$B \bar{B}$ and $B \bar{B}$ static potentials and heavy tetraquarks from lattice QCD

Bethe Forum on methods in lattice field theory – Rheinische Friedrich-Wilhelms-Universität Bonn

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Goals, motivation (1)

- Study tetraquarks/mesonic molecules by combining lattice QCD and phenomenology/model calculations.

- Basic idea:
  1. Compute the potential of two heavy quarks in the presence of two light quarks using lattice QCD.
  2. Explore, whether the potentials are sufficiently attractive to generate a bound state (a rather stable tetraquark/mesonic molecule) using phenomenology/model calculations.

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Why are such investigations important?

Quite a number of mesons are only poorly understood.

- Charged bottomonium states, e.g. $Z_b(10610)^{±}$ and $Z_b(10650)^{±}$ ... must be four quark states.
- Charged charmonium states, e.g. $Z_c(3940)^{±}$ and $Z_c(4430)^{±}$ ... must be four quark states.
- $X(3872)$ ($\bar{c}c$ state): mass not as expected from quark models; could be a $D-D^*$ molecule, a bound diquark-antidiquark, ...
Outline

- A brief introduction to lattice QCD hadron spectroscopy.
  - QCD (quantum chromodynamics).
  - Hadron spectroscopy.
  - Lattice QCD.
- Heavy-heavy-light-light tetraquarks.
- $BB$ static potentials.
- $BB$ tetraquarks.
- $B\bar{B}$ static potentials.
- Inclusion of $B/B^*$ mass splitting.
- Outlook.
QCD (quantum chromodynamics)

- Quantum field theory of quarks (six flavors $u, d, s, c, t, b$, which differ in mass) and gluons.

- Part of the standard model explaining the formation of hadrons (usually mesons = $q\bar{q}$ and baryons = $qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.

- Definition of QCD simple:

  \[
  S = \int d^4x \left( \sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}(f) \left( \gamma_\mu \left( \partial_\mu - iA_\mu \right) + m^{(f)} \right) \psi(f) + \frac{1}{2g^2} \text{Tr} \left( F_{\mu\nu}F^{\mu\nu} \right) \right)
  \]

  \[
  F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].
  \]

- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).

  → Solve QCD numerically by means of lattice QCD.
Hadron spectroscopy

• Proceed as follows:

(1) Compute the temporal correlation function $C(t)$ of a suitable hadron creation operator $O$ (an operator $O$, which generates the quantum numbers of the hadron of interest, when applied to the vacuum $|\Omega\rangle$).

(2) Determine the corresponding hadron mass from the asymptotic exponential decay in time.

• Example: $D$ meson mass $m_D$ (valence quarks $\bar{c}$ and $u$, $J^P = 0^-$),

$$O \equiv \int d^3r \; \bar{c}(\vec{r}) \gamma_5 u(\vec{r})$$

$$C(t) \equiv \langle \Omega | O(t) O(0) | \Omega \rangle \xrightarrow{t \to \infty} \propto \exp \left( -m_D t \right).$$

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Lattice QCD (1)

- To compute a temporal correlation function $C(t)$, use the path integral formulation of QCD,

\[
C(t) = \langle \Omega | O(t) O(0) | \Omega \rangle = \frac{1}{Z} \int \left( \prod_f D\psi^{(f)} D\bar{\psi}^{(f)} \right) DA_\mu O(t) O(0) e^{-S[\psi^{(f)}, \bar{\psi}^{(f)}, A_\mu]}.
\]

- $|\Omega\rangle$: ground state/vacuum.
- $O(t), O(0)$: functions of the quark and gluon fields (cf. previous slides).
- $\int \left( \prod_f D\psi^{(f)} D\bar{\psi}^{(f)} \right) DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(x, t)$ and $A_\mu(x, t)$.
- $e^{-S[\psi^{(f)}, \bar{\psi}^{(f)}, A_\mu]}$: weight factor containing the QCD action.
Lattice QCD (2)

• Numerical implementation of the path integral formalism in QCD:
  
  – Discretize spacetime with sufficiently small lattice spacing
    \[ a \approx 0.05 \text{ fm} \ldots 0.10 \text{ fm} \]
    \[ \rightarrow \text{“continuum physics”}. \]
  
