Adjoint string breaking in the pseudoparticle approach

Christian Szasz
University of Erlangen-Nürnberg
christian.szasz@theorie3.physik.uni-erlangen.de

Marc Wagner
Humboldt University Berlin
mcwagner@physik.hu-berlin.de
http://people.physik.hu-berlin.de/~mcwagner/

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Outline

Part I: introduction to the pseudoparticle approach

• Basic principle, pseudoparticle ensembles and their building blocks.

Part II: adjoint string breaking in the pseudoparticle approach

• Pure Wilson loop static potentials, Casimir scaling.
• The gluelump spectrum.
• The static adjoint potential, string breaking.

Summary, further results, ongoing projects
Part I: introduction to the pseudoparticle approach
Basic principle (1)

- Pseudoparticle approach (PP approach; F. Lenz, M.W., 2005):
  - A numerical technique to approximate Euclidean path integrals.
  - In this talk: application to SU(2) Yang-Mills theory,

\[
\langle \mathcal{O} \rangle = \frac{1}{Z} \int DA \mathcal{O}[A] e^{-S[A]}
\]

\[
S[A] = \frac{1}{4g^2} \int d^4 x \, F_{\mu\nu}^a F_{\mu\nu}^a, \quad F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c.
\]

- Goals:
  * Build a model for SU(2) Yang-Mills theory with a small number of physically relevant degrees of freedom.
  * Analyze the importance of certain classes of gauge field configurations with respect to confinement and other essential properties of SU(2) Yang-Mills theory.
Basic principle (2)

- Related work:
  - Ensembles of calorons with non-trivial holonomy (P. Gerhold, E.-M. Ilgenfritz, M. Müller-Preussker, 2006).
  - Ensembles of dyons (D. Diakonov, V. Petrov, 2007).
Basic principle (3)

- PP: any gauge field configuration $a^a_{\mu}$, which is localized in space and in time.

- Consider only those gauge field configurations, which can be written as a sum of a fixed number ($\approx 500$) of PPs:

$$A^a_{\mu}(x) = \sum_j \rho^{ab}(j)a^b_{\mu}(x - z(j))$$

($j$: PP index; $\rho^{ab}(j)$: degrees of freedom of the $j$-th PP, i.e. amplitude and color orientation; $z(j)$: position of the $j$-th PP).

- Define the functional integration as an integration over the PP degrees of freedom:

$$\int DA \ldots \rightarrow \int \left( \prod_j d\rho^{ab}(j) \right) \ldots$$
Building blocks of PP ensembles


\[
a_{\mu,\text{instanton}}(x) = \frac{\eta_{\mu\nu} x_\nu}{x^2 + \lambda^2}
\]
\[
a_{\mu,\text{antiinstanton}}(x) = \frac{\bar{\eta}_{\mu\nu} x_\nu}{x^2 + \lambda^2}
\]
\[
a_{\mu,\text{akyron}}(x) = \delta^{a1} \frac{x_\mu}{x^2 + \lambda^2}.
\]

- Instantons, antiinstantons and akyrons form a basis of all gauge field configurations in the “continuum limit”.

- Degrees of freedom: amplitudes $A(i)$, color orientations $C^{ab}(i)$, positions $z(i)$.

\[
A_{\mu}^a(x) = A(i) C^{ab}(i) a_{\mu,\text{instanton}}^a(x - z(i))
\]
\[
A_{\mu}^a(x) = A(i) C^{ab}(i) a_{\mu,\text{antiinstanton}}^a(x - z(i))
\]
\[
A_{\mu}^a(x) = A(i) C^{ab}(i) a_{\mu,\text{akyron}}^a(x - z(i)).
\]
PP ensembles (1)

- **PP ensemble**: a fixed number of PPs (in this talk: 625) inside a periodic spacetime hypercube (in this talk: extension 5.04).

- **Gauge field**:

\[
A_{\mu}^{a}(x) = \sum_{i} A(i) C^{ab}(i) a_{\mu,\text{instanton}}^{b}(x - z(i)) + \sum_{j} A(j) C^{ab}(j) a_{\mu,\text{antiinstanton}}^{b}(x - z(j)) + \sum_{k} A(k) C^{ab}(k) a_{\mu,\text{akyron}}^{b}(x - z(k)).
\]

- Choose color orientations \( C^{ab}(i) \) and positions \( z(i) \) randomly.

