Plasma Astrophysics
Chapter 8:Outflow and Accretion

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Outflow and Accretion

- In the universe, outflow and accretion are common feature.
- Outflow
  - Solar wind, stellar wind, Pulsar wind.
  - Galactic disk wind
  - Outflow/jet from accretion disk
- Accretion: the gravitational attraction of gas onto a central object.
  - Galaxy, AGN (supper-massive BH)
  - Binaries (from remnant star to compact object)
  - Isolated compact object (white dwarf, neutron star, BH)
  - T-Tauri star (protostar), protoplanet

Solar wind

- The solar corona cannot remain in static equilibrium but is continually expanding. The continual expansion is called the solar wind.
- Solar wind velocity ~ 300-900 km/s near the earth
- Temperature 10^5-10^6 K

- Steady flow: solar wind
- Transient flow: coronal mass ejection

LASCO observation (white light)

Movie here

Parker wind model

- Parker (1958): gas pressure of solar corona can drive the wind
- Assume: the expanding plasma which is isothermal and steady (thermal-driven wind).
- Start with 3D HD equations with spherical symmetry and time steady ($\partial/\partial t = 0$)

$$\nabla \cdot (\rho \boldsymbol{v}) = 0, \quad (8.1)$$

$$\rho(\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla p + \rho \boldsymbol{g}, \quad (8.2)$$

$$p = \frac{R}{\mu}\rho T, \quad (8.3)$$

$$T = T_0 \quad (8.4)$$

- We restrict our attention to the spherically symmetric solution. The velocity $\boldsymbol{v}$ is taken as purely radial $\boldsymbol{v} = v\hat{r}$ and the gravitational acceleration $\boldsymbol{g} = g\hat{r}$ obeys the inverse square law,

$$g = -\frac{GM_\odot}{r^2} \quad (8.5)$$

Parker wind model (cont.)

- From isothermal, we have constant sound speed,

$$c_s^2 = p/\rho \qquad (8.6)$$

- For simplicity, we are interested in the dependence on the radial direction only.
- The expressions for the differential operators in the spherical coordinates are

$$\nabla a = \frac{da}{dr}, \ \nabla \cdot A = \frac{1}{r^2}\frac{d}{dr}\left(r^2 A_r\right)$$

- In the spherical geometry, the governing equations are

$$\rho v \frac{dv}{dr} = -\frac{dp}{dr} - \frac{GM_\odot \rho}{r^2}, \qquad (8.7)$$

$$\frac{d}{dr}(r^2 \rho v) = 0 \rightarrow r^2 \rho v = \text{const.} \qquad (8.8)$$

Parker wind model (cont.)

- Substituting eq (8.6) and (8.7), exclude pressure from equations

$$v\frac{dv}{dr} = -c_s^2 \frac{1}{\rho}\frac{d\rho}{dr} - \frac{GM_\odot}{r^2} \qquad (8.9)$$

- To exclude ρ, using eq (8.8),

$$\frac{d}{dr}(r^2 \rho v) = \rho \frac{d}{dr}(r^2 v) + r^2 v \frac{d\rho}{dr} = 0$$

- And obtain

$$\frac{1}{\rho}\frac{d\rho}{dr} = -\frac{1}{r^2 v}\frac{d}{dr}(r^2 v) \qquad (8.10)$$

- Now eq (8.9) becomes

$$v\frac{dv}{dr} = \frac{c_s^2}{r^2 v}\frac{d}{dr}(r^2 v) - \frac{GM_\odot}{r^2}$$

Parker wind model (cont.)

- Rewriting this equation, we obtain

$$\left(v - \frac{c_s^2}{v}\right)\frac{dv}{dr} = \frac{2c_s^2}{r} - \frac{GM_\odot}{r^2}$$

- And, then

$$\left(v - \frac{c_s^2}{v}\right)\frac{dv}{dr} = 2\frac{c_s^2}{r^2}(r - r_c)$$

- Where $r_c = GM_\odot/(2c_s^2)$ is the *critical radius* (*critical point* or *sonic point*) showing the position where the wind speed reaches the sound speed, $v = c_s$

Parker wind model (cont.)

- This is a separable ODE, which can readily be integrated,

$$\int \left(v - \frac{c_s^2}{v} \right) dv = \int 2 \frac{c_s^2}{r^2} (r - r_c) dr$$

