The continuum limit of the static-light meson spectrum

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Abstract

We investigate the continuum limit of the low lying static-light meson spectrum using Wilson twisted mass lattice QCD with $N_f = 2$ dynamical quark flavours. We consider three values of the lattice spacing $a \approx 0.051\,\mathrm{fm}$, $0.064\,\mathrm{fm}$, $0.080\,\mathrm{fm}$ and various values of the pion mass in the range $280\,\mathrm{MeV} \lesssim m_{\mathrm{PS}} \lesssim 640\,\mathrm{MeV}$. We present results in the continuum limit for light cloud angular momentum $j = 1/2$, $3/2$, $5/2$ and for parity $\mathcal{P} = +$, $-$. We extrapolate our results to physical quark masses, make predictions regarding the spectrum of B and B_s mesons and compare with available experimental results.

1 Introduction

A systematic way to study B and B_s mesons from first principles is with lattice QCD. Since $am_b > 1$ at currently available lattice spacings for large volume simulations, one needs to use for the b quark a formalism such as Heavy Quark Effective Theory (HQET) [1, 2] or Non-Relativistic QCD [3]. An alternative procedure has recently been proposed [4] which is based on HQET but does not make use of the static point. Here we follow the standard HQET route, which enables all sources of systematic error to be controlled.

In the static limit a heavy-light meson will be the "hydrogen atom" of QCD. Since in this limit there are no interactions involving the heavy quark spin, states are doubly degenerate, i.e. there is no hyperfine splitting. Therefore, it is common to label static-light mesons by parity $\mathcal{P}$ and the total angular momentum of the light degrees of freedom j with $j = |l \pm 1/2|$, where l and $\pm 1/2$ denote respectively angular momentum and spin. An equivalent notation is given by $l_\pm$, which reads $S \equiv (1/2)^-$, $P_- \equiv (1/2)^+$, $P_+ \equiv (3/2)^+$, $D_- \equiv (3/2)^-$, $D_+ \equiv (5/2)^-$, $F_- \equiv (5/2)^+$, $F_+ \equiv (7/2)^+$ ... The total angular momentum of a static-light meson is either $J = j + 1/2$ or $J = j - 1/2$, where both states are of the same mass. Note that in contrast to parity, charge conjugation is not a good quantum number, since static-light mesons are made from non-identical quarks.

The static-light meson spectrum has been studied comprehensively by lattice methods in the quenched approximation with a rather coarse lattice spacing [5]. Lattice studies with $N_f = 2$ flavours of dynamical sea quarks have also explored this spectrum [6, 7, 8, 9, 10, 11, 12]. Here following our initial study [13, 14], we use $N_f = 2$ flavours and are able to reach lighter dynamical quark masses, which are closer to the physical u/d quark mass, so enabling a more reliable extrapolation. Note that in our formalism, maximally twisted mass lattice QCD, mass differences in the static-light spectrum are $\mathcal{O}(a)$ improved so that the continuum limit is more readily accessible. We now extend our study to include three different lattice spacings, which gives us confidence that we are indeed extracting the continuum limit.

In this paper, we approach the B meson spectrum by concentrating on the unitary sector, where valence quarks and sea quarks are of the same mass. This is appropriate for static-light mesons with a light quark, which is u or d.

We also estimate masses of B_s mesons with s quarks of physical mass, where the s quark is treated as a valence quark in the sea of light u and d quarks (so this is a partially quenched study). We took our s quark mass values from ETMC studies of strange mesons [15, 16].

Within the twisted mass formalism, it is feasible to use $N_f = 2 + 1 + 1$ flavours of dynamical sea quarks, which would give a more appropriate focus on the static-strange meson spectrum if strange quark sea effects were significant. This is under study by ETMC.

In HQET the leading order is just the static limit. The next correction will be of order $1/m_Q$, where m_Q is the mass of the heavy quark. This correction is expected be relatively small for b quarks, but larger for c quarks. Lattice methods to evaluate these $1/m_Q$ contributions to the B meson hyperfine splittings have been established and tested in quenched studies [17, 18, 19, 20, 21]. We intend to explore these contributions using lattice techniques subsequently. An alternative way to predict the spectrum for B and B_s mesons is to interpolate between D and D_s states, where the experimental spectrum is rather well known, and the static limit obtained by lattice QCD assuming a dependence as $1/m_Q$. Thus the splittings among B and B_s mesons

should be approximately $m_c/m_b \approx 1/3$ of those among the corresponding D and D_s mesons.

For excited D_s mesons, experiment has shown that some of the states have very narrow decay widths [22]. This comes about, since the hadronic transitions to DK and D_sM (where M is a flavour singlet mesonic system, e.g. η', $\pi\pi$ or f_0) are not allowed energetically. The isospin violating decay to $D_s\pi$ together with electromagnetic decay to $D_s\gamma$ are then responsible for the narrow width observed. A similar situation may exist for B_s decays and we investigate this here using our lattice mass determinations of the excited states. This will enable us to predict whether narrow excited B_s mesons should be found.

As well as exploring this issue of great interest to experiment, we determine the excited state spectrum of static-light mesons as fully as possible. This will help the construction of phenomenological models and will shed light on questions such as, whether there is an inversion of the level ordering with l_+ lighter than l_- at larger l or for radial excitations as has been predicted [23, 24, 25, 26, 27].

Since we measure the spectrum for a range of values of the bare quark mass parameter μ_q for the light quark, we could also compare with chiral effective Lagrangians appropriate to HQET. This comparison would be most appropriate applied to heavy-light decay constants in the continuum limit (see ref [28]). Since that study awaits more precise renormalization constants, we do not discuss it further here.

Since we have discussed the basic methods in a previous paper [14], in this paper we present only briefly the details of our computation of static-light meson mass differences. We give a full discussion of our extrapolation to the continuum and to physical light quark masses. We also discuss the interpolation to the physical b quark mass.

2   Lattice details

We use $N_f = 2$ flavour gauge configurations produced by the European Twisted Mass Collaboration (ETMC). The gauge action is tree-level Symanzik improved [29], while the fermionic action is Wilson twisted mass at maximal twist (cf. e.g. [30] and references therein). As argued in [14] this ensures automatic $\mathcal{O}(a)$ improvement for static-light spectral quantities, e.g. mass differences of static-light mesons, the quantities we are focusing on in this work.

