Static-light meson masses from twisted mass lattice QCD

and

The 1/2 versus 3/2 puzzle

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Heavy-light mesons

- Heavy-light meson: a meson made from a heavy quark ($b, c$) and a light quark ($u, d, s$), e.g. $B = \{\bar{b}u, \bar{b}d\}$, $B_s = \bar{b}s$, $D = \{\bar{c}u, \bar{c}d\}$, $D_s = \bar{c}s$.

- Static limit, i.e. $m_b, m_c \to \infty$:
  - No interactions involving the static quark spin.
  - Classify states according to parity $P$ and total angular momentum of the light cloud $j$.

- $m_b, m_c$ finite, but heavy:
  - Classify states according to parity $P$ and total angular momentum $J$.

<table>
<thead>
<tr>
<th>$j^P$</th>
<th>$J^P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1/2)^-$</td>
<td>$0^-$</td>
</tr>
<tr>
<td></td>
<td>$H$</td>
</tr>
<tr>
<td></td>
<td>$H^*$</td>
</tr>
<tr>
<td>$(1/2)^+$</td>
<td>$0^+$</td>
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<tr>
<td></td>
<td>$H_0^*$</td>
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<tr>
<td></td>
<td>$H_0^{1/2}$</td>
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<tr>
<td></td>
<td>$1^+$</td>
</tr>
<tr>
<td></td>
<td>$H_1^*$</td>
</tr>
<tr>
<td></td>
<td>$H_1^{1/2}$</td>
</tr>
<tr>
<td>$(3/2)^+$</td>
<td>$1^+$</td>
</tr>
<tr>
<td></td>
<td>$H_1$</td>
</tr>
<tr>
<td></td>
<td>$H_1^{3/2}$</td>
</tr>
<tr>
<td></td>
<td>$2^+$</td>
</tr>
<tr>
<td></td>
<td>$H_2^*$</td>
</tr>
<tr>
<td></td>
<td>$H_2^{3/2}$</td>
</tr>
</tbody>
</table>
Static-light meson masses from twisted mass lattice QCD

in collaboration with
Karl Jansen, Chris Michael, Andrea Shindler
Basic principle (1)

- Let $O(x)$ be a suitable “static-light meson creation operator”, i.e. an operator such that $O(x)|\Omega\rangle$ is a state containing a static-light meson at position $x$ ($|\Omega\rangle$: vacuum).

- Determine the mass of the ground state of the corresponding static-light meson from the exponential behavior of the corresponding correlation function $C$ at large Euclidean times $T$:

$$C(T) = \langle \Omega| \left( O(x, T) \right)^\dagger O(x, 0)|\Omega\rangle =$$

$$= \langle \Omega| e^{+HT} \left( O(x, 0) \right)^\dagger e^{-HT} O(x, 0)|\Omega\rangle =$$

$$= \sum_n \left| \langle n| O(x, 0)|\Omega\rangle \right|^2 \exp \left( - \left( E_n - E_\Omega \right) T \right) \approx \quad \text{(for } T \gg 1)$$

$$\approx \left| \langle 0| O(x, 0)|\Omega\rangle \right|^2 \exp \left( - \left( E_0 - E_\Omega \right) T \right).$$

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Basic principle (2)

- **General form of a static-light meson creation operator**:

\[
O(x) = \bar{Q}(x) \int d\hat{n} \Gamma(\hat{n}) U(x; x + d\hat{n}) q(x + d\hat{n}).
\]

- \( \bar{Q}(x) \) creates an infinitely heavy i.e. static antiquark at position \( x \).
- \( q(x + d\hat{n}) \) creates a light quark at position \( x + d\hat{n} \) separated by a distance \( d \) from the static antiquark.
- The spatial parallel transporter

\[
U(x; x + d\hat{n}) = P \left\{ \exp \left( +i \int_{x}^{x+d\hat{n}} dz_j A_j(z) \right) \right\}
\]

connects the antiquark and the quark in a gauge invariant way via gluons.
- The integration over the unit sphere \( \int d\hat{n} \) combined with a suitable weight factor \( \Gamma(\hat{n}) \) yields well defined total angular momentum \( J \) and parity \( \mathcal{P} \) (\( \Gamma(\hat{n}) \) is a combination of spherical harmonics \( \rightarrow \) angular momentum] and \( \gamma \)-matrices \( \rightarrow \) spin]; Wigner-Eckart theorem).
Basic principle (3)

