Relativistic Hydrodynamics

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Errata/Corrige

Notes

- All of the typos reported in black have been fixed in the revised **paperback** version, but are still present in the **hardback** version till a new version is published.
- The page and equation numbering varies slightly between the **paperback** and the **hardback** versions. All of the numbering reported in this errata refers to the **hardback** version.
- All of the typos reported in **blue** have been found after the **paperback** version was published and hence are present only on the **paperback** version. The page numbering refers therefore to the **paperback** version.

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1
A Brief Review of General Relativity

- Page 28, Eq. (1.115). Change

$$v^x = \frac{v^{x'} + V}{1 + v^{x'}V}\,, \qquad v^y = \frac{Wv^{y'}}{1 + v^{x'}V}\,, \qquad v^z = \frac{Wv^{z'}}{1 + v^{x'}V}\,.$$

$\rightarrow$

$$v^x = \frac{v^{x'} + V}{1 + v^{x'}V}\,, \qquad v^y = \frac{v^{y'}}{W(1 + v^{x'}V)}\,, \qquad v^z = \frac{v^{z'}}{W(1 + v^{x'}V)}\,.$$

- Page 32, Eq. (1.140). Change

$$\mathscr{L}_{\phi \boldsymbol{V}} \boldsymbol{T} = \phi \mathscr{L}_{\boldsymbol{V}} \boldsymbol{T}\,,$$

$\rightarrow$

$$\mathscr{L}_{\phi \boldsymbol{V}} \boldsymbol{T} = \phi \mathscr{L}_{\boldsymbol{V}} \boldsymbol{T} - \boldsymbol{V} \mathscr{L}_{\boldsymbol{T}} \phi\,,$$

- Page 32, Eq. (1.141). Change

$$\mathscr{L}_{\boldsymbol{V}} \phi = V^\nu \partial_\nu \phi_\nu = \frac{d\phi}{d\lambda}\,,$$

$\rightarrow$

$$\mathscr{L}_{\boldsymbol{V}} \phi = V^\nu \partial_\nu \phi = \frac{d\phi}{d\lambda}\,,$$

- Page 33, four lines before Eq. (1.147). Change
...not all bases are such that $\boldsymbol{e}_\mu \cdot \boldsymbol{e}_\nu \neq \eta_{\mu\nu}$
$\rightarrow$
...not all bases are such that $\boldsymbol{e}_\mu \cdot \boldsymbol{e}_\nu = \eta_{\mu\nu}$
- Page 38, Eq. (1.174). Change

$$\mathscr{L}_{\boldsymbol{\eta}}\, \boldsymbol{\xi} = \mathscr{L}_{\boldsymbol{\xi}}\, \boldsymbol{\eta} = 0\,,$$

$\rightarrow$

$$\mathscr{L}_{\boldsymbol{\eta}}\, \boldsymbol{\xi} = -\mathscr{L}_{\boldsymbol{\xi}}\, \boldsymbol{\eta} = 0\,,$$

- Page 40, Eq. (1.183). Change

$$\mathcal{S} := \int_{\mathscr{P}_1}^{\mathscr{P}_2} 2\mathcal{L} d\lambda = \ldots$$

$\rightarrow$

$$\mathcal{S} := \int_{\mathscr{P}_1}^{\mathscr{P}_2} \mathcal{L} d\lambda = \ldots$$

- Page 43, Eq. (1.196). Change

$$\frac{d^2(x^\mu + \xi^\mu)}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}(x+\xi)\frac{d(x^\alpha + \xi^\mu)}{d\lambda}\frac{d(x^\beta + \xi^\mu)}{d\lambda} = 0\,.$$

$\rightarrow$

$$\frac{d^2(x^\mu + \xi^\mu)}{d\lambda^2} + \Gamma^\mu_{\alpha\beta}(x^\mu+\xi^\mu)\frac{d(x^\alpha + \xi^\alpha)}{d\lambda}\frac{d(x^\beta + \xi^\beta)}{d\lambda} = 0\,.$$

- Page 44, Eq. (1.202). Change

$$R_{\alpha[\beta\gamma\delta]} = 2(R_{\alpha\beta\gamma\delta} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta}) = 0\,.$$

$\rightarrow$

$$3!R_{\alpha[\beta\gamma\delta]} = 2(R_{\alpha\beta\gamma\delta} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta}) = 0\,.$$

- Page 45, Eq. (1.207). Change

$$\Gamma^r_{\theta\theta} = -\sin\theta\cos\theta\,, \qquad \Gamma^\theta_{r\theta} = \cot\theta\,.$$

$\rightarrow$

$$\Gamma^\theta_{\phi\phi} = -\sin\theta\cos\theta\,, \qquad \Gamma^\phi_{\theta\phi} = \cot\theta\,.$$

- Page 45, Eq. (1.208). Change

$$R^r{}_{\theta\theta r} = -\frac{1}{R_s^2}g_{\theta\theta} = -\sin^2\theta\,, \qquad R^r{}_{\theta\theta r} = \frac{1}{R_s^2}g_{rr} = 1\,,$$

$\rightarrow$

$$R^\theta{}_{\phi\phi\theta} = -\frac{1}{R_s^2}g_{\phi\phi} = -\sin^2\theta\,, \qquad R^\phi{}_{\theta\phi\theta} = \frac{1}{R_s^2}g_{\phi\phi} = 1\,,$$

- Page 45, after Eq. (1.208). Change
  while the Ricci scalar is simply $R = 1/R_s^2$.
  $\rightarrow$
  while the Ricci tensor is $R_{ij} = g_{ij}/R_s^2$ and the Ricci scalar is simply given by $R = 2/R_s^2$.