- \( A_{\mu}^{a} \) is **not a classical solution** (not even close to a classical solution), i.e. the PP approach is not a semiclassical method.
  
  - Long range interactions between PPs.
  - Variable amplitudes \( A(i) \).

PP ensembles (2)

- Approximation of the path integral:

\[ \langle \mathcal{O} \rangle = \frac{1}{Z} \int \left( \prod_i dA(i) \right) \mathcal{O}(A(i)) e^{-S(A(i))} \]

(integration over PP amplitudes).

- Solve this multidimensional integral via standard Monte-Carlo methods.
Part II: adjoint string breaking in the pseudoparticle approach
String breaking

- **Static potential** $V(R)$: the energy of the lowest state containing two static sources $\phi$ and $\phi^\dagger$ ("two infinitely heavy quarks") at separation $R$ (+ light particles [gluons, dynamical quarks, ...]).

- **String breaking in QCD**: when two static sources are separated adiabatically beyond a certain distance, the connecting gauge string breaks and they are screened by dynamical quarks; a pair of (essentially) non-interacting static-light mesons is formed.

- **String breaking in pure SU(2) Yang-Mills theory**:  
  - Fundamental representation $(\phi^{(1/2)} = (\phi_1, \phi_2))$: no string breaking, since gluons are not able to screen sources in the fundamental representation.
  - Adjoint representation $(\phi^{(1)} = (\phi_1, \phi_2, \phi_3)$ or $\phi^{(1)} = \phi_a \sigma_a / 2)$: when two static sources are separated adiabatically beyond a certain distance, the connecting gauge string breaks and they are screened by gluons; a pair of (essentially) non-interacting gluelumps is formed.

The starting point to extract the energy of the lowest state containing two static sources $\phi^{(J)}$ and $(\phi^{(J)})^\dagger$ in spin-$J$-representation are “string trial states”

$$S^{(J)}(x, y)|\Omega\rangle = (\phi^{(J)}(x))^{\dagger}U^{(J)}(x; y)\phi^{(J)}(y)|\Omega\rangle, \quad |x - y| = R.$$  

We consider temporal correlations

$$C^{(J)}_{\text{string}}(T) = \langle \Omega|\left(S^{(J)}(x, y, T)\right)^{\dagger}S^{(J)}(x, y, 0)|\Omega\rangle$$

and compute the corresponding potential values from effective mass plateaus,

$$m^{(\ldots)}_{\text{effective, string}}(T) = -\frac{1}{a} \ln \frac{C^{(\ldots)}_{\text{string}}(T)}{C^{(\ldots)}_{\text{string}}(T - a)}.$$
Integrating out the static sources yields expectation values of polynomials of fundamental representation Wilson loops $W_{(R,T)}$:

\[
\begin{align*}
C_{\text{string}}^{(1/2)}(T) & \propto \langle W_{(R,T)}(1/2) \rangle \\
C_{\text{string}}^{(1)}(T) & \propto \langle W_{(R,T)}(1) \rangle = \langle \frac{4}{3}(W_{(R,T)})^2 - \frac{1}{3} \rangle \\
C_{\text{string}}^{(3/2)}(T) & \propto \langle W_{(R,T)}(3/2) \rangle = \langle 2(W_{(R,T)})^3 - W_{(R,T)} \rangle \\
C_{\text{string}}^{(2)}(T) & \propto \langle W_{(R,T)}(2) \rangle = \langle \frac{16}{5}(W_{(R,T)})^4 - \frac{12}{5}(W_{(R,T)})^2 + \frac{1}{5} \rangle \\
C_{\text{string}}^{(5/2)}(T) & \propto \langle W_{(R,T)}(5/2) \rangle = \langle \frac{16}{3}(W_{(R,T)})^5 - \frac{16}{3}(W_{(R,T)})^3 + W_{(R,T)} \rangle \\
\ldots
\end{align*}
\]
Pure Wilson loop static potentials (3)

- Numerical results for the fundamental representation ($J = 1/2$):
  - The potential is linear for large separations, i.e. confinement.
  - The scale is set by fitting $V(R) = V_0 + \sigma R$ and by identifying $\sigma$ with $\sigma_{\text{physical}} = 4.2/\text{fm}^2$.
  - A coupling constant of $g = 12.5$ (standard choice for results shown in this talk) corresponds to a spacetime region of $L^4 = (1.85 \text{ fm})^4$.
  - Like in lattice gauge theory, the scale can be changed by changing the value of $g$, e.g. $g = 9.5 \ldots 18.5$ corresponds to $L = 1.55 \text{ fm} \ldots 2.31 \text{ fm}$.