- **General form of a static-light meson creation operator:**

\[ O(x) = \bar{Q}(x) \int d\hat{n} \Gamma(\hat{n}) U(x; x + d\hat{n}) q(x + d\hat{n}). \]

- **List of operators** (*J*: total angular momentum; *j*: total angular momentum of the light cloud; *P*: parity):

<table>
<thead>
<tr>
<th>( \Gamma(\hat{n}) )</th>
<th>( J^P )</th>
<th>( j^P )</th>
<th>( O_h )</th>
<th>lattice ( j^P )</th>
<th>notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_5, \gamma_5 \gamma_j \hat{n}_j ) 1, ( \gamma_j \hat{n}_j )</td>
<td>0(^-) [1(^-)]</td>
<td>(1/2(^-))</td>
<td>( A_1 )</td>
<td>(1/2(^-)), (7/2(^-)), ...</td>
<td>( S )</td>
</tr>
<tr>
<td>( \gamma_1 \hat{n}_1 - \gamma_2 \hat{n}_2 ) (and cyclic) ( \gamma_5(\gamma_1 \hat{n}_1 - \gamma_2 \hat{n}_2) ) (and cyclic)</td>
<td>2(^+) [1(^+)]</td>
<td>(3/2(^+))</td>
<td>( E )</td>
<td>(3/2(^+)), (5/2(^+)), ...</td>
<td>( P_+ )</td>
</tr>
<tr>
<td>( \gamma_1 \hat{n}_2 \hat{n}_3 + \gamma_2 \hat{n}_3 \hat{n}_1 + \gamma_3 \hat{n}_1 \hat{n}_2 ) ( \gamma_5(\gamma_1 \hat{n}_2 \hat{n}_3 + \gamma_2 \hat{n}_3 \hat{n}_1 + \gamma_3 \hat{n}_1 \hat{n}_2) )</td>
<td>3(^-) [2(^-)]</td>
<td>(5/2(^-))</td>
<td>( A_2 )</td>
<td>(5/2(^-)), (7/2(^-)), ...</td>
<td>( D_+ )</td>
</tr>
</tbody>
</table>

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Twisted mass lattice QCD

- Twisted mass action (two degenerate flavors, “continuum version”):

\[
S_{\text{fermionic}} = \int d^4x \bar{\chi} \left( \gamma_\mu D_\mu + m + i \mu \gamma_5 \tau_3 - \frac{a \Box}{2} \right) \chi
\]

\[
\psi = e^{i \omega \gamma_5 \tau_3 / 2} \chi
\]

(\(\psi\): physical basis quark fields; \(\chi\): twisted basis quark fields; \(\mu\): twisted mass; \(\tau_3\): third Pauli matrix acting in flavor space; \(a\): lattice spacing).

- Wilson term: removes fermionic doublers.
- Twisted mass term: automatic \(O(a)\) improvement, when tuned to maximal twist \((\omega = \pi/2)\).

+ Automatic \(O(a)\) improvement.
+ Numerically cheap, i.e. large lattices and small lattice spacings possible.

- Explicit breaking of parity and flavor symmetry.
Simulation setup

• $24^3 \times 48$ lattices.

• Twisted mass Dirac operator with two degenerate flavors:

\[ Q^{(x)} = \gamma_\mu D_\mu + m + i\mu\gamma_5 + \frac{a}{2}\Box, \quad m + 4 = \frac{1}{2\kappa} \]

with $\kappa = 0.160856$.

• Tree-level Symanzik improved gauge action with $\beta = 3.9$.

• Lattice spacing $a \approx 0.0855(5)$ fm, spatial lattice extension $24 \times a \approx 2.05$ fm.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$m_{PS}$ in MeV</th>
<th>number of gauges</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0040</td>
<td>314(2)</td>
<td>1400</td>
</tr>
<tr>
<td>0.0064</td>
<td>391(1)</td>
<td>1450</td>
</tr>
<tr>
<td>0.0085</td>
<td>448(1)</td>
<td>1350</td>
</tr>
<tr>
<td>0.0100</td>
<td>485(1)</td>
<td>900</td>
</tr>
<tr>
<td>0.0150</td>
<td>597(2)</td>
<td>1000</td>
</tr>
</tbody>
</table>

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• Linear extrapolation in \((m_\pi)^2\) to physical light quark masses:
  
  – “\(B\) mesons”: \(u/d\) quark extrapolation \((m_{PS} = 139.6\, \text{MeV})\).
  
