$B B$, $B \bar{B}$ and hybrid static potentials from lattice QCD

Effective Field Theory Seminar – Technische Universität München, Germany

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Goals, motivation (1)

- Study exotic mesons (tetraquarks/mesonic molecules, hybrid mesons) by combining lattice QCD and phenomenology/model calculations.

- Compute the potential of two heavy valence quarks
  - in the presence of two additional light valence quarks (tetraquarks/mesonic molecules),
  - in the presence of gluonic excitations (hybrid mesons)

  using lattice QCD.

- Explore, whether the potentials are sufficiently attractive to generate a bound state (a rather stable exotic meson) using phenomenology/model calculations.
Goals, motivation (2)

- Why are such investigations important?
  
  Quite a number of mesons are only poorly understood.

  - Example $X(3872)$ ($\bar{c}c$ state): mass not as expected from quark models; could be a $D-D^*$ molecule, a bound diquark-antidiquark, ...

  - Example $D_{s0}^*(2317)$, $D_{s1}(2460)$: masses significantly lower than expected from quark models, almost equal or even lower than the corresponding $D$ mesons; could be tetraquarks, ...

  - Charged bottomonium states, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$ ... must be four quark states.

  - Charged charmonium states, e.g. $Z_c(3940)^\pm$ and $Z_c(4430)^\pm$ ... must be four quark states.

  - Mesons with non-quark model quantum numbers, e.g. $\pi_1(1400)$, $\pi_1(1600)$ ... candidates for hybrid mesons.
Goals, motivation (3)

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Outline

- A brief introduction to lattice QCD hadron spectroscopy.
  - QCD (quantum chromodynamics).
  - Hadron spectroscopy.
  - Lattice QCD.

- Ongoing lattice projects:
  1. $B\bar{B}$ static potentials.
  2. $B\bar{B}$ static potentials.
  3. Hybrid static potentials.

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QCD (quantum chromodynamics)

- Quantum field theory of quarks (six flavors $u, d, s, c, t, b$, which differ in mass) and gluons.

- Part of the standard model explaining the formation of hadrons (usually mesons = $q\bar{q}$ and baryons = $qqq/\bar{q}\bar{q}\bar{q}$) and their masses; essential for decays involving hadrons.

- Definition of QCD simple:

  $$S = \int d^4x \left( \sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}(f) \left( \gamma_\mu \left( \partial_\mu - iA_\mu \right) + m(f) \right) \psi(f) + \frac{1}{2g^2} \text{Tr} \left( F_{\mu\nu}F_{\mu\nu} \right) \right)$$

  $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu].$$

- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of the absence of any small parameter (i.e. perturbation theory not applicable).
  
  → Solve QCD numerically by means of lattice QCD.
Hadron spectroscopy

- Proceed as follows:
  1. Compute the temporal correlation function $C(t)$ of a suitable hadron creation operator $O$ (an operator $O$, which generates the quantum numbers of the hadron of interest, when applied to the vacuum $|\Omega\rangle$).
  2. Determine the corresponding hadron mass from the asymptotic exponential decay in time.

- Example: $D$ meson mass $m_D$ (valence quarks $\bar{c}$ and $u$, $J^P = 0^-$),

  $$O \equiv \int d^3r \bar{c}(r)\gamma_5 u(r)$$

  $$C(t) \equiv \langle \Omega|O^\dagger(t)O(0)|\Omega\rangle \underset{t \to \infty}{\overset{\propto}{\propto}} \exp \left( -m_D t \right).$$

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![Graph showing $O^+(t)$ vs. $t$](example: D meson)
Lattice QCD (1)

- To compute a temporal correlation function $C(t)$, use the path integral formulation of QCD,

$$C(t) = \langle \Omega | O^\dagger(t)O(0) | \Omega \rangle = \frac{1}{Z} \int \left( \prod_f D\psi^{(f)} D\bar{\psi}^{(f)} \right) DA_\mu O^\dagger(t)O(0) e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}.$$

