Topological objects in gauge theories – Problem Set 1

April 23, 2010 (will be discussed on April 30, 2010)

Problem 1

Derive the Euler-Lagrange equations from the principle of least action, $\delta S = 0$, for theories containing n real scalar fields $\phi^1, \ldots, \phi^n$, i.e. $\mathcal{L} = \mathcal{L}(\phi^1, \partial_\mu \phi^1, \ldots, \phi^n, \partial_\mu \phi^n)$.

Problem 2

Derive the equations of motion and interpret the particle content (How many types of particles? What are their masses? What kind of interactions?) of the following theories:

(a)

$$\mathcal{L} \;=\; \frac{1}{2}(\partial^\mu \vec{\phi})(\partial_\mu \vec{\phi}) - V(|\vec{\phi}|) \quad , \quad V(|\vec{\phi}|) \;=\; \frac{m^2}{2}|\vec{\phi}|^2,$$

where $\vec{\phi} = (\phi_1 \, , \, \phi_2)$ and ϕ_1 and ϕ_2 are real scalar fields.

(b)

$$\mathcal{L} \;=\; \frac{1}{2}(\partial^\mu \vec{\phi})(\partial_\mu \vec{\phi}) - V(\vec{\phi}) \quad , \quad V(\vec{\phi}) \;=\; \frac{1}{2}\Big(m^2(\phi_1)^2 + 2\epsilon\phi_1\phi_2 + m^2(\phi_2)^2\Big),$$

where $\vec{\phi} = (\phi_1 \, , \, \phi_2)$ and ϕ_1 and ϕ_2 are real scalar fields.

(c)

$$\mathcal{L} \;=\; (\partial^\mu \phi)^\dagger(\partial_\mu \phi) - V(|\phi|) \quad , \quad V(|\phi|) \;=\; \frac{\lambda}{4}\Big(|\phi|^2 - a^2\Big)^2,$$

where ϕ is a complex scalar field.