  – “\(B_s\) mesons”: \(s\) quark extrapolation \((m_{PS} = 700.0\, \text{MeV})\).

  • However: sea of two degenerate \(s\) instead of \(u\) and \(d\).
Results (2)

- Prediction for excited $B$ states $B_0^*$, $B_1^*$, $B_1$ and $B_2^*$ ($P$ wave states):
  - Linear interpolation in $m_c/m_Q$ to physical $b$ quark mass (input: $u/d$ extrapolated lattice data for $m_Q = \infty$, experimental data for $m_Q = m_c$).

- Experimental results:
  - CDF and DØ (both $j^P = (3/2)^+$ states, i.e. $B_1$ and $B_2^*$).
  - PDG (unknown $j^P$, denoted by $B_J^*$).

\begin{tabular}{|c|c|c|c|c|}
\hline
   & $m - m(B^0_0)$ in MeV & \\
state & lattice & CDF & DØ & PDG \\
\hline
$B_0^*$ & 406(19) & & & \\
$B_1^*$ & 428(19) & & & \\
$B_1$ & 508(8) & 454(5) & 441(4) & \\
$B_2^*$ & 520(8) & 458(6) & 467(4) & \\
$B_J^*$ & & & & 418(8) \\
\hline
\end{tabular}
Results (3)

- Prediction for excited $B_s$ states $B_{s0}^*$, $B_{s1}^*$, $B_{s1}$ and $B_{s2}^*$ ($P$ wave states):
  - Linear interpolation in $m_c/m_Q$ to physical $b$ quark mass (input: $s$ extrapolated lattice data for $m_Q = \infty$, experimental data for $m_Q = m_c$).

- Experimental results:
  - CDF and DØ (both $j^P = (3/2)^+$ states, i.e. $B_{s1}$ and $B_{s2}^*$).
  - PDG (unknown $j^P$, denoted by $B_{sJ}^*$).

<table>
<thead>
<tr>
<th>state</th>
<th>$m - m(B_s)$ in MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_{s0}^*$</td>
<td>$493(16)$</td>
</tr>
<tr>
<td>$B_{s1}^*$</td>
<td>$535(16)$</td>
</tr>
<tr>
<td>$B_{s1}$</td>
<td>$510(13)$</td>
</tr>
<tr>
<td>$B_{s2}^*$</td>
<td>$521(13)$</td>
</tr>
<tr>
<td>$B_{sJ}^*$</td>
<td>$487(16)$</td>
</tr>
</tbody>
</table>

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Summary

- Static-light meson masses have been computed via twisted mass lattice QCD at a small value of the lattice spacing \( a = 0.0855 \text{ fm} \) and at small values of the pion mass \( 300 \text{ MeV} \lesssim m_{\pi S} \lesssim 600 \text{ MeV} \):
  - Total angular momentum of the light cloud \( j = 1/2, 3/2, 5/2 \).
  - Parity \( \mathcal{P} = +, - \).

- Interpolation/extrapolation to physical quark masses allows predictions for the spectrum of \( B \) mesons and \( B_s \) mesons:
  - Agreement up to 15\% with experimental \( P \) wave \( B \) meson results.
  - Agreement up to 10\% with experimental \( P \) wave \( B_s \) meson results.