- $|\Omega\rangle$: ground state/vacuum.
- $O^\dagger(t), O(0)$: functions of the quark and gluon fields (cf. previous slides).
- $\int \left( \prod_f D\psi^{(f)} D\bar{\psi}^{(f)} \right) DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(x,t)$ and $A_\mu(x,t)$.
- $e^{-S[\psi^{(f)},\bar{\psi}^{(f)},A_\mu]}$: weight factor containing the QCD action.
Lattice QCD (2)

• Numerical implementation of the path integral formalism in QCD:
  
  – Discretize spacetime with sufficiently small lattice spacing
    \[ a \approx 0.05 \text{ fm} \ldots 0.10 \text{ fm} \]
    \rightarrow \text{“continuum physics”.}
  
  – “Make spacetime periodic” with sufficiently large extension
    \[ L \approx 2.0 \text{ fm} \ldots 4.0 \text{ fm} \text{ (4-dimensional torus)} \]
    \rightarrow \text{“no finite size effects”}.

\[
x_{\mu} = (n_0, n_1, n_2, n_3) \in \mathbb{Z}^4
\]
Lattice QCD (3)

- Numerical implementation of the path integral formalism in QCD:
  - After discretization the path integral becomes an ordinary multidimensional integral:
    \[
    \int D\psi D\bar{\psi} DA \ldots \rightarrow \prod_{\mu} \left( \int d\psi(x_\mu) d\bar{\psi}(x_\mu) dU(x_\mu) \right) \ldots
    \]
  - Typical present-day dimensionality of a discretized QCD path integral:
    * \( x_\mu \): \( 32^4 \approx 10^6 \) lattice sites.
    * \( \psi = \psi^{a,(f)}_A \): 24 quark degrees of freedom for every flavor \((\times 2 \text{ particle/antiparticle, } \times 3 \text{ color, } \times 4 \text{ spin})\), 2 flavors.
    * \( U = U^{ab}_\mu \): 32 gluon degrees of freedom \((\times 8 \text{ color, } \times 4 \text{ spin})\).
    * In total: \( 32^4 \times (2 \times 24 + 32) \approx 83 \times 10^6 \) dimensional integral.
    → standard approaches for numerical integration not applicable
    → sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).
Heavy-heavy-light-light tetraquarks (1)

- Study possibly existing $\bar{Q}Qqq$ and $\bar{Q}Q\bar{q}q$ tetraquark states ($q \in \{u,d,s,c\}$):
  - Use the static approximation for the heavy quarks $\bar{Q}Q$ and $\bar{Q}Q$ (reduces the necessary computation time significantly).
  - Most appropriate for $\bar{Q}Q \equiv \bar{b}b$ and $\bar{Q}Q \equiv \bar{b}b$, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$.  
  - Could also yield information about $\bar{Q}Q \equiv \bar{c}c$ and $\bar{Q}Q \equiv \bar{c}c$, e.g. $Z_c(3940)^\pm$ and $Z_c(4430)^\pm$.

- Proceed in two steps:
  1. Compute the potential of two heavy quarks $\bar{Q}Q$ and $\bar{Q}Q$ in the background of two light quarks $qq$ and $\bar{q}q$ by means of lattice QCD.
  2. Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $\bar{Q}Q$ and $\bar{Q}Q$; a bound state would indicate a tetraquark state.

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Since heavy $b$ quarks are treated in the static approximation, their spins are irrelevant (mesons are labeled by the spin of the light degrees of freedom $j$).

Consider only pseudoscalar/vector mesons ($j^P = (1/2)^{-}$, PDG: $B, B^*$) and scalar/pseudovector mesons ($j^P = (1/2)^{+}$, PDG: $B_0^*, B_1^*$), which are among the lightest static-light mesons.

Study the dependence of the mesonic potential $V(R)$ on

- the “light” quark flavors $u, d, s$ and/or $c$ (isospin),
- the “light” quark spin (the static quark spin is irrelevant),
- the type of the meson $S$ and/or $P_-$.

→ Many different channels/quantum numbers ... attractive, repulsive ...

\[
\overline{Q} \uparrow_{u} \quad V(R) = ? \quad \overline{Q} \uparrow_{d}
\]

\[
P = - \quad R \quad P = +
\]
In the following $\bar{Q}\bar{Q}qq$, i.e. "BB" (not $\bar{Q}Q\bar{q}q$, i.e. "B$\bar{B}"").