We use three different values of the lattice spacing $a \approx 0.051\,\text{fm}$, $0.064\,\text{fm}$, $0.080\,\text{fm}$ and various values of the pion mass in the range $280\,\text{MeV} \lesssim m_{\text{PS}} \lesssim 640\,\text{MeV}$. All lattice volumes are big enough to fulfill $m_{\text{PS}}L > 3.2$. The ensembles we are considering are listed in Table 1. Details regarding the generation of gauge configurations and analysis procedures for standard quantities (e.g. lattice spacing, pion mass) can be found in [31, 32].

In Table 1 we also list the number of gauges, on which we have computed static-light correlation functions, and the number and type of inversions performed to estimate light quark propagators stochastically. Note that in contrast to our previous work [13, 14] we treat B_s mesons in a partially quenched approach, where the mass of the valence quark is approximately the mass of the physical s quark, taken from the study of strange mesons using the same configurations [15, 16],

- $\beta = 3.90 \quad \rightarrow \quad \mu_{q,\text{valence}} = \mu_{q,s} = 0.022,$

β	$L^3 \times T$	μ_{q}	a in fm	m_{PS} in MeV	# of gauges	# and type of inversions
3.90	$24^3 \times 48$	0.0040	0.0801(14)	336(6)	1420/580	(spin)/(4rand)
		0.0064		417(7)	1480/-	(spin)/-
		0.0085		478(8)	1360/480	(spin)/(4rand)
		0.0100		517(9)	460/480	(6rand)/(4rand)
		0.0150		637(11)	1000/-	(1rand)/-
4.05	$32^3 \times 64$	0.0030	0.0638(10)	321(5)	240/240	(4rand)/(4rand)
		0.0060		443(7)	500/500	(4rand)/(4rand)
4.20	$48^3 \times 96$	0.0020	0.0514(8)	284(5)	420/420	(spin)/(4rand)

Table 1: ensembles (a and m_{PS} have been taken from [32]; # of gauges considered for B/B_s mesons; # and type of inversions for B/B_s mesons: (spin) four spin diluted timeslice sources on the same randomly chosen timeslice; (1rand) a single timeslice source on a randomly chosen timeslice; (4rand) four timeslice sources on four randomly chosen timeslices; (6rand) six timeslice sources on six randomly chosen timeslices).

- $\beta = 4.05$ → $\mu_{\mathrm{q,valence}} = \mu_{\mathrm{q},s} = 0.017$,

- $\beta = 4.20$ → $\mu_{\mathrm{q,valence}} = \mu_{\mathrm{q},s} = 0.015$,

while the sea is considerably lighter (cf. the listed μ_{q} values in Table 1).

3 Static-light mass differences

The determination of static-light mass differences is essentially identical to what we have done in [13, 14].

For each of our ensembles characterised by the gauge coupling β and the twisted light quark mass μ_{q} (cf. Table 1) and each of the lattice angular momentum representations A_1, E and A_2 we compute 6×6 static-light correlation matrices. The corresponding meson creation operators differ in their (twisted mass) parity, in their γ matrix structure and in their spatial size. They are precisely the same we have been using before and are explained in detail in [14], section 3, Table 3.

From these correlation matrices we compute effective mass plateaux using variational methods [33, 34] (cf. [35] for exemplary plots showing the quality of our plateaus). We extract mass differences by fitting constants to these plateaus at sufficiently large temporal separations $T_{\min} \ldots T_{\max}$. We determine $T_{\min}$ and $T_{\max}$ by requiring that the reduced χ^2 is $\mathcal{O}(1)$. $T_{\min}$ values are listed in Table 2, while $T_{\max} = 11$ for $\beta = 3.90$ and $\beta = 4.05$ and $T_{\max} = 17$ for $\beta = 4.20$ in most cases (for some of the excited states smaller values had to be chosen, because the signal was lost in statistical noise). Note, however, that the choice of $T_{\max}$ is essentially irrelevant for the resulting mass (on the "$T_{\max}$ side" of the effective mass plateau statistical errors are rather large and, therefore, data points only have a very weak effect on the fit). Since we are only interested in mass differences $\Delta M(j^{\mathcal{P}}) = M(j^{\mathcal{P}}) - M(S)$, the jackknife analysis has

been applied directly to the mass difference and not to the individual masses. The samples for $M(j^{\mathcal{P}})$ and $M(S)$ entering for such a mass difference have been obtained with the same value of $T_{\min}$.

β	μ_{q}	B mesons						B_s mesons					
		P_-	P_+	$D_\pm$	D_+	$F_\pm$	S^*	P_-	P_+	$D_\pm$	D_+	$F_\pm$	S^*
3.90	0.0040	6	6	5	4	4	4	6	6	5	5	4	4
	0.0064	6	6	5	4	4	4	-	-	-	-	-	-
	0.0085	6	6	5	4	4	4	5	5	4	4	4	4
	0.0100	6	6	5	4	4	4	5	5	4	4	4	4
	0.0150	6	6	5	4	4	4	-	-	-	-	-	-
4.05	0.0030	7	6	6	5	5	6	7	7	6	5	5	7
	0.0060	7	6	6	5	5	5	7	7	6	5	5	7
4.20	0.0020	10	10	8	7	7	9	11	9	9	8	8	11

Table 2: $T_{\min}$ for fitting constants to effective mass plateaus.

The resulting mass differences $\Delta M(j^{\mathcal{P}})a$ (in lattice units), where $j^{\mathcal{P}} \in \{P_-, P_+, D_\pm, D_+, F_\pm, S^*\}$, together with the pion masses $m_{\mathrm{PS}}a$ (in lattice units; cf. Table 1 and [32]) and the lattice spacings a (in physical units; cf. Table 1) serve as input for the extrapolation procedure to physical u/d quark masses described in the next section.

We checked the stability of our results by varying $T_{\min}$ by ± 1 as well as by fitting superpositions of exponentials to the elements of the correlation matrices (as done in [14]) instead of solving generalised eigenvalue problems. We found consistency within statistical errors.