- Page 48, Eq. (1.220). Change

$$R_{\mu\nu} = 8\pi \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} + \frac{1}{4\pi} \Lambda g_{\mu\nu} \right) ,$$

$\rightarrow$

$$R_{\mu\nu} = 8\pi \left(T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu} + \frac{1}{8\pi} \Lambda g_{\mu\nu} \right) ,$$

- Page 49, one line after Eq. (1.223). Change
  ...the coordinate time runs slower than the proper time.
  $\rightarrow$
  ...the proper time runs slower than the coordinate time.

- Page 51, Eq. (1.229). Change

$$\frac{d}{d\tau} \left[\left(1 - \frac{2M}{r} \right)^{-1} \frac{dr}{d\tau} \right] = r \left[\left(\frac{d\theta}{d\tau} \right)^2 + \sin^2\theta \left(\frac{d\phi}{d\tau} \right)^2 \right] ,$$

$\rightarrow$

$$\begin{aligned} \frac{d}{d\tau} \left[\left(1 - \frac{2M}{r} \right)^{-1} \frac{dr}{d\tau} \right] = {} & r \left[\left(\frac{d\theta}{d\tau} \right)^2 + \sin^2\theta \left(\frac{d\phi}{d\tau} \right)^2 \right] \\ & - \frac{M}{r^2} \left[\left(\frac{dt}{d\tau} \right)^2 + \left(1 - \frac{2M}{r} \right)^{-2} \left(\frac{dr}{d\tau} \right)^2 \right] , \end{aligned}$$

2

A Kinetic-Theory Description of Fluids

- Page 77, Eq. (2.30). Change

$$S(t) := -k_{\mathrm{B}} V H(t) = -k_{\mathrm{B}} V \int f(t, \vec{\boldsymbol{x}}, \vec{\boldsymbol{u}}) \; \ln(f(t, \vec{\boldsymbol{x}}, \vec{\boldsymbol{u}})) \; d^3u \,,$$

$\rightarrow$

$$S(t) := -k_{\mathrm{B}} V H(t) = -k_{\mathrm{B}} V \int f(t, \vec{\boldsymbol{u}}) \; \ln(f(t, \vec{\boldsymbol{u}})) \; d^3u \,,$$

- Page 79, after Eq. (2.39). Change
"...through the evolution of the *momentum flux* ρv_j (*i.e.*, the rate of change of linear momentum per unit time and unit area), ..."
$\rightarrow$
"...through the flux of the *momentum density tensor* $\rho v_i v_j + P_{ij}$ (*i.e.*, the rate of change per unit time and unit area orthogonal to the i-th direction of the j-th component of the linear momentum), ..."
- Page 80, after Eq. (2.42). Change
"...unlike the *kinetic energy*, $\frac{1}{2}\rho v^i v_i$,..."
$\rightarrow$
"...unlike the *kinetic energy density*, $\frac{1}{2}\rho v^i v_i$,..."
- Page 81, Eq. (2.47). Change

$$\epsilon = \frac{3}{2}\frac{k_{\mathrm{B}} T}{m} \,, \qquad\qquad p = \frac{2}{3}\frac{\epsilon}{nm} = n k_{\mathrm{B}} T \,,$$

$\rightarrow$

$$\epsilon = \frac{3}{2}\frac{k_{\mathrm{B}} T}{m} \,, \qquad\qquad p = \frac{2}{3} n\, m\, \epsilon = n k_{\mathrm{B}} T \,,$$

- Page 82. Change Eq. (2.51)

$$\langle \vec{\boldsymbol{u}}^2 \rangle = \frac{3k_{\mathrm{B}} T}{m} - \langle \vec{\boldsymbol{u}} \rangle^2 = \frac{3k_{\mathrm{B}} T}{m} - \vec{\boldsymbol{v}}^2 \,,$$

$\rightarrow$

$$\langle \vec{\boldsymbol{u}}^2 \rangle = \frac{3k_{\mathrm{B}} T}{m} + \langle \vec{\boldsymbol{u}} \rangle^2 = \frac{3k_{\mathrm{B}} T}{m} + \vec{\boldsymbol{v}}^2 \,,$$