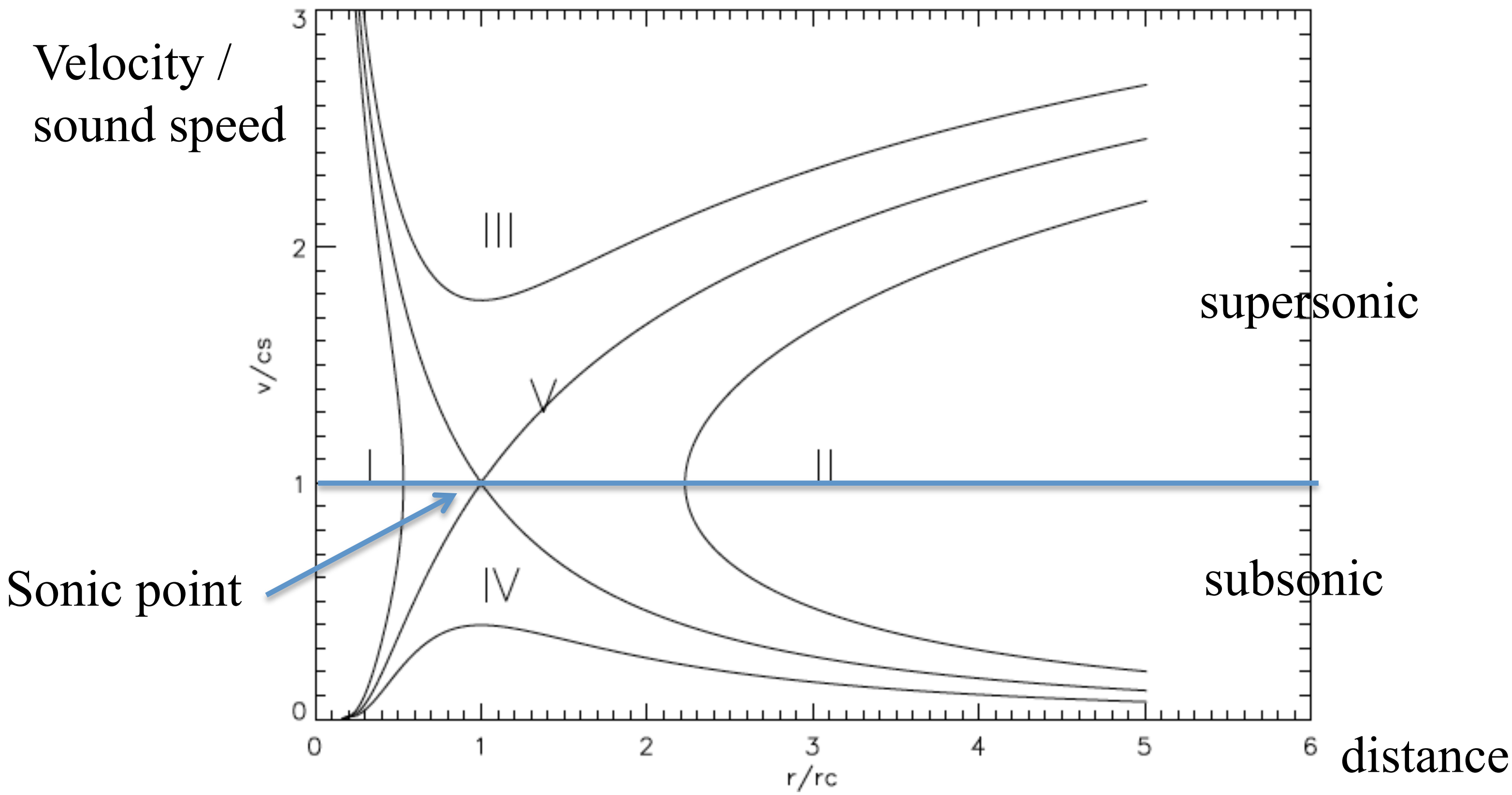
- The solution is

$$\left(\frac{v}{c_s} \right)^2 - \log \left(\frac{v}{c_s} \right)^2 = 4 \log \left(\frac{r}{r_c} \right) + 4 \frac{r_c}{r} + C$$

- The constant of integration C can be determined from boundary conditions, and it determines the specific solution.

Parker wind model (cont.)

- Several types of solution are present



- Type I & II: double valued (two values of the velocity at the same distance), non-physical.
- Type III: has initially supersonic speeds at the Sun which are not observed

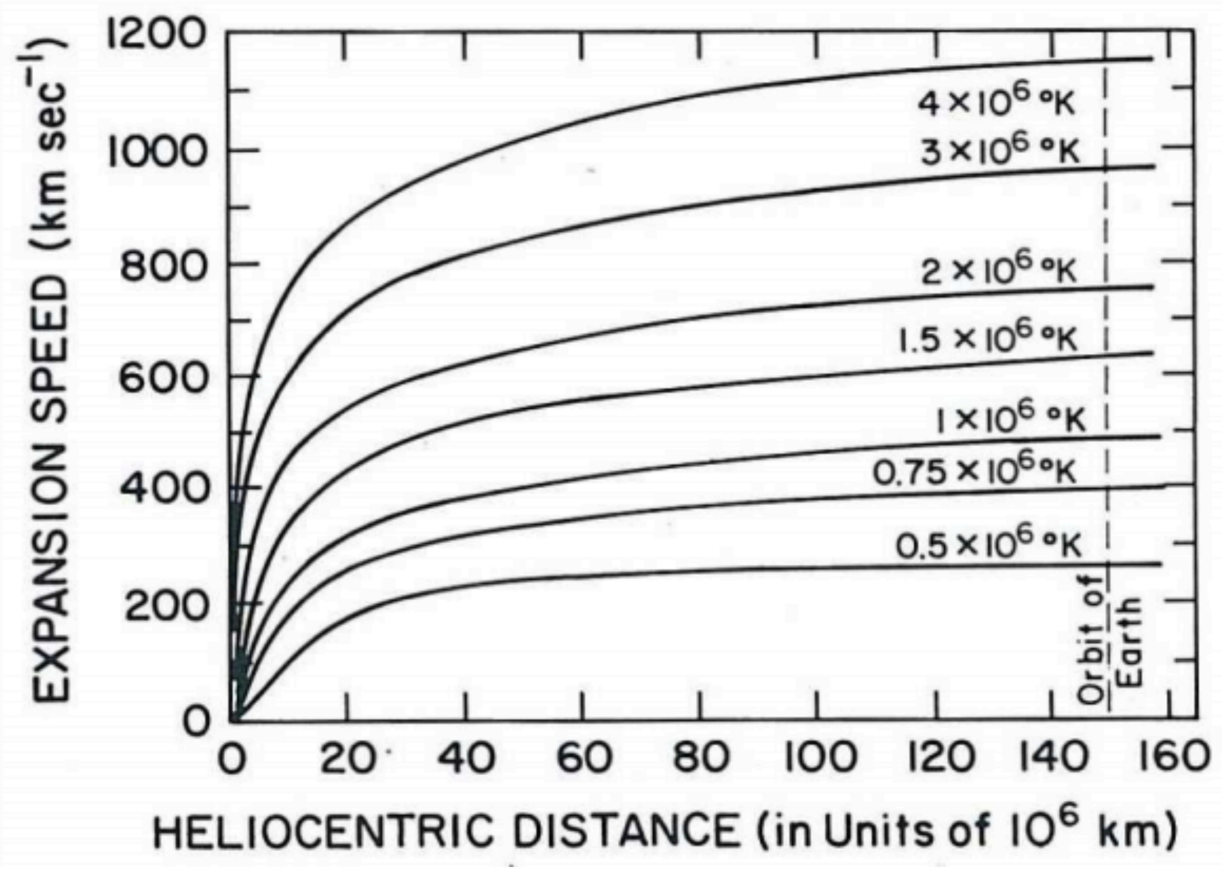
Parker wind model (cont.)

- Type IV (subsonic => subsonic): seem also be physically possible (The "solar breeze" solutions). But not fit observation.
- The unique solution of type V passes through the critical point ($r = r_c$, $v = c_s$) and is given by C = -3. This is the "solar wind" solution. So the solar wind is transonic flow.
- For a typical coronal sound speed of about 10^5 m/s and the critical radius is

$$r_c = \frac{GM_\odot}{2c_s^2} \approx 6 \times 10^9 \mathrm{m} \approx 9 - 10R_\odot$$

- At the Earth's orbit, the solar wind speed can be obtained by using r = 214R_sun, which gives v = 310 km/s.