4 Continuum limit and extrapolation to physical u/d quark masses

4.1 Numerical results

The mass differences $\Delta M(j^{\mathcal{P}})$ obtained for all our ensembles are plotted against $(m_{\mathrm{PS}})^2$ in Figure 1 (unitary, i.e. "B mesons") and Figure 2 (partially quenched, i.e. "B_s mesons"). Note that, although we use three different values of the lattice spacing, points corresponding to the same mass difference fall on a single curve. This is reassuring, since we use Wilson twisted mass lattice QCD at maximal twist, where static-light mass differences are $\mathcal{O}(a)$ improved [14]. In Table 3 and Table 4 we collect the values of the mass differences in MeV[1] for all simulation points for B and B_s mesons respectively.

For the extrapolation to physical light quark masses, we could use an effective field theory approach (Chiral HQET for instance) as used to study the decay constants [28] of the ground state. This approach has not been developed to discuss mass differences between excited states

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[1]The scale has been set by the pion decay constant f_π as explained in detail in [32].
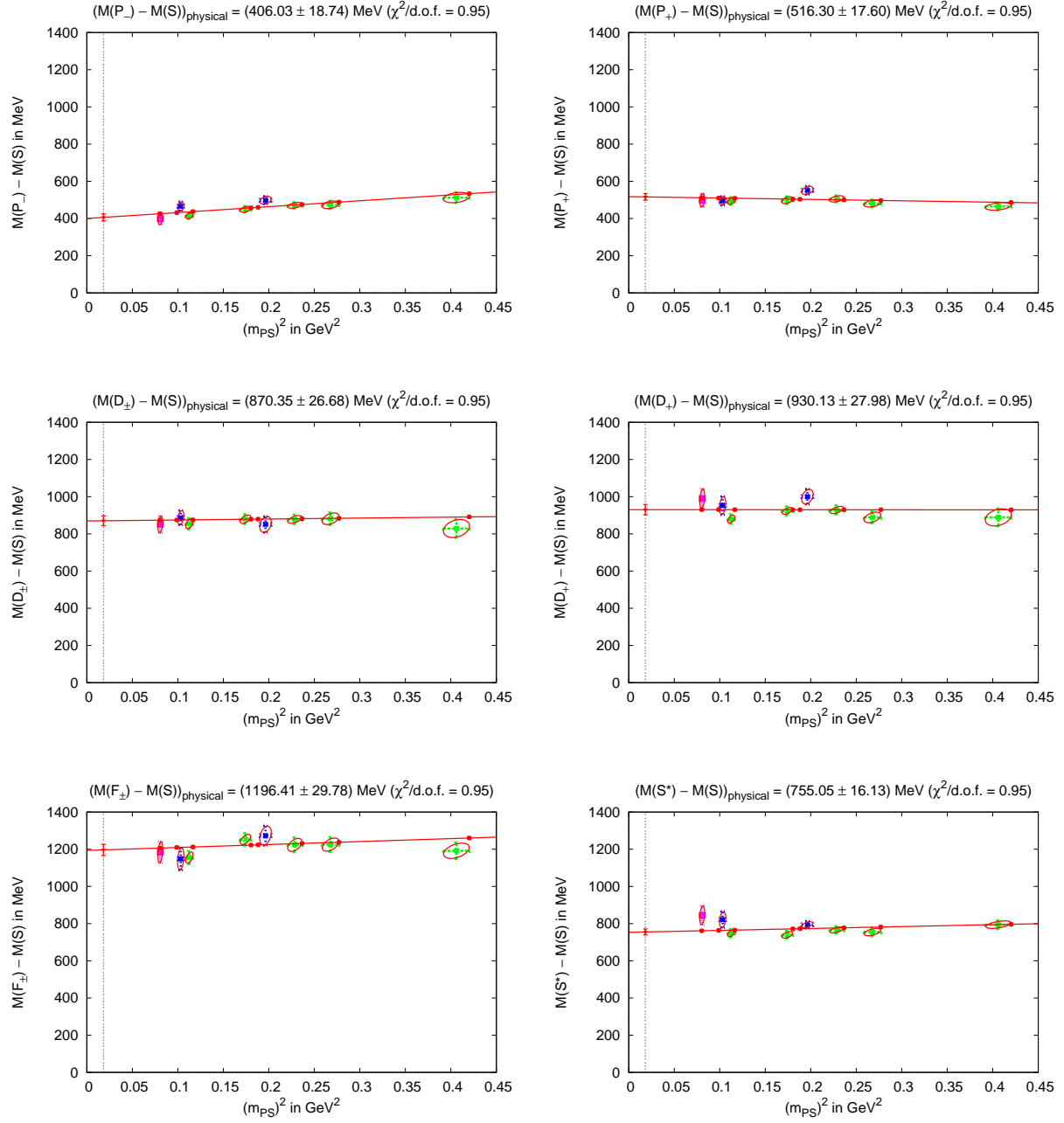
Figure 1: static-light mass differences linearly extrapolated to the physical u/d quark mass (unitary, i.e. B mesons).
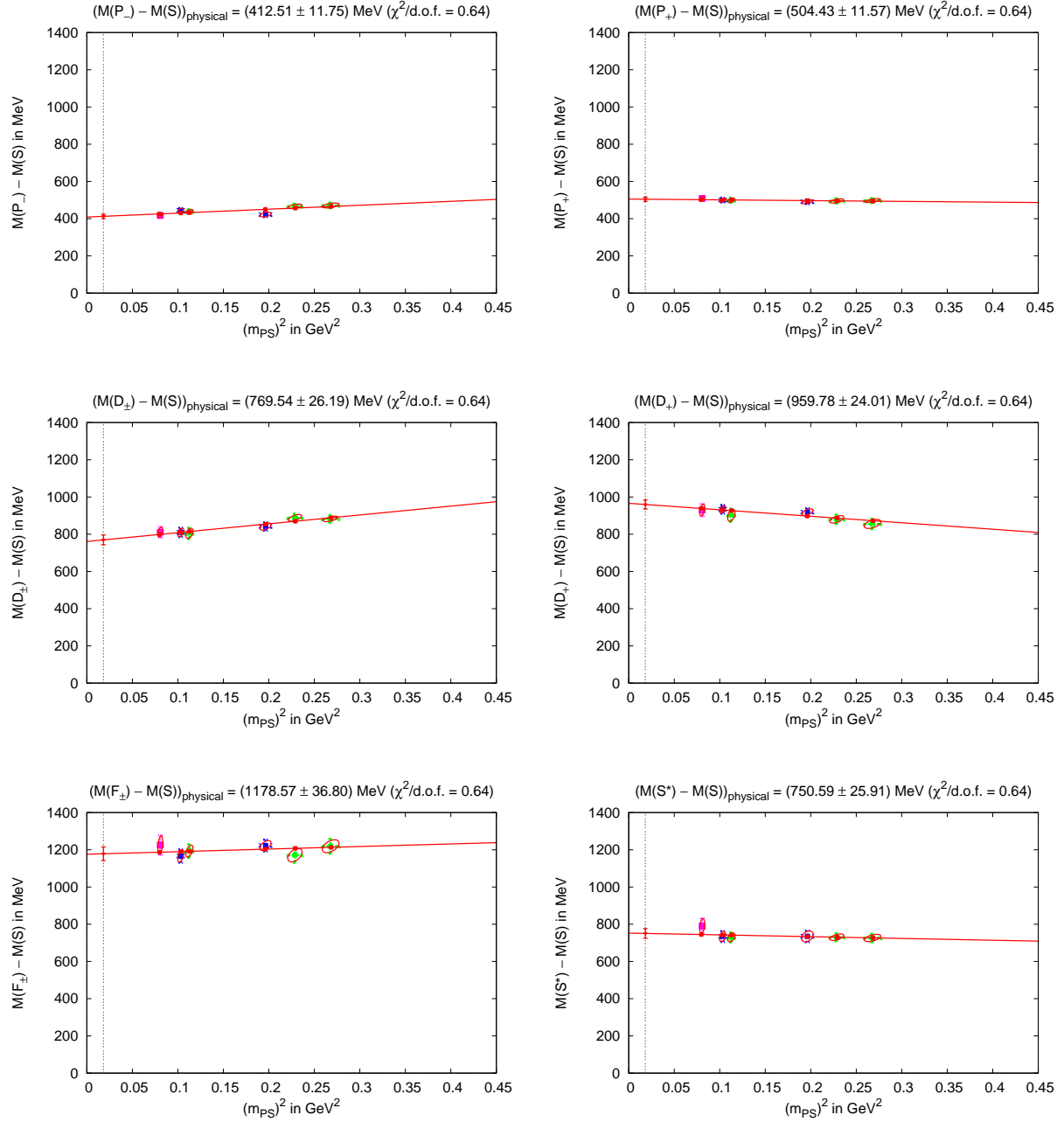
Figure 2: static-light mass differences linearly extrapolated to the physical u/d quark mass (partially quenched, i.e. B_s mesons).