  – “Make spacetime periodic” with sufficiently large extension
    \[ L \approx 2.0 \text{ fm} \ldots 4.0 \text{ fm} \text{ (4-dimensional torus)} \]
    \[ \rightarrow \text{“no finite volume effects”}. \]

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Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:
    \[
    \int D\psi D\bar{\psi} DA \ldots \rightarrow \prod_{x_\mu} \left( \int d\psi(x_\mu) d\bar{\psi}(x_\mu) dU(x_\mu) \right) \ldots
    \]
  - Typical present-day dimensionality of a discretized QCD path integral:
    * \( x_\mu : 32^4 \approx 10^6 \) lattice sites.
    * \( \psi = \psi^{a,(f)}_A \): 24 quark degrees of freedom for every flavor
      (\( \times 2 \) particle/antiparticle, \( \times 3 \) color, \( \times 4 \) spin), 2 flavors.
    * \( U = U^{ab}_{\mu} \): 32 gluon degrees of freedom (\( \times 8 \) color, \( \times 4 \) spin).
    * In total: \( 32^4 \times (2 \times 24 + 32) \approx 83 \times 10^6 \) dimensional integral.

→ standard approaches for numerical integration not applicable
→ sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).
Heavy-heavy-light-light-light tetraquarks (1)

- Study possibly existing $\bar{Q}Qqq$ and $\bar{Q}Q\bar{q}q$ tetraquark states ($q \in \{u, d, s, c\}$):
  - Use the static approximation for the heavy quarks $\bar{Q}Q$ and $\bar{Q}Q$ (reduces the necessary computation time significantly).
  - Most appropriate for $\bar{Q}Q \equiv \bar{b}b$ and $\bar{Q}Q \equiv \bar{b}b$, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$.
  - Could also provide information about $\bar{Q}Q \equiv \bar{c}c$ and $\bar{Q}Q \equiv \bar{c}c$, e.g. $Z_c(3940)^\pm$ and $Z_c(4430)^\pm$.

- Proceed in two steps:
  1. Compute the potential of two heavy quarks $\bar{Q}Q$ and $\bar{Q}Q$ in the presence of two “light quarks” $qq$ and $\bar{q}q$ ($q \in \{u, d, s, c\}$) using lattice QCD.
  2. Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $\bar{Q}Q$ and $\bar{Q}Q$; the existence of a bound state would indicate a rather stable tetraquark state.
Since heavy $b$ quarks are treated in the static approximation, their spins are irrelevant (mesons are labeled by the spin of the light degrees of freedom $j$).

Consider only pseudoscalar/vector mesons ($j^P = (1/2)^-$, PDG: $B, B^*$) and scalar/pseudovector mesons ($j^P = (1/2)^+$, PDG: $B_0^*, B_1^*$), which are among the lightest static-light mesons.

Study the dependence of the mesonic potential $V(R)$ on

- the “light” quark flavors $u, d, s$ and/or $c$ (isospin),
- the “light” quark spin (the static quark spin is irrelevant),
- the type of the meson $B, B^*$ and/or $B_0^*, B_1^*$.

→ Many different channels/quantum numbers ... attractive, repulsive ...

\[ V(R) = ? \]
In the following $\bar{Q}\bar{Q}qq$, i.e. “$BB$” (not $\bar{Q}Q\bar{q}q$, i.e. “$B\bar{B}$”).

To extract the potential(s) of a given sector $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$, compute the temporal correlation function of the trial state

$$(C\Gamma)_{AB}(C\tilde{\Gamma})_{CD}\left(\bar{Q}_C(-R/2)q^{(1)}_A(-R/2)\right)\left(\bar{Q}_D(+R/2)q^{(2)}_B(+R/2)\right)|\Omega\rangle.$$ 

- $C = \gamma_0\gamma_2$ (charge conjugation matrix).
- $q^{(1)}q^{(2)} \in \{ud - du, uu, dd, ud + du, ss, cc\}$ (isospin $I, I_z$).
- $\Gamma$ is an arbitrary combination of $\gamma$ matrices (spin $|j_z|$, parity $\mathcal{P}, \mathcal{P}_x$).
\[ I = 0 \text{ (left) and } I = 1 \text{ (right); } |j_z| = 0 \text{ (top) and } |j_z| = 1 \text{ (bottom).} \]
Why are certain channels attractive and others repulsive? (1)

- Wave function of two identical fermions (light quarks $q^{(1)}q^{(2)}$) must be antisymmetric (Pauli principle); in qft/QCD automatically realized on the level of states.
- $q^{(1)}q^{(2)}$ isospin: $I = 0$ antisymmetric, $I = 1$ symmetric.
- $q^{(1)}q^{(2)}$ spin: $j = 0$ antisymmetric, $j = 1$ symmetric.
- $q^{(1)}q^{(2)}$ color:
  
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$: must be antisymmetric, i.e. a triplet $\bar{3}$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$: must be symmetric, i.e. a sextet $6$.