To extract the potential(s) of a given sector $(I, I_z, |j_z|, \mathcal{P}, \mathcal{P}_x)$, compute the temporal correlation function of the trial state

$$(C\Gamma)_{AB} \left( \bar{Q}_C(-R/2)q_A^{(1)}(-R/2) \right) \left( \bar{Q}_C(+R/2)q_B^{(2)}(+R/2) \right) |\Omega\rangle.$$

- $C = \gamma_0\gamma_2$ (charge conjugation matrix).
- $q^{(1)}q^{(2)} \in \{ud - du, uu, dd, ud + du, ss, cc\}$ (isospin $I, I_z$).
- $\Gamma$ is an arbitrary combination of $\gamma$ matrices (spin $|j_z|$, parity $\mathcal{P}, \mathcal{P}_x$).
\* \( I = 0 \) (left) and \( I = 1 \) (right); \(|j_z| = 0 \) (top) and \(|j_z| = 1 \) (bottom).
Focus on the two attractive channels between ground state static-light mesons “$B$ and/or $B^*$” (probably the best candidates to find a tetraquark):

- Scalar isosinglet (more attractive):
  \[ qq = (ud - du) / \sqrt{2}, \quad \Gamma = \gamma_5 + \gamma_0 \gamma_5, \]
  quantum numbers \( (I, |j_z|, \mathcal{P}, \mathcal{P}_x) = (0, 0, -, +). \)

- Vector isotriplet (less attractive):
  \[ qq \in \{ uu, (ud + du) / \sqrt{2}, dd \}, \quad \Gamma = \gamma_j + \gamma_0 \gamma_j, \]
  quantum numbers \( (I, |j_z|, \mathcal{P}, \mathcal{P}_x) = (1, \{0, 1\}, -, \pm). \)

Computations for \( qq = ll, ss, cc \ (l \in \{u, d\}) \) to study the mass dependence.
Two competing effects:

- The potential for light quarks is wider/deeper, i.e. favors the existence of a bound state (a tetraquark).
- Heavier quarks correspond to heavier mesons \((m(B) < m(B_s) < m(B_c))\), which form more readily a bound state (a tetraquark), i.e. require a less wide/deep potential for a bound state.

\[\text{BB \ static potentials/tetraquarks (4)}\]

**$BB$ static potentials/tetraquarks (5)**

- Solve the non-relativistic Schrödinger equation for the relative coordinate of the heavy quarks $\bar{Q}Q$,

\[
\left( -\frac{1}{2\mu} \nabla + V(r) \right) \psi(r) = E\psi(r), \quad \mu = \frac{m(B_{(s,c)})}{2};
\]

a bound state, i.e. $E_0 < 0$, would be an indication for a tetraquark state.

- Clear indication for a bound state for the scalar isosinglet and $qq = ll$ (i.e. $BB$), binding energy $E \approx -50$ MeV, confidence level $\approx 2\sigma$.

- No binding for the vector isotriplet or for $qq = ss, cc$ (i.e. $B_sB_s, B_cB_c$).

\[\mu = \frac{m_B}{2}, \quad a = 0.079 \text{ fm} \]
\[\mu = \frac{m_B}{2}, \quad a = 0.096 \text{ fm} \]
\[\mu = \frac{m_B}{2}, \quad a = 0.079 \text{ fm} \]
\[\mu = \frac{m_B}{2}, \quad a = 0.096 \text{ fm} \]
To quantify “no binding”, we list for each channel the factor, by which the effective mass $\mu$ in Schrödinger’s equation has to be multiplied, to obtain binding with confidence level $1\sigma$ and $2\sigma$ (the potential is not changed).

