The SU(2) quark-antiquark potential in the pseudoparticle approach

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Outline

- Basic principle.
- Building blocks of PP ensembles.
- PP ensembles.
- Quark-antiquark potential.
- Quantitative results.
- Summary.
- Outlook.
Basic principle (1)

• Pseudoparticle approach (PP approach):
  – A numerical technique to approximate Euclidean path integrals (in this talk: SU(2) Yang-Mills theory \( \approx \) QCD with infinitely heavy quarks):

\[
\langle \mathcal{O} \rangle = \frac{1}{Z} \int D A \mathcal{O}[A] e^{-S[A]}
\]

\[
S[A] = \frac{1}{4g^2} \int d^4 x \ F_{\mu\nu}^a F_{\mu\nu}^a , \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c .
\]

  – A tool to analyze the importance of gauge field configurations with respect to confinement.
  – A method, from which we can get a better understanding of the Yang-Mills path integral.
Basic principle (2)

- PP: any gauge field configuration $a^a_{\mu}$, which is localized in space and time.

- Consider only those gauge field configurations, which can be written as a sum of a fixed number ($\approx 400$) of PPs:

$$A^a_{\mu}(x) = \sum_i \rho^{ab}(i)a^b_{\mu}(x - z(i)).$$

($i$: PP index; $\rho^{ab}(i)$: degrees of freedom of the $i$-th PP, i.e. amplitude and color orientation; $z(i)$: position of the $i$-th PP).

- Approximate the path integral by an integration over PP degrees of freedom:

$$\int DA \ldots \rightarrow \int \left( \prod_i d\rho^{ab}(i) \right) \ldots$$
Building blocks of PP ensembles


\[
\begin{align*}
a^a_{\mu,\text{instanton}}(x) &= \eta^a_{\mu\nu} \frac{x_\nu}{x^2 + \lambda^2} \\
a^a_{\mu,\text{antiinstanton}}(x) &= \bar{\eta}^a_{\mu\nu} \frac{x_\nu}{x^2 + \lambda^2} \\
a^a_{\mu,\text{akyron}}(x) &= \delta^{a1} \frac{x_\mu}{x^2 + \lambda^2}.
\end{align*}
\]

- Degrees of freedom: amplitudes $A^a(i)$, color orientations $C^{ab}(i)$, positions $z(i)$.

\[
\begin{align*}
A^a_\mu(x) &= A(i)C^{ab}(i)a^a_{\mu,\text{instanton}}(x - z(i)) \\
A^a_\mu(x) &= A(i)C^{ab}(i)a^a_{\mu,\text{antiinstanton}}(x - z(i)) \\
A^a_\mu(x) &= A(i)C^{ab}(i)a^a_{\mu,\text{akyron}}(x - z(i)).
\end{align*}
\]

- Instantons, antiinstantons and akyrons form a basis of all gauge field configurations in the “continuum limit”.

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• PP ensemble: a fixed number of PPs inside a “spacetime hypersphere”.

• Gauge field:

\[ A^a_\mu(x) = \sum_i A(i) C^{ab}(i) a^b_\mu,\text{instanton}(x - z(i)) + \]
\[ \sum_j A(j) C^{ab}(j) a^b_\mu,\text{antiinstanton}(x - z(j)) + \]
\[ \sum_k A(k) C^{ab}(k) a^b_\mu,\text{akyron}(x - z(k)). \]

• Choose color orientations \( C^{ab}(i) \) and positions \( z(i) \) randomly.

• \( A^a_\mu \) is no classical solution (not even close to a classical solution)!!!

• Long range interactions between PPs.
PP ensembles (2)

- Approximation of the path integral:

\[
\langle \mathcal{O} \rangle = \frac{1}{Z} \int \left( \prod_i dA(i) \right) \mathcal{O}(A(i)) e^{-S(A(i))}.
\]

- Solve this multidimensional integral via Monte-Carlo simulations.

- Exclude boundary effects: observables have to be “measured” sufficiently far away from the boundary.
Quark-antiquark potential (1)

- Common tool to determine the potential of a static quark-antiquark pair: Wilson loops.