- Page 83, Eq. (2.54). Change

$$f_0(u) = 4\pi n u^2 \left(\frac{m}{2\pi k_{\rm B} T}\right)^{3/2} \exp\left(-\frac{mu^2}{2k_{\rm B} T}\right) .$$

$\to$

$$4\pi u^2 f_0(u) = 4\pi n u^2 \left(\frac{m}{2\pi k_{\rm B} T}\right)^{3/2} \exp\left(-\frac{mu^2}{2k_{\rm B} T}\right) .$$

- Page 84, Fig. 2.5. Change the labels on the axes: $v \to u$.
- Page 84, Eq. (2.59). Change

$$T = \frac{mn}{3k_{\rm B}} \int (\vec{\boldsymbol{u}} - \vec{\boldsymbol{v}})^2 f_0 \, d^3u = \frac{m}{3k_{\rm B}} \langle (\vec{\boldsymbol{u}} - \vec{\boldsymbol{v}})^2 \rangle .$$

$\to$

$$T = \frac{m}{3nk_{\rm B}} \int (\vec{\boldsymbol{u}} - \vec{\boldsymbol{v}})^2 f_0 \, d^3u = \frac{m}{3k_{\rm B}} \langle (\vec{\boldsymbol{u}} - \vec{\boldsymbol{v}})^2 \rangle .$$

- Page 84, last paragraph of Sect. 2.2.4. Change
"(Problem 1 is dedicated to showing...)"
$\to$
"(Problem 4 is dedicated to showing...)"
- Page 86, after Eq. (2.70). Change
"...represents the flux of energy per unit surface and unit time, *i.e.*, the *Newtonian energy flux density vector*."
$\to$
"...represents the flux of energy per per unit time and unit area, *i.e.*, the *Newtonian energy-density flux vector*."
- Page 90, after Eq. (2.82). Change
"...and recalling that $p'_x = 0$ in the local Lorentz rest frame..."
$\to$
"...and recalling that $p_{x'} = 0$ in the local Lorentz rest frame..."
- Page 90, last line. Change
"The *relativistic Maxwell–Boltzmann* equation can then be obtained..."
$\to$
"The *relativistic Boltzmann* equation can then be obtained..."
- Page 91, Eq. (2.88). Change

$$K := \sqrt{(p_1)^\alpha (p_2)_\alpha - m^4 c^4} \, .$$

$\to$

$$K := \sqrt{(p_1)^\alpha (p_2)_\alpha - m^2 c^2} \, .$$

- Page 91, after Eq. (2.88). Change
"Note that the collisionless Maxwell-Boltzmann equation, namely (2.86) with..."
$\to$
"Note that the relativistic collisionless Boltzmann equation, namely (2.86) with..."

- Page 95, second line of Sec. 2.3.4. Change

  "...we multiply the relativistic Maxwell–Boltzmann equation (2.86) by..."
  $\rightarrow$
  "...we multiply the relativistic Boltzmann equation (2.86) by..."
- Page 95, after Eq. (2.110). Change

  "...can be transformed into a volume integral in momentum space..."
  $\rightarrow$
  "...can be transformed into a surface integral in momentum space..."
- Page 97, second line of Sec. 2.3.6. Change

  "...is a solution of the relativistic Maxwell–Boltzmann equation (2.86)..."
  $\rightarrow$
  "...is a solution of the relativistic Boltzmann equation (2.86)..."
- Page 106, Eq. (2.161). Change

$$\boxed{c_p - c_V = -T\left(\frac{\partial p}{\partial T}\right)_p^2 \Big/ \left(\frac{\partial p}{\partial V}\right)_T > 0\,,}$$

$\rightarrow$

$$\boxed{c_p - c_V = -T\left(\frac{\partial p}{\partial T}\right)_V^2 \Big/ \left(\frac{\partial p}{\partial V}\right)_T > 0\,,}$$

- Page 107, Eq. (2.168). Change

$$\boxed{c_s^2 := \left(\frac{\partial p}{\partial e}\right)_s\,,}$$

$\rightarrow$

$$\boxed{c_s^2 := c^2\left(\frac{\partial p}{\partial e}\right)_s\,,}$$

- Page 108, Eq. (2.171). Change

$$c_s^2 = \frac{1}{h}(c_s^2)_{\mathrm{N}}\,.$$

$\rightarrow$

$$c_s^2 = \frac{c^2}{h}(c_s^2)_{\mathrm{N}}\,.$$

- Page 108, Eq. (2.175). Change

$$\mathscr{G} > \frac{3}{2}c_s^2\,, \qquad\qquad \left(\mathscr{G} < \frac{3}{2}c_s^2\right)\,.$$

$\rightarrow$

$$\mathscr{G} > \frac{3}{2}\frac{c_s^2}{c^2}\,, \qquad \left(\mathscr{G} < \frac{3}{2}\frac{c_s^2}{c^2}\right)\,.$$