<table>
<thead>
<tr>
<th>flavor</th>
<th>light</th>
<th>strange</th>
<th>charm</th>
</tr>
</thead>
<tbody>
<tr>
<td>confidence level for binding</td>
<td>$1\sigma$</td>
<td>$2\sigma$</td>
<td>$1\sigma$</td>
</tr>
<tr>
<td>scalar isosinglet</td>
<td>0.8</td>
<td>1.9</td>
<td>3.1</td>
</tr>
<tr>
<td>vector isotriplet</td>
<td>1.9</td>
<td>2.5</td>
<td>3.4</td>
</tr>
</tbody>
</table>

- **Factors $\leq 1.0$ indicate binding.**
- **Light quarks ($u/d$) are unphysically heavy (correspond to $m_\pi \approx 340$ MeV); physically light $u/d$ quarks are expected to yield stronger binding for the scalar isosinglet, might lead to binding also for the vector isotriplet (computations in progress).**
- **Mass splitting $m(B^*) - m(B) \approx 50$ MeV, neglected at the moment, is expected to weaken binding (coupled channel analysis in progress).**

$B\bar{B}$ static potentials/tetraquarks (1)

- Experimentally more interesting case: $\bar{Q}Q\bar{q}q$, i.e. “$B\bar{B}$”, trial states

$$\gamma_5,_{AB}\Gamma_{CD}\left(\bar{Q}_A(-R/2)q_D^{(1)}(-R/2)\right)\left(\bar{q}_C(+R/2)Q_B^{(2)}(+R/2)\right)|\Omega\rangle.$$  

- At the moment only preliminary results for $\bar{q}q = \bar{c}c$, “$I = 1$”.

- Qualitative difference to $\bar{Q}\bar{Q}qq$: all channels are attractive (for $\bar{Q}\bar{Q}qq$ half of them are attractive, half of them are repulsive).
  
  - Can be understood by comparing the potential of $\bar{Q}Q$ and of $\bar{Q}\bar{Q}$ generated by one-gluon exchange.
  
  - For $\bar{Q}\bar{Q}$ the Pauli principle applied to $qq$ implies either a symmetric (sextet) or an antisymmetric (triplet) color orientation of the static quarks corresponding to a repulsive or attractive interaction, respectively.

  - For $\bar{Q}Q$ no such restriction is present, i.e. all channels contain contributions of the attractive color singlet, which dominates the repulsive color octet.

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Heavy-heavy-light-light-light tetraquarks (3)

- Future plans for $BB$ and $B\bar{B}$:
  - Computations with light $u/d$ quarks of physical mass ($m_\pi \approx 140$ MeV instead of $m_\pi \approx 340$ MeV).
  - Light quarks of different mass: $BB_s$, $BB_c$ and $B_sB_c$ potentials.
  - Refined model calculations with the resulting static-static-light-light potentials: take mass splitting $m(B^*) - m(B) \approx 50$ MeV into account (coupled channel analysis).

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Heavy-heavy-light-light tetraquarks (4)

- Future plans for $BB$ and $B\bar{B}$:
  - Study the structure of the states corresponding to the computed potentials:
    - In a lattice computation two different creation operators generating the same quantum numbers yield the same potential.
    - At the moment exclusively creation operators of mesonic molecule type.
    - For $BB$ use also
      - creation operators of diquark-antidiquark type.
    - For $B\bar{B}$ use also
      - creation operators of diquark-antidiquark type,
      - creation operators of bottomonium + pion type ($Q\bar{Q}$ string + $\pi$),
      - for $I = 0$ creation operators of bottomonium type ($Q\bar{Q}$ string).
    - Resulting correlation matrices provide information about the structure.
Hybrid static potentials (1)

- Hybrid mesons:
  - Quark antiquark states with excited gluonic fields.
  - Not restricted to quark model quantum numbers $J^{PC}$, where $P = (-1)^{L+1}$ and $C = (-1)^{L+S}$ ($L$: angular momentum, $S$: spin).
  - Exotic states with $J^{PC} = 0^{-+}, 0^{--}, 1^{--}, \ldots$ can be realized by excited gluonic fields.
  - Examples for $J^{PC} = 1^{-+}$ states: $\pi_1(1400)$, $\pi_1(1600)$. 

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Hybrid static potentials (2)

- Quantum numbers of states with a static quark and a static antiquark:
  - Angular momentum \( j_z \) with respect to the axis of separation; states with \( j_z = 0, \pm 1, \pm 2, \ldots \) are also labeled by \( \Sigma, \Pi, \Delta, \ldots \).
  - The combination of parity and charge conjugation \( P \circ C \); states with \( P \circ C = +, - \) are also labeled by \( g, u \).
  - Rotational invariant \( \Sigma \) states are either symmetric or antisymmetric with respect to spatial reflections along an axis perpendicular to the axis of separation denoted by \( P_x = +, - \).

