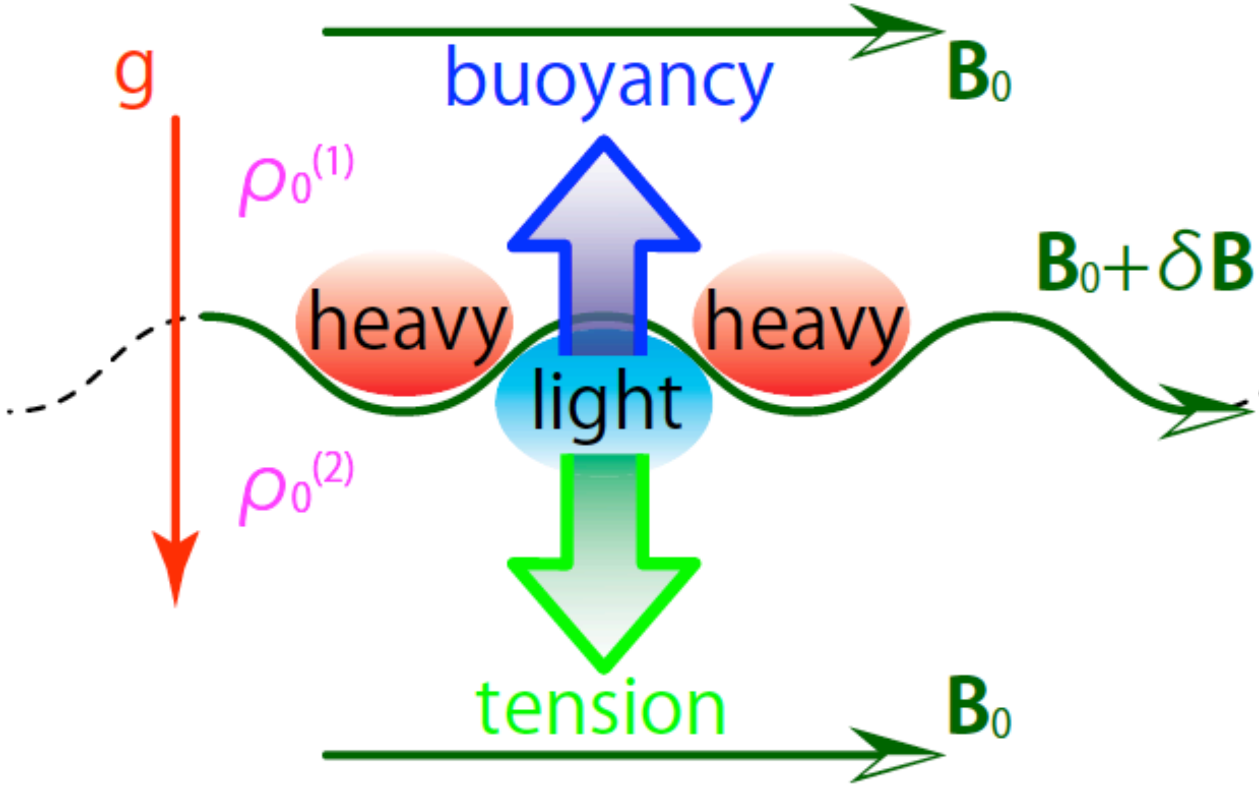
RTI in uniform magnetic field (cont.)

- This dispersion relation implies
- When no magnetic field, interface is unstable ($w^2 < 0$) provided heavy plasma rests on top of light one ($\rho_0^{(1)} > \rho_0^{(2)}$)
- This instability is so-called Rayleigh-Taylor Instability
- If perturbations are uniform along the field direction (k_x=0), the magnetic field has no effect on stability
- If perturbations are purely along the field (k_y=0, k_x=k): 2nd term is positive, so allow a stabilizing effect
- When $\rho_0^{(1)} > \rho_0^{(2)}$, the interface is unstable for wavelength, 0< k<k_c
  where

$$k_c = \frac{(\rho_0^{(1)} - \rho_0^{(2)})g\mu_0}{2B_0^2}$$

- Fastest growing mode wavelength is 1/2 k_c

RTI in uniform magnetic field (cont.)



