Studying tetraquark candidates using lattice QCD

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• The nonet of light scalar mesons ($J^P = 0^+$)

$\sigma \equiv f_0(500), I = 0, 400 \ldots 550 \text{ MeV}$ \hspace{1cm} ($\bar{s}s \ldots?$),

$\kappa \equiv K_0^*(800), I = 1/2, 682 \pm 29 \text{ MeV}$ \hspace{1cm} ($\bar{s}u, \bar{s}d, \bar{u}s, \bar{d}s \ldots?$),

$a_0(980), I = 1, 980 \pm 20 \text{ MeV}$ \hspace{1cm} ($\bar{u}d, \bar{d}u, \bar{u}u - \bar{d}d \ldots?$)

$f_0(980), I = 0, 990 \pm 20 \text{ MeV}$ \hspace{1cm} ($\bar{u}u + \bar{d}d \ldots?$)

is poorly understood:

– All nine states are unexpectedly light (should rather be close to the corresponding $J^P = 1^+, 2^+$ states around 1200 $\ldots$ 1500 MeV).

– The ordering of states is inverted compared to expectation:

  * E.g. in a $q\bar{q}$ picture the $I = 1$ $a_0(980)$ states must necessarily be formed by two $u/d$ quarks, while the $I = 1/2$ $\kappa$ states are made from an $s$ and a $u/d$ quark; since $m_s > m_{u/d}$ one would expect $m(\kappa) > m(a_0(980))$. 
In a tetraquark picture the quark content could e.g. be the following:

\[ \kappa \equiv \bar{s}u(\bar{u}u + \bar{d}d) \] (one \( s \) quark, three light quarks)

\[ a_0(980) \equiv \bar{s}ud\bar{s} \] (two \( s \) quarks, two light quarks);

this would naturally explain the observed ordering.

− Certain decays also support a tetraquark interpretation: e.g. \( a_0(980) \) readily decays to \( K + \bar{K} \), which indicates that besides the two light quarks required by \( I = 1 \) also an \( s\bar{s} \) pair is present.

→ Study such states by means of lattice QCD to confirm or to rule out their interpretation in terms of tetraquarks.
• Examples of heavy mesons, which are tetraquark candidates:
  
  – $D_{s0}^*(2317)^\pm$, $D_{s1}(2460)^\pm$,
  
  – charmonium states $X(3872)$, $Z(4430)^\pm$, $Z(4050)^\pm$, $Z(4250)^\pm$, ...
  
  – $\bar{c}c\bar{c}c$ (experimentally not yet observed, predicted by theory) ...
    
  
  – $bb(\bar{u}\bar{d} - \bar{d}\bar{u})$ (experimentally not yet observed, predicted by theory) ...
    
(1) Wilson twisted mass study of $a_0(980)$:

[C. Alexandrou et al. [ETM Collaboration], JHEP 1304, 137 (2013) [arXiv:1212.1418 [hep-lat]]]

- Wilson twisted mass fermions (generated by the ETM Collaboration).
  [R. Baron et al., JHEP 1006, 111 (2010) [arXiv:1004.5284 [hep-lat]]]
- Computations at light $u/d$ quark masses corresponding to $m_\pi \gtrsim 280$ MeV.
- No disconnected diagrams/closed fermion loops.

(2) Recent technical advances:

- Wilson + clover fermions (generated by the PACS-CS Collaboration).
- Computations close to physically light $u/d$ quark masses.
- Inclusion of disconnected diagrams/closed fermion loops.

(3) Exploring a possibly existing $\bar{c}c\bar{c}c$ tetraquark.

(4) Static-static-light-light tetraquarks (close to $bb(\bar{u}d - \bar{d}u)$).
Lattice QCD hadron spectroscopy (1)

• Lattice QCD: discretized version of QCD,

\[
S = \int d^4 x \left( \sum_{\psi\in\{u,d,s,c,t,b\}} \bar{\psi}(\gamma^\mu (\partial_\mu - iA_\mu) + m(\psi)) \psi + \frac{1}{2g^2} \text{Tr}(F_{\mu\nu}F_{\mu\nu}) \right)
\]

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].
\]

• Let \( \mathcal{O} \) be a suitable “hadron creation operator”, i.e. an operator formed by quark fields \( \psi \) and gluonic fields \( A_\mu \) such that \( \mathcal{O}|\Omega\rangle \) is a state containing the hadron of interest (\(|\Omega\rangle\): QCD vacuum).