- Page 108, Eq. (2.172) and (2.173). Change

$$\begin{aligned} c_s^2 &= \frac{1}{h}\left(\frac{dp}{d\rho}\right)_s = \left(\frac{d\ln h}{d\ln\rho}\right)_s\,, \\ &= \frac{1}{h}\left[\left(\frac{\partial p}{\partial\rho}\right)_\epsilon + \frac{d\epsilon}{d\rho}\left(\frac{\partial p}{\partial\epsilon}\right)_\rho\right] = \frac{1}{h}\left[\left(\frac{\partial p}{\partial\rho}\right)_\epsilon + \frac{p}{\rho^2}\left(\frac{\partial p}{\partial\epsilon}\right)_\rho\right]\,. \end{aligned}$$

$\rightarrow$

$$\begin{aligned} c_s^2 &= \frac{c^2}{h}\left(\frac{dp}{d\rho}\right)_s = c^2\left(\frac{d\ln h}{d\ln\rho}\right)_s\,, \\ &= \frac{c^2}{h}\left[\left(\frac{\partial p}{\partial\rho}\right)_\epsilon + \frac{d\epsilon}{d\rho}\left(\frac{\partial p}{\partial\epsilon}\right)_\rho\right] = \frac{c^2}{h}\left[\left(\frac{\partial p}{\partial\rho}\right)_\epsilon + \frac{p}{\rho^2}\left(\frac{\partial p}{\partial\epsilon}\right)_\rho\right]\,. \end{aligned}$$

- Page 116, Eq. (2.232). Change

$$c_s^2 = \frac{p(5\rho h - 8p)}{3\rho h(\rho h - p)}\,,$$

$\rightarrow$

$$c_s^2 = c^2\frac{p(5\rho h - 8p)}{3\rho h(\rho h - p)}\,,$$

- Page 117, Eq. (2.234). Change

$$c_s^2 = \frac{\gamma\epsilon(\gamma-1)}{c^2+\gamma\epsilon} = \left(\frac{h-c^2}{h}\right)(\gamma-1) = \frac{\gamma p}{\rho h}\,.$$

$\rightarrow$

$$c_s^2 = \frac{c^2\gamma(\gamma-1)\epsilon}{c^2+\gamma\epsilon} = \frac{c^2(h-c^2)(\gamma-1)}{h} = \frac{c^2\gamma p}{\rho h}\,.$$

- Page 118, footnote 34. Change

"A fluid obeying the ideal-fluid equation of state with $\epsilon = 0$ would also have a zero temperature and could provide a reasonable model for a cold and old neutron star."
$\rightarrow$
"A fluid obeying a general polytropic equation of state can have, at least mathematically, $\epsilon = 0$, although such a choice would be difficult to justify from a physical point of view. However, if the polytropic transformation is *isentropic*, then the specific internal energy is fully determined and is proportional to the rest-mass density [cf. Eq. (2.248) and discussion around it]. A polytropic and isentropic equation of state is often used to obtain a reasonable approximation of the description of matter of a cold and old neutron star.

- Page 119, after Eq. (2.247)
  Put differently, a polytropic equation of state is equivalent to an ideal-fluid equation of state *only* under those *isentropic transformations* for which the *adiabatic index* of the fluid γ is the same as the *adiabatic index of the polytrope* Γ."
  $\rightarrow$
  "Put differently, if a fluid obeys the ideal-fluid equation of state and is isentropic, then its equation of state can also be written in a polytropic form [*cf.*, Eq. (2.242)], with polytropic exponent $\Gamma = \gamma$; in this case, the polytropic exponent is also the adiabatic index. On the other hand, if a fluid obeys the polytropic equation of state and is isentropic, then it is at least formally possible to express the pressure as $p = \rho\epsilon(\Gamma - 1)$ [*cf.*, Eq. (2.228)]. However, this does not necessarily mean that such a fluid obeys an ideal-fluid equation of state. For this to be the case, Γ must be the ratio of the specific heats c_p/c_V *and* the specific internal energy must be a function of the temperature only."
- Page 119, Eq. (2.249). Change

$$c_s^2 = \frac{\Gamma p}{\rho h} = \frac{\Gamma(\Gamma-1)p}{\rho(\Gamma-1)+\Gamma p} = \left(\frac{1}{\Gamma K \rho^{\Gamma-1}} + \frac{1}{\Gamma-1}\right)^{-1} .$$

$\rightarrow$

$$c_s^2 = c^2\frac{\Gamma p}{\rho h} = c^2\frac{\Gamma(\Gamma-1)p}{\rho(\Gamma-1)+\Gamma p} = c^2\left(\frac{1}{\Gamma K \rho^{\Gamma-1}} + \frac{1}{\Gamma-1}\right)^{-1} .$$

3

Relativistic Perfect Fluids

- Page 139, before Eq. (3.28). Change The simplest quantity to determine is the *rest-mass density current*, namely $\to$ The simplest quantity to determine is the *rest-mass current*, namely
- Page 139, before Eq. (3.28). Change

$$J^{\hat{\mu}}: \quad \text{flux of rest-mass current density in the } \hat{\mu}\text{-direction},$$