- For large wavelength ($\lambda > 2\pi/k_c$), magnetic tension is insufficient to counteract gravity
- For short wavelength ($\lambda < 2\pi/k_c$), magnetic tension strong enough to make stable

RTI simulations

2D hydro
(single mode)

3D hydro (random
perturbation)

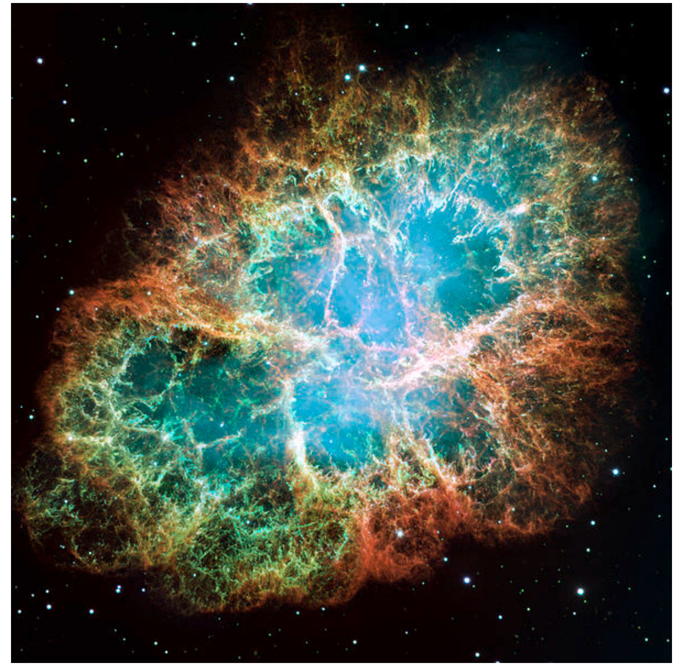
Movie in here

Movie in here

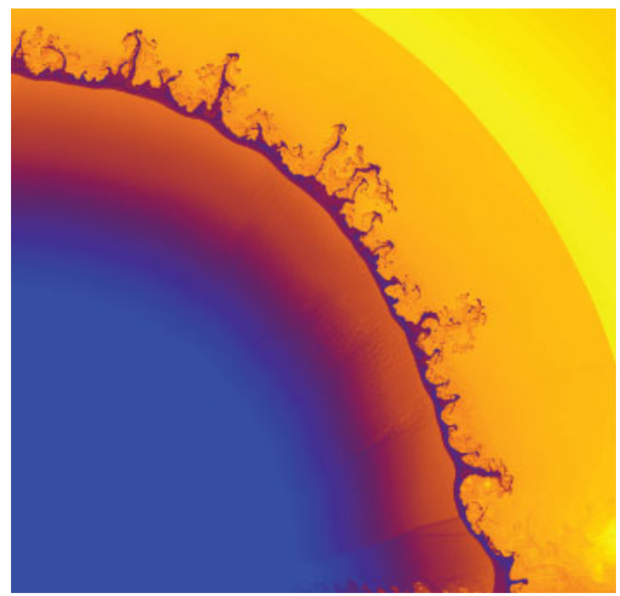
RTI in nature

Experiment of RTI

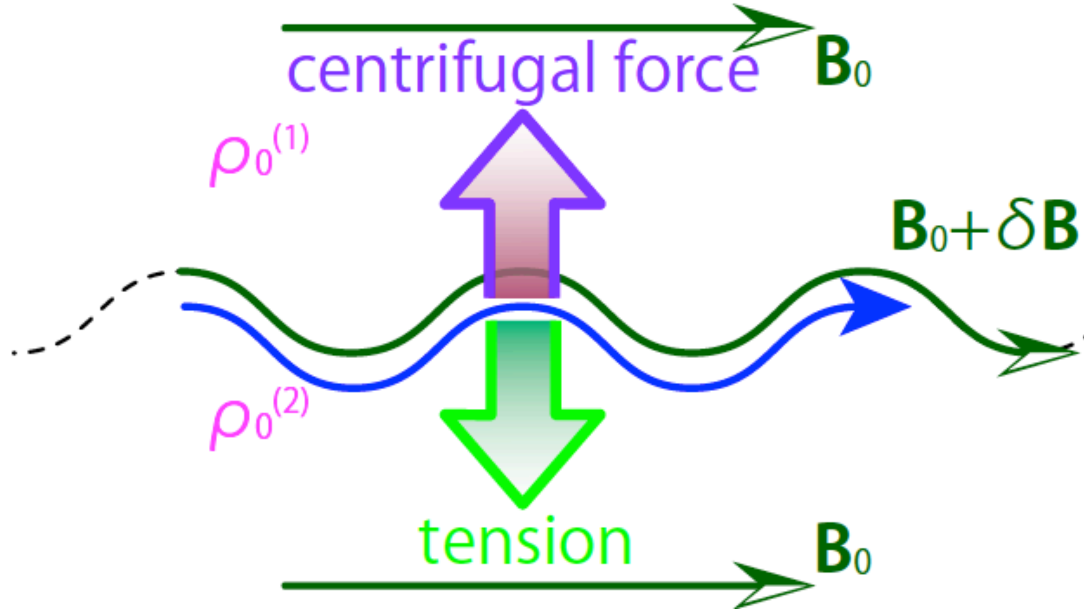
Crab nebula (SNR)