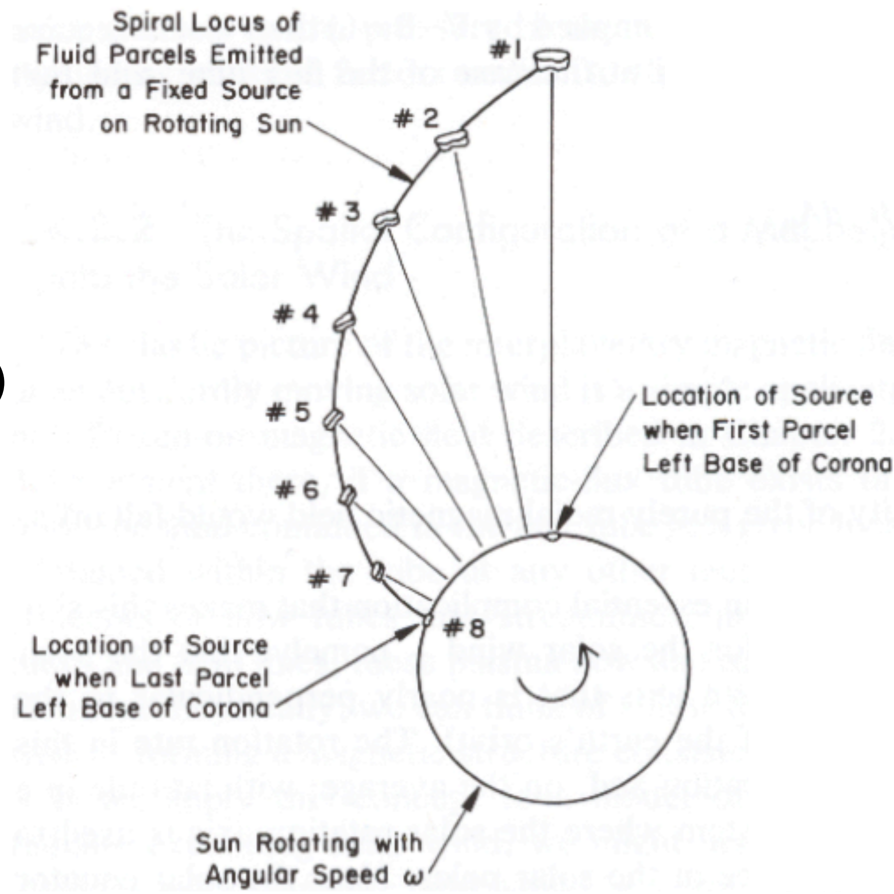
Parker wind model (cont.)



- Parker wind speed depends on temperature.
- High temperature corona makes faster wind
- But this trend is not consistent with recent observation => need other acceleration mechanism.

Parker spiral

- Solar atmosphere is high conductivity- flux ‘frozen-in’
- In photosphere/lower corona, fields frozen in fluid rotate with the sun
- In outer corona, plasma (solar wind) carries magnetic field outward with it
- For the radial flow, the rotation of the Sun makes the solar magnetic field twist up into a spiral, so-called the *Parker spiral*.



Parker spiral (cont.)

- Magnetic field near the pole region can be treated as radial field.
- From magnetic flux conservation

$$B_r A = B_0 A_0$$

- Where A and A_0 are cross sectional area of magnetic field at distance r and bases
- Here, $A = 4\pi r^2$ and $A_0 = 4\pi r_0^2$

$$B_r 4\pi r^2 = B_0 4\pi r_0^2 \rightarrow B_r = B_0 (r_0^2 / r^2)$$

Parker spiral (cont.)

- At lower latitudes, the initial magnetic field at surface is radial
- The foot point of magnetic field rotates with Sun, ω_s
- As sun rotates and solar wind expands radially, it gets toroidal component of magnetic field

$$B_\phi = -B_r \left(\frac{\omega_s r}{v_{sw}} \right)$$

- Using $B_r = B_0 (r_0^2 / r^2)$

$$B_\phi = -B_0 \left(\frac{r_0^2}{r^2} \right) \left(\frac{\omega_s r}{v_{sw}} \right)$$

- Resulting field is called *Parker spiral*

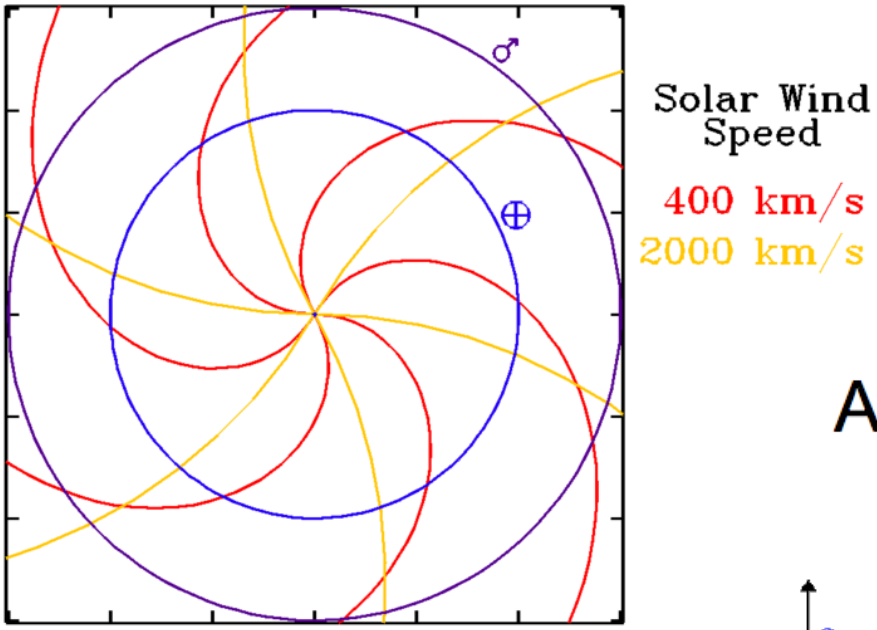
Parker spiral (cont.)

- Average angle of equatorial magnetic field is

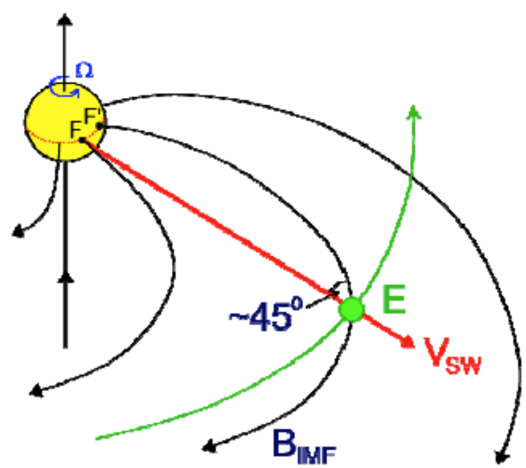
$$\tan\theta = B_\phi/B_r = r\omega_s/v_{sw}$$

- Magnetic field is more tangled with larger radius
- Angular velocity of Sun is ω_s = 2.87 x 10^6 s^{-1}.
- At the earth (1 AU = 1.50 x 10^8 km), the co-rotating velocity is $r\omega_s$= 429 km/s
- From v_{sw}~400-450 km/s, the angle of interplanetary magnetic field at the earth is ~ 45 degree

Parker spiral (cont.)