- Outlook:
  - Extrapolate to the continuum.
  - Include a sea of \( u/d \) quarks for \( B_s \) computations by using 2+1+1 flavor twisted mass lattice QCD.
The 1/2 versus 3/2 puzzle

in collaboration with

Benoit Blossier, Karl Jansen, Olivier Pène
1/2 versus 3/2: experimental side

- Consider the semileptonic decay $B \rightarrow X_c l \nu$.
- Experiments, which have studied this decay: ALEPH, BaBar, BELLE, CDF, DELPHI, DØ.
- What is $X_c$?
  - $\approx 75\%$ $D$ and $D^*$, i.e. $S$ wave states (agreement with theory).
  - $\approx 10\%$ $D^{3/2}_1$ and $D^{3/2}_2$, i.e. $j = 3/2$ $P$ wave states (agreement with theory).
  - For the remaining $\approx 15\%$ the situation is not clear:
    * A “natural candidate” would be $D^{1/2}_0$ and $D^{1/2}_1$, i.e. $j = 1/2$ $P$ wave states.
    * This would imply $\Gamma(B \rightarrow D^{1/2}_{0,1} l \nu) > \Gamma(B \rightarrow D^{3/2}_{1,2} l \nu)$, which is in “conflict” with theory.
    * This “conflict” between experiment and theory is called the “1/2 versus 3/2 puzzle”.

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1/2 versus 3/2: theory side (1)

• Static limit \((m_b, m_c \to \infty)\) with both \(b\) and \(c\) quark at rest:

\[
\langle D_0^{1/2} | \bar{c} \gamma_5 \gamma_j D_k b | B \rangle = -ig_{jk} \left( m(D_0^{1/2}) - m(B) \right) \tau_{1/2}
\]

\[
\langle D_2^{3/2} | \bar{c} \gamma_5 \gamma_j D_k b | B \rangle = +i\sqrt{3}\epsilon_{jk} \left( m(D_2^{3/2}) - m(B) \right) \tau_{3/2}
\]

and

\[
\frac{\Gamma(B \to D_{0,1}^{1/2} l \nu)}{\Gamma(B \to D_{1,2}^{3/2} l \nu)} = \frac{|\tau_{1/2}|^2}{|\tau_{3/2}|^2}.
\]

\((\tau_{1/2}, \tau_{3/2}: \text{Isgur-Wise form factors}).\)
Phenomenological models:

- $|\tau_{1/2}| < |\tau_{3/2}|$, which is in “conflict” with experiment.

OPE:

- Uraltsev sum rule:

$$\sum_n |\tau_{3/2}^{(n)}|^2 - |\tau_{1/2}^{(n)}|^2 = \frac{1}{4}$$

$$(\tau_{1/2} \equiv \tau_{1/2}^{(0)} \text{ and } \tau_{3/2} \equiv \tau_{3/2}^{(0)})$$

- From experience with sum rules one would expect approximate saturation from the ground states, i.e.

$$|\tau_{3/2}^{(0)}|^2 - |\tau_{1/2}^{(0)}|^2 \approx \frac{1}{4},$$

which also implies $|\tau_{1/2}| < |\tau_{3/2}|$, which is in “conflict” with experiment.
\[ \frac{1}{2} \text{ versus } \frac{3}{2}: \text{ possible explanations} \]

- **Experiment:**
  - The signal for the remaining 15\% of \( X_c \) is rather vague; therefore, only a small part might be \( D_{0,1}^{1/2} \).

- **Phenomenological models:**
  - Models might give a wrong answer.

- **OPE:**
  - Sum rules hold in the static limit and might change significantly for finite quark masses.
  - Sum rules might not be saturated by the ground states.

- **A lattice computation of** \( \tau_{1/2} \) and \( \tau_{3/2} \) could shed some light on this puzzle.
Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (1)

- Simulation setup:
  - As before, but only a single value of the light quark mass ($\mu = 0.0040$, corresponding to $m_{PS} \approx 300\, \text{MeV}$).
  - Preliminary results (computations have been performed on $\approx 100$ gauge configurations only [1400 available]).
Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (2)

- "Effective form factors",

$$\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) = \frac{N(\tilde{P}_-) N(\tilde{S}) \langle \tilde{P}_-(T_0) | (\bar{Q} \gamma_5 \gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \langle \tilde{P}_-(T_0) | \tilde{P}_-(T_1) \rangle \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle}$$

$$\tau_{3/2,\text{effective}}(T_0 - T_1, T_1 - T_2) = \sqrt{\frac{1}{6}} \frac{N(\tilde{P}_+) N(\tilde{S}) \langle \tilde{P}_+(T_0) | (\bar{Q} \gamma_5 (\gamma_1 D_1 - \gamma_2 D_2) Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_+) - m(\tilde{S})) \langle \tilde{P}_+(T_0) | \tilde{P}_+(T_1) \rangle \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle}.$$  