- The four quarks $\bar{Q}\bar{Q}q^{(1)}q^{(2)}$ must form a color singlet:
  
  - $q^{(1)}q^{(2)}$ in a color triplet $\bar{3}$ $\rightarrow$ static quarks $\bar{Q}\bar{Q}$ also in a triplet $3$.
  - $q^{(1)}q^{(2)}$ in a color sextet $6$ $\rightarrow$ static quarks $\bar{Q}\bar{Q}$ also in a sextet $\bar{6}$.
Why are certain channels attractive and others repulsive? (2)

- Attractive/repulsive behavior of $Q\bar{Q}$ at small separations $r$ is mainly due to 1-gluon exchange (the static quarks $Q\bar{Q}$ are rather close, inside a large light quark cloud formed by $q^{(1)}q^{(2)}$, i.e. no color screening of the color charges $Q\bar{Q}$ due to $q^{(1)}q^{(2)}$):
  - color triplet $3$ is attractive, $V(r) = -2\alpha_s/3r$,
  - color sextet $\bar{6}$ is repulsive, $V(r) = +\alpha_s/3r$

(easy to calculate in LO perturbation theory).

- Summary:
  - $(I = 0, j = 0)$ and $(I = 1, j = 1)$ → attractive $Q\bar{Q}$ potential $V(r)$.
  - $(I = 0, j = 1)$ and $(I = 1, j = 0)$ → repulsive $Q\bar{Q}$ potential $V(r)$.

This expectation is consistent with the obtained lattice results.
**BB** static potentials (5)

- Focus on the two attractive channels between ground state static-light mesons "B and $B^*$" (probably the best candidates to find a tetraquark):
  
  - Scalar isosinglet (more attractive):
    \[ qq = (ud - du)/\sqrt{2}, \Gamma = \gamma_5 + \gamma_0\gamma_5, \]
    quantum numbers \((I, |j_z|, P, P_x) = (0, 0, -, +)\).
  
  - Vector isotriplet (less attractive):
    \[ qq \in \{uu, (ud + du)/\sqrt{2}, dd\}, \Gamma = \gamma_j + \gamma_0\gamma_j, \]
    quantum numbers \((I, |j_z|, P, P_x) = (1, \{0, 1\}, -, \pm)\).

- Computations for \(qq = ll, ss, cc\) \((l \in \{u, d\})\) to study the mass dependence.
Two competing effects:

- The potential for light quarks is wider/deeper, i.e. favors the existence of a bound state (a tetraquark).
- Heavier quarks correspond to heavier mesons \((m(B) < m(B_s) < m(B_c))\), which form more readily a bound state (a tetraquark), i.e. require a less wide/deep potential for a bound state.

**BB tetraquarks (1)**

- Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $\bar{Q}Q$,

$$\left( -\frac{1}{2\mu} \Delta + V(r) \right) \psi(r) = E\psi(r), \quad \mu = \frac{m(B_{(s,c)})}{2};$$

a bound state, i.e. $E_0 < 0$, would be an indication for a tetraquark state.

- There is a bound state for the scalar isosinglet and $qq = ll$ (i.e. $BB$), binding energy $E \approx -50$ MeV, confidence level $\approx 2\sigma$.

- No binding for the vector isotriplet or for $qq = ss, cc$ (i.e. $B_sB_s, B_cB_c$).
To quantify “no binding”, we list for each channel the factor, by which the effective mass $\mu$ in Schrödinger’s equation has to be multiplied, to obtain binding with confidence level $1\sigma$ and $2\sigma$ (the potential is not changed).