![Diagram of effective masses](image)

![Diagram of fundamental potential](image)
Numerical techniques (1)

• Latticization of pseudoparticle gauge field configurations: allows the use of efficient lattice techniques, while the underlying model is still a continuum model.

• Smearing techniques:
  – HYP2 smearing of temporal links (3 iterations): removes UV fluctuations and reduces the self energy of the static sources; the signal-to-noise ratio is significantly improved (e.g. for the adjoint potential computation time is reduced by a factor of $\approx 200$).
  – APE smearing of spatial links (5, 15 and 35 iterations): increases ground state overlaps of trial states; effective mass plateaus are reached at smaller temporal separations.
Numerical techniques (2)

- **Variational technique:**
  - Instead of a single trial state $|\Phi\rangle$ consider a set of trial states \{|$\Phi^{(1)}\rangle$, \ldots, $|\Phi^{(N)}\rangle$\} differing e.g. by their APE smearing levels.
  - Diagonalize the corresponding correlation matrices
    \[
    C_{AB}(T) = \langle \Phi^{(A)}(T) | \Phi^{(B)}(0) \rangle
    \]
    according to
    \[
    C_{AB}(T_0)v_B^{(n)} = C_{AB}(T_0 - a)v_B^{(n)} \lambda^{(n)}.
    \]
  - Approximations of low lying states are then given by
    \[
    |n\rangle \approx v_A^{(n)}|\Phi^{(A)}\rangle,
    \]
    the corresponding effective masses by
    \[
    m_{\text{effective}}^{(n)}(T) = -\frac{1}{a} \ln \frac{(v_A^{(n)})^\dagger C_{AB}(T)v_B^{(n)}}{(v_A^{(n)})^\dagger C_{AB}(T-a)v_B^{(n)}}.
    \]
Pure Wilson loop static potentials (4)

- Numerical results for higher representations ($J = 1, \ldots, J = 5/2$):
  
  - Higher representation potentials exhibit Casimir scaling:
    
    $$V^{(1/2)}(R) \approx \frac{V^{(1)}(R)}{8/3} \approx \frac{V^{(3/2)}(R)}{5} \approx \frac{V^{(2)}(R)}{8} \approx \frac{V^{(5/2)}(R)}{35/3}.$$  

  - The adjoint potential ($J = 1$) shows no sign of string breaking even at separations $R \approx 1.6$ fm.
Why is string breaking elusive?

- Static potential at small separations $R$: the string trial state has good overlap to the physical ground state, which is expected to be a string state.

- Static potential at large separations $R$: the string trial state has poor overlap to the physical ground state, which is expected to be a two gluelump state.

- Solution: Extend the set of trial states by “two-gluelump trial states”, which are supposed to have good overlap to the physical ground state at large $R$,

$$G^{(...)}_j(x)G^{(...)}_j(y)|\Omega\rangle, \quad |x - y| = R$$

with

$$G^{(\text{magnetic}, J=1)}_j(x) = \text{Tr}\left(\phi^{(1)}(x)B_j(x)\right), \quad j = x, y, z$$

$$G^{(\text{electric}, J=1)}_z(x) = \text{Tr}\left(\phi^{(1)}(x)(D_xB_y(x) - D_yB_x(x))\right) \quad \text{(and cyclic)}$$

$$G^{(\text{electric}, J=2)}_z(x) = \text{Tr}\left(\phi^{(1)}(x)(D_xB_y(x) + D_yB_x(x))\right) \quad \text{(and cyclic)}.$$
The gluelump spectrum (1)

- Symmetry group of states constrained by a single static source $\phi^{(1)}(x)$: $\text{SO}(3) \otimes P$.