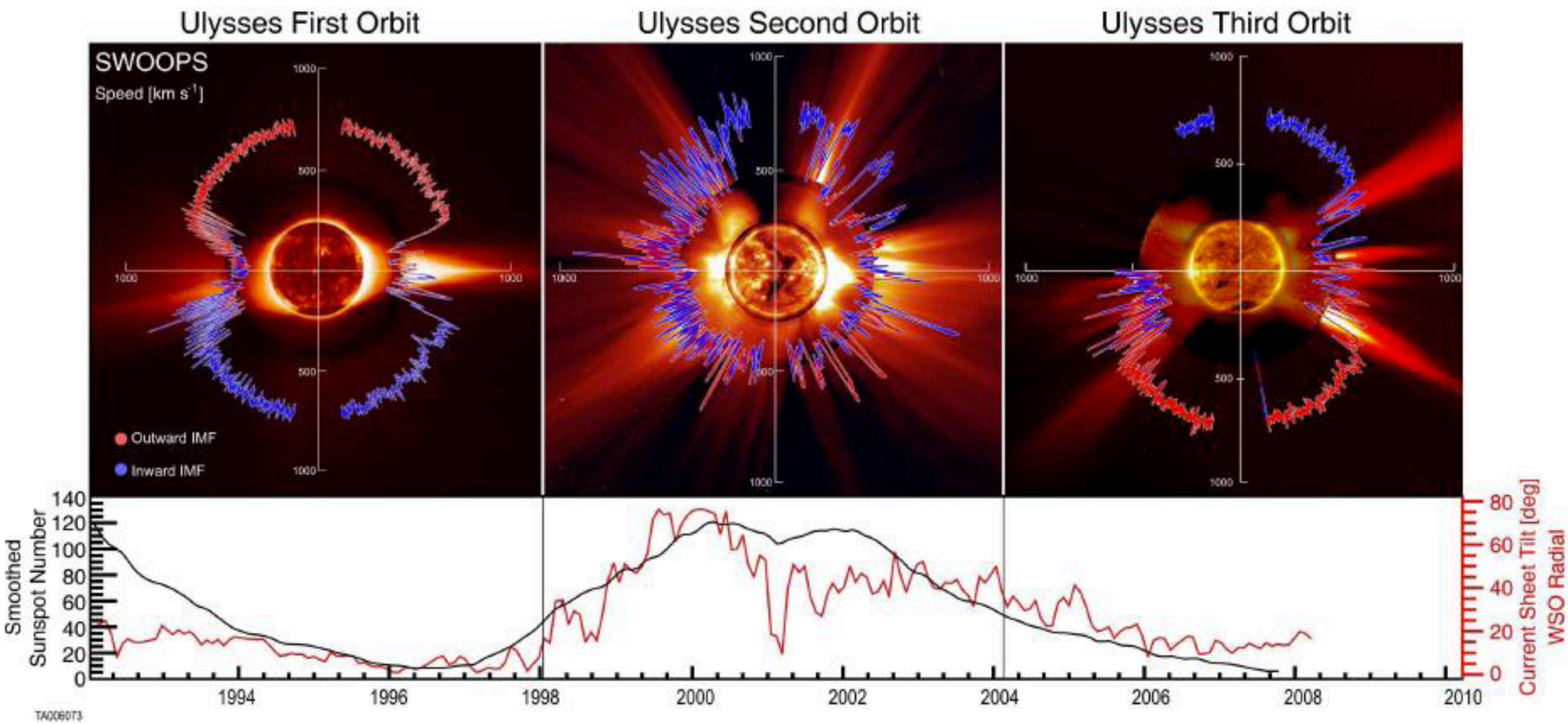


Average IMF Strength and Direction



At:	Angle:	Strength:
Mercury	21°	35 nT
Earth	45°	7 nT
Mars	56°	4 nT
Jupiter	80°	1 nT
Neptune	88°	0.2 nT

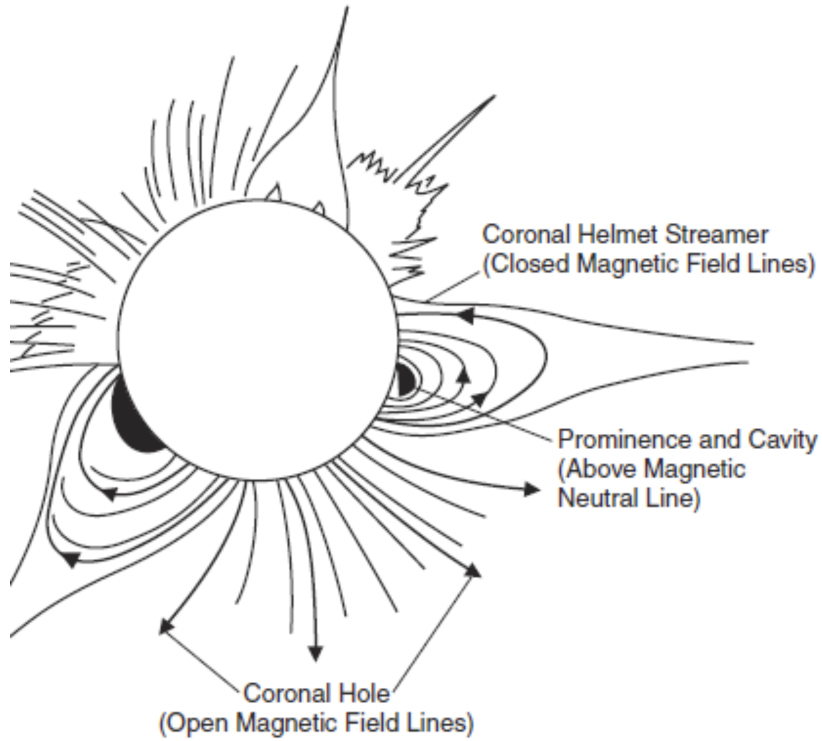
Current status of Solar wind observation

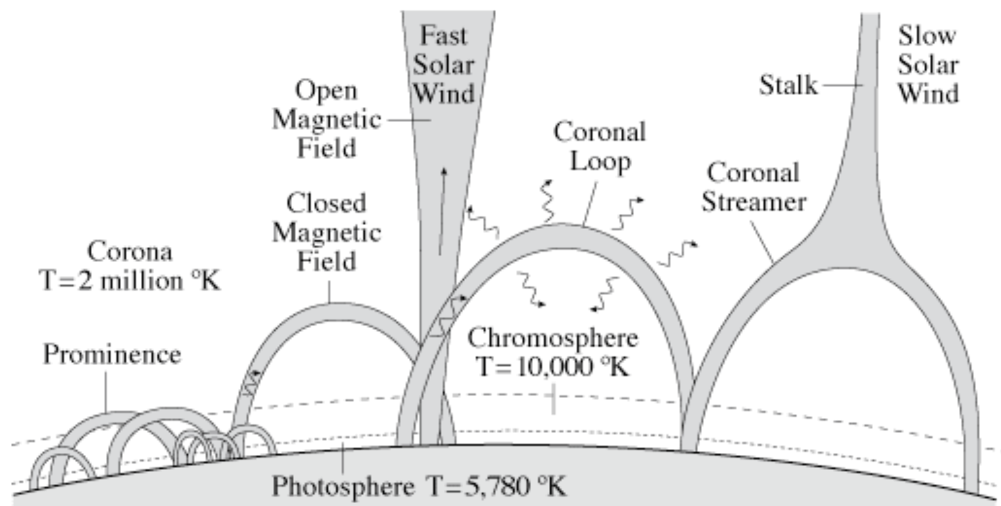


- There are two type of solar wind, fast wind (~700-800 km/s) and slow wind (~ 300-400 km/s).
- Wind speed varies to solar activity.