β	μ_{q}	$\Delta M(P_-)$	$\Delta M(P_+)$	$\Delta M(D_\pm)$	$\Delta M(D_+)$	$\Delta M(F_\pm)$	$\Delta M(S^*)$
3.90	0.0040	415(17)	494(20)	855(30)	879(25)	1155(35)	749(22)
	0.0064	449(17)	499(20)	879(26)	924(24)	1253(33)	740(21)
	0.0085	471(17)	506(19)	878(25)	928(24)	1223(40)	766(20)
	0.0100	474(22)	481(21)	881(34)	889(32)	1225(40)	755(23)
	0.0150	513(29)	465(21)	829(50)	889(48)	1192(45)	794(24)
4.05	0.0030	465(26)	495(24)	887(39)	952(49)	1148(60)	821(44)
	0.0060	498(22)	551(23)	851(44)	1000(41)	1273(53)	794(20)
4.2	0.0020	399(31)	498(35)	851(45)	990(53)	1184(58)	845(51)

Table 3: static-light mass differences in MeV (unitary, i.e. B mesons) for all simulation points; details on the analysis procedure of the correlation functions are given in section 3.

β	μ_{q}	$\Delta M(P_-)$	$\Delta M(P_+)$	$\Delta M(D_\pm)$	$\Delta M(D_+)$	$\Delta M(F_\pm)$	$\Delta M(S^*)$
3.90	0.0040	438(13)	499(14)	805(30)	902(35)	1193(37)	729(26)
	0.0085	466(14)	495(14)	888(23)	880(24)	1171(41)	730(21)
	0.0100	471(15)	497(13)	882(20)	855(28)	1219(40)	726(22)
4.05	0.0030	444(13)	500(13)	810(26)	934(24)	1167(36)	734(29)
	0.0060	422(14)	491(13)	842(23)	918(22)	1223(32)	735(31)
4.2	0.0020	417(13)	509(13)	811(29)	930(34)	1226(52)	790(41)

Table 4: static-light mass differences in MeV (partially quenched, i.e. B_s mesons) for all simulation points; details on the analysis procedure of the correlation functions are given in section 3.

and the ground state (e.g. $M(P_-) - M(S)$), so is not appropriate here. Instead we use the simplest assumption which is supported by our results: a linear dependence.

Because our ground state mass values enter into all of the mass differences we study, we simultaneously fit to all the meson mass differences we have computed. We find that fits which are independent of the lattice spacing and which are linear in the light quark mass (represented by the mass squared of the light-light pseudoscalar meson) are acceptable, i.e. yield $\chi^2/\mathrm{dof} \lesssim 1$.

For the B_s mesons, our results depend on the strange quark mass we choose. We have taken these values from studies of strange-light mesons [15, 16] as discussed above. The possible systematic error arising from an incorrect value for the strange quark mass is very small: because the mass differences we measure turn out to be very weakly dependent on that mass. This will be seen when we compare our results for the B and B_s mesons extrapolated to physical light quark masses.

The details of our fitting procedure are collected in appendix A.

As already mentioned both fits (one for B mesons, the other for B_s mesons) are of good quality in a sense that $\chi^2/\mathrm{dof} \lesssim 1$. This shows that at the present level of statistical accuracy the continuum limit has already been reached at our largest value of the lattice spacing $a \approx 0.080\,\mathrm{fm}$. Moreover,

these fits enable us to extrapolate to physical u/d quark masses.

Extrapolations of static-light mass differences to physical u/d quark masses are listed in Table 5 in MeV both for B mesons and for B_s mesons. Note that both fits give $\chi^2/\text{d.o.f.} \approx 1$, i.e. are consistent with our assumption that static-light meson mass differences as functions of $(m_{\text{PS}})^2$ can be parameterised by straight lines.