- $N(X)$: norm of state $|X\rangle$.
- $m(X)$: mass of state $|X\rangle$.
- Three-point functions ($T_0$, $T_1$ and $T_2$).
- Two-point functions ($T_0$ and $T_1$ or $T_1$ and $T_2$).

$$\tau_{1/2} = \lim_{T_0-T_1, T_1-T_2 \to \infty} \tau_{1/2,\text{effective}}, \quad \tau_{3/2} = \lim_{T_0-T_1, T_1-T_2 \to \infty} \tau_{3/2,\text{effective}}.$$  

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Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (3)

- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) = \frac{N(\tilde{P}_-) N(\tilde{S}) \langle \tilde{P}_-(T_0) | (\bar{Q} \gamma_5 \gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \langle \tilde{P}_-(T_0) | \tilde{P}_-(T_1) \rangle \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle}$, ... 

- Two-point function $\langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle$: a standard lattice computation.

- Determine the norm of $|\tilde{S}\rangle$, $N(\tilde{S})$, by performing a $\chi^2$ minimizing fit with 

$$f(T) = N(\tilde{S})^2 e^{-m(S)T}$$

- to $\langle \tilde{S}(T) | \tilde{S}(0) \rangle$ at large $T$.

- Analogously for the others.
Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (4)

- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2) = \left| \frac{N(\tilde{P}_-) N(\tilde{S}) \langle \tilde{P}_-(T_0) | (\bar{Q} \gamma_5 \gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle}{(m(\tilde{P}_-) - m(\tilde{S})) \langle \tilde{P}_-(T_0) | \tilde{P}_-(T_1) \rangle \langle \tilde{S}(T_1) | \tilde{S}(T_2) \rangle} \right|$, \ldots

- Three-point functions $\langle \tilde{P}_-(T_0) | (\bar{Q} \gamma_5 \gamma_3 D_3 Q)(T_1) | \tilde{S}(T_2) \rangle$:

- Analogously for the other three-point functions.

- Mass differences $m(P_-) - m(S)$ and $m(P_+) - m(S)$:
  cf. the first part of this talk.
Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (5)

- $\tau_{1/2,\text{effective}}(T_0 - T_1, T_1 - T_2)$ and $\tau_{3/2,\text{effective}}(T_0 - T_1, T_1 - T_2)$ exhibit nice plateaus due to “optimized” trial states $|\tilde{S}\rangle$, $|\tilde{P}_-\rangle$ and $|\tilde{P}_+\rangle$.

- $T_0 - T_2 = 8$:
  - $\tau_{1/2} = 0.32$, $\tau_{3/2} = 0.47$.
  - $(\tau_{3/2})^2 - (\tau_{1/2})^2 = 0.12$.

- $T_0 - T_2 = 10$:
  - $\tau_{1/2} = 0.30$, $\tau_{3/2} = 0.54$.
  - $(\tau_{3/2})^2 - (\tau_{1/2})^2 = 0.20$.

- $\tau_{3/2} > \tau_{1/2}$, i.e. theoretical expectation confirmed.

- “Consistent” with Uraltsev sum rule:
  $$\sum_n |\tau_{3/2}^{(n)}|^2 - |\tau_{1/2}^{(n)}|^2 = \frac{1}{4}.$$
Lattice computation of $\tau_{1/2}$ and $\tau_{3/2}$ (6)

- Comparison with the only existing lattice study (quenched, exploratory):
  - $16^3 \times 40$ lattice, $m_{\text{sea}} = \infty$, $m_{\text{PS}} = 800$ MeV.
  - $\tau_{1/2} = 0.38(4)$, $\tau_{3/2} = 0.53(8)$.
Conclusions

- $\tau_{1/2}$ and $\tau_{3/2}$ have been computed on dynamical ETMC gauge field configurations.

- Preliminary results indicate that in the static limit

\[
\Gamma(B \to D_{0,1}^{1/2} l \nu) < \Gamma(B \to D_{1,2}^{3/2} l \nu)
\]

(as expected from OPE and phenomenological models).

- “To do list”:
  - Improve statistics.
  - Consider different light quark masses to extrapolate to $u/d$ masses.
  - Perform the continuum limit.
  - Compute HQET $1/m_Q$ corrections.