Z states with lattice QCD

“Workshop on Z states”, Gießen

Marc Wagner
Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik
mwagner@th.physik.uni-frankfurt.de
http://th.physik.uni-frankfurt.de/~mwagner/

in collaboration with Abdou Abdel-Rehim, Constantia Alexandrou, Joshua Berlin, Pedro Bicudo, Krzysztof Cichy, Mattia Dalla Brida, Antje Peters, Jonas Scheunert, Johann Schneider, Björn Wagenbach

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Introduction

• Goal:
  – Study tetraquark candidates (= 4-quark states) using lattice QCD.
  – In particular study tetraquarks with a heavy $\bar{c}c$ or $\bar{b}b$ pair.

• Motivation: Experimentally measured
  – charged bottomonium states, e.g. $Z_b(10610)^+$ and $Z_b(10650)^+$ ... must be four quark states,
  – charged charmonium states, e.g. $Z_c(3900)^\pm$, $Z_c(4020)^\pm$, $Z_c(4050)^\pm$, $Z_c(4250)^\pm$, $Z_c(4430)^\pm$ ... must be four quark states,
  – ...

• A very challenging problem in lattice QCD
  $\rightarrow$ no solid results yet.
(1) Introduction to QCD and lattice QCD.

(2) Two lattice QCD approaches to study $Z$ states:

- Creation operators with 4 quarks of finite mass
  $\rightarrow$ more suited for $Z_c$ states.

- Creation operators with 2 quarks of finite mass and 2 static quarks
  $\rightarrow$ more suited for $Z_b$ states.
Part 1: Introduction to QCD and lattice QCD
QCD (quantum chromodynamics)

- Quantum field theory of quarks (six flavors u, d, s, c, t, b, which differ in mass) and gluons.

- Part of the standard model explaining the formation of hadrons (mesons with integer spin, usually $qq$; baryons with half-integer spin, usually $qqq/qqq$) and their masses; essential for decays involving hadrons.

- Definition of QCD simple:

  \[ S = \int d^4 x \left( \sum_{f \in \{u,d,s,c,t,b\}} \bar{\psi}^{(f)}(\gamma_\mu(\partial_\mu - iA_\mu) + m^{(f)})\psi^{(f)} + \frac{1}{2g^2} \text{Tr}\left( F_{\mu\nu} F^{\mu\nu} \right) \right) \]

  \[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]. \]

- However, no analytical solutions for low energy QCD observables, e.g. hadron masses, known, because of
  - non-linear field equations,
  - the absence of any small parameter/coupling constant (i.e. perturbation theory not applicable).

→ Solve QCD numerically by means of lattice QCD.
Hadron spectroscopy

- Proceed as follows:

1. **Compute the temporal correlation function** $C(t)$ **of a suitable hadron creation operator** $O$.
   - An operator $O$, which generates the quantum numbers (flavor, $J^{PC}$) of the hadron of interest, when applied to the vacuum $|\Omega\rangle$.
   - An operator $O$, which crudely generates the hadron of interest (in particular same number of quarks), when applied to the vacuum $|\Omega\rangle$.

2. **Determine the corresponding hadron mass from the asymptotic exponential decay of** $C(t)$ **in time**.

- Example: $D$ meson mass $m_D$ (valence quarks $\bar{c}$ and $u$, $J^P = 0^-$),

$$
O \equiv \int d^3 r \, \bar{c}(r) \gamma_5 u(r)
$$

$$
C(t) \equiv \langle \Omega | O(t) O(0) | \Omega \rangle \xrightarrow{t \to \infty} e^{-m_D t}.
$$
To compute a temporal correlation function $C(t)$, use the path integral formulation of QCD,

$$C(t) = \langle \Omega | O^\dagger(t) O(0) | \Omega \rangle = \frac{1}{Z} \int \left( \prod_f D\psi^{(f)} D\bar{\psi}^{(f)} \right) DA_\mu O^\dagger(t) O(0) e^{-S[\psi^{(f)}, \bar{\psi}^{(f)}, A_\mu]}.$$  