	P_-	P_+	$D_\pm$	D_+	$F_\pm$	S^*	$\chi^2/\text{d.o.f.}$
B mesons	406(19)	516(18)	870(27)	930(28)	1196(30)	755(16)	0.95
B_s mesons	413(12)	504(12)	770(26)	960(24)	1179(37)	751(26)	0.64

Table 5: $M(j^{\mathcal{P}}) - M(S)$ in MeV extrapolated to physical light quark masses.

To check the stability of these fits, we have varied $T_{\min}$ by ± 1. Within statistical errors mass differences obtained with $T_{\min} - 1$, with $T_{\min}$ and with $T_{\min} + 1$ are in agreement.

The extrapolations are shown in Figure 1 and Figure 2. The red dots represent the maximum likelihood estimates of $\bar{\mathbf{z}} = ((m_{\text{PS}})^2, \Delta M(j^{\mathcal{P}}))$ obtained during the fitting procedure. In addition to x-y-error bars we also plot covariance ellipses, which reflect the correlations between $(m_{\text{PS}})^2$ and $\Delta M(j^{\mathcal{P}})$ induced by the lattice spacing a, that is they are generated from the inverses of the corresponding 2×2 submatrices of the covariance matrix C.

4.2 Contamination of static-light meson masses by multi particle states

The radially and orbitally excited static-light mesons P_-, P_+, D_-, D_+, F_-, F_+ and S^* can decay into multi particle states $S + n \times \pi$ with relative angular momentum such that quantum numbers $j^{\mathcal{P}}$ are identical. In particular the P_- static-light meson is not protected by angular momentum, i.e. it can decay via an S wave into $S + \pi$, whose wave function is not suppressed at the origin. In the following we argue that the effect of $S + \pi$ states on our P_- mass is small compared to its statistical error. To this end we resort to a model presented and to numerical results obtained in [36, 37, 38].

We consider the P_- static-light meson at $\beta = 3.90$ and our lightest u/d quark mass at this β value ($\mu_q = 0.0040$). In that ensemble the masses of the P_- state and of the $S + \pi$ state are quite similar: $m(P_-)a \approx 0.57$ and $(m(S) + m(\pi))a \approx 0.53$ (we consider the case, where the pion has zero momentum). Therefore, we expect mixing of P_- and $S + \pi$ with respect to the eigenstates of the Hamiltonian H, mixing which will be different in different spatial volumes. Consequently, we do not focus on the eigenvalues of these states, but rather on $m(P_-) = \langle P_- | H | P_- \rangle$ ($| P_- \rangle$ is a state with $j^{\mathcal{P}} = (1/2)^+$ created by single particle operators, e.g. operators of type $\bar{Q} u$ or $\bar{Q} d$, which we have used in the construction of trial states). At very large temporal separation the correlators we are studying will inevitably yield the eigenvalues of the Hamiltonian. At intermediate temporal separations, however, one can expect to read off $m(P_-)$ as we will explain in the following.

In [38] the effective coupling strength of the decay $P_- \to S + \pi$ has been estimated by a lattice computation: $\Gamma/k \approx 0.46$. Moreover, some evidence has been obtained that this quantity is fairly independent of the light quark mass. Using this result one can determine the mixing

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element xa of the energy matrix via eqn. (5) in [38] for our situation ($L/a = 24$, $m_\pi a \approx 0.14$):

$$xa \quad = \quad \left(\frac{2\pi(\Gamma/k)}{3(L/a)^3(m_\pi a)} \right)^{1/2} \quad \approx \quad 0.023. \tag{1}$$

As detailed in [36, 37, 38] for large temporal separations the P_- correlator is of the form

$$C_{P_-}(t/a) \quad \propto \quad e^{-(m_{P_-} a)(t/a)} \cosh((xa)(t/a)), \tag{2}$$

while the corresponding effective mass is

$$m_{\text{effective},P_-}(t/a)a \quad = \quad -\frac{d}{d(t/a)} \ln\left(C_{P_-}(t/a) \right) \quad =$$

$$= \quad \frac{d}{d(t/a)}\left((m_{P_-}a)(t/a) - \ln\left(\cosh((xa)(t/a)) \right) \right) \quad = \quad m_{P_-}a - \tanh((xa)(t/a))xa. \tag{3}$$

At $t/a = 12$ (the maximum temporal separation we have considered) the estimated systematic error of m_{P_-} coming from mixing with $S + \pi$ is $\tanh((xa)(t/a))xa \approx 0.0063$, i.e. roughly a 1% effect. This correction is significantly smaller than the statistical error of $m_{\text{effective},P_-}a$ in that t region.

For the other temporal separations and/or ensembles we obtain similar estimates. We, therefore, expect that at the present level of statistical accuracy the effect of multi particle states on our static-light meson masses, in particular on P_-, is negligible.

Our conclusions are in agreement with those obtained in [9], where a study of the static-light meson spectrum with similar techniques has been performed using two different lattice volumes. No volume dependence of the eigenvectors of static-light meson states has been observed, which is a sign that contributions of multi particle states are strongly suppressed.

5   Extrapolation to the physical b quark mass

To make contact with experimentally available results on the spectrum of B mesons, we need to correct for the non-infinite mass of the b quark. In Heavy Quark Effective Theory, the leading correction will be of order $1/m_H$, where m_H is the heavy quark mass. It is possible, in principle, to evaluate the coefficients of this correction from first principles on a lattice [19, 20]. This we intend to explore in the future, but here we use a more direct method to establish the size of this small correction between static quarks and b quarks of realistic mass. These $1/m_H$ terms will break the degeneracy of mesonic states found in the static limit.

We evaluate for physical b quarks by interpolating between static heavy quarks and the charm quark, where experimental data is available. As a measure of the heavy quark mass, we take the mass of the ground state heavy-light meson (D or B). This measure is equivalent to another (such as using quark masses in some scheme) to the order $1/m_H$ we are using. One test of this interpolation can be made. The hyperfine splitting between D^* and D of 141 MeV when interpolated from the static limit (namely zero) gives for B^* and B a splitting reduced by $m(D)/m(B) = 0.35$ to 49 MeV which agrees with the observed splitting [22] of 46 MeV to within 6%.

For the fine splitting, the kinetic term (rather than the chromo-magnetic) is relevant and the experimental results for the spectrum are rather incomplete - indeed this current study is to establish the spectrum from a theoretical input. Lattice studies do confirm [19, 20] that a $1/m_H$ behavior is dominant down to masses near the charm quark mass.