Solar wind (standard paradigm)

- Fast solar wind (steady)
  - Emerges from open field lines
- Slow solar wind (steady)
  - Escapes intermittently from the streamer belt
- Other sources (transient event)
  - Coronal mass ejections (CMEs)





Magneto-centrifugal wind

- Waver &Davis (1964): consider wind driven by magneto-centrifugal force to model solar wind.
- (But) From current status, it does not apply to solar wind model because the rotation speed of sun is slow.
- However, we can apply other astrophysical object to fast rotator (magnetic rotator) or disk

- Start with 3D MHD equations with spherical coordinate (r, ϕ, θ)
- Assume: time steady ($\partial/\partial t = 0$), axisymmetry ($\partial/\partial\phi = 0$), magnetic field and velocity field are radial & toroidal i.e., $\boldsymbol{B}=(B_r, B_\phi, 0)$, $\boldsymbol{v}=(v_r, v_\phi, 0)$, ideal (adiabatic) MHD, and 1D ($\partial/\partial\theta = 0$) on the equatorial plane ($\theta = \pi/2$)

Magneto-centrifugal wind (cont.)

- Conservation of mass requires that

$$\rho v_r r^2 = f = \text{const} \qquad (8.11)$$

where *f* is mass flux.

- Wind is perfect conductor, thus ***E***=-***v*** x ***B***. From Maxwell's equations

$$(\nabla \times \boldsymbol{E})_\phi = \frac{1}{r}\frac{d}{dr}[r(v_r B_\phi - v_\phi B_r)] = 0$$

- But in a perfectly conducting fluid, ***v*** is parallel to ***B*** in a frame that rotates with the Sun (or any rotating body).

$$r(v_r B_\phi - v_\phi B_r) = \text{const.} = -\Omega r^2 B_r \qquad (8.12)$$

- Where Ω is the angular velocity of the Sun (or any rotating body) from which wind or jet comes out.

Magneto-centrifugal wind (cont.)

- Since div ***B***=0,

$$r^2 B_r = \text{const.} = r_0^2 B_0 = \Phi \qquad (8.13)$$

where Φ is the magnetic flux.

- From toroidal component of equation of motion,

$$\rho \frac{v_r}{r}\frac{d}{dr}(r v_\phi) = (\boldsymbol{J} \times \boldsymbol{B})_\phi = \frac{1}{\mu_0}[(\nabla \times \boldsymbol{B}) \times \boldsymbol{B}]_\phi = \frac{B_r}{\mu_0 r}\frac{d}{dr}(r B_\phi)$$

- But

$$\frac{B_r}{\mu_0 \rho v_r} = \frac{B_r r^2}{\mu_0 \rho v_r r^2} = \text{const.}$$

- Which allows to integrate the toroidal component of equation of motion and obtained

$$r\left(v_\phi - \frac{B_r B_\phi}{\mu_0 \rho v_r}\right) = \text{const.} = \Omega r_A^2 \quad (8.14)$$

Magneto-centrifugal wind (cont.)

- From equation of state,

$$p = K\rho^{\gamma} \qquad (8.15)$$

- From total energy conservation law, we get

$$\frac{1}{2}v_r^2 + \frac{1}{2}(v_\phi^2 - \Omega r)^2 + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} - \frac{GM}{r} - \frac{\Omega^2 r^2}{2} = \text{const.} = E \qquad (8.16)$$

- Where E is total energy of the wind. This is Bernoulli's equation in rotational frame (including potential from centrifugal force).
- The basic MHD equations are integrated into six conservation equations eq (8.11) – (8.16).
- These six parameter, f, Φ, Ω, $r_A{}^2$, K, E are integral constant.
- The unknown variables are also six, ρ, v_r, B_r, v_ϕ, B_ϕ, p
- Hence, if these six constants are given, the equations are solved so that six unknown physical quantities are determined at each r

Magneto-centrifugal wind (cont.)

- Eliminating v_ϕ in eq (8.12) and (8.14), we find

$$\frac{B_\phi}{B_r} = -\frac{r\Omega}{v_r}\frac{(1-r_A^2/r^2)}{(1-v_{Ar}^2/v_r^2)}$$

- It follows that r must be equal to r_A when v_r is equal to v_{Ar}.
- Here $v_{Ar} = B_r/(\mu_0\rho)^{1/2}$ is the Alfven velocity due to the radial component of magnetic field.
- r_A is called *Alfven radius* or *Alfven point*

Magneto-centrifugal wind (cont.)

- Before solving equations, it will be useful to calculate the asymptotic behavior of the physical quantities in this wind.
- As $r \to \infty$, we find

$$B_r \propto r^{-2}$$

- Since in adiabatic wind, wind velocity v_r should tend to be constant terminal velocity v_∞ from energy conservation, i.e.,

$$v_r \to v_\infty$$

- Then we obtain

$$\rho \propto r^{-2},$$
$$v_{Ar} \propto B_r/\rho^{1/2} \propto r^{-1},$$
$$B_\phi/B_r \propto r,$$
$$B_\phi \propto r^{-1}$$

- Hence, the degree of magnetic twist is increases with distance r

Magneto-centrifugal wind (cont.)

- Calculate singular points in this wind. We put eqs (8.11)-(8.15) into eq(8.16) then get following equation only r and ρ

$$H(r,\rho) = \frac{f^2}{2}\frac{1}{\rho^2 r^4}\frac{\gamma K}{\gamma - 1}\rho^{\gamma-1} - \frac{GM}{r} + \frac{\Omega^2 r^2}{2}\left[\frac{(1 - r_A^2/r^2)^2}{(1-\rho/\rho_A)^2} - 1\right]$$

Where $\frac{\rho}{\rho_A} = \frac{v_r r^2}{v_{Ar} r_A^2} = M_A^2$ (8.17)

M_A: *Alfven Mach number*

- Since the eq (8.11) is written as

$$\frac{1}{v_r}\frac{dv_r}{dr} = -\frac{1}{\rho}\frac{d\rho}{dr} - \frac{2}{r} = -\frac{\frac{\partial H}{\partial r} + \frac{2\rho}{r}\frac{\partial H}{\partial \rho}}{\rho\frac{\partial H}{\partial \rho}}$$

Wind equation

Magneto-centrifugal wind (cont.)

