Forces between heavy mesons

ETMC meeting – Bonn

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Ongoing static-light projects

- Continuum limit of the static-light meson spectrum.
  [Chris Michael, Andrea Shindler, Marc Wagner]
  → Almost finished, draft circulated last week.

- $f_B$ and $f_{B_s}$.
  [Chris Michael, Andrea Shindler]

- Static-light baryon spectrum.
  [Marc Wagner, Christian Wiese]
  → Talk by Christian Wiese.

- Forces between static-light mesons.
  [Marc Wagner]
  → This talk.

- String breaking.
  [Karl Jansen, Attila Nagy, Marc Wagner]
Goal: compute the potential of (or equivalently the force between) two $B$ mesons:

- Treat the $b$ quark in the static approximation.
- Consider only pseudoscalar mesons ($j^P = (1/2)^-$, denoted by $S$) and scalar mesons ($j^P = (1/2)^+$, denoted by $P_-$), which are among the lightest static-light mesons.
- Study the dependence of the mesonic potential $V(R)$ on
  * the light quark flavor $u$ and/or $d$ (isospin),
  * the light quark spin (the static quark spin is irrelevant),
  * the type of the meson $S$ and/or $P_-$.
Motivation:

- First principles computation of a hadronic force.
- Until now it has only been studied in the quenched approximation.
(Pseudo)scalar $B$ mesons

- Symmetries and quantum numbers of static-light mesons:
  - Isospin: $I = 1/2$, $I_z = \pm 1/2$, i.e. $B \equiv \bar{Q}u$ or $B \equiv \bar{Q}d$.
  - Parity: $\mathcal{P} = \pm$.
  - Rotations:
    * Light cloud angular momentum $j = 1/2$ (for $S$ and $P_-$), $j_z = \pm 1/2$.
    * Static quark spin: irrelevant (static quarks can also be treated as spinless color charges).

- Static-light meson creation operators:
  - $\bar{Q}\gamma_5q$ (pseudoscalar, i.e. $S$), $q \in \{u, d\}$,
  - $\bar{Q}q$ (scalar, i.e. $P_-$)

($j_z$ is not well-defined, when using these operators).
Symmetries and quantum numbers of a pair of static-light mesons (separated along the $z$-axis):

- Isospin: $I = 0, 1, I_z = -1, 0, +1$.
- Rotations around the $z$-axis:
  * Angular momentum of the light degrees of freedom $j_z = -1, 0, +1$.
  * Static quark spin: irrelevant (the static quarks can be treated as spinless color charges).
- Parity: $\mathcal{P} = \pm$.
- If $j_z = 0$, reflection along the $x$-axis: $\mathcal{P}_x = \pm$.
- Instead of using $j_z = \pm 1$ one can also label states by $|j_z| = 1, \mathcal{P}_x = \pm$.

→ Label $BB$ states by $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$. 
To extract the potential(s) of a given sector (characterized by 
$(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$), compute the temporal correlation function of the trial state

$$(C\Gamma)_{AB}\left(\bar{Q}_C(-R/2)q_A^{(1)}(-R/2)\right)\left(\bar{Q}_C(+R/2)q_B^{(2)}(+R/2)\right)|\Omega\rangle,$$

where

- $C = \gamma_0\gamma_2$ (charge conjugation matrix),
- $q^{(1)}q^{(2)} \in \{ud - du, uu, dd, ud + du\}$ (isospin $I, I_z$),
- $\Gamma$ is an arbitrary combination of $\gamma$ matrices (spin $|j_z|$, parity $\mathcal{P}, \mathcal{P}_x$).

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• Symmetries of twisted mass lattice QCD compared to QCD:
  – SU(2) isospin breaks down to U(1): $I_z$ is still a good quantum number, $I$ is not.
  – Parity $P$ is replaced by $P^{(tm)}$, which is parity combined with light flavor exchange.
  – Twisted mass $B\bar{B}$ sectors:
    * $I_z = \pm 1$: \((I_z, |j_z|, \frac{P^{(tm)} P^{(tm)}_x}{P P_x})\),
    * $I_z = 0$: \((I_z, |j_z|, \frac{P^{(tm)} P^{(tm)}_x}{P x}, \frac{P^{(tm)} P^{(tm)}_x}{P x})\).
  \[
  \rightarrow \text{QCD sectors } (I, I_z, |j_z|, P, P_x) \text{ are pairwise combined.}
  \]
**BB systems (4)**