• More precisely: ... an operator such that \( \mathcal{O}|\Omega\rangle \) has the same quantum numbers (\( J^{PC} \), flavor) as the hadron of interest.

• Examples:
  
  – Pion creation operator: \( \mathcal{O} = \int d^3 x \bar{u}(x)\gamma_5 d(x). \)
  
  – Proton creation operator: \( \mathcal{O} = \int d^3 x \epsilon^{abc} u^a(x)(u^b T(x)C\gamma_5 d^c(x)). \)
Lattice QCD hadron spectroscopy (2)

- Determine the mass of the ground state of the hadron of interest from the exponential behavior of the corresponding correlation function $C$ at large Euclidean times $t$:

$$C(t) = \langle \Omega | \mathcal{O}(t) \mathcal{O}(0) | \Omega \rangle = \langle \Omega | e^{+Ht} \mathcal{O}(0) e^{-Ht} \mathcal{O}(0) | \Omega \rangle =$$

$$= \sum_n \left| \langle n | \mathcal{O}(0) | \Omega \rangle \right|^2 \exp \left( - (E_n - E_\Omega) t \right) \approx \left( \text{for } "t \gg 1" \right)$$

$$\approx \left| \langle 0 | \mathcal{O}(0) | \Omega \rangle \right|^2 \exp \left( - \overbrace{(E_0 - E_\Omega) t}^{m(\text{hadron})} \right).$$

- Usually the exponent is determined by identifying the plateau value of a so-called effective mass:

$$m_{\text{effective}}(t) = \frac{1}{a} \ln \left( \frac{C(t)}{C(t+a)} \right) \approx \left( \text{for } "t \gg 1" \right)$$

$$\approx E_0 - E_\Omega = m(\text{hadron}).$$
Part 1:
Wilson twisted mass study of $a_0(980)$
Tetraquark creation operators

- $a_0(980)$:
  - Quantum numbers $I(J^P) = 1(0^+)$.  
  - Mass $980 \pm 20$ MeV.

- Tetraquark creation operators:
  - Two light quarks needed, due to $I = 1$, e.g. $u\bar{d}$.
  - $a_0(980)$ decays to $K\bar{K}$ ... suggests an additional $s\bar{s}$ pair.
  - $K\bar{K}$ molecule type (models a bound $K\bar{K}$ state):
    $$\mathcal{O}_{a_0(980)}^{K\bar{K} \text{ molecule}} = \sum_x \left( \bar{s}(x)\gamma_5 u(x) \right) \left( \bar{d}(x)\gamma_5 s(x) \right).$$
  - Diquark type (models a bound diquark-antidiquark):
    $$\mathcal{O}_{a_0(980)}^{\text{diquark}} = \sum_x \left( \epsilon^{abc} \bar{s}^b(x) C\gamma_5 \bar{d}^{c,T}(x) \right) \left( \epsilon^{ade} u^{d,T}(x) C\gamma_5 s^e(x) \right).$$

Marc Wagner, “Studying tetraquark candidates using lattice QCD”, Feb 17, 2014
Wilson twisted mass lattice setup

- Gauge link configurations generated by the ETM Collaboration.
  
  [R. Baron et al., JHEP 1006, 111 (2010) [arXiv:1004.5284 [hep-lat]]]

- 2+1+1 dynamical Wilson twisted mass quark flavors, i.e. $u$, $d$, $s$ and $c$ sea quarks (twisted mass lattice QCD isospin and parity are slightly broken).

- Various light $u/d$ quark masses corresponding pion masses $m_\pi \approx 280 \ldots 460$ MeV.