- Hence the point where $\partial H/\partial\rho = 0$ becomes the singular point.
- From eq (8.17), we obtain

$$\rho\frac{\partial H}{\partial\rho} = -\frac{(v_r^2 - v_{sr}^2)(v_r^2 - v_{fr}^2)}{v_r^2 - v_{Ar}^2}$$

- Here

$$v_{sr}^2 = \frac{1}{2}\left[c_s^2 + v_{Ar}^2 + v_{A\phi}^2 - \sqrt{(c_s^2 + v_{Ar}^2 + v_{A\phi}^2)^2 - 4c_s^2 v_{Ar}^2}\right]$$

$$v_{fr}^2 = \frac{1}{2}\left[c_s^2 + v_{Ar}^2 + v_{A\phi}^2 + \sqrt{(c_s^2 + v_{Ar}^2 + v_{A\phi}^2)^2 - 4c_s^2 v_{Ar}^2}\right]$$

- Similarly,

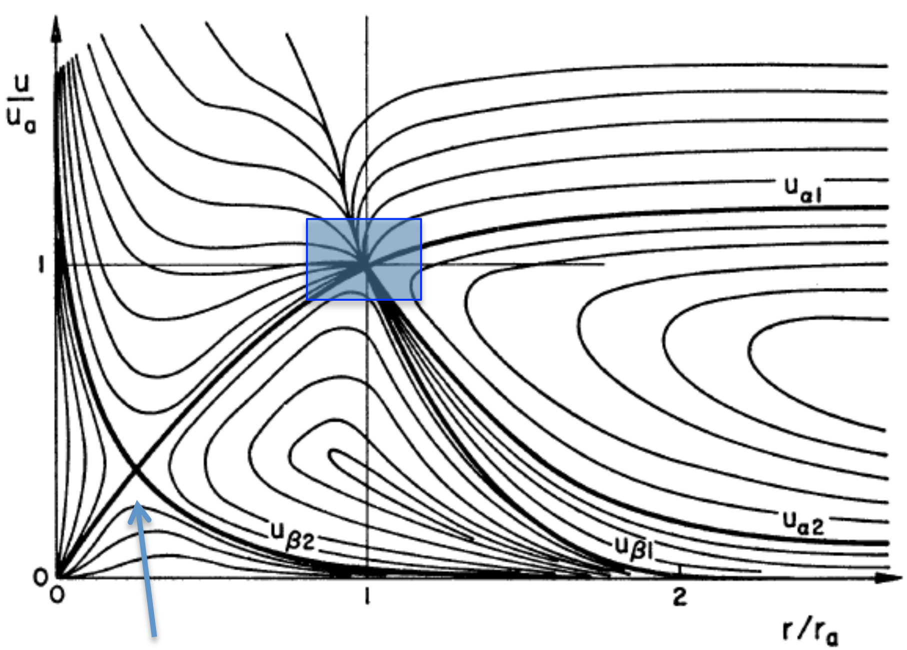
$$\rho\frac{\partial H}{\partial r} = -\frac{2v_r^2}{r} + \frac{GM}{r^2} - r\Omega^2\left[1 - \frac{v_r^4(1 - r_A^4/r^4)}{(v_r^2 - v_{Ar}^2)}\right]$$

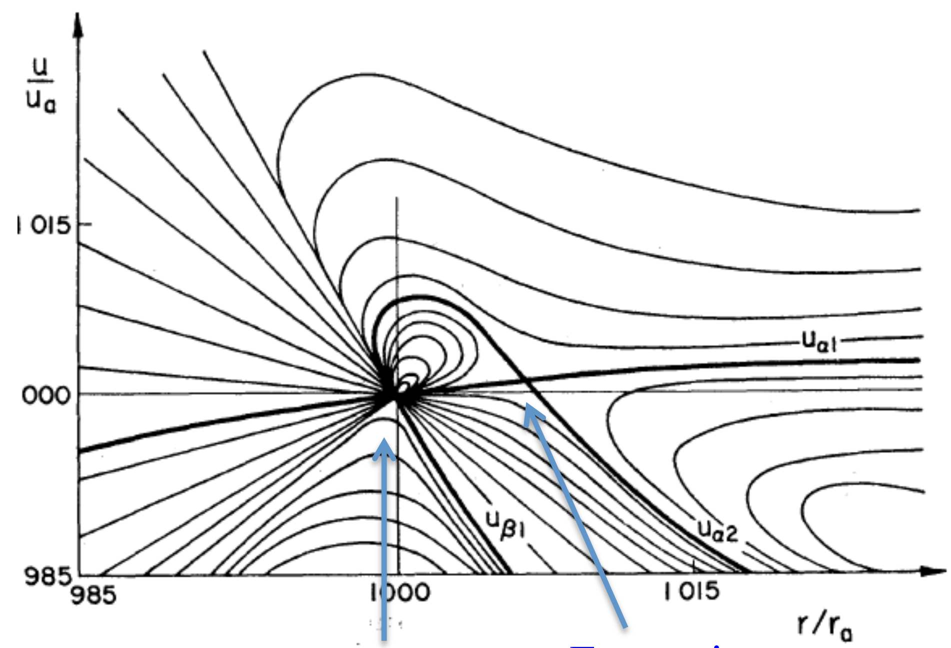
Magneto-centrifugal wind (cont.)

- From these equation, we find when $\partial H/\partial \rho = 0$ (i.e., $v_r = v_{sr}$ or $v_r = v_{fr}$), $\partial H/\partial r = 0$ must be equal to zero. The point where $\partial H/\partial r = 0$ are called slow point ($r = r_{sr}$) and fast point ($r = r_{fr}$).

Solution curve of 1D magneto-centrifugal wind (weber & Davis 1967)

Radial velocity





Slow point

Radial distance

Alfven point

Fast point

Magneto-centrifugal wind (cont.)

- Weber-Davis model is considered equatorial plane.
- But it can be applied any 2D field configuration which assume that trans-field direction (perpendicular to poloidal field line) is balanced and solve (poloidal) field aligned flow.

- If we consider more realistic situation in 2D, we need to solve additional equation, so-called *Grad-Shafranov equation* (*trans-field equation*) which describing force balance perpendicular to poloidal field line coupling with wind equations.
- In general, GS equation is very complicated (second-order quasi-linear partial differential equation) and difficult to find the solution.
- This kind of study is applied to stellar outflows, astrophysical jets from accretion disk and pulsar wind.

Bondi accretion

- Consider spherically-symmetric steady accretion under the gravitational field.
- Spherical accretion onto gravitating body was first studied by Bondi (1952), and is often called *Bondi accretion*
- Spherical outflow is *Parker wind*.
- Analogy is similar to that in Parker wind (only view point is different).

- Far from the accreting gravitating object, the plasma has a uniform density and a uniform pressure (ρ_∞ and p_∞)
- The sound speed far from the gravitating object has the value

$$c_{s\infty} = \sqrt{\gamma p_\infty / \rho_\infty}$$

Bondi accretion (cont.)