We interpolate our lattice results for static-light mass differences of P and S wave states to the physical b quark mass at $m(D)/m(B) = 0.35$ linearly in $m(D)/m_H$, making use of experimental data on D and D_s mesons as input [22]. For details regarding this method of extrapolation cf. [14]. Results are listed and compared to experimental results in Table 6. The corresponding extrapolations are shown in Figure 3.

For D mesons the assignment of the two $J^{\mathcal{P}} = 1^+$ states to B_1^* and B_1 is easy, because their widths differ by more than an order of magnitude (we associate the narrow state with B_1 [one of the two degenerate $j^{\mathcal{P}} = (3/2)^+$ states in the static limit, which can only decay to $S + \pi$ via a D wave and is, therefore, protected by angular momentum]; the wide state with B_1^* [one of the two degenerate $j^{\mathcal{P}} = (1/2)^+$ states in the static limit, which can readily decay to $S + \pi$ via an S wave]). In contrast to that the situation is less clear for D_s mesons, where both $J^{\mathcal{P}} = 1^+$ states have similar (narrow) widths. Therefore, we show both possibilities in Table 6 and in Figure 3.

	$M - M(B)$ in MeV			$M - M(B_s)$ in MeV	
state	lattice	experiment	state	lattice	experiment
B_0^*	443(21)		B_{s0}^*	391(8)	
B_1^*	460(22)		B_{s1}^*	440(8)/467(8)	
B_1	530(12)	444(2)	B_{s1}	526(8)/499(8)	463(1)
B_2^*	543(12)	464(5)	B_{s2}^*	539(8)	473(1)
B_J^*		418(8)	B_{sJ}^*		487(15)

Table 6: lattice and experimental results for P wave B and B_s states (B_J^* and B_{sJ}^* denote rather vague experimental signals, which can be interpreted as stemming from several broad and narrow resonances possibly including the $j = 1/2$ P wave states B_0^*, B_1^*, B_{s0}^* and B_{s1}^*; the two lattice values listed for B_{s1}^* and B_{s1} correspond to the two possibilities of assigning experimental $J^{\mathcal{P}} = 1^+$ D results [cf. text for more details]).

Compared to our previous study [13, 14] at a single lattice spacing, the above results are similar for the B (unitary) case. For B_s mesons we now employ a partially quenched s quark which allows a more realistic treatment of the light quark sea. So our new results supersede those obtained previously for B_s. Indeed we find a significant dependence on the sea quark mass (cf. Figure 2), which is now the physical u/d quark mass, while it previously corresponded to the significantly heavier s quark mass.

In our lattice study we have extracted the continuum limit and have extrapolated to physical light quarks using a linear dependence. We have then interpolated to the physical b quark assuming that a $1/m_H$ behavior is valid down to the charm quark mass. These assumptions induce systematic errors and, in principle, they can be quantified by further lattice studies.

The assumption of a linear extrapolation to physical light quarks is sensitive to possible admixtures of two body states which become more important at lighter quark masses as thresholds
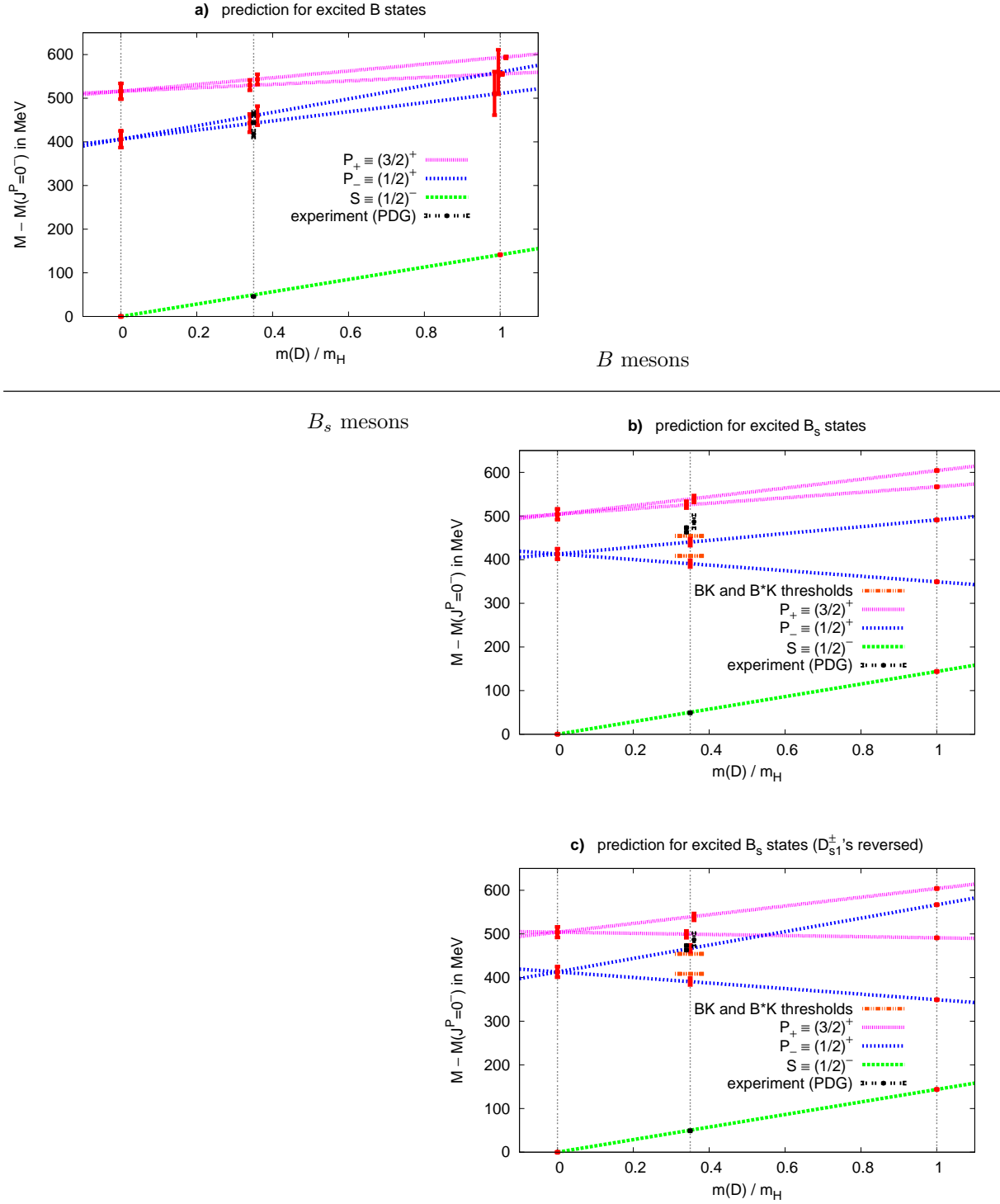
Figure 3: Static-light mass differences linearly extrapolated to the physical b quark mass. **a)** Unitary, i.e. B mesons. **b)**, **c)** Partially quenched, i.e. B_s mesons.