Simulation of SNR



Kelvin-Helmholtz instability



- Consider the velocity shear between the boundary ($v_0^{(0)} \neq v_0^{(1)}$) with no gravity force
- If displacement is occurred at the boundary, the plasma flowing parallel to the boundary is bended and feel a centrifugal force so that this bending is growing up.
- Then two plasmas are mixing each other.
- The magnetic field parallel to flow is affected to stabilized the instability by magnetic tension force

Kelvin-Helmholtz instability (cont.)

- Consider a uniform horizontal magnetic field ($B_0\hat{x}$) parallel to the interface that separates uniform plasma with $\rho_0^{(1)},\ v_0^{(1)}$ and $\rho_0^{(2)},\ v_0^{(2)}$

- The dispersion relation can be described as (same as eq. 7.43)

$$(\rho_0^{(1)} + \rho_0^{(2)})\omega^2 - 2k(\rho_0^{(1)} v_0^{(1)} + \rho_0^{(2)} v_0^{(2)})\omega + k^2(\rho_0^{(1)} v_0^{(1)2} + \rho_0^{(2)} v_0^{(2)2})$$
$$-\frac{2k^2 B_0^2}{\mu_0} + kg(\rho_0^{(1)} - \rho_0^{(2)}) = 0$$

Kelvin-Helmholtz instability (cont.)

- If no gravity, the interface is unstable when

$$v_{A,S}^2 < \frac{\rho_0^{(1)}\rho_0^{(2)}}{(\rho_0^{(1)}+\rho_0^{(2)})^2}(v_0^{(1)}-v_0^{(2)})^2$$

Where $v_{A,S}^2 \equiv \dfrac{2B_0^2}{\mu_0(\rho_0^{(1)}+\rho_0^{(2)})}$

- This is so-called *Kelvin-Helmholtz instability*
- Here, similar to RTI, the magnetic field has affected to stabilize the system.
- If no B-field and no gravity, all wavelengths are unstable as long as $v_0^{(1)} \neq v_0^{(2)}$

Kelvin-Helmholtz instability (cont.)

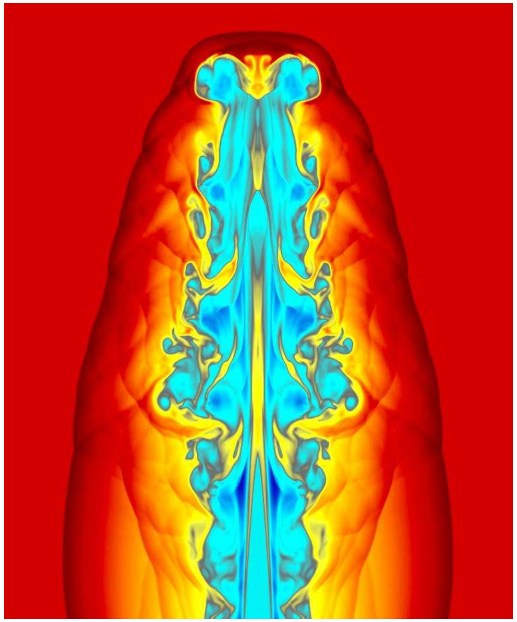
- If no B-field but with gravity and stable against RTI ($\rho_0^{(2)} > \rho_0^{(1)}$), the interface is unstable when

$$k > \frac{g(\rho_0^{(2)2} - \rho_0^{(1)2})}{\rho_0^{(1)}\rho_0^{(2)}(v_0^{(1)} - v_0^{(2)})^2}$$

- Gravity is stabilize at long wavelength for KH instability

KHI simulations

2D jet propagation



2D hydro

2D MHD

Movie in here

Movie in here

KHI in nature

cloud

Movie in here

Jupiter's great red spot

Relativistic jet in M87

Summary

- There are many potentially growing instabilities in the universe.
- These instabilities are strongly related the dynamics in the universe.

- Important:
  - what system is stable/unstable against instabilities (condition for stable/unstable of instability)
  - What is the time scale of growing instabilities (growth rate). Does it affects the dynamics of system?