Plasma Astrophysics (ASTR6880)
Exercise 2

Return the solutions until lecture on Wednesday, October 23, 2013

1. Starting from ieal MHD equations (including pressure equation),

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$
$$\rho \left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla) \boldsymbol{v} \right] = -\nabla p + \frac{1}{\mu_0} (\nabla \times \boldsymbol{B}) \times \boldsymbol{B},$$
$$\frac{\partial p}{\partial t} + (\boldsymbol{v} \cdot \nabla) p = -\gamma p \nabla \cdot \boldsymbol{v},$$
$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = 0,$$
$$\nabla \cdot \boldsymbol{B} = 0,$$
$$p = (\gamma - 1) \rho e,$$

derive the conservation form of total energy,

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho v^2 + \rho e + \frac{B^2}{2\mu_0} \right) + \nabla \cdot \left[\left(\frac{1}{2} \rho \boldsymbol{v}^2 + \rho e + p + \frac{B^2}{\mu_0} \right) \boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{B}) \frac{\boldsymbol{B}}{\mu_0} \right] = 0.$$

[Hint: adding the separate contributions of kinetic, internal and magnetic energy from equation of motion, pressure equation and induction equation.]

2. Calculate the Alfvén wave fundamental oscillation period in a coronal loop of length 5×10^7 m with background magnetic field of 10^{-3} T and particle density of 10^{15} m^{-3} (fully ionized hydrogen gas, $m = 0.5 m_i$, $\mu = 0.5$).

[Hint: fundamental oscillation mode has $\lambda = L/2$.]

3. Starting from generalized wave equation,

$$\begin{aligned} \frac{\omega^2 \boldsymbol{v}_1}{v_A^2} &= k^2 \cos^2(\theta_{kB_0}) \boldsymbol{v}_1 - (\boldsymbol{k} \cdot \boldsymbol{v}_1) k \cos(\theta_{kB_0}) \hat{\boldsymbol{B}}_0 \\ &+ \left[\left(1 + \frac{c_s^2}{v_A^2} \right) (\boldsymbol{k} \cdot \boldsymbol{v}_1) - k \cos(\theta_{kB_0}) (\hat{\boldsymbol{B}}_0 \cdot \boldsymbol{v}_1) \right] \boldsymbol{k}, \end{aligned}$$

derive the magnetoacoustic dispersion relation using two dot-product ($\boldsymbol{k}$ and $\hat{\boldsymbol{B}}_0$),

$$\omega^4 - \omega^2 k^2 (c_s^2 + v_A^2) + c_s^2 v_A^2 k^4 \cos^2(\theta_{kB_0}) = 0$$

[Hint: From these two equations obtained from two dot-product, deleted $(\boldsymbol{k} \cdot \boldsymbol{v}_1)$ and $(\hat{\boldsymbol{B}}_0 \cdot \boldsymbol{v}_1)$]

4. Calculate the magnetoacoustic fast-mode wave phase speed in a 4×10^6 K stellar corona with particle density of 10^{14} m^{-3} (fully ionized hydrogen gas, $m = 0.5m_i$, $\mu = 0.5$), consider a horizontal wave propagating at right angle (90°) to a background magnetic field of 10^{-3} T.

   [Hint: using ideal equation of state, $p = nk_BT$]

Please write the solutions in English.