- **BB creation operators for** $I_z = +1$: 16 operators of type

\[
(C\Gamma)_{AB}\left(\bar{Q}_C(-R/2)^{(u)}(A,-R/2)\right)\left(\bar{Q}_C(+R/2)^{(u)}(B,+R/2)\right).
\]

| $\Gamma$ twisted | $|j_z|$, $P^{(tm,light)}$, $P_x^{(tm,light)}$ | $\Gamma$ pseudo physical | $|j_z|$, $P^{(light)}$, $P_x^{(light)}$ |
|-------------------|---------------------------------|---------------------|---------------------------------|
| $\gamma_5$       | 0, +                            | $\mp i$             | 0, −, −                         |
| $\gamma_0\gamma_5$ | 0, +                            | $+\gamma_0\gamma_5$ | 0, +, +                         |
| 1                 | 0, +                            | $\mp i\gamma_5$     | 0, +, +                         |
| $\gamma_0$       | 0, −                            | $+\gamma_0$         | 0, +, −                         |
| $\gamma_3$       | 0, +                            | $+\gamma_3$         | 0, −, −                         |
| $\gamma_0\gamma_3$ | 0, +                           | $\mp i\gamma_0\gamma_3\gamma_5$ | 0, +, +                  |
| $\gamma_3\gamma_5$ | 0, −                          | $+\gamma_3\gamma_5$ | 0, −, +                         |
| $\gamma_0\gamma_3\gamma_5$ | 0, +            | $\mp i\gamma_0\gamma_3$ | 0, −, −                        |
| $\gamma_1$       | 1, −                            | $+\gamma_1$         | 1, −, +                         |
| $\gamma_0\gamma_1$ | 1, −                           | $\mp i\gamma_0\gamma_1\gamma_5$ | 1, +, −                  |
| $\gamma_1\gamma_5$ | 1, +                         | $+\gamma_1\gamma_5$ | 1, −, −                         |
| $\gamma_0\gamma_1\gamma_5$ | 1, −             | $\mp i\gamma_0\gamma_1$ | 1, −, +                        |
| …                 | …                               | …                   | …                               |

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**BB systems (5)**

- **BB** creation operators for $I_z = 0$: 32 operators of type

\[
(C\Gamma)_{AB}\left(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2)\right)\left(\bar{Q}_C(+R/2)\chi_B^{(d)}(+R/2)\right) \pm (u \leftrightarrow d).
\]

| $\Gamma$ twisted, $\pm$ | $|j_z|$, $P_{(tm,\text{light})}$, $P_{x_{(tm,\text{light})}}$ | $\Gamma$ pseudo physical, $\pm$ | $|j_z|$, $I$, $P_{(\text{light})}$, $P_{x_{(\text{light})}}$ |
|-------------------------|------------------------------------|------------------------|-----------------------------------|
| $\gamma_5$, $+$        | $0$, $+$, $+$                      | $+\gamma_5$, $+$       | $0$, $1$, $+$, $+$               |
| $\gamma_0\gamma_5$, $+$| $0$, $+$, $+$                      | $+i\gamma_0$, $-$      | $0$, $0$, $-$, $-$               |
| $1$, $-$               | $0$, $-$, $+$                      | $+1$, $-$              | $0$, $0$, $+$, $-$               |
| $\gamma_0$, $-$        | $0$, $+$, $+$                      | $+i\gamma_0\gamma_5$, $+$| $0$, $1$, $+$, $+$               |
| $\gamma_5$, $-$        | $0$, $+$, $-$                      | $+\gamma_5$, $-$       | $0$, $0$, $-$, $+$               |
| $\gamma_0\gamma_5$, $-$| $0$, $+$, $-$                      | $+i\gamma_0$, $+$      | $0$, $1$, $+$, $-$               |
| $1$, $+$               | $0$, $-$, $-$                      | $+1$, $+$              | $0$, $1$, $-$, $-$               |
| $\gamma_0$, $+$        | $0$, $+$, $-$                      | $+i\gamma_0\gamma_5$, $-$| $0$, $0$, $-$, $+$               |
| $\gamma_3$, $+$        | $1$, $-$, $-$                      | $+i\gamma_3\gamma_5$, $-$| $1$, $0$, $+$, $+$               |
| $\gamma_0\gamma_3$, $+$| $1$, $-$, $-$                      | $+\gamma_0\gamma_3$, $+$| $1$, $1$, $-$, $-$               |
| $\gamma_3\gamma_5$, $-$| $1$, $-$, $-$                      | $+i\gamma_3$, $+$      | $1$, $1$, $-$, $-$               |
| $\gamma_0\gamma_3\gamma_5$, $-$| $1$, $+$, $-$                    | $+\gamma_0\gamma_3\gamma_5$, $-$| $1$, $0$, $-$, $+$               |

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Simulation setup

- $\beta = 3.90$, $L^3 \times T = 24^3 \times 48$, $\mu = 0.0040$
  - lattice spacing $a \approx 0.080$ fm
  - pion mass $m_{PS} \approx 336$ MeV.

- Preliminary results on 18 gauge configurations:
  - Effective mass plateaus are fitted at rather small temporal separation
  - contamination by excited states.

- 12 $u$ and 12 $d$ inversions per gauge configuration (stochastic timeslice sources located on the same timeslice).

- APE smearing of spatial links and Gaussian smearing of light quark fields to “optimize” the ground state overlap of trial states.

- Wilson lines of static quarks are discretized by path ordered products of ordinary links, i.e. no HYP static action.
Discussion of results (1)

- Four “types of potentials”:
  - Two attractive, two repulsive.
  - Two have asymptotic values, which are larger by $\approx 400$ MeV.

- There are cases, where two potentials with identical quantum numbers are completely different (i.e. of different type)
  → at least one of the corresponding trial states must have very small ground state overlap
  → physical understanding, i.e. interpretation of trial states needed.