$\to$

$$J^{\hat{\mu}}: \quad \text{flux of rest-mass in the } \hat{\mu}\text{-direction},$$

- Page 139, before Eq. (3.29). Change

$$\begin{aligned}
T^{\hat{0}\hat{0}} &: \quad \text{total energy density},\\
T^{\hat{0}\hat{i}} &: \quad \text{flux of energy density in } \hat{i}\text{-th direction},\\
T^{\hat{i}\hat{0}} &: \quad \text{flux of } \hat{i}\text{-momentum in } \hat{0}\text{-th direction} \quad (\hat{i}\text{-momentum density}),\\
T^{\hat{j}\hat{i}} &: \quad \text{flux of } \hat{j}\text{-th component of momentum density in } \hat{i}\text{-th direction}\,.
\end{aligned}$$

$\to$

$$\begin{aligned}
T^{\hat{0}\hat{0}} &: \quad \text{total energy density},\\
T^{\hat{0}\hat{i}} &: \quad \text{flux of energy in } \hat{i}\text{-th direction},\\
T^{\hat{i}\hat{0}} &: \quad \text{flux of } \hat{i}\text{-momentum in } \hat{0}\text{-th direction}\,,\\
T^{\hat{j}\hat{i}} &: \quad \text{flux of } \hat{j}\text{-momentum in } \hat{i}\text{-th direction}\,.
\end{aligned}$$

- Page 139, (3.29). Change the second line as follows:

$$T^{\hat{0}\hat{i}} = T^{\hat{0}\hat{i}} = 0\,,$$

$\to$

$$T^{\hat{0}\hat{i}} = T^{\hat{i}\hat{0}} = 0\,,$$

- Page 146, Eq. (3.69). Change

$$\mathscr{L}_{\boldsymbol{u}}(hu_\mu) = -\frac{1}{\rho}\nabla_\mu p - \nabla_\mu h\,.$$

$\to$

$$\mathscr{L}_{\boldsymbol{u}}(hu_\mu) = -\frac{1}{\rho}\nabla_\mu p = -\nabla_\mu h\,.$$

- Page 146, Eq. (3.71). Change

$$\mathscr{L}_{\boldsymbol{u}}(hu_\mu\xi^\mu) = -\frac{\xi^\mu\nabla_\mu p}{\rho} - \xi^\mu\nabla_\mu h = -\frac{1}{\rho}\mathscr{L}_{\boldsymbol{\xi}}p - \mathscr{L}_{\boldsymbol{\xi}}h\,,$$

$\rightarrow$

$$\mathscr{L}_{\boldsymbol{u}}(hu_\mu\xi^\mu) = -\frac{\xi^\mu\nabla_\mu p}{\rho} = -\frac{1}{\rho}\mathscr{L}_{\boldsymbol{\xi}}p = -\mathscr{L}_{\boldsymbol{\xi}}h\,,$$

- Page 146, before Eq. (3.72). Change
  "and thus use the condition (3.65) with $\mathscr{L}_{\boldsymbol{u}}p = 0$ and $\mathscr{L}_{\boldsymbol{u}}h = 0$, to finally obtain"
  $\rightarrow$
  "and thus use the condition (3.65) with $\mathscr{L}_{\boldsymbol{\xi}}p = 0$ to finally obtain"
- Page 146, after Eq. (3.72). Change
  Note the similarity between expression (3.72) and the corresponding equation (1.185) along geodesic trajectories, *i.e.*, $\mathscr{L}_{\boldsymbol{u}}(u_\mu\xi^\mu)$.
  $\rightarrow$
  Note the similarity between expression (3.72) and the corresponding equation (1.185) along geodesic trajectories, *i.e.*, $\mathscr{L}_{\boldsymbol{u}}(u_\mu\xi^\mu) = 0$.
- Page 155, caption of Fig. 3.4. Change
  ... Show with blue solid lines
  $\rightarrow$
  ... Shown with blue solid lines

- Page 177, Eq. (3.253). Change

$$P^{\alpha\beta}_{R} := \int I_\nu N^\alpha N^\beta d\nu d\Omega\,.$$

$\rightarrow$

$$P^{\alpha\beta}_{R} := h^\alpha_{\ \gamma}\, h^\beta_{\ \delta}\, T^{\gamma\delta}_{R} = h^\alpha_{\ \gamma}\, h^\beta_{\ \delta}\int I_\nu N^\gamma N^\delta d\nu d\Omega\,.$$

- Page 179, footnote 26. Change
  "Multifluids of this type as sometimes also referred to as..."
  $\rightarrow$
  "Multifluids of this type are sometimes also referred to as..."