- Consider a spherically symmetric flow around an object of mass M.
- The flow is supposed to be steady and 1D in radial direction.
- The flow is assumed to be inviscid and adiabatic, and magnetic and radiation fields are ignored.
- The continuity equations and equation of motion are

$$\frac{1}{4\pi r^2}\frac{d}{dr}(4\pi r^2 \rho v) = 0, \qquad (8.18)$$

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM}{r^2}. \qquad (8.19)$$

- Where v is flow velocity (positive for wind and negative for accretion.)
- The polytropic relation is assumed, $p=K\rho^{\gamma}$

Bondi accretion (cont.)

- Integrating the eq (8.18) & (8.19) yields

$$-4\pi r^2 \rho v = \text{const.} = \dot{M}, \quad (8.20)$$

$$\frac{1}{2}v^2 + \frac{\gamma}{\gamma-1}\frac{p}{\rho} - \frac{GM}{r} = \text{const.} = E. \quad (8.21)$$

- Where $\dot{M}$ is *mass accretion rate* (which is constant in the present case) and E is *the Bernoulli constant*.

- Let us introduce the sound speed and rewrite the basic equation as

$$-4\pi r^2 c_s^{\frac{2}{\gamma-1}} v = (K\gamma)^{\frac{1}{\gamma-1}} \dot{M}, \quad (8.22)$$

$$\frac{1}{2}v^2 + \frac{1}{\gamma-1}c_s^2 - \frac{GM}{r} = E. \quad (8.23)$$

Bondi accretion (cont.)

- From the logarithmic differentiation of eq (8.20) we have

$$\frac{2}{r} + \frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{v}\frac{dv}{dr} = 0$$

- Eliminating dρ/dr from eq (8.19), we obtain

$$(v^2 - c_s^2)\frac{1}{v}\frac{dv}{dr} = \frac{2}{r}c_s^2 - \frac{GM}{r^2}$$

- Here the sound speed is expressed as

$$c_s^2 = (\gamma - 1)\left(E + \frac{GM}{r} - \frac{1}{2}v^2\right)$$

Bondi accretion (cont.)

- In the adiabatic case, considering regularity condition $v_c = -c_{sc}$ and $r_c = GM/2c_{sc}^2$ at critical point, from continuity and Bernoulli equations, we have

$$\begin{aligned} (K\gamma)^{1/(\gamma-1)}\dot{M} &= 4\pi r_c^2 |v_c|^{(\gamma+1)/(\gamma-1)}, \\ E &= \frac{5-3\gamma}{2(\gamma-1)} v_c^2 \end{aligned}$$

- These give the relations between the quantities at the critical point and flow parameter. Furthermore, critical radius r_c is expressed in terms of γ and E as

$$r_c = \frac{GM}{2c_s^2} = \frac{(5-3\gamma)GM}{4(\gamma-1)E}$$

- From this critical radius is determined by Mass of central object and flow energy.

Bondi accretion (cont.)

- Moreover, in order for the steady transonic solution to exist, E must be positive. Hence, the condition

$$1 < \gamma < 5/3$$

- Should be satisfied in the case of spherically symmetric adiabatic flow.
- In adiabatic case, γ=5/3 does not make transonic flow. To satisfy γ<5/3, we should consider *non-adiabatic effect* such as thermal conduction or radiation cooling.

- (Parker wind is assumed isothermal, therefore does not effect this problem)

Bondi accretion (cont.)

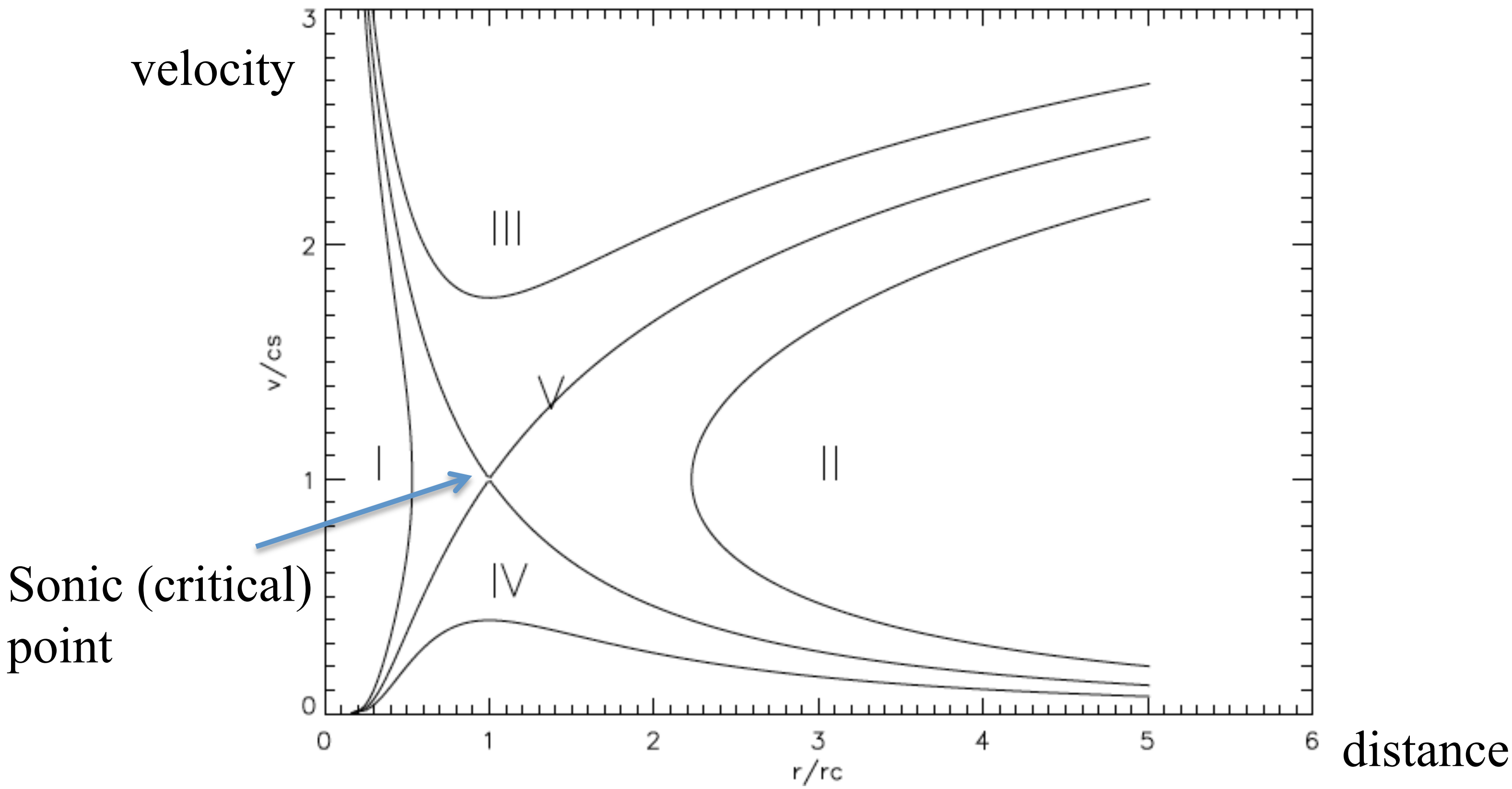
- Let us introduce the Mach number $\mathcal{M} \equiv v/c_s$ and derive the wind equation
- In adiabatic case, we easily derive

$$\frac{d\mathcal{M}}{dr} = \frac{\mathcal{N}}{\mathcal{D}}$$

$$\begin{aligned} \mathcal{D} &= \mathcal{M}^2 - 1 \\ \mathcal{N} &= \mathcal{M}\left(\frac{\gamma-1}{2}\mathcal{M}^2 + 1\right)\left[\frac{2}{r} - \frac{\gamma+1}{2(\gamma-1)}\frac{1}{E+\frac{GM}{r}}\frac{GM}{r^2}\right] \end{aligned}$$

Bondi accretion (cont.)