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Discussion of results (2)

- Expectation at large meson separation $R$: $V(R) \approx 2 \times \text{meson mass.}$
  - Potentials with smaller asymptotic value at $\approx 2 \times m(S)$.
  - $m(P_-) - m(S) \approx 400$ MeV: approximately the observed difference between different types of potentials.

→ Two types correspond to $S-S$ potentials.
→ Two types correspond to $S-P_-$ potentials.

- Can this be understood in detail on the level of the used $BB$ creation operators?

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![Graph showing potentials and meson separation](image)
• Rotate the $BB$ creation operators to the pseudo physical basis and express them in terms of static-light meson creation operators (use suitable spin and parity projectors for the light quarks).

  – Examples:

  * $uu$, $\Gamma = \gamma_5 \rightarrow \Gamma^{(ppb)} = -i \rightarrow \mathcal{P} = -, \mathcal{P}_x = -$:

    $$(C\gamma_5)_{AB}\left(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2)\right)\left(\bar{Q}_C(+R/2)\chi_B^{(u)}(+R/2)\right) =$$

    $$= +i(S_\uparrow P_\downarrow - S_\downarrow P_\uparrow + P_\uparrow S_\downarrow - P_\downarrow S_\uparrow).$$

  * $uu$, $\Gamma = \gamma_3 \rightarrow \gamma_3^{(ppb)} = \gamma_3 \rightarrow \mathcal{P} = -, \mathcal{P}_x = -$:

    $$(C\gamma_3)_{AB}\left(\bar{Q}_C(-R/2)\chi_A^{(u)}(-R/2)\right)\left(\bar{Q}_C(+R/2)\chi_B^{(u)}(+R/2)\right) =$$

    $$= -i(S_\uparrow S_\downarrow + S_\downarrow S_\uparrow - P_\uparrow P_\downarrow - P_\downarrow P_\uparrow).$$

  – $SS/SP_-$ content and asymptotic values in agreement for all $12 + 24$ independent potentials $\rightarrow$ asymptotic differences understood.
Discussion of results (4)

- Is there a general rule, about when a potential is repulsive and when attractive?

  - $S$–$S$ potentials:
    * $(I = 0, s = 0)$ or $(I = 1, s = 1)$, i.e. $I = s$ $\rightarrow$ attractive
    $(I = 0, s = 1)$ or $(I = 1, s = 0)$, i.e. $I \neq s$ $\rightarrow$ repulsive
      ($s$: combined angular momentum of the two mesons).
    * Example: $uu$, $\Gamma = \gamma_3$ $\rightarrow$ $\Gamma^{(ppb)} = \gamma_3$ $\rightarrow$ $P = -$, $P_x = -$:
      $-i \left( S_{\uparrow}S_{\downarrow} + S_{\downarrow}S_{\uparrow} - P_{\uparrow}P_{\downarrow} - P_{\downarrow}P_{\uparrow} \right)$,
      i.e. $I = 1$, $s = 1$; the numerically obtained potential is attractive, i.e. in agreement with the above stated rule.
    * All $6 + 12$ independent $S$–$S$ potentials fulfill the rule.
    * Agreement with similar quenched lattice studies.
      
Discussion of results (5)

- $S-P_-$ potentials:

  * Do not obey the above stated rule.

  * It can, however, easily be generalized by including parity, i.e. symmetry or antisymmetry under exchange of $S$ and $P_-$:
    trial state symmetric under meson exchange $\rightarrow$ attractive
    trial state antisymmetric under meson exchange $\rightarrow$ repulsive
    (meson exchange $\equiv$ exchange of flavor, spin and parity).

  * Example: $uu, \Gamma = \gamma_0 \rightarrow \Gamma^{(ppb)} = \gamma_0 \rightarrow \mathcal{P} = +, \mathcal{P}_x = -$:
    
    $-\left( S_{\uparrow}P_{\downarrow} - S_{\downarrow}P_{\uparrow} - P_{\uparrow}S_{\downarrow} + P_{\downarrow}S_{\uparrow} \right)$,

    i.e. $I = 1$ (symmetric), $s = 0$ (antisymmetric), antisymmetric with respect to $S \leftrightarrow P_-; \text{ the numerically obtained potential is attractive, i.e. in agreement with the above stated general rule.}$

  * All $6 + 12$ independent $S-P_-$ potentials (and all $6 + 12$ independent $S-S$ potentials) fulfill the general rule.
Summary and future plans

• Contraction and analysis code for computation of $BB$ potentials (arbitrary flavor, spin and parity) ready.

• Preliminary results promising:
  – Qualitative agreement with existing quenched results for $S–S$ potentials.
  – Computation of $S–P_–$ potentials seems feasible.

• To do list:
  – Implement off-axis separations.
  – Increase statistics by a factor of $\approx 50$.
  – Other $\beta, L^3 \times T, \mu$ values.
  – Partially quenched computations, to obtain $B_s B_s$ and/or $B_s B$ potentials.
  – Improve lattice meson potentials at small separations (where the suppression of UV fluctuations due to the lattice cutoff yields wrong results) with corresponding perturbative potentials.