<table>
<thead>
<tr>
<th>flavor</th>
<th>confidence level for binding</th>
<th>light</th>
<th>strange</th>
<th>charm</th>
</tr>
</thead>
<tbody>
<tr>
<td>scalar isosinglet</td>
<td>$1\sigma$</td>
<td>$2\sigma$</td>
<td>$1\sigma$</td>
<td>$2\sigma$</td>
</tr>
<tr>
<td>vector isotriplet</td>
<td>$0.8$</td>
<td>$1.0$</td>
<td>$1.9$</td>
<td>$2.2$</td>
</tr>
<tr>
<td></td>
<td>$1.9$</td>
<td>$2.1$</td>
<td>$2.5$</td>
<td>$2.7$</td>
</tr>
</tbody>
</table>

- Factors $\leq 1.0$ indicate binding.
- Light quarks ($u/d$) are unphysically heavy (correspond to $m_\pi \approx 340$ MeV); physically light $u/d$ quarks are expected to yield stronger binding for the scalar isosinglet, might lead to binding also for the vector isotriplet (computations in progress).
- Mass splitting $m(B^*) - m(B) \approx 50$ MeV, neglected at the moment, is expected to weaken binding (coupled channel analysis; see later slides).

What are the quantum numbers of the $\bar{b}\bar{b}ll$ tetraquark (light scalar isosinglet)?

- Light scalar isosinglet: $I = 0$, $j = 0$, $ll$ in a color $3$, $\bar{b}\bar{b}$ in a color $3$ (antisymmetric) ... as discussed above.

- Wave function of $\bar{b}\bar{b}$ must also be antisymmetric (Pauli principle); in the lattice QCD computation not automatically realized (static quarks are spinless color charges, which can be distinguished by their positions).
  
  - $\bar{b}\bar{b}$ is flavor symmetric.
  
  - $\bar{b}\bar{b}$ spin must also be symmetric, i.e. $j_b = 1$.

- The $\bar{b}\bar{b}ll$ tetraquark has quantum numbers isospin \( I = 0 \), spin \( J = 1 \), parity \( \mathcal{P} = + \) (parity not obvious).
$B \bar{B}$ static potentials

- Experimentally more interesting case: $QQ\bar{q}q$, i.e. “$B \bar{B}$”, trial states

$$\Gamma_{AB} \tilde{\Gamma}_{CD} (\bar{Q}_C(-R/2)q_B^{(1)}(-R/2))(\bar{q}_A^{(2)}(+R/2)Q_D(+R/2))|\Omega\rangle.$$ 

- At the moment only preliminary results for $\bar{q}q = \bar{c}c$, “$I = 1$”.

- Qualitative difference to $Q\bar{Q}qq$: all channels are attractive (for $Q\bar{Q}qq$ half of them are attractive, half of them are repulsive).

- Can again be understood by the 1-gluon exchange potential of $\bar{Q}Q$:
  - No Pauli principle for $\bar{q}^{(1)}q^{(2)}$ (particle and antiparticle are not identical).
  - $\bar{q}^{(1)}q^{(2)}$ can be in a symmetric color singlet 1 for any isospin/spin orientation.
  - $\bar{q}^{(1)}q^{(2)}$ in a color singlet 1 $\rightarrow$ static quarks $\bar{Q}Q$ also in a singlet 1.
  - Color singlet is attractive, $V(r) = -4\alpha_s/3r$ (LO perturbation theory).
Inclusion of $B/B^*$ mass splitting (1)

- Mass splitting $m_{B^*} - m_B \approx 50$ MeV has been neglected so far.
- Mass splitting $m_{B^*} - m_B$ is, however, of the same order of magnitude as the previously obtained binding energy $E \approx -50$ MeV.
- Moreover, two competing effects:
  - An attractive $Q\bar{Q}qq$ channel correspond to a linear combination of $BB$, $BB^*$ and/or $B^*B^*$, e.g.
    \[
    \text{scalar isosinglet} \equiv BB + B_xB_x^* + B_yB_y^* + B_zB_z^*.
    \]
  - The $BB$ interaction is a superposition of attractive and repulsive $Q\bar{Q}qq$ potentials.
- Goal: take mass splitting $m_{B^*} - m_B \approx 50$ MeV into account
  $\rightarrow$ refined model calculation with the computed $Q\bar{Q}qq$ potentials.
- Will there still be a bound state?

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Inclusion of $B/B^*$ mass splitting (2)

Solve a coupled channel Schrödinger equation (1)

• Previously:
  – A wave function $\psi$ with 1 component corresponding to $BB$ ($B \equiv B^*$).

