The adjoint potential in the pseudoparticle approach: string breaking and Casimir scaling

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Part I: introduction to the pseudoparticle approach

- Basic principle, pseudoparticle ensembles and their building blocks.

Part II: adjoint string breaking in the pseudoparticle approach

- Pure Wilson loop static potentials, Casimir scaling.
- The static adjoint potential, string breaking.
Part I: introduction to the pseudoparticle approach
Basic principle (1)

- Pseudoparticle approach (PP approach; F. Lenz, M.W., 2005):
  - A numerical technique to approximate Euclidean path integrals.
  - In this talk: application to SU(2) Yang-Mills theory,

\[
\langle \mathcal{O} \rangle = \frac{1}{Z} \int D A \mathcal{O}[A] e^{-S[A]}
\]

\[
S[A] = \frac{1}{4g^2} \int d^4x F^a_{\mu\nu} F^a_{\mu\nu}, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \epsilon^{abc} A^b_\mu A^c_\nu.
\]

- Goals:
  * Build a model for SU(2) Yang-Mills theory with a small number of physically relevant degrees of freedom.
  * Analyze the importance of certain classes of gauge field configurations with respect to confinement and other essential properties of SU(2) Yang-Mills theory.
Basic principle (2)

- Related work:
  - Ensembles of calorons with non-trivial holonomy (P. Gerhold, E.-M. Ilgenfritz, M. Müller-Preussker, 2006).
  - Ensembles of dyons (D. Diakonov, V. Petrov, 2007).
Basic principle (3)

- PP: any gauge field configuration $a_\mu^a$, which is localized in space and in time.
- Consider only those gauge field configurations, which can be written as a sum of a fixed number of PPs:

$$A^a_\mu(x) = \sum_j A(j) C^{ab}(j) a^b_\mu(x - z(j))$$

($j$: PP index; $A(j) \in \mathbb{R}$: amplitude of the $j$-th PP; $C^{ab}(j) \in SO(3)$: color orientation of the $j$-th PP; $z(j) \in \mathbb{R}^4$: position of the $j$-th PP).

- Define the functional integration as an integration over PP amplitudes and color orientations:

$$\int DA \ldots \rightarrow \int \left( \prod_j dA(j) dC(j) \right) \ldots$$
Basic principle (4)

- PP ensemble: 625 long range PPs ("instantons", "antiinstantons", akyrons) inside a periodic spacetime hypercube (extension $5.0^4$):

\[
\begin{align*}
    a_{\mu,\text{instanton}}(x) &= \eta_{\mu\nu}^{a} \frac{x_{\nu}}{x^2 + \lambda^2}, \\
    a_{\mu,\text{antiinstanton}}(x) &= \tilde{\eta}_{\mu\nu}^{a} \frac{x_{\nu}}{x^2 + \lambda^2}, \\
    a_{\mu,\text{akyron}}(x) &= \delta^{a1} \frac{x_{\mu}}{x^2 + \lambda^2},
\end{align*}
\]

i.e.

\[
A_{\mu}^{a}(x) = \sum_{i} A(i) C^{ab}(i) a_{\mu,\text{instanton}}(x - z(i)) + \sum_{j} A(j) C^{ab}(j) a_{\mu,\text{antiinstanton}}(x - z(j)) + \sum_{k} A(k) C^{ab}(k) a_{\mu,\text{akyron}}(x - z(k)).
\]
Basic principle (5)

- $A^{\alpha}_{\mu}$ is not a classical solution (not even close to a classical solution), i.e. the PP approach is not a semiclassical method.
  - Long range interactions between PPs.
  - Variable amplitudes $A(i)$. 

Part II: adjoint string breaking in the pseudoparticle approach
String breaking

- **Static potential** $V(R)$: the energy of the lowest state containing two static charges $\phi$ and $\phi^\dagger$ (“two infinitely heavy quarks”) at separation $R$ (+ light particles [gluons, dynamical quarks, ...]).

- **String breaking in QCD**: when two static charges are separated adiabatically beyond a certain distance, the connecting gauge string breaks and they are screened by dynamical quarks; a pair of (essentially) non-interacting static-light mesons is formed.

- **String breaking in pure SU(2) Yang-Mills theory**:
  - Fundamental representation $(\phi^{(1/2)} = (\phi_1, \phi_2))$: no string breaking, since gluons are not able to screen charges in the fundamental representation.
  - Adjoint representation $(\phi^{(1)} = (\phi_1, \phi_2, \phi_3) \text{ or } \phi^{(1)} = \phi_a \sigma_a / 2)$: when two static charges are separated adiabatically beyond a certain distance, the connecting gauge string breaks and they are screened by gluons; a pair of (essentially) non-interacting gluelumps is formed.
Pure Wilson loop static potentials (1)

- The starting point to extract the energy of the lowest state containing two static charges \( \phi^{(J)} \) and \( (\phi^{(J)})^\dagger \) in spin-\( J \)-representation are “string trial states”

\[
S^{(J)}(x, y)|\Omega\rangle = (\phi^{(J)}(x))^\dagger U^{(J)}(x; y)\phi^{(J)}(y)|\Omega\rangle, \quad |x - y| = R.
\]