- Several types of solution are present



Bondi accretion (cont.)

- If the accretion is transonic, then we can uniquely determine the accretion rate $\dot{M}_t$ in terms of the mass M of the accreting object and the density ρ_∞ and the sound speed $c_{s\infty}$ at infinity (ambient value).
- From eq (8.23),

$$\frac{v^2}{2} + \frac{c_s^2}{\gamma - 1} - \frac{GM}{r} = \frac{c_{s\infty}^2}{\gamma - 1},$$

or

$$c_{sc} = c_{s\infty} \left(\frac{2}{5 - 3\gamma} \right)^{1/2}$$

- This implies that

$$r_c = \frac{5 - 3\gamma}{4} \frac{GM}{c_{s\infty}^2}$$

or

$$\rho_{rc} = \rho_\infty \left(\frac{2}{5 - 3\gamma} \right)^{1/(\gamma - 1)}$$

Bondi accretion (cont.)

- Using the relation $\dot{M} = 4\pi r_c^2 \rho_{rc} c_{sc}$, we find that the transonic accretion rate is

$$\dot{M}_t = 4\pi q_c \frac{G^2 M^2 \rho_\infty}{c_{s\infty}^3}$$

- Where

$$q_c(\gamma) = \frac{1}{4}\left(\frac{2}{5-3\gamma}\right)^{(5-3\gamma)/(2\gamma-2)}$$

- The numerical value of q_c ranges from $q_c = 1/4$ at γ=5/3 to $q_c = e^{3/2}/4 \sim$ 1.12 when γ=1.
- If accreting medium is ionized hydrogen, the transonic accretion rate has

$$\dot{M}_t = 1.2 \times 10^{10}\ \mathrm{g\ sec}^{-1}\left(\frac{M}{M_\odot}\right)^2\left(\frac{\rho_\infty}{10^{-24}\mathrm{g\ cm}^{-3}}\right)\left(\frac{T_\infty}{10^4\mathrm{K}}\right)^{-3/2}$$

- This amounts to about $10^{-16} M_\odot \mathrm{yr}^{-1}$ for a $1 M_\odot$ gravitating body.

Bondi accretion (cont.)

- The relation between the bulk velocity $v(r)$ and the sound speed $c_s(r)$ can be computed from the equation

$$-v = \frac{\dot{M}}{4\pi r^2 \rho(r)} = \frac{\dot{M}}{4\pi r^2 \rho_\infty} \left(\frac{c_{s\infty}}{c_s(r)} \right)^{2/(\gamma-1)}$$

- Thus

$$v = -\frac{q_c G^2 M^2}{r^2 c_{s\infty}^3} \left(\frac{c_s(r)}{c_{s\infty}} \right)^{-2/(\gamma-1)}$$

- Or

$$\frac{v}{c_{s\infty}} = -\frac{q_c}{4} \left(\frac{r}{r_a} \right)^{-2} \left(\frac{c_s(r)}{c_{s\infty}} \right)^{-2/(\gamma-1)}$$

- Where $r_a \equiv 2GM/c_{s\infty}^2$ *accretion radius* or *Bondi radius*
  The radius at which the density and sound speed start to significantly increase from their ambient values of ρ_∞ and $c_{s\infty}$

Bondi accretion (cont.)

- The relation between the critical radius and the accretion radius is

$$r_c = [(5-3\gamma)/8] r_a$$

- At large radius ($r >> r_a$)

$$v \approx -\frac{q_c c_{s\infty}}{4}\left(\frac{r}{r_a}\right)^{-2}\left[1 - \frac{1}{2}\frac{r_a}{r}\right]$$

$$c_s \approx c_{s\infty}\left[1 + \frac{\gamma-1}{4}\frac{r_a}{r}\right]$$

$$\rho \approx \rho_\infty\left[1 + \frac{1}{2}\frac{r_a}{r}\right]$$

- From gas with γ=5/3, at small radius ($r << r_a$)

$$v \approx -c_s \approx -\frac{c_{s\infty}}{2}\left(\frac{r}{r_a}\right)^{-1/2}$$

$$\rho \approx \frac{\rho_\infty}{8}\left(\frac{r}{r_a}\right)^{-3/2}$$

Bondi accretion (cont.)

- If $1 < \gamma < 5/3$, the infall at $r << r_c$ is supersonic, and the infalling gas is in free fall. From Bernoulli integral, we find $v^2/2 \sim GM/r$ or

$$v \approx -c_{s\infty} \left(\frac{r}{r_a} \right)^{-1/2}$$

$$\rho \approx \frac{q_c \rho_\infty}{4} \left(\frac{r}{r_a} \right)^{-3/2}$$

- Spherical accretion of gas thus has a characteristic density profile, with $\rho^{-3/2}$ at small radius and ρ = constant at large radius.
- The infall velocity profile is $v^{-1/2}$ at small radius

Bondi accretion (cont.)

- If accreting body has a constant velocity V with respect to ambient medium, the transonic accretion rate is

$$\dot{M}_t = 4\pi\tilde{q}\frac{G^2M^2\rho_\infty}{(c_{s\infty}^2 + V^2)^{3/2}}$$

- Where $\tilde{q}$ is a order of unity. When $V > c_{s\infty}$, a bow shock forms in front of the accreting object which increases the temperature and decreases the bulk infalling velocity relative to accreting central object.
- At $r \ll r_a \sim 2GM/(V^2 + c_{s\infty}^2)$, the flow of the gas is approximately radial, and takes the form of the spherically symmetric Bondi solution.

Summary

- Study the steady spherically outflow and accretion.
- The solution of wind equation with integral constants shows variety of flow profile (outflow and accretion).
- Transonic solution (pass through the sonic point) is the most favorable solution for accretion and outflow.
- In MHD case, there are three critical points (slow, Alfven and fast).
- The solution should pass through all three critical points.
- The twist of magnetic field is proportional to distance, i.e., in far region, toroidal (azimuthal) magnetic field is dominant.