- Page 190, Exercise 1. should read:
  Use the definitions (3.11)–(3.13) to show that

$$\sigma_{\alpha\beta}\sigma^{\alpha\beta} = \frac{1}{2}\left(\nabla_\mu a^\mu - 2u_\nu\nabla_\mu\nabla^{(\mu}u^{\nu)} + a^2 - \frac{2}{3}\Theta^2\right)\,, \tag{3.1}$$

$$\omega_{\alpha\beta}\omega^{\alpha\beta} = \frac{1}{2}\left(a^2 - \nabla_\mu a^\mu - 2u_\nu \nabla_\mu \nabla^{[\mu} u^{\nu]}\right) , \tag{3.2}$$

thus concluding that the term $2(\omega^2 - \sigma^2)$ entering the Raychaudhuri equation (3.27) can be written as

$$2(\omega^2 - \sigma^2) = \frac{1}{3}\Theta^2 - \nabla_\mu a^\mu + u_\nu \nabla_\mu \nabla^\nu u^\mu \, . \tag{3.3}$$

4

Linear and Nonlinear Hydrodynamic Waves

- Page 200, Eq. (4.54). Change

$$\det(\mathcal{A}^t - \lambda \mathcal{A}^x) = 0\,,$$

$\rightarrow$

$$\det(\mathcal{A}^x - \lambda \mathcal{A}^t) = 0\,,$$

- Page 213, Fig. 4.5. The tangents to the fluidlines on either side of the rarefaction tail should be exactly the same and not as shown.
- Page 216, below Eq. (4.114). Change
" It is also convenient to rewrite the continuity equation (4.112) and the conservation of energy (4.113)"
$\rightarrow$
" It is also convenient to rewrite the continuity equation (4.112) and the conservation of momentum (4.113)"
- Page 216. Change Eq. (4.117) as follows

$$(h_b W_b v_b)^2 - (h_a W_a v_a)^2 = -\left(\frac{h_a}{\rho_a} + \frac{h_b}{\rho_b}\right) [\![p]\!]\ .$$

$\rightarrow$

$$(h_b W_b v_b)^2 - (h_a W_a v_a)^2 = \left(\frac{h_a}{\rho_a} + \frac{h_b}{\rho_b}\right) [\![p]\!]\ .$$

- Page 217, first paragraph. Change
"The classical Hugoniot adiabat is readily obtained from (4.118) after recalling that in the Newtonian limit $h_{\rm N} = 1 + \epsilon + p/\rho \approx 1$"
$\rightarrow$
"The classical Hugoniot adiabat is readily obtained from (4.118) after recalling that in the Newtonian limit $h = 1 + \epsilon + p/\rho \approx 1$"
- Page 221, Eq. (4.138). Change

$$W_{ab}^2 = \frac{(3e_a + e_b)(3e_b + e_a)}{16 e_1 e_2} = \frac{4}{9} W_a^2 W_b^2\,,$$

$\rightarrow$

$$W_{ab}^2 = \frac{(3e_a + e_b)(3e_b + e_a)}{16 e_a e_b} = \frac{4}{9} W_a^2 W_b^2\,,$$

- Page 222, after Eq. (4.141), the expression for the shock velocity should be modified as follows:

$$V_{\scriptscriptstyle S}^{\pm} = \rho_b W_b v_b / (\rho_b W_b \pm \rho_a)$$

$\rightarrow$

$$V_{\scriptscriptstyle S}^{\pm} = \rho_b W_b v_b / (\rho_b W_b \mp \rho_a)$$

- Page 225, Fig. 4.11, panel on bottom right. The labels in the spacetime diagram should be corrected as follows:
  $\mathscr{R}_{\leftarrow} \rightarrow \mathscr{R}_{\rightarrow}$ and $\mathscr{S}_{\rightarrow} \rightarrow \mathscr{S}_{\leftarrow}$.
- Page 228, last sentence in the first paragraph should be modified as follows:

  These values mark the transition from one wave pattern to another one, and that are directly computed from the initial conditions (Rezzolla and Zanotti, 2001).
  $\rightarrow$
  These values mark the transition from one wave pattern to another one, and are directly computed from the initial conditions (Rezzolla and Zanotti, 2001).

5

Reaction Fronts: Detonations and Deflagrations

- Page 284, problem 2. Change
  "Derive the inequalities (5.4)–(5.6) across a reaction front [Hint: start from the laws of conservation of momentum and energy (4.114)–(4.113)].
  $\rightarrow$
  "Derive the inequalities (5.4)–(5.6) across a reaction front [Hint: start from the laws of conservation of momentum and energy (4.113)–(4.114)].

6
Relativistic Non-Perfect Fluids

- Page 297, Eq. (6.73). Change: $\chi \to \chi_t$
- Page 301, Eq. (6.85). Change:

$$q_\nu = -\kappa \left[\mathcal{D}_\nu \ln T + a_\nu + \beta_1 \dot{q}_\nu + \frac{1}{2} T \nabla_\mu \left(\frac{\beta_1}{T} u^\mu \right) q_\nu \right] ,$$

$\rightarrow$

$$q_\nu = -\kappa T \left[\mathcal{D}_\nu \ln T + a_\nu + \beta_1 \dot{q}_\nu + \frac{1}{2} T \nabla_\mu \left(\frac{\beta_1}{T} u^\mu \right) q_\nu \right] ,$$

- Page 306: invert the inequalities in Eqs. (6.115) and (6.116).

