Problem 1 [“Higher order” perturbation theory, combinatorial (Feynman) rules]

Consider Euclidean \( \phi^4 \)-theory,

\[
S[\phi] = \int d^4x \left( \frac{1}{2} (\partial_\mu \phi(x))(\partial_\mu \phi(x)) + \frac{m^2}{2} \phi^2(x) + \frac{\lambda}{4!} \phi^4(x) \right).
\]

(a) Draw all diagrams of order \( \lambda^3 \) contributing to the connected 2-point function \( G_2(x_1, x_2) \).

(b) Draw all diagrams of order \( \lambda^3 \) contributing to the connected 4-point function \( G_4(x_1, x_2, x_3, x_4) \).

(c) Truncate the diagrams drawn in (b). Which of them are 1-particle-irreducible?

Problem 2 [Feynman rules in axial gauge]

Write down the effective action (gauge-fixing term, ghost term) for QCD (i.e. the \( SU(3) \) gauge theory) in the axial gauge:

\[ n_\mu A^a_\mu = 0 \ , \ \text{for some fixed } n_\mu : n_\mu n_\mu = 1 ; \]

What happens to the ghost? Derive the gluon propagator (in momentum space) and sketch the corresponding Feynman rules for the quark-gluon interactions.

Hint: when inverting the gluon operator, the symmetry-motivated Ansatz

\[
\left( -k^2 g_{\mu\nu} + k_\mu k_\nu - \frac{1}{\alpha} n_\mu n_\nu \right)^{-1} = A g_{\mu\nu} + B k_\mu k_\nu + C (k_\nu n_\mu + k_\mu n_\nu)
\]

will suffice.