- $|\Omega\rangle$: ground state/vacuum.
- $O^\dagger(t), O(0)$: functions of the quark and gluon fields (cf. previous slide).
- $\int \left( \prod_f D\psi^{(f)} D\bar{\psi}^{(f)} \right) DA_\mu$: integral over all possible quark and gluon field configurations $\psi^{(f)}(r, t)$ and $A_\mu(r, t)$.
- $e^{-S[\psi^{(f)}, \bar{\psi}^{(f)}, A_\mu]}$: weight factor containing the QCD action.
Lattice QCD (2)

- Numerical implementation of the path integral formalism in QCD:
  - Discretize spacetime with sufficiently small lattice spacing \( a \approx 0.05 \text{ fm} \ldots 0.10 \text{ fm} \rightarrow \) “continuum physics”.
  - “Make spacetime periodic” with sufficiently large extension \( L \approx 2.0 \text{ fm} \ldots 4.0 \text{ fm} \) (4-dimensional torus)
    \( \rightarrow \) “no finite volume effects”.

\[
x_\mu = (n_0, n_1, n_2, n_3) \in \mathbb{Z}^4
\]
Lattice QCD (3)

• Numerical implementation of the path integral formalism in QCD:
  
  – After discretization the path integral becomes an ordinary multidimensional integral:

\[
\int D\psi D\bar{\psi} DA \ldots \rightarrow \prod_{x_\mu} \left( \int d\psi(x_\mu) d\bar{\psi}(x_\mu) dU(x_\mu) \right) \ldots
\]

  – Typical present-day dimensionality of a discretized QCD path integral:

* \(x_\mu\): 32\(^4\) \(\approx 10^6\) lattice sites.

* \(\psi = \psi_A^{(f)}\): 24 quark degrees of freedom for every flavor
  \((\times 2\text{ particle/antiparticle, } \times 3\text{ color, } \times 4\text{ spin}), 2\text{ flavors.}\)

* \(U = U_{\mu}^{ab}\): 32 gluon degrees of freedom \((\times 8\text{ color, } \times 4\text{ spin})\).

* In total: \(32^4 \times (2 \times 24 + 32) \approx 83 \times 10^6\) dimensional integral.

→ Standard approaches for numerical integration not applicable.

→ Sophisticated algorithms mandatory (stochastic integration techniques, so-called Monte-Carlo algorithms).
Part 2a: Creation operators with 4 quarks of finite mass ($Z_C$ states)


[C. Alexandrou et al. [ETM Collaboration], JHEP 1304, 137 (2013) [arXiv:1212.1418 [hep-lat]]]
[J. Schneider, bachelor thesis, Goethe University Frankfurt am Main (2014)]
Why are $Z_c$’s difficult? (1)

- Lattice QCD = QCD ... i.e. a quantum field theory.
  - E.g. quark numbers are not fixed (quark-antiquark creation and annihilation possible).
  - The only input to determine a meson state, e.g. the mass, are its QCD quantum numbers (flavor, $J^{PC}$).
  - No additional input or ansatz possible.
  - Quark numbers and structure of a state (like spin $S$, angular momentum $L$, spatial width, ...) are part of the lattice QCD result, i.e. dictated by QCD dynamics.

- In lattice QCD one provides quantum numbers (flavor, $J^{PC}$) ... and obtains the $n$ lowest states from that sector (typically $n = 1$, sometimes $n = 2, 3, 4$ ... the larger $n$, the more difficult the computation).

- $Z_c$ states are close in mass to 2-meson states.
  - E.g. $Z_c(3900)^+$, quantum numbers $J^P = 1^+$ (needs confirmation)
    
    - $m_{Z_c(3900)^+} = 3889$ MeV
    - $m_{J/\psi} + m_\pi = (3097 + 139)$ MeV = 3236 MeV
    - $m_{\eta_c} + m_\rho = (2984 + 775)$ MeV = 3759 MeV
    - $m_D + m_{D^*} = (1870 + 2007)$ MeV = 3877 MeV

    ... further 2-meson states ... additionally states with relative momentum ...