7

Formulations of the Einstein–Euler Equations

- Page 337, Eq. (7.100). Change

$$\tilde{\Gamma}^i_{jk} = \Gamma^i_{jk} - \frac{1}{3}(\delta^i_j \Gamma^m_{km} + \delta^i_k \Gamma^m_{jm} - \gamma_{jk}\gamma^{il}\Gamma^m_{lm}) \quad = \Gamma^i_{jk} + 2(\delta^i_j \partial_k \ln\phi + \delta^i_k \partial_j \ln\phi - \gamma_{jk}\gamma^{il}\partial_l \ln\phi)\,,$$

$\to$

$$\tilde{\Gamma}^i_{jk} = \Gamma^i_{jk} - \frac{1}{3}(\delta^i_j \Gamma^m_{km} + \delta^i_k \Gamma^m_{jm} - \gamma_{jk}\gamma^{il}\Gamma^m_{lm}) \quad = \Gamma^i_{jk} + \delta^i_j \partial_k \ln\phi + \delta^i_k \partial_j \ln\phi - \gamma_{jk}\gamma^{il}\partial_l \ln\phi\,,$$

- Page 339, Eq. (7.108). Change

$$^{(3)}R + K^2 = K^{ij}K_{ij} + 4\pi E = \tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2 + 4\pi E\,,$$

$\to$

$$^{(3)}R + K^2 = K^{ij}K_{ij} + 4\pi E = \tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2 + 16\pi E\,,$$

- Page 339, last two lines:
(note that $\tilde{\gamma}_{ij}$ and $\tilde{A}_{ij}$, have only five independent components each since they are traceless)
$\to$
(note that $\tilde{\gamma}_{ij}$ and $\tilde{A}_{ij}$, have only five independent components each since they have traces that are equal to three or zero, respectively)

- Page 342, correct sign in third term of Eq. (7.113). Change

$$R_{\mu\nu} + 2\nabla_{(\mu}Z_{\nu)} + \kappa_1[2n_{(\mu}Z_{\nu)} - (1+\kappa_2)g_{\mu\nu}n_\sigma Z^\sigma] = 8\pi\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)\,,$$

$\to$

$$R_{\mu\nu} + 2\nabla_{(\mu}Z_{\nu)} + \kappa_1[2n_{(\mu}Z_{\nu)} + (1+\kappa_2)g_{\mu\nu}n_\sigma Z^\sigma] = 8\pi\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)\,,$$

- Page 343, first term on the right-hand-side of Eq. (7.116). Change
$$D_j\alpha_j$$
$\to$
$$D_iD_j\alpha$$
- Page 345, first term in Eq (7.131). Change
$$D_j\Theta_{ij} = 0\,,$$
$\to$
$$D^j\Theta_{ij} = 0\,,$$
- Page 345, Eq (7.132). Change
$$\Sigma_{ij} := \Theta_{ij} - \frac{1}{3}\gamma_{ij}\Theta_{kl}\Theta^{kl} = \frac{1}{2}\gamma^{1/3}\mathscr{L}_{\boldsymbol{t}}\bar{\gamma}_{ij}\,,$$
$\to$
$$\Sigma_{ij} := \Theta_{ij} - \frac{1}{3}\gamma_{ij}\Theta_{kl}\gamma^{kl} = \frac{1}{2}\gamma^{1/3}\mathscr{L}_{\boldsymbol{t}}\tilde{\gamma}_{ij}\,,$$
- Page 345, after Eq (7.132). Change
where $\bar{\gamma}_{ij} := \gamma^{-1/3}\gamma_{ij}$ is the conformal metric.
$\to$
where $\tilde{\gamma}_{ij} := \gamma^{-1/3}\gamma_{ij}$ is the conformal metric.
- Page 345, first term in Eq (7.133). Change
$$D_j\Sigma_{ij} = 0\,,$$
$\to$
$$D^j\Sigma_{ij} = 0\,,$$
- Page 354, first line. Change
"constraint decouples from the Hamiltonian constraint and it is possible to solve the latter to obtain the three vectors $\bar{V}^i$"
$\to$
"constraint decouples from the Hamiltonian constraint and it is possible to solve the former to obtain the three vectors $\bar{V}^i$"
- Page 354, fourth line. Change
"The calculation of initial data via the solution of the constrains simplifies considerably if"
$\to$
"The calculation of initial data via the solution of the constraints simplifies considerably if"
- Page 356, first sentence before Eq. (7.180). Change
"we further introduce the conformal metric $\bar{\boldsymbol{\gamma}}$ [*cf.*, Eq. (7.152)], such that"
$\to$
"we further introduce the conformal metric $\tilde{\boldsymbol{\gamma}}$ [*cf.*, Eq. (7.152), although we here use a tilde rather than a bar to be closer to the notation of Bonazzola *et al.* (2004)], such that"