- "Gluelump trial states" with well defined quantum numbers:
  - Magnetic gluelump: $J = 1$, $P = +$,
    \[ G^{(\text{magnetic}, J=1)}_{j}(x)|\Omega\rangle = \text{Tr}\left(\phi^{(1)}(x)B_{j}(x)\right)|\Omega\rangle, \quad j = x, y, z. \]
  - Electric gluelump: $J = 1$, $P = -$,
    \[ G^{(\text{electric}, J=1)}_{z}(x)|\Omega\rangle = \text{Tr}\left(\phi^{(1)}(x)\left(D_{x}B_{y}(x) - D_{y}B_{x}(x)\right)\right)|\Omega\rangle \]
    (and cyclic).
  - Electric gluelump: $J = 2$, $P = -$,
    \[ G^{(\text{electric}, J=2)}_{z}(x)|\Omega\rangle = \text{Tr}\left(\phi^{(1)}(x)\left(D_{x}B_{y}(x) + D_{y}B_{x}(x)\right)\right)|\Omega\rangle \]
    (and cyclic).
The gluelump spectrum (2)

- We consider temporal correlations of gluelump trial states,

\[ C_{\text{gluelump}}^{(...)}(T) = \langle \Omega | \left( G^{(...)}_{j}(x, T) \right)^{\dagger} G^{(...)}_{j}(x, T) | \Omega \rangle, \]

and compute the corresponding gluelump masses from effective mass plateaus,

\[ m_{\text{effective,gluelump}}^{(...)}(T) = -\frac{1}{a} \ln \frac{C_{\text{gluelump}}^{(...)}(T)}{C_{\text{gluelump}}^{(...)}(T - a)}. \]
The gluelump ...

- Numerical results related to gluelump masses:
  - Gluelump masses depend on the regularization and tend to infinity in the continuum limit, i.e. gluelump masses are not physically meaningful.
  - Differences between gluelump masses are physical observables.
  - The string breaking distance $R_{sb}$ can be estimated by solving $V^{(1)}(R_{sb}) = 2m(...)$.

<table>
<thead>
<tr>
<th>gluelump</th>
<th>$m(...)$</th>
<th>$m - m^{(\text{electric}, J=1)}$</th>
<th>estimated $R_{sb}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>magnetic, $J = 1$</td>
<td>1037(21) MeV</td>
<td>172(38) MeV</td>
<td>1.01(3) fm</td>
</tr>
<tr>
<td>electric, $J = 1$</td>
<td>865(17) MeV</td>
<td>$-$</td>
<td>0.85(2) fm</td>
</tr>
<tr>
<td>electric, $J = 2$</td>
<td>1123(23) MeV</td>
<td>257(40) MeV</td>
<td>1.09(3) fm</td>
</tr>
</tbody>
</table>

Scaling

- Like in lattice gauge theory the physical scale can be changed by changing the value of the coupling constant $g$ (the scale is set by identifying $\sigma$ with $\sigma_{\text{physical}} = 4.2/\text{fm}^2$).

- Previous “pseudoparticle papers”: the dimensionless ratios $\chi^{1/4}/\sigma^{1/2}$ and $T_{\text{critical}}/\sigma^{1/2}$ are essentially independent of $g$.

- The estimated string breaking distances $R_{\text{sb}}(\text{magnetic, } J = 1)$, $R_{\text{sb}}(\text{electric, } J = 1)$ and $R_{\text{sb}}(\text{electric, } J = 2)$ vary by $\approx 5\%$, $\approx 10\%$ and $\approx 20\%$, while the extension of the spacetime region is increased by $\approx 50\%$. 

![Graphs showing estimated string breaking distances and the extension of spacetime as functions of $g$.](image-url)
The static adjoint potential (1)

- Symmetry group of states constrained by a pair of static sources $\phi^{(1)}(x)$ and $\phi^{(1)}(y)$:
  
  - $J_z = 0 \quad \rightarrow \quad SO(2) \otimes P_z \otimes P_x.$
  
  - $J_z \neq 0 \quad \rightarrow \quad SO(2) \otimes P_z.$

- The string trial states

  $$(\phi^{(1)}(x))^\dagger U^{(1)}(x; y) \phi^{(1)}(y) \coset{\Omega}, \quad |x - y| = R$$

  have quantum numbers $J_z = 0$, $P_z = +$ and $P_x = +$.