- Wilson loop ($z$: closed spacetime curve):

\[ W_z[A] = \frac{1}{2} \text{Tr} \left( P \left\{ \exp \left( i \oint dz \mu A_\mu(z) \right) \right\} \right). \]


- Wilson loops $\leftrightarrow$ quark-antiquark potential ($R$: quark-antiquark separation):

\[ V_{q\bar{q}}(R) = -\lim_{T \to \infty} \frac{1}{T} \ln \langle W_{(R,T)} \rangle. \]

- Assumption: the potential can be parameterized according to

\[ V_{q\bar{q}}(R) = V_0 - \frac{\alpha}{R} + \sigma R. \]
Method 1: Determine the string tension $\sigma$ and the Coulomb coefficient $\alpha$

- “Guess” the functional dependence of ensemble averages of Wilson loops:

$$- \ln \left\langle W_{(R,T)} \right\rangle = V_0 (R + T) - \alpha \left( \frac{R}{T} + \frac{T}{R} \right) + \beta + \sigma RT.$$ 

- Determine the string tension $\sigma$ and the Coulomb coefficient $\alpha$ by fitting the “Wilson loop ansatz” to Monte-Carlo data for $- \ln \left\langle W_{(R,T)} \right\rangle$.

- Several approaches:
  - Area perimeter fits.
  - Creutz ratios.
  - Generalized Creutz ratios.
  - ...
Method 1: Determine the string tension $\sigma$ and the Coulomb coefficient $\alpha$

- Results for PP ensembles containing $\approx 400$ PPs:
  - Coulomb coefficient $\alpha > 0$
    $\rightarrow$ attractive “Coulomb-like” interaction at small quark-antiquark separations.
  - String tension $\sigma > 0$
    $\rightarrow$ linear potential for large quark-antiquark separations, confinement.
  - $\sigma$ is an increasing function of the coupling constant $g$
    $\rightarrow$ adjust the physical scale by choosing appropriate values for $g$. 

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Quantitative results

• For quantitative results, including the string tension, we need other dimensionful quantities:
  – Topological susceptibility $\chi = \langle Q^2 \rangle / V$.
  – Critical temperature $T_{\text{critical}}$.

• Dimensionless quantities (physically meaningful): $\chi^{1/4}/\sigma^{1/2}$, $T_{\text{critical}}/\sigma^{1/2}$, $\alpha$.

• Consider different $g = 2.0 \ldots 5.5$ (diameter of the spacetime hypersphere 0.9 fm $\ldots$ 2.0 fm).

• Results are in qualitative agreement with results from lattice calculations.

• Consistent scaling behavior of $\sigma$, $\chi$ and $T_{\text{critical}}$.

• $\alpha$ should be constant.
Method 2: Calculate the quark-antiquark potential directly

- For large $T$:

$$V_{q\bar{q}}(R)T \approx -\ln \left\langle W_{(R,T)} \right\rangle.$$  

- From the slope of $-\ln \left\langle W_{(R,T)} \right\rangle |_{R=\text{constant}}$ we can read off $V_{q\bar{q}}(R)$.

- Results are in agreement with our previous results.
The PP approach with $\approx 400$ instantons, antiinstantons and akyrons is able to reproduce many essential features of SU(2) Yang-Mills theory:

- Quark-antiquark potential:
  - Linear potential for large quark-antiquark separations (confinement).
  - “Coulomb-like” attractive force for small quark-antiquark separations.

- Consistent scaling behavior of $\sigma$, $\chi$ and $T_{\text{critical}}$.

- Dimensionless quantities $\chi^{1/4}/\sigma^{1/2}$, $T_{\text{critical}}/\sigma^{1/2}$ and $\alpha$ are in qualitative agreement with results from lattice calculations.
Outlook

- Compare different PP ensembles to analyze, which gauge field configurations are responsible for confinement:
  - Pure akyron ensembles (no topological charge density) → deconfinement.
  - Gaussian localized PPs (PPs of limited size) → deconfinement for small PP size, confinement for large PP size.

- Overall picture: topological charge and long range interactions between PPs are important for confinement.