- Singly disconnected contributions/closed fermion loops neglected, i.e. no $s$ quark propagation within the same timeslice (“no quark antiquark pair creation/annihilation”).
Numerical results $a_0(980)$ (1)

- Effective mass, molecule type operator:

$$O^{K\bar{K} \text{ molecule}}_{a_0(980)} = \sum_x \left( \bar{s}(x) \gamma_5 u(x) \right) \left( \bar{d}(x) \gamma_5 s(x) \right).$$

- The effective mass plateau indicates a state, which is roughly consistent with the experimentally measured $a_0(980)$ mass 980 ± 20 MeV.

- Conclusion: $a_0(980)$ is a tetraquark state of $K\bar{K}$ molecule type ...

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Numerical results $a_0(980)$ (2)

- Effective mass, diquark type operator:

$$O_{a_0(980)}^{\text{diquark}} = \sum_x \left( \epsilon^{abc} \bar{s}^b(x) C \gamma_5 \bar{d}^{c,T}(x) \right) \left( \epsilon^{ade} u^{d,T}(x) C \gamma_5 s^e(x) \right).$$

- The effective mass plateau indicates a state, which is roughly consistent with the experimentally measured $a_0(980)$ mass $980 \pm 20$ MeV.

- Conclusion: $a_0(980)$ is a tetraquark state of diquark type ...? Or a mixture of $K\bar{K}$ molecule and a diquark-antidiquark pair?

![Graph of effective mass vs. t/a](image)
Numerical results $a_0(980)$ (3)

- Study both operators at the same time, extract the two lowest energy eigenstates by diagonalizing a $2 \times 2$ correlation matrix ("generalized eigenvalue problem"): 

$$
O_{a_0(980) \text{ molecule}} = \sum_x \left( \bar{s}(x) \gamma_5 u(x) \right) \left( \bar{d}(x) \gamma_5 s(x) \right) 
$$

$$
O_{a_0(980) \text{ diquark}} = \sum_x \left( \epsilon^{abc} \bar{s}^b(x) C \gamma_5 \bar{d}^c,T(x) \right) \left( \epsilon^{ade} u^d,T(x) C \gamma_5 s^e(x) \right). 
$$

- Now two orthogonal states roughly consistent with the experimentally measured $a_0(980)$ mass $980 \pm 20$ MeV ...?
Two-particle creation operators (1)

- Explanation: there are two-particle states, which have the same quantum numbers as $a_0(980)$, $I(J^{PC}) = 1(0^{++})$,

  \[- K + \bar{K} \ (m(K) \approx 500 \text{ MeV}), \]
  \[- \eta_s + \pi \ (m(\eta_s \equiv \bar{s}\gamma_5 s) \approx 700 \text{ MeV}, m(\pi) \approx 300 \text{ MeV in our lattice setup}), \]

  which are both around the expected $a_0(980)$ mass $980 \pm 20 \text{ MeV}$. 

- What we have seen in the previous plots might actually be two-particle states (our operators are of tetraquark type, but they nevertheless generate overlap [possibly small, but certainly not vanishing] to two-particle states).

- To determine, whether there is a bound $a_0(980)$ tetraquark state, we need to resolve the above listed two-particle states $K + \bar{K}$ and $\eta_s + \pi$ and check, whether there is an additional 3rd state in the mass region around $980 \pm 20 \text{ MeV}$; to this end we need operators of two-particle type.
Two-particle creation operators (2)

- Two-particle operators:
  - Two-particle $K + \bar{K}$ type:
    \[
    \mathcal{O}_{a_0(980)}^{K + \bar{K} \text{ two-particle}} = \left( \sum_x \bar{s}(x) \gamma_5 u(x) \right) \left( \sum_y \bar{d}(y) \gamma_5 s(y) \right).
    \]
  - Two-particle $\eta_s + \pi$ type:
    \[
    \mathcal{O}_{a_0(980)}^{\eta_s + \pi \text{ two-particle}} = \left( \sum_x \bar{s}(x) \gamma_5 s(x) \right) \left( \sum_y \bar{d}(y) \gamma_5 u(y) \right).
    \]
Numerical results $a_0(980)$ (4)

- Study all four operators ($K\bar{K}$ molecule, diquark, $K+\bar{K}$ two-particle, $\eta_s+\pi$ two-particle) at the same time, extract the four lowest energy eigenstates by diagonalizing a $4 \times 4$ correlation matrix (left plot).