- Example: the ordinary static potential has quantum numbers \( J_{P \circ C}^P = \Sigma_g^+ \).
- Hybrid static potentials: quantum numbers different from \( \Sigma_g^+ \).
Hybrid static potentials (3)

- Hybrid creation operators:

\[ O \equiv \bar{Q}(-R/2)U(-R/2; 0) \text{ insertion } U(0; +R/2)Q(+R/2). \]

- \( Q(+R/2), \bar{Q}(-R/2) \): static quark antiquark pair at separation \( R \).
- \( U(z_1, z_2) \): gluonic parallel transporter along the axis of separation,

\[ U(z_1, z_2) \equiv P\left( \exp \left( i \int_{z_1}^{z_2} dz A_z(z) \right) \right). \]

- “insertion”: cf. table.

<table>
<thead>
<tr>
<th>quantum numbers</th>
<th>( J^{P_x}_{P\circ C} )</th>
<th>operator insertions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma^+_g )</td>
<td>1, ( R \cdot E ), ( R \cdot (D \times B) )</td>
<td></td>
</tr>
<tr>
<td>( \Pi_g )</td>
<td>( R \times E ), ( R \times (D \times B) )</td>
<td></td>
</tr>
<tr>
<td>( \Sigma^-_u )</td>
<td>( R \cdot B ), ( R \cdot (D \times E) )</td>
<td></td>
</tr>
<tr>
<td>( \Pi_u )</td>
<td>( R \times B ), ( R \times (D \times E) )</td>
<td></td>
</tr>
<tr>
<td>( \Sigma^-_g )</td>
<td>( (R \cdot D)(R \cdot B) )</td>
<td></td>
</tr>
</tbody>
</table>

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Hybrid static potentials (4)

- Preliminary SU(2) results.
- Lattice setup:
  - More than 700 essentially independent gauge link configurations.
  - \(24^4\) lattice sites.
  - Lattice spacing \(a \approx 0.073\) fm (when identifying \(r_0\) with \(0.46\) fm).
- Extract a potential value \(V(R)\) from the plateau of the corresponding effective mass \(V(R) = \ln(C(R, t + a)/C(R, t))/a\), where \(C(R, t)\) are Wilson loops with the previously discussed insertions.
Hybrid static potentials (5)

- Quantum numbers $\Sigma^+_g$, $\Pi_u$, $\Sigma^-_u$ and $\Sigma^-_g$ (two different hybrid creation operators for $\Sigma^+_g$, $\Pi_u$ and $\Sigma^-_u$):
  - Resulting potentials identical within statistical errors.
  - $\Sigma^+_g$ (ordinary static potential):
    Wilson loops (green) superior to $R \cdot E$ (yellow).
  - $\Pi_u$:
    $R \times B$ (magenta) superior to $R \times (D \times E)$ (orange).
  - $\Sigma^-_u$:
    $R \cdot B$ (blue) superior to $R \cdot (D \times E)$ (black).

→ Certain information about the gluonic string.

Hybrid static potentials (6)

- Statistical errors of hybrid static potentials quite large → local insertions might generate structures rather different from those of the corresponding physical states.

- Implement spatially extended creation operators generating the same quantum numbers ($\Sigma^+_g, \Pi_u, \Sigma^-_u, \Sigma^-_g, \ldots$) → corresponding correlation functions could be dominated by the ground state already at small temporal separations → smaller statistical errors expected.

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Hybrid static potentials (7)

• Goals:

  – Precise results for hybrid static potentials for SU(3) Yang-Mills theory and QCD.

  – Use these results to estimate masses of hybrid mesons by solving a Schrödinger-like equation with the computed hybrid static potentials.

  – In the context of effective field theories like pNRQCD there might be interest in the short distance behavior of hybrid static potentials, which is related to gluelump masses ...?

Conclusions

- Lattice QCD computations with static quarks combined with model calculations could provide interesting qualitative and to some extent also quantitative insights.