Problem 1 [Gamma matrices] Check the anticommutation relations for the Dirac gamma matrices:

\[ \{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu\nu}. \] (1)

Problem 2 [Dirac and Klein-Gordon] Show that the Dirac equation

\[ (i\gamma^\mu \partial_\mu - m)\psi = 0 \] (2)

implies the Klein-Gordon equation:

\[ (\partial^\mu \partial_\mu + m^2)\psi = 0. \] (3)

Problem 3 [Continuity equation (Dirac)] Use the Dirac equation and its adjoint to derive a continuity equation. Show that the four-current is given by

\[ j^\mu = \bar{\psi} \gamma^\mu \psi. \] (4)

Is \( j^0 \equiv \rho \) positive? Prove it.

Problem 4 [Solutions of the Dirac equation] Using the spinors \( u_1, u_2, v_1 \) and \( v_2 \) from the lecture, write the full solution of the Dirac equation (including the appropriate exponential) for:

a) particle with spin \( s_z = +1/2 \) moving along the z-axis,

b) particle with spin \( s_z = -1/2 \) moving along the z-axis,

c) antiparticle with spin \( s_z = +1/2 \) moving along the z-axis,

\[ d) \text{antiparticle with spin } s_z = -1/2 \text{ moving along the } z \text{-axis}. \]

Is it possible to write a solution for particles/antiparticles with definite \( s_x \) and \( s_y \)?

Problem 5 [Spin sums] Calculate the so-called spin sums:

\[ \sum_{s=1,2} u_s(p) \bar{u}_s(p), \] (5)

\[ \sum_{s=1,2} v_s(p) \bar{v}_s(p). \] (6)