→ These states must be determined with a single computation at the same time.
Why are $Z_C$’s difficult? (2)

- Such a computation requires a sizable number of hadron creation operators.
  - Each of the states, which one intends to compute, has to be crudely approximated by one of these operators.
    * E.g. if you are interested to study tetraquarks, you need 4-quark operators ...
    * ... if you cannot exclude a 2-quark structure, you also need 2-quark operators ...
    * ... for the 2-meson states of similar mass you need additional 4-quark operators of 2-meson structure.
  - E.g. for $Z_C(3900)^+$ one should/could consider the following operators:

  \[
  O_{\text{diquark}} = \int d^3r \left( e^{abc} \bar{d}(r) C\gamma_5 \bar{c}^{\nu},T(r) \right) \left( e^{ade} c^{\nu},T(r) C\gamma_{\mu} u^e(r) \right), \ldots
  \]

  \[
  \equiv \text{antidiquark}\quad \equiv \text{diquark}
  \]

  \[
  O_{\text{mesonic molecule}} = \int d^3r \left( \bar{d}(r) \gamma_5 c(r) \right) \left( \bar{c}(r) \gamma_{\mu} u(r) \right), \ldots
  \]

  \[
  \equiv D\quad \equiv D^*
  \]

  \[
  O_{J/\psi+\pi} = \int d^3r \left( \bar{c}(r) \gamma_{\mu} c(r) \right) \int d^3\tilde{r} \left( \bar{d}(\tilde{r}) \gamma_5 u(\tilde{r}) \right), \ldots
  \]

  \[
  \equiv J/\psi\quad \equiv \pi
  \]

• Then one has to compute the temporal correlation matrix from these operators,

\[ C_{jk}(t) = \langle \Omega | O_j^\dagger(t) O_k(0) | \Omega \rangle. \]

The extraction of several states (the \( Z_c \) state and the 2-meson states of similar mass) requires very high precision.

– 4-quark operators tend to increase statistical errors drastically.
– \( \bar{c}c \) pairs in operators (\( \rightarrow \) quark-antiquark creation and annihilation, closed quark loops, disconnected diagrams) drastically increase statistical errors.
Why are $Z_c$’s difficult? (4)

- Until now we assumed that the $Z_c$ state of interest is a stable state (i.e. corresponds to an eigenstate of the QCD Hamiltonian).

- If the $Z_c$ state of interest is unstable, i.e. readily decays into a lighter meson-meson pair, the computation is even more difficult.
  → One needs to compute the masses of 2-meson states as functions of the spatial volume ...
  distortions compared to the spectrum of non-interacting mesons provide information about resonance parameters (mass, width).

Current status of $Z_c$’s

• A very nice recent lattice QCD paper about charged $Z_c$ states:

• Current status of our computations:
  - Developing and testing numerical methods to compute the corresponding correlation matrix sufficiently precise.
  - No physics results yet. ("At unphysically heavy $u/d$ quark masses, and when neglecting $s\bar{s}$ creation and annihilation, $a_0(980)$ is not a stable tetraquark."

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Study of the $Z_c^+$ channel using lattice QCD

Sasa Prelovsek,1,2, C. B. Lang,3, Luka Leskovec,2, and Daniel Mohler1,4,5

1Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia
2Jozef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia
3Institute of Physics, University of Graz, A–8010 Graz, Austria
4Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, Illinois 60510-5011, USA

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Recently experimentalists have discovered several charged charmonium-like hadrons $Z_c^+$ with unconventional quark content $cc\bar{d}u$. We perform a search for $Z_c^+$ with mass below 4.2 GeV in the channel $I^G(J^{PC}) = 1^+(1^-^+)$ using lattice QCD. The major challenge is presented by the two-meson states $J/\psi\pi$, $\psi_2\pi$, $\eta_{1D}\pi$, $D\bar{D}^*$, $D^*\bar{D}^*$, $\eta_{c}\rho$ that are inevitably present in this channel. The spectrum of eigenstates is extracted using a number of meson-meson and diquark-antidiquark interpolating fields. For our pion mass of 266 MeV we find all the expected two-meson states but no additional candidate for $Z_c^+$ below 4.2 GeV. Possible reasons for not seeing an additional eigenstate related to $Z_c^+$ are discussed. We also illustrate how a simulation incorporating interpolators with a structure resembling low-lying two-mesons states seems to render a $Z_c^+$ candidate, which is however
Part 2b: Creation operators with 2 quarks of finite mass and 2 static quarks ($Z_b$ states)

\[ Z_b' s \ (\bar{b}bqq \ \text{tetraquarks}) \ (1) \]

- **Basic idea**: Investigate the existence of heavy tetraquarks \( \bar{b}bqq \) in two steps.

