Problem 1 [\phi^4 theory, perturbation theory]

Given the action of the Euclidean \( \phi^4 \)-theory,

\[
S_E[\phi] = \int d^4x \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{m^2}{2} \phi^2 + \frac{\lambda}{4!} \phi^4 \right],
\]

in the Lecture you were given the relation

\[
Z[J] := \frac{1}{Z(0)} \int D\phi \exp (-S_E + (J, \phi)) = \frac{Z_0(0)}{Z(0)} \exp \left( -S_I \left[ \frac{\delta}{\delta J} \right] \right) \exp \left( \frac{1}{2} (J, \Delta_F J) \right) \bigg|_{J=0},
\]

where

\[
\exp \left( -S_I \left[ \frac{\delta}{\delta J} \right] \right) \exp \left( \frac{1}{2} (J, \Delta_F J) \right) = \exp \left( \frac{1}{2} (J, \Delta_F J) \right) \{ 1 + \lambda W_1[J] + \lambda^2 W_2[J] + O(\lambda^3) \}.
\]

(i) We derived \( W_1[J] \) and parts of \( W_2[J] \) during the Lecture; now compute the full expression for \( W_2[J] \), where

\[
W_2[J] = \frac{1}{2} \left( \frac{1}{4!} \right)^2 \exp \left( -\frac{1}{2} (J, \Delta_F J) \right) \left\{ \int d^4x \left[ \frac{\delta}{\delta J} \right]^4 \right\}^2 \exp \left( \frac{1}{2} (J, \Delta_F J) \right).
\]

(ii) Translate the expression obtained above into a diagrammatic language.

(iii) Use the resulting \( W_2[J] \) from (i) to derive \( G_2(y_1, y_2) \) up to order \( O(\lambda^2) \).