  - Still only two low-lying states around $980 \pm 20$ MeV, the 2nd and 3rd excitation are $\approx 750$ MeV heavier.
  - The signal of the low-lying states is of much better quality than before (when we only considered tetraquark operators) → suggests that the observed low-lying states have much better overlap to the two-particle operators and are most likely of two-particle type.
When determining low-lying eigenstates from a correlation matrix, one does not only obtain their mass, but also information about their operator content, i.e. which percentage of which operator is present in an extracted state:

→ The ground state is a $\eta_s + \pi$ state ($\gtrsim 95\%$ two-particle $\eta_s + \pi$ content).
→ The first excitation is a $K + \bar{K}$ state ($\gtrsim 95\%$ two-particle $K + \bar{K}$ content).
Numerical results $a_0(980)$ (6)

- What about the 2nd and 3rd excitation? ... Are these tetraquark states? ... What is their nature?

- Two-particle states with one relative quantum of momentum (one particle has momentum $+p_{\text{min}} = +2\pi/L$ the other $-p_{\text{min}}$) also have quantum numbers $I(J^{PC}) = 1(0^{++})$; their masses can easily be estimated:
  
  - $p_{\text{min}} = 2\pi/L \approx 715$ MeV (the results presented correspond to a small lattice with spatial extension $L = 1.73$ fm);
  
  - $m(K(+p_{\text{min}}) + \bar{K}(-p_{\text{min}})) \approx 2\sqrt{m(K)^2 + p_{\text{min}}^2} \approx 1750$ MeV;
  
  - $m(\eta(+p_{\text{min}}) + \pi(-p_{\text{min}})) \approx \sqrt{m(\eta)^2 + p_{\text{min}}^2} + \sqrt{m(\pi)^2 + p_{\text{min}}^2} \approx 1780$ MeV;

  these estimated mass values are consistent with the observed mass values of the 2nd and 3rd excitation → suggests to interpret these states as two-particle states.
Numerical results $a_0(980)$ (7)

- **Summary:**
  - In the $a_0(980)$ sector (quantum numbers $I(J^{PC}) = 1(0^{++})$) we do not observe any low-lying (mass $\lesssim 1750$ MeV) tetraquark state, even though we employed operators of tetraquark structure ($K\bar{K}$ molecule, diquark).
  - The experimentally measured mass for $a_0(980)$ is $980 \pm 20$ MeV.
  - **Conclusion:** $a_0(980)$ does not seem to be a strongly bound tetraquark state (either of molecule or of diquark type) ... maybe an ordinary quark-antiquark state or a rather unstable resonance.
Part 2: Recent technical advances
Wilson + clover lattice setup

- Gauge link configurations generated by the PACS-CS Collaboration.

- 2+1 dynamical Wilson + clover quark flavors, i.e. $u$, $d$ and $s$ sea quarks.
  $\rightarrow$ In contrast to twisted mass parity and isospin are exact symmetries, i.e. no pion and kaon mass splitting, easy separation of $P = +, -$ states, ...

- Light $u/d$ quark masses corresponding to pion masses $m_\pi \approx 150$ MeV and $m_\pi \approx 300$ MeV.
  $\rightarrow$ Computations close to physically light $u/d$ quark masses possible.