  1. Compute potentials of two heavy antiquarks \( \bar{b}b \) in the presence of two lighter quarks \( qq \in \{ud, ss, cc\} \) using lattice QCD.
  2. Check, whether these potentials are sufficiently attractive, to host a bound state by solving a corresponding Schrödinger equation. (\( \rightarrow \) This would indicate a stable \( \bar{b}bqq \) tetraquark.)

- (1) + (2) \( \rightarrow \) **Born-Oppenheimer approximation**:
  - Proposed in 1927 for molecular and solid state calculations.  
    \[ \text{[M. Born, R. Oppenheimer, "Zur Quantentheorie der Molekeln," Annalen der Physik 389, Nr. 20, 1927]} \]
  - In our computations step (1) not quantum mechanics, but lattice QCD.
  - Approximation valid, if \( m_q \ll m_b \) (most appropriate for \( qq = ud \)).
$Z_b$'s ($\bar{b}bqq$ tetraquarks) (2)

- **B.-O., step (1):** Lattice QCD computation of $\bar{b}b$ potentials $V_{\bar{b}b}(r)$.

  1. Use $\bar{b}bqq$ creation operators

     $$O_{\bar{b}bqq} \equiv (C\Gamma)_{AB}(C\tilde{\Gamma})_{CD}\left(\bar{b}_C(-r/2)q_A^{(1)}(-r/2)\right)\left(\bar{b}_D(+r/2)q_B^{(2)}(+r/2)\right)$$

     (different light quark flavors $qq \in \{ud, ss, cc\}$ and quark spin/parity).

  2. Compute temporal correlation functions.

  3. Determine $V_{\bar{b}b}(r)$ from the exponential decays of the correlation functions.
• **B.-O., step (2):** Solve the Schrödinger equation for the relative coordinate $r$ of the two $\bar{b}$ quarks,

\[
\left( -\frac{1}{2\mu} \Delta + V_{\bar{b}\bar{b}}(r) \right) \psi(r) = \frac{E}{r} \psi(r), \quad \mu = \frac{m_{b}}{2};
\]

possibly existing bound states, i.e. $E < 0$, indicate $\bar{b}\bar{b}qq$ tetraquarks.

• **A single bound state for one specific potential $V_{\bar{b}\bar{b}}(r)$ and light quarks $qq = ud$:**

  – Binding energy $E = -93^{+47}_{-43}$ MeV, i.e. confidence level $\approx 2\sigma$.
  – Quantum numbers of the $\bar{b}bud$ tetraquark: $I(J^P) = 0(1^+)$.  

  \textbf{→ Prediction of a tetraquarks.} (No experimental results for $\bar{b}bqq$ available.)

• No further bound states, in particular not for $qq = ss$ or $qq = cc$.

• **Work in Progress:**

  – Including effects due to the $\bar{b}b$ spins  
    \textbf{→ binding energy reduced, $\bar{b}bud$ persists (preliminary).}

  – The experimentally more relevant case $\bar{b}b\bar{q}q$ (e.g. $Z_b(10610)^+$, $Z_b(10650)^+$).
Conclusions

• The study of tetraquarks, resonances, etc. is currently a very popular, but also one of the most challenging problems in lattice QCD.

• In my opinion solid $Z$ state results from lattice QCD, i.e. results
  – at physical quark masses,
  – including a continuum extrapolation,
  – without approximations, e.g. neglect of closed quark loops, disconnected diagrams, etc., will require more time ... (Perhaps $O(5\text{ years})$ ...?)

• Not meant as a negative statement ... lattice QCD has made continuous and significant progress over the last decades:
  
  $\sim 1980 \ldots 1990$ Only gluons (Yang-Mills theory with infinitely heavy quarks).
  $\sim 1990 \ldots 2000$ First dynamical, but unphysically heavy $u/d$ quarks.
  $\sim 2000 \ldots 2015$ Decreasing $m_{u/d}$ to its physical value.
    $\sim 2000$ First crude results for simple $\bar{q}q$ mesons (uncontrolled systematic errors).
    $\sim 2010$ First solid results for simple $\bar{q}q$ mesons.
    $\sim 2020$ First solid results for mesons of more complicated structure, e.g. $Z$’s ...?