Advanced General Relativity

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Abstract

This is a course on advanced general relativity and provides an introduction to the study of solutions of Einstein equations, either in fixed or dynamical spacetimes. The course also provides an introduction to the mathematical and numerical techniques presently employed for the accurate solution of the Einstein equations together with those of relativistic hydrodynamics. The first part of the course will concentrate on the mathematical aspects of the solutions of the Einstein equations for compact objects (i.e., black holes, neutron stars), either as static/stationary solutions or when evolved (perturbation theory, gravitational collapse). A second part of the course will provide an introduction to numerical relativity, reviewing the 3+1 formulation of the equations, be it the field equations or those of relativistic hydrodynamics, the definition of hyperbolic system of partial differential equations and the development of nonlinear waves in hydrodynamics. A final part of the course, will concentrate on the numerical aspects and the most advanced techniques for the numerical solution of these equations. The students are expected to be familiar with the theory of General Relativity and to be proficient in differential geometry and tensor calculus. A series of exercises parallels the course. A detailed syllabus and a list of references can be found here. A series of exercises will parallel the course. The content of the lectures can be found in a series of books [1, 2, 3, 4, 5].

Syllabus and plan of the lectures

1.a Review of the curvature tensor
1.b Review of the Einstein equations

2.a Black hole solution: Schwarzschild
2.a Black hole solution: particle motion around Schwarzchild

3.a Black hole solution: Kerr
3.a Black hole solution: particle motion around Kerr

4.a Nonrotatitng relativistic stars
4.b Rotatitng relativistic stars

5.a Gravitational collapse to black hole
5.b Apparent and event horizons

6.a Perturbation theory: black holes
6.b Perturbation theory: relativistic stars

7.a The 3+1 decomposition of spacetime
7.b Formulations of the Einstein equations. Lagrangian formulations

8.a The ADM formulation
8.b Conformal traceless formulations

9.a Gauge conditions in 3+1 formulations
9.b Constraint equations. initial data and constrained evolution

10.a Hyperbolic systems of partial differential equations
10.b Quasi-linear formulation. Conservative formulation

11.a Characteristic equations for linear systems. Riemann invariants
11.b Characteristics and caustics. Domain of determinacy. Region of influence

12.a Linear hydrodynamic waves. Sound waves
12.b Nonlinear hydrodynamic waves. Rarefaction waves. Shock waves

13.a Contact discontinuities. The Riemann problem
13.b Solution of the one-dimensional Riemann problem

14.a Finite-Difference Methods. The discretisation process
14.b Numerical errors. Consistency. Convergence and Stability.

15.a The upwind scheme. The FTCS scheme. The Lax-Friedrichs scheme
15.b The leapfrog scheme. The Lax-Wendroff scheme. Kreiss-Oliger dissipation. Artificial-viscosity approaches.

References

[1] Misner C W, Thorne K S and Wheeler J A 1973 *Gravitation* (San Francisco: W. H. Freeman)

[2] Rezzolla L and Zanotti O 2013 *Relativistic Hydrodynamics* (Oxford, UK: Oxford University Press)

[3] Alcubierre M 2008 *Introduction to* $3+1$ *Numerical Relativity* (Oxford, UK: Oxford University Press)

[4] Baumgarte T W and Shapiro S L 2010 *Numerical Relativity: Solving Einstein's Equations on the Computer* (Cambridge University Press, Cambridge UK)

[5] Gourgoulhon E 2012 *3+1 Formalism in General Relativity* (*Lecture Notes in Physics, Berlin Springer Verlag* vol 846)