- Singly disconnected contributions/closed fermion loops included.
  $\rightarrow$ $s$ quark propagation within the same timeslice ("quark antiquark pair creation/annihilation taken into account").
Closed fermion loops (1)

- In our previous Wilson twisted mass study of \(a_0(980)\) we neglected singly disconnected contributions/closed fermion loops:

  \[\mathcal{O}_{a_0(980)}^{q\bar{q}} = \sum_x \left( \bar{d}(x) u(x) \right),\]

  because cross correlations between this operator and any of the four-quark operators \(\mathcal{O}_{a_0(980)}^{K\bar{K} \text{ molecule}}, \mathcal{O}_{a_0(980)}^{\text{diquark}}, \mathcal{O}_{a_0(980)}^{K+\bar{K} \text{ two-particle}}\) or \(\mathcal{O}_{a_0(980)}^{\eta_s+\pi \text{ two-particle}}\) correspond to closed fermion loops.

  → Also correlations between the four-quark operators include closed fermion loops; therefore, we introduced a source of systematic error, which is difficult to estimate or to control.
Closed fermion loops (2)

- Technical aspects of disconnected diagrams/closed fermion loops:
  - Blue: point-to-all propagators applicable.
  - Red: due to \( \sum_x \), timeslice-to-all propagators needed.
  - Timeslice-to-all propagators can be estimated stochastically.
  - Using several stochastic timeslice-to-all propagators results in a poor signal-to-noise ratio.
  
  → Combine three point-to-all (blue) and one stochastic timeslice-to-all (red) propagator.

\[ \sum_x \text{ needed, to have } \mathbf{p} = 0 \]

\[ \sum_x \text{ can be omitted, because of translational invariance} \]
Closed fermion loops (3)

- Effective masses from a $4 \times 4$ correlation matrix ($\mathcal{O}_{q\bar{q}}^{a_0(980)}$, $\mathcal{O}_{K\bar{K}}^{a_0(980)}$, $\mathcal{O}_{a_0(980)}^{\eta_s\pi}$ molecule, $\mathcal{O}_{a_0(980)}^{\text{diquark}}$) at $m_\pi \approx 300\text{ MeV}$:
  - Lowest (two) energy level(s) roughly consistent with $K + \bar{K}$, $\eta + \pi$ and a possibly existing additional $a_0(980)$ state.
  - For physically interesting statements we need smaller errors and to include $\mathcal{O}_{a_0(980)}^{K\bar{K} \text{ two-particle}}$ and $\mathcal{O}_{a_0(980)}^{\eta_s+\pi \text{ two-particle}}$ (work in progress).
Work in progress, outlook

- Enlarge correlation matrices such that
  - $q\bar{q}$ operators,
  - tetraquark operators (mesonic molecules, diquark-antidiquark pairs),
  - two-meson operators

  are included.

- Perform computations at pion mass $m_\pi \approx 150$ MeV.

- Address various physical questions/systems (tetraquark candidates with different flavor structure, search for additional bound states, ...).
Part 3:
Exploring a possibly existing $\bar{c}c\bar{c}c$ tetraquark
\( \bar{c}c\bar{c}c \) tetraquark ...? (1)

- Recently a \( \bar{c}c\bar{c}c \) tetraquark has been predicted
  - using a coupled system of covariant Bethe-Salpeter equations,
  - mass \( m(\bar{c}c\bar{c}c) = (5.3 \pm 0.5) \text{ GeV} \),
  - predominantly of mesonic molecule type (two \( \eta_c \) mesons),
  - rather strongly bound \( (2 \times m(\eta_c) = 6.0 \text{ GeV}) \), binding energy \( \Delta E = m(\bar{c}c\bar{c}c) - 2 \times m(\eta_c) \approx -(0.7 \pm 0.5) \text{ GeV} \).


- Should be within experimental reach (PANDA experiment).
  \( \rightarrow \) Investigate the existence of this \( \bar{c}c\bar{c}c \) state using lattice QCD.
\( \bar{c}c\bar{c}c \) tetraquark …? (2)

- Use the same techniques and setup as discussed for the \( a_0(980) \) meson.
- First attempt:
  - Molecule type \( \bar{c}c\bar{c}c \) creation operator (models a bound \( \eta_c\eta_c \) state):
    \[
    \mathcal{O}_{\bar{c}c\bar{c}c}^{\eta_c\eta_c \text{ molecule}} = \sum_x \left( \bar{c}(x)\gamma_5 c(x) \right) \left( \bar{c}(x)\gamma_5 c(x) \right).
    \]
  - Inconclusive results:
    * Neither an indication for a \( \bar{c}c\bar{c}c \) state significantly below \( 2 \times m(\eta_c) \) …
    * … nor can the existence of such a state be ruled out