- We consider temporal correlations

\[
C^{(J)}_{\text{string}}(T) = \langle \Omega | \left( S^{(J)}(x, y, T) \right)^\dagger S^{(J)}(x, y, 0)|\Omega\rangle = \langle W^{(J)}_{(R, T)} \rangle
\]

and compute the corresponding potential values from effective mass plateaus,

\[
m^{(J)}_{\text{effective, string}}(T) = -\frac{1}{a} \ln \frac{C^{(J)}_{\text{string}}(T)}{C^{(J)}_{\text{string}}(T - a)}.
\]
Pure Wilson loop static potentials (2)

- Numerical results for the fundamental representation ($J = 1/2$):
  - The potential is linear for large separations, i.e. confinement.
  - The scale is set by fitting $V^{(1/2)}(R) = V_0 + \sigma R$ and by identifying $\sigma$ with $\sigma_{\text{physical}} = 4.2/\text{fm}^2$.
  - A coupling constant of $g = 12.5$ (standard choice for results shown in this talk) corresponds to a spacetime region of $L^4 = (1.85 \text{ fm})^4$.
  - Like in lattice gauge theory, the scale can be changed by changing the value of $g$, e.g. $g = 9.5\ldots18.5$ corresponds to $L = 1.55 \text{ fm}\ldots2.31 \text{ fm}$.

![Diagram of effective masses and fundamental potential]
• Numerical results for higher representations ($J = 1, \ldots, J = 5/2$):

  – Higher representation potentials exhibit Casimir scaling:

    $$V^{(1/2)}(R) \approx \frac{V^{(1)}(R)}{8/3} \approx \frac{V^{(3/2)}(R)}{5} \approx \frac{V^{(2)}(R)}{8} \approx \frac{V^{(5/2)}(R)}{35/3}.$$ 

  – The adjoint potential ($J = 1$) shows no sign of string breaking even at separations $R \approx 1.6$ fm.
Why is string breaking elusive?

- Static potential at small separations $R$: the string trial state has good overlap to the physical ground state, which is expected to be a string state.

- Static potential at large separations $R$: the string trial state has poor overlap to the physical ground state, which is expected to be a two gluelump state.

- Solution: Extend the set of trial states by “two-gluelump trial states”, which are supposed to have good overlap to the physical ground state at large $R$,

$$\sum_{j=x,y,z} G^{(...)}_j(x) G^{(...)}_j(y) |\Omega\rangle , \quad |x - y| = R$$

with

$$G^{(J=1,P=+)}_j(x) = \text{Tr} \left( \phi^{(1)}(x) B_j(x) \right) \quad \text{(magnetic gluelump)}$$

$$G^{(J=1,P=-)}_j(x) = \text{Tr} \left( \phi^{(1)}(x) \epsilon_{jkl} D_k B_l(x) \right) \quad \text{(electric gluelump)}$$

$$G^{(J=2,P=-)}_j(x) = \text{Tr} \left( \phi^{(1)}(x) |\epsilon_{jkl}| D_k B_l(x) \right).$$

The static adjoint potential (1)

- We extract the adjoint potential from correlation matrices containing both string trial states and magnetic two gluelump trial states via effective masses.
  - The potential saturates at $V^{(1)} \approx 2m^{(J=1,P=+)}$.
  - The string breaking distance $R_{sb}^{(J=1,P=+)} \approx 1.0$ fm and the level ordering are in qualitative agreement with lattice results ($R_{sb,lattice}^{(J=1,P=+)} = 1.0 \text{ fm} \ldots 1.25 \text{ fm}$).

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**magnetic trial states**

**magnetic two-gluelump trial states**

- second excited state
- first excited state
- ground state

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**lattice result**

(P. de Forcrand, O. Philipsen, hep-lat/9912050)
The static adjoint potential (2)

- Mixing analysis to investigate, whether the string really breaks:
  - During the computation of effective masses we obtain approximations
    \[ |0\rangle \approx a_{\text{string}}^0 |\text{string}\rangle + a_{\text{two-gluelump}}^0 |\text{two-gluelump}\rangle \]
    \[ |1\rangle \approx a_{\text{string}}^1 |\text{string}\rangle + a_{\text{two-gluelump}}^1 |\text{two-gluelump}\rangle, \]
    where \( |\text{string}\rangle \) and \( |\text{two-gluelump}\rangle \) are normalized trial states.
  - The amplitudes \( a_j \) indicate a smooth but rapid transition between string and two-gluelump states.


- Overlaps of the ground state approximation
- Overlaps of the first excited state approximation

lattice result
(P. de Forcrand, O. Philipsen, hep-lat/9912050)
Summary

- The static potential in the fundamental representation is linear for large separations (→ confinement).
- Static potentials in higher representations exhibit Casimir scaling.
- The static potential in the adjoint representation is in qualitative agreement with lattice results:
  - String breaking at ≈ 1.0 fm.
  - Correct level ordering: the “first excited string state” is below the “two gluelump ground state”.
  - A mixing analysis indicates a smooth transition between a string and a two gluelump state, when two static charges are separated adiabatically; the transition region is very narrow.
Outlook

- Apply the PP approach to fermionic theories:
  - First steps regarding the inclusion of fermionic fields in the PP approach have been successful (phase diagram of the GN model, M.W., 2007).
  - Apply the PP approach to QCD:
    * Cheap computation of exact all-to-all propagators.
    * Identification of properties of physically relevant fermionic field configurations.
  - Apply the PP approach to supersymmetric theories:
    * Exact supersymmetry is possible due to exact translational invariance.