• Now:
  – A static light meson can correspond to $B$ or $B^* = (B^*_x, B^*_y, B^*_z)$.
  – Therefore, a wave function $\vec{\psi}$ with 16 components corresponding to $(BB, BB^*_x, BB^*_y, BB^*_z, B^*_xB, B^*_yB, B^*_zB, \ldots, B^*_zB)$.

• Coupled channel Schrödinger equation $H\vec{\psi}(r_1, r_2) = E\vec{\psi}(r_1, r_2)$,

\[
H = M \otimes 1 + 1 \otimes M + \frac{p_1^2}{2} (M \otimes 1)^{-1} + \frac{p_2^2}{2} (1 \otimes M)^{-1} + V(|r_1 - r_2|),
\]

where $M = \text{diag}(m_B, m_{B^*}, m_{B^*}, m_{B^*})$ and $V$ is a 16 × 16 non-diagonal matrix containing the $\bar{Q}Qqq$ potentials (both attractive and repulsive).

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Inclusion of $B/B^*$ mass splitting (3)

Solve a coupled channel Schrödinger equation (2)

- Coupled channel Schrödinger equation $H\vec{\psi}(r_1, r_2) = E\vec{\psi}(r_1, r_2)$,

\[
H = M \otimes 1 + 1 \otimes M + \frac{p_1^2}{2} (M \otimes 1)^{-1} + \frac{p_2^2}{2} (1 \otimes M)^{-1} + V(|r_1 - r_2|),
\]

where $M = \text{diag}(m_B, m_B^*, m_B^*, m_B^*)$ and $V$ is a $16 \times 16$ non-diagonal matrix containing the $\bar{Q}\bar{Q}qq$ potentials (both attractive and repulsive).

- Specific limits:
  - $V = 0$, i.e. no interactions:
    \[
    E = m_B + m_B + \frac{p_1^2}{2m_B} + \frac{p_2^2}{2m_B}, \quad m_B + m_B^* + \frac{p_1^2}{2m_B} + \frac{p_2^2}{2m_{B^*}}, \ldots
    \]
  - $m_{B^*} = m_B$, i.e. “old 1-component SE calculation”:
    \[
    E \approx 2m_B - 50 \text{ MeV}.
    \]
Inclusion of $B/B^*$ mass splitting (4)

Solve a coupled channel Schrödinger equation (3)

- Transform the $16 \times 16$ Schrödinger equation to block diagonal structure:
  - Total spin $J = 0$: $2 \times 2$ structure.
  - Total spin $J = 1$: $3 \times 3$ structure ($3 \times$ due to $J_z$ degeneracy).
  - Total spin $J = 2$: $1 \times 1$ structure ($5 \times$ due to $J_z$ degeneracy).

- Work in progress ...
  - First very preliminary results indicate that for $J = 0$ the bound state does not exist anymore (however, still very close to a bound state).
  - However:
    * More realistic/relevant $J = 1$ equation not yet investigated.
    * Unphysically heavy $u/d$ quarks ($m_\pi \approx 340$ MeV) ... physically light quarks will lead to more attractive $\bar{Q}Qqq$ potentials.
    * $M = \text{diag}(m_B, m_{B^*}, m_{B^*}, m_{B^*})$; for small $\bar{Q}Q$ separation $M = m_b$ would be more appropriate ... should enhance binding.
Outlook (1)

- Future plans for $BB$ and $B\bar{B}$:
  - Computations with light $u/d$ quarks of physical mass ($m_\pi \approx 140$ MeV instead of $m_\pi \approx 340$ MeV).
  - Light quarks of different mass: $BB_s$, $BB_c$ and $B_sB_c$ potentials.
Outlook (2)

• Future plans for \( BB \) and \( B\bar{B} \):
  
  – Study the structure of the states corresponding to the computed potentials:
    
    * In a lattice computation two different creation operators generating the same quantum numbers yield the same potential.
    * At the moment exclusively creation operators of mesonic molecule type.
    * For \( BB \) use also
      · creation operators of diquark-antidiquark type.
    * For \( B\bar{B} \) use also
      · creation operators of diquark-antidiquark type,
      · creation operators of bottomonium + pion type (\( Q\bar{Q} \) string + \( \pi \)),
      · for \( I = 0 \) creation operators of bottomonium type (\( Q\bar{Q} \) string).
    * Resulting correlation matrices provide information about the structure.