- Since we want to observe the decay of a string state into a two-gluelump state, we need two-gluelump trial states with the same quantum numbers:

  $$(G_x^{(\cdots)}(x)G_y^{(\cdots)}(y) + G_y^{(\cdots)}(x)G_y^{(\cdots)}(y) + G_z^{(\cdots)}(x)G_z^{(\cdots)}(y)) \coset{\Omega}$$

  $$(G_x^{(\cdots)}(x)G_x^{(\cdots)}(y) + G_y^{(\cdots)}(x)G_y^{(\cdots)}(y) - 2G_z^{(\cdots)}(x)G_z^{(\cdots)}(y)) \coset{\Omega}.$$
The static adjoint potential (2)

- We extract the adjoint potential from correlation matrices containing both string trial states and two gluelump trial states via effective masses.
  - The potential saturates at $V \approx 2m(\ldots)$.
  - The string breaking distances
    * $R_{sb}^{(\text{magnetic},J=1)} \approx 1.0 \text{ fm}$
    * $R_{sb}^{(\text{electric},J=1)} \approx 0.85 \text{ fm}$
  and the level ordering in qualitative agreement with lattice results ($R_{sb,\text{lattice}}^{(\text{magnetic},J=1)} = 1.0 \text{ fm} \ldots 1.25 \text{ fm}$).

![Graph](image-url)
The static adjoint potential (3)

- Mixing analysis to investigate, whether the string really breaks:
  - During the computation of effective masses we obtain approximations:
    
    \[|0\rangle \approx a_{\text{string}}^0 |\text{string}\rangle + a_{\text{two-gluelump}}^0 |\text{two-gluelump}\rangle\]
    
    \[|1\rangle \approx a_{\text{string}}^1 |\text{string}\rangle + a_{\text{two-gluelump}}^1 |\text{two-gluelump}\rangle,\]
    
    where \(|\text{string}\rangle\) and \(|\text{two-gluelump}\rangle\) are normalized trial states.
  - The amplitudes \(a_j\) indicate a smooth but rapid transition between string and two-gluelump states.

![Graphs showing ground state and first excited state](image_url)
Summary

• The static potential in the fundamental representation is linear for large separations (→ confinement).

• Static potentials in higher representations exhibit Casimir scaling.

• The static potential in the adjoint representation is in qualitative agreement with lattice results:
  – String breaking at $R_{sb} \approx 1.0 \text{ fm}$.
  – Level ordering correct: the “first excited string state” is below the “two gluelump ground state”.
  – Mixing analysis indicates a smooth transition between a string and a two gluelump state, when two static charges are separated adiabatically; the transition region is very narrow.
Further results and ongoing projects (1)

- SU(2) Yang-Mills theory:
  - Dimensionless ratios $\chi^{1/4}/\sigma^{1/2}$ ($\chi$: topological susceptibility) and $T_{\text{critical}}/\sigma^{1/2}$ ($T_{\text{critical}}$: critical temperature of the confinement deconfinement phase transition) exhibit excellent scaling behaviors and are in qualitative agreement with lattice results (F. Lenz, M.W., 2005).
  - Identification of typical properties of physically relevant gauge field configurations: extended structures, transverse degrees of freedom (M.W., 2006).
  - Glueball spectrum in qualitative agreement with lattice results (F. Lenz, J. W. Negele, M. Thies, 2007).
  - Cluster formation of topological charge seems to be in agreement with lattice results (E.-M. Ilgenfritz, S. Solbrig, 2008).
Further results and ongoing projects (2)

- 1+1 dimensional Gross-Neveu model:
  - Fermionic fields successfully treated within the pseudoparticle approach (M.W., 2007).

- Schwinger model (ongoing):
  - Goal: identification of physically relevant fermionic field configurations (E. Radatz, M.W.).

- 1+1 dimensional Wess-Zumino model (ongoing):
  - Goal: pseudoparticle regularization of supersymmetric theories, which are difficult to handle on the lattice, due to lack of translational invariance.