for decay open. We have explored this possibility and found no evidence of such effects, so it is difficult to estimate the magnitude of a possible systematic error from this. If there was a significant difference between the light quark behavior for the ground state and an excited state, this would introduce an error on our extrapolation to the physical value which could be as large as 10 MeV.

The test of the $1/m_H$ assumption for the chromo-magnetic term, discussed above, was found to be valid within 6%. This suggests that an estimate of the systematic errors for the B and B_s meson mass splittings coming from $1/m$ effects should also be at least of order 6%. Since the $1/m$ correction to the P wave states is of order 100 MeV, this implies a systematic error of order 6 MeV.

One further possible source of systematic error is from our neglect of the strange contribution to the sea. This will be addressed in a future study making use of the $N_f = 2 + 1 + 1$ sea which includes dynamical s quarks from ETMC [39, 40].

Overall, it seems prudent to assign systematic errors on our mass differences (for P_- and P_+ relative to S) of order 20 MeV from these effects, even though we have little evidence for such effects.

The experimental determination of the spectrum of excited B and B_s mesons is quite limited [22]. Assuming that the relatively narrow states seen correspond to our P_+ state (since a $J^\mathcal{P} = 2^+$ state must have that assignment), the mass difference we see of over 500 MeV does not agree closely with the experimental results of around 450 MeV. We do get a mass difference of around 450 MeV from our P_- states, although such states cannot have $J^\mathcal{P} = 2^+$.

In view of this discrepancy with experimental results, it is also interesting to compare with independent existing lattice computations, in particular with the rather recent study reported in [12]. There the light quark extrapolation is only performed in the valence quark mass (from which static-light mass differences essentially seem to be independent, as can be seen by comparing our B and B_s results and also from corresponding plots and numbers presented in [12]), while the sea quark mass is kept fixed. More generally, a comparison of the dependence of static-light mass differences on the sea quark mass, which we have computed down to $m_{\mathrm{PS}} \approx 280\,\mathrm{MeV}$, with existing lattice studies is not possible: there the number of investigated sea quark masses is rather small and they are quite heavy, around the mass of the s quark. What one can do, however, is to compare meson mass differences for a given value of the sea quark mass. Before comparing results (in physical units) with those quoted in [12] it should be noted that in [12] the scale is set by identifying r_0 with 0.49 fm, while our result for this quantity is $r_0 = 0.42\,\mathrm{fm}$ [32]. Therefore, to perform a meaningful comparison, one should express all quantities in units of r_0 or equivalently scale all masses in physical units listed in [12] by a factor of around $0.49/0.42 \approx 1.14$. For the lightest sea quark mass considered in [12] corresponding to $m_{\mathrm{PS}} \approx 461\,\mathrm{MeV}$ it is most appropriate to compare with our results at $\beta = 3.90$, $\mu_{\mathrm{q}} = 0.0100$ ($m_{\mathrm{PS}} \approx 517\,\mathrm{MeV}$). For the P wave mass differences one finds

$$\frac{(m(P_-) - m(S))_{\mathrm{ETMC}}}{(m(P_-) - m(S))_{[12]}} \quad \approx \quad \frac{474(29)\,\mathrm{MeV}}{454(19)(9)\,\mathrm{MeV}} \quad \approx \quad 1.04(11) \tag{4}$$

$$\frac{(m(P_+) - m(S))_{\mathrm{ETMC}}}{(m(P_+) - m(S))_{[12]}} \quad \approx \quad \frac{481(27)\,\mathrm{MeV}}{446(17)(9)\,\mathrm{MeV}} \quad \approx \quad 1.08(11), \tag{5}$$

ratios, which are within statistical errors fully consistent with the expected factor 1.14.

It is interesting to note that the ratios between our lattice results and the experimental values (see Table 6) are on the same ballpark of the ratio between two values of r_0 used above, i.e. ≈ 1.14. While there is no reason to doubt the precise determination of the lattice spacing performed in [32], it would be interesting, although beyond the scope of this paper, to investigate, whether simulations at lighter quark masses and/or with $N_f = 2+1+1$ dynamical flavours will improve the agreement with experimental results.

One interesting issue is whether the B_s states are stable to the strong decay to BK. This decay has a threshold at $408\,\mathrm{MeV}$ above the ground state B_s meson. Our P_- states (the upper two in Table 6) do indeed have masses which are close to (or below) this threshold. That would imply that these two states (B_{s0}^* and B_{s1}^*) should have a very small decay width. This is consistent with the experimental observation that only two candidate P wave B^* states have been seen so far: corresponding to the heavier P_+ states. All the other states B_s we study, including the S^*, lie higher than this BK threshold and so would have a strong decay open.

Moreover, our findings clearly indicate that there is no inversion of level ordering for P wave states, neither for B mesons nor for B_s mesons. B_0^* and B_1^* (B_{s0}^* and B_{s1}^*) are considerably lighter than B_1 and B_2^* (B_{s1} and B_{s2}^*) as can be read off from Table 6 and Figure 3. This is in contrast to predictions obtained from certain phenomenological models [23, 24, 25, 26, 27] and, therefore, might provide valuable input for future model building.

6 Conclusions

We have determined the continuum limit for static-light mesons on a lattice using $N_f = 2$ flavours of light quarks. The removal of $\mathcal{O}(a)$ effects by using maximally-twisted mass fermions for meson mass differences in the static limit is confirmed.

We have investigated the light sea quark mass dependence of B and B_s mesons down to $m_{\mathrm{PS}} \approx 280\,\mathrm{MeV}$, which is significantly lighter than what has been achieved in previous studies of static-light mesons. We find that our results are compatible with a linear extrapolation in the light quark mass to its physical value. We see no sign of any mixing with two body effects and this is consistent with our estimate that such effects should be too small to see on our lattices.