- Page 356, Eq. (7.180). Change

$$f := \det(f_{ij}) = \bar{\gamma} := \det(\bar{\gamma}_{ij}),$$

$\to$

$$f := \det(f_{ij}) = \tilde{\gamma} := \det(\tilde{\gamma}_{ij}),$$

- Page 356, Eq. (7.181). Change

$$\psi = (\gamma/\bar{\gamma})^{1/12} = (\gamma/f)^{1/12}.$$

$\to$

$$\psi = (\gamma/\tilde{\gamma})^{1/12} = (\gamma/f)^{1/12}.$$

- Page 384, second equation in Exercise 7. Change

$$D_i D_j \phi = -\frac{1}{2\phi}\tilde{D}_i \tilde{D}_j + \frac{1}{2\phi^2}\partial_i \phi \, \partial_j \phi.$$

$\to$

$$D_i D_j \phi = \tilde{D}_i \tilde{D}_j \phi + \frac{2}{\phi}\partial_i \phi \, \partial_j \phi - \frac{1}{\phi}\gamma_{ij}\partial^k \phi \, \partial_k \phi.$$

8
Numerical Relativistic Hydrodynamics: Finite-Difference Methods

- Page 393, Eq. (8.16). Change

$$\epsilon_j^{(h)} = \tilde{C}h^{\tilde{p}_j} + \mathcal{O}(h^{\tilde{p}_j+1}) \,,$$

$\rightarrow$

$$\epsilon_j^{(h)} = Ch^{p_j} + \mathcal{O}(h^{p_j+1}) \,,$$

- Page 393, below Eq. (8.16). Change “with $\tilde{C}$ a constant” to “with C a constant”.
- Page 393, Eq. (8.30). Change

$$\tilde{p} \coloneqq \frac{\log R(h,k)}{\log(h/k)} \,,$$

$\rightarrow$

$$\tilde{p} \coloneqq \frac{\log |R(h,k)|}{\log(h/k)} \,,$$

- Page 395, the 7th line before Eq. (8.37). Change

“...its application across a time interval Δt introduces an associate truncation error $\epsilon_j(h)$.”
$\rightarrow$
“...its application across a time interval Δt introduces an associated truncation error $\epsilon(h)$.”

- Page 406, Eq. (8.87). Change

$$\tilde{u}(x,t) = e^{-\varepsilon k^2 t} e^{ik\left[x-\left(v+\beta k^2\right)t\right]} \,,$$

$\rightarrow$

$$\tilde{u}(x,t) = e^{-\varepsilon k^2 t} e^{ik\left[x-\left(\lambda+\beta k^2\right)t\right]} \,,$$

10

Numerical Relativistic Hydrodynamics: High-Order Methods

- Page 464, Eq. (10.14). Change

$$A_{ik} \coloneqq \int_0^1 \Psi_k(\xi) d\xi \,, \qquad \forall\, I_i \in S^l_j \,.$$

$\rightarrow$

$$A_{ik} \coloneqq \int_{I_i} \Psi_k(\xi) d\xi \,, \qquad \forall\, I_i \in S^l_j \,.$$

11

Relativistic Hydrodynamics of Non-Selfgravitating Fluids

- Page 518, Eq. (11.84). Change

$$\frac{d\mathcal{W}}{\mathcal{W}} = \frac{M}{r^2\mathcal{W}}dr + \frac{u}{\mathcal{W}}du\,.$$

$\rightarrow$

$$\frac{d\mathcal{W}}{\mathcal{W}} = \frac{M}{r^2\mathcal{W}^2}dr + \frac{u}{\mathcal{W}^2}du\,.$$

12

Relativistic Hydrodynamics of Selfgravitating Fluids

- Page 596, 12th line after Eq. (12.13)
  "$M = 2.01 \pm 0.4\,M_\odot$" $\rightarrow$ "$M = 2.01 \pm 0.04\,M_\odot$"
- Page 597, caption of Fig. 12.1
  "$M = 2.01 \pm 0.4\,M_\odot$" $\rightarrow$ "$M = 2.01 \pm 0.04\,M_\odot$"
- Page 601, second term in Eq. (12.31). Change

$$H_0^2 = \frac{1}{R_{\mathrm{i}}^2}\left[1 - \frac{\varepsilon(1+w_{\mathrm{R}})^2}{w_{\mathrm{i}}}\right] .$$

$\rightarrow$

$$H_0^2 = \frac{1}{R_{\mathrm{i}}^2}\left[1 - \frac{\varepsilon(1+w_{\mathrm{i}})^2}{w_{\mathrm{i}}}\right] .$$

- Page 601, second paragraph, change:
  ... the energy density $\rho(r)$ and the pressure $p(r)$ ...
  $\rightarrow$
  ... the energy density $e(r)$ and the pressure $p(r)$ ...
- Page 606, caption of Fig. 12.4:
  "$K = 100$" $\rightarrow$ "$K = 164$"
- Page 606, second but last line:
  "$K = 100$" $\rightarrow$ "$K = 164$"