Numerical freeze-out hyper-surface extraction in four dimensions with the STEVE algorithm

Bernd R. Schlei, FAIR Synchrotrons
Introduction
Numerical Implementation of FOHS Elements

Manifolds of Co-Dimension 1 are numerically approximated best by:

- Line-Elements in 2 D → Contours
  - “DICONEX”
- Triangles in 3 D → Surfaces
  - “VESTA”
- Tetrahedrons in 4 D → Hyper-Surfaces
  - “STEVE”

Such Choices may lead to Size Correction Factors for the particular Tiling.

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2 Dimensions
DICONEX – Dilated CONtour EXtraction

1: Initial Distribution of “Active” Pixels.
2 / 3: Left / Right-Turning Vectors.
4 / 5: Dilated Contours.

Fast 3-Step Algorithm:
“Vectors – Connect – Shift”

B. R. Schlei,
SPIE Vol. 4794 (2002) 63;
Image and Vision Computing
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Features of DICONEX Contours

1. Only **four** Pixel Neighbors have to be considered during DICONEX Contour Construction.

2. DICONEX Contours are **never** self-intersecting or degenerated!

**4 Neighbors**

Smoothing of DICONEX Contours:

Initially given Gray-Level Images provide additional Spectral Information, which allow the **Dislocation** of the Points, which support the DICONEX Contours:

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Example: Extraction of 1+1 D FOHS (Part 1)

Temperature Field
(Color Coded Gray-Level Image)

Objective: Provide FOHS for
\( T_{\text{Pixel}} = T_{\text{Fluid}} \geq T_f \approx 139 \text{ MeV} \).

Dilated Contours (DICONEX)
with Shifted Point Normal Vectors

Example: Extraction of 1+1 D FOHS (Part 2)

Dilated Contours (DICONEX) Shifted with Respect to Temperatures along Shifted Point Normal Vectors

Final 1+1 D Freeze-Out Hyper-Surface after Removal of unphysical Edges

3 Dimensions
VESTA – Volume-Enclosing Surface exTraction Algorithm

Fast 3-Step Algorithm: “Faces – Connect – Substitute”

1: Initial Voxel Faces and Voxel Face Centers.
2: Vector Cycle, connecting Voxel Faces.
3: Reduced Vector Cycle, connecting Voxel Face Centers. (Only N-Cycles with N = 3, 4, 5, 6, 7 are possible.)
4: Rendered VESTA Surface for Single Voxel.

Voxels, which are in Contact with another Voxel only through one single Edge may be disconnected or connected.

B. R. Schlei (Copyright © 2002 - 2009), Los Alamos Preprint LA-UR-02-7733.
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VESTA - Surface Cycle Break-Up

3-Cycle

4-Cycle

4-Cycle

5-Cycle

7-Cycle

6-Cycle

6-Cycle

6-Cycle

6-Cycle

Such a Break-Up into Triangles allows for Point Dislocations!
Fluid Expansion and Cooling in HYLANDER-C (e.g., S+S @ 200A GeV)


DICONEX Contours enclose Pixels (Fluid Grid Points) with
\[ T_{\text{Pixel}} = T_{\text{Fluid}} \geq T_f \approx 139 \text{ MeV}. \]

Note the Break-Up of ONE Contour into TWO Contours between Images \( t_7 \) and \( t_8 \).
While Building Temporal Correlations between the Contours, one may encounter Correspondence Problems.

Discontinue Approach. STOP!

Consider a Temporal Stack of Blob-Pixels (➔ Voxels). Use only Voxel Faces, which separate the Shapes Interior from its exterior Regions.

Rather Begin Here. GO!
While using the actual Temperature Values within a Voxel and one of its Neighbors, one may dislocate the Face Centers of the VESTA Surface with respect to $T_f$ (or $\varepsilon_f$, or $n_f$, … etc.). This usually leads to a Surface Smoothing. 

Surface Tiles with all of their Components $t \leq 0$, or $r \leq 0$, have no meaning for 2+1 D Hydrodynamics. They are therefore removed.
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**FOHS and Normal Vectors**

- \( d\sigma_\mu \) are the Normal Vectors for each FOHS Triangle; their Lengths are equal to their corresponding Triangle Area.
- \( \mathbf{x}_\mu \) are the Centers of Mass for each FOHS Triangle; all related Field Quantities are evaluated at these Points.

\[ r \]

\[ z \]

\[ t \]

\[ r \]

\[ z \]

\[ t \]

\[ r \]
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4 Dimensions
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4D Analogies with 2D and 3D (Part 1)

The Generalization

DICONEX → VESTA → STEVE

has been quite straightforward.

(STEVE = Space-Time Enclosing Volume Extraction,
B. R. Schlei (Copyright © 2003 - 2009))

Toxel, Volumes, Volume Centers, … etc.
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4D Analogies with 2D and 3D (Part 2)

2D: 4 Edges
3D: 6 Faces
4D: 8 Volumes
nD: 2^n Hyper-Faces

2D: 4 Line Segments
3D: 8 Triangles
4D: 16 Tetrahedrons
nD: 2^n Hyper-Triangles

Furthermore:

2D: 2 Pixels, which touch in only one Point may be connected or disconnected.
3D: 2 Voxels, which touch in only one Edge may be connected or disconnected.
4D: 2 Toxels, which touch in only one Face may be connected or disconnected.
nD: 2 Hyper-Voxels, which touch in only one Hyper-Edge may be connected or disconnected.
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**Time-Sequence of FOHS Projections**

VESTA Rendering of FOHS in 2+1 D Hydrodynamics at fixed Times \((t_1 < \ldots < t_8)\).
NOTE: in the field of digital image processing, the y-direction is arranged from top to bottom, i.e., in negative spatial y-direction.

Numbering Convention:
always start with “0” !!!

2x2x2x2 Toxel
(time varying voxel)

Neighborhood

Sample Input File:
(W1 denotes ASCII)

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0-Site Positions (t/x/y/z) will be counted:
- from past to future (t)
- from left to right (x)
- from top to bottom (y)
- from front to back (z)

min/max values are measured always at the Site Centers !!!

More columns (# of columns>1) may be added for further field values at the given sites.

The first column will always be evaluated with respect to the above given iso-value.
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Definition of FOHS 4-Normals


Freeze-Out Hyper-Surface Element 4-Normal: \[ d\sigma^\mu = \sqrt{-g} \, dS^\mu, \quad \mu = 0, 1, 2, 3. \]

\[ g = \det(g^{ik}), \quad \text{with} \quad g^{ik}, \quad \text{i.e., the “Metric Tensor”}, \quad \text{and} \quad dS^i = -\frac{1}{6} \varepsilon^{iklm} dS_{klm}, \]

where \( \varepsilon^{iklm} \) is the “Completely Antisymmetric Unit Tensor” of fourth rank.

Conversely, we also have \( dS_{klm} = \epsilon_{nklm} dS^n \).

The Projections of the Volume of a Parallelepiped are spanned by three 4-Vectors:

\[ dx_1^\mu, dx_2^\mu, dx_3^\mu, \quad \mu = 0, 1, 2, 3. \]

They are given by the Determinants

\[ dS^{ikl} = \begin{vmatrix} dx_1^i & dx_2^i & dx_3^i \\ dx_1^k & dx_2^k & dx_3^k \\ dx_1^l & dx_2^l & dx_3^l \end{vmatrix} \]

\[ d\sigma_\mu \] points to the Exterior of an enclosed Space-Time Region.
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Movie: Example of a Propagating Surface

Initial Surface Tile

Final Surface Tiles
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Q & A