Problem 9

Show that the $n$-sphere $S^n = \{ r \in \mathbb{R}^{n+1} | |r| = 1 \}$ and the punctured $\mathbb{R}^{n+1}$, $\mathbb{R}^{n+1} \setminus \{ \vec{0} \}$, are homotopically equivalent.

Problem 10

Consider SU(2) Yang-Mills-Higgs theory with the Higgs field in the adjoint representation in $d = 3$:

$$\mathcal{L} = \text{Tr} \left( (D^\mu \phi)(D_\mu \phi) \right) - V(\text{Tr}(\phi \phi)) - \frac{1}{2} \text{Tr}(F^\mu_\nu F_{\mu\nu})$$

$$V(\text{Tr}(\phi \phi)) = \frac{\lambda}{4} \left( 2\text{Tr}(\phi \phi) - a^2 \right)^2.$$

Give an example of the asymptotic form ($|r| \to \infty$) of a finite energy field configuration $\phi$, $A_\mu$ with

(a) $\text{deg}(\phi) = -1$,

(b) $\text{deg}(\phi) = +2$.

Perform an explicit calculation of the degree. Determine the corresponding asymptotic magnetic field.