We have determined masses for a wide variety of excited states in the continuum limit and this will be a valuable resource for model builders.

We have employed the assumption of a $1/m_H$ dependence on the heavy quark mass together with experimental results for charm-light mesons to allow us to estimate the spectrum that one would obtain for physical b quarks.

Our results imply that there will be a $J^{\mathcal{P}} = 0^+$ and $J^{\mathcal{P}} = 1^+$ B_s meson which has a narrow width since its strong decay to BK is suppressed (or zero) due to phase space effects.

Future directions include (i) determination of f_B and f_{B_s} (for a preliminary result cf. [28]); (ii) a similar investigation regarding static-light baryons; (iii) extending these computations to $N_f = 2+1+1$ flavour ETMC gauge configurations [39, 40].

A   Details of the fitting procedure

Data points $(m_{\mathrm{PS}})^2$ and $\Delta M(j^{\mathcal{P}})$, $j^{\mathcal{P}} \in \{P_-\,,\ P_+\,,\ D_\pm\,,\ D_+\,,\ F_\pm\,,\ S^*\}$ corresponding to the same β are correlated via the lattice spacing a. We take that into account via a covariance matrix, which we estimate by resampling $m_{\mathrm{PS}}a$, $\Delta M(j^{\mathcal{P}})a$ and a $(100,000$ samples$)$. Consequently, we do not fit straight lines to the Data points $((m_{\mathrm{PS}})^2\,,\ \Delta M(j^{\mathcal{P}}))$ individually for every static-light state $j^{\mathcal{P}}$, but perform a single correlated fit of six straight lines to the six mass differences of interest. During the fitting we take statistical errors both along the horizontal axis (errors in $(m_{\mathrm{PS}})^2$) and along the vertical axis (errors in $\Delta M(j^{\mathcal{P}})$) into account.

The method of performing the two-dimensional fits is based on what has been used in [41].

To be able to express the corresponding equations in a compact way, we introduce the following notation:

- $\mathbf{z} = (\mathbf{x}\,,\ \mathbf{y}(1)\,,\ \mathbf{y}(2)\,,\ \ldots)$.

- $\mathbf{x} = (((m_{\mathrm{PS}})^2)^{(1)}\,,\ ((m_{\mathrm{PS}})^2)^{(2)}\,,\ \ldots)$ (the upper index $^{(\cdots)}$ refers to both the lattice spacing and to the light quark mass).

- $\mathbf{y}(j) = ((\Delta M)^{(1)}(j)\,,\ (\Delta M)^{(2)}(j)\,,\ \ldots)$ (the upper index $^{(\cdots)}$ refers to both the lattice spacing and to the light quark mass, the index (j) refers to $j^{\mathcal{P}}$).

- C denotes the estimated covariance matrix for $\mathbf{z}$ (a 56×56 matrix for B mesons, a 42×42 matrix for B_s mesons).

- The linear fits $y(j) = a(j)x + b(j)$ are parameterised by $a(j)$ and $b(j)$ (the quantities, which will finally allow the extrapolation to physical u/d quark masses).

The basic idea of the method is a maximum likelihood determination of the "true values" $\bar{\mathbf{z}} = (\bar{\mathbf{x}}\,,\ \bar{\mathbf{y}}(1)\,,\ \bar{\mathbf{y}}(2)\,,\ \ldots)$. This amounts to minimizing

$$\frac{1}{2}\Big(\mathbf{z} - \bar{\mathbf{z}}\Big)^T C^{-1} \Big(\mathbf{z} - \bar{\mathbf{z}}\Big) - \sum_{j,n} \lambda_n(j)\Big(a(j)\bar{x}_n + b(j) - \bar{y}_n(j)\Big) \tag{6}$$

with respect to $\bar{\mathbf{z}}$, $a(j)$, $b(j)$ and $\vec{\lambda}(j)$ under the constraints $\bar{y}_n(j) = a(j)\bar{x}_n + b(j)$. For $\mathbf{z}$ we use the same resampling procedure as for estimating the covariance matrix (this is necessary, because $z_A \equiv \langle((m_{\mathrm{PS}})^2)^{(n)}\rangle \neq \langle(m_{\mathrm{PS}})^{(n)}a\rangle^2/\langle a\rangle^2$ and $z_A \equiv \langle(\Delta M)^{(n)}(j)\rangle \neq \langle(\Delta M)^{(n)}(j)a/\langle a\rangle\rangle$.

The constraint minimization is equivalent to solving a system of non-linear equations, which we do by means of the scaled-hybrid algorithm of the GSL library [42]. It needs initial parameters, which should preferably be close to the global extremum. Such initial parameters can be obtained by individual standard one-dimensional straight line fits:

- $\lambda_n(j) = 0$,

- $a(j)$ and $b(j)$ minimizing

$$\sum_n \frac{\Big(a(j)x_n + b(j) - y_n(j)\Big)^2}{C_{y_n(j),y_n(j)}}, \tag{7}$$

- $\bar{\mathbf{x}} = \mathbf{x}$ and $\bar{\mathbf{y}}(j) = \mathbf{y}(j)$.

To judge the quality of the resulting fit, we define a "reduced χ^2" via

$$
\frac{\chi^2}{\text{d.o.f.}} = \frac{\left(\mathbf{z} - \bar{\mathbf{z}}\right)^T C^{-1} \left(\mathbf{z} - \bar{\mathbf{z}}\right)}{\text{d.o.f.}}, \tag{8}
$$

where d.o.f. is the number of entries of $\bar{\mathbf{z}}$ minus the number of $a(j)$ and $b(j)$, i.e. d.o.f. = 44 for B mesons and d.o.f. = 32 for B_s mesons respectively.

The resulting straight lines allow an extrapolation to physical u/d quark masses (corresponding to $m_{\text{PS}} = 135\,\text{MeV}$). The corresponding statistical errors are obtained by repeating this fitting and extrapolation procedure 100 times with randomly sampled sets z_A (we randomly sample the input data and compute $z_A \equiv ((m_{\text{PS}})^{(n)}a)^2/a^2$ and $z_A \equiv ((\Delta M)^{(n)}(j)a)/a)$ and taking the variance.

Acknowledgments

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