(the effective mass still decreases at large temporal separations \( t \), which signals a trial state \( \mathcal{O}_{\bar{c}c\bar{c}c}^{\eta_c\eta_c \text{ molecule}} |\Omega\rangle \), which has a poor ground state overlap; the ground state could be \( |\eta_c + \eta_c\rangle \) or \( |\bar{c}c\bar{c}c\rangle \) of different structure).
The molecule type $\bar{c}c\bar{c}c$ creation operator used generates a trial state with the two $\eta_c$ mesons essentially on top of each other.

In a possibly existing $\bar{c}c\bar{c}c$ tetraquark state the two $\eta_c$ mesons could be quite far separated, which would imply a poor overlap of the above trial state with the $\bar{c}c\bar{c}c$ state.

Therefore, we also employed an improved molecule type $\bar{c}c\bar{c}c$ creation operator:

$$O_{\eta_c\eta_c \text{molecule}}^{\bar{c}c\bar{c}c}(d) = \sum_x \left( \bar{c}(x) \gamma_5 c(x) \right) \sum_{n=\pm e_x, \pm e_y, \pm e_z} \left( \bar{c}(x + d n) \gamma_5 c(x + r n) \right)$$

($d$ models the size of the mesonic molecule, the separation of the two $\eta_c$ mesons).
\( \bar{c}c\bar{c}c \) tetraquark …? (4)

- Still no sign of a \( \bar{c}c\bar{c}c \) state significantly below \( 2 \times m(\eta_c) \) …
  - Left plot: \( d \approx 0.00 \text{ fm} , \ 0.45 \text{ fm} , \ 0.72 \text{ fm} \).
  - Right plot: solving a generalized eigenvalue problem.

- We plan to explore the dependence of the results on the quark masses, in particular the existence of a bound four-quark state (lattice results strongly indicate that two \( B \) mesons can form a bound \( bb(\bar{u}\bar{d} - \bar{d}u) \) state) …

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Part 4:
Static-static-light-light tetraquarks
Static-static-light-light tetraquarks (1)

- Study possibly existing $QQ\bar{q}\bar{q}$ (heavy-heavy-light-light) tetraquark states:
  - Use the static approximation for the heavy quarks $QQ$ (reduces the necessary computation time significantly).
  - Most appropriate for $QQ \equiv bb$.
  - Could also yield information for $QQ \equiv cc$.

- Proceed in two steps:
  1. Compute the potential of two heavy quarks $QQ$ in the background of two light antiquarks $\bar{q}\bar{q}$ by means of lattice QCD

\[
\mathcal{O}_{QQ\bar{q}\bar{q}} = (C\Gamma)_{AB} \left( Q_C(x_1)\bar{q}_A^{(1)}(x_1) \right) \left( Q_C(x_2)\bar{q}_B^{(2)}(x_2) \right)
\]

($R = |x_1 - x_2|$, $\bar{q}^{(1)}\bar{q}^{(2)} \in \{ud - du, uu, dd, ud + du\}$, $C =$ charge conjugation matrix, $\Gamma =$ any $\gamma$ combination) → many different channels/quantum numbers.


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Static-static-light-light-light tetraquarks (2)

(2) Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $QQ$.

• Clear indication for a bound state for $QQ \equiv bb$ in a specific channel:
  
  – Quantum numbers: $I(J^P) = 0(0^+) , 0(1^+) \quad$ (degeneracy with respect to the heavy spin).
  
  – Binding energy: $E \approx -50 \text{ MeV}$.


• No four-quark binding in other channels.

• Next steps:
  
  – Extend from $QQ\bar{q}\bar{q}$ to $Q\bar{Q}q\bar{q}$ (experimentally more realistic/interesting).
  
  – Establish connection to computations with four quarks of finite mass.

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