The vortex mechanism of generation of astrophysical jets

M.G. Abrahamyan[1]

Yerevan State University, Armenia.

Abstract

On the basis of exact solutions of the hydrodynamic equations with generalization of the Rankin vortex, the vortex mechanism of generation of astrophysical streams is offered. It is shown, that occurrence of the Rankin vortex in the polar stratum of a rotating protostar causes dilatational and converging to a trunk of a vortex streams the substances providing exponential growth of angular velocity of gyration of a trunk and a pressure drop on its axis. Growth of angular velocity of gyration of a trunk of a vortex and a pressure drop on its axis stop, when jump of rotational velocity on a trunk surface reaches the sound velocity. It occurs in time after vortex occurrence during which the vortex motion sweeps more and more deep stratums of a cloud. Dilatational velocity of a stream along a vortex trunk thus accrues, causing the outflow of mass from a surface of a protostar as a stream eruption.

Keywords: hydrodynamics, vortex, generation, jet eruption.

1. Introduction

Jet eruptions - the universal phenomenon in the Universe: it sweeps a major gamut of astrophysical objects, from nuclear of active galaxies (AGNs) to young star formations of small masses (YSOs) within our Galaxy.

Questions of an origin of streams and mechanisms of their formation are not understood to the extremity (see the review [1]). The strong collimation of streams, considering partial ionizing of thrown up substance, contacts a magnetic field [2-4].

In the present article the vortex mechanism of generation, acceleration and collimation of astrophysical jets on the basis of exact vortex solutions of the hydrodynamic equations is offered.

2. Non linear instability of Polar Regions of protostar against the Rankin vortex perturbations. Let's examine protostellar object of the mass *M,* of radius R in the framework of incompressible fluid model [5]. Let's because the torsion oscillations of protostar on their polar regions the Rankin vortex was formed with azimuthally velocity profile [6-9],

$$v_j(r) = \begin{cases} V_{in} r, & r \quad r_0, \\ V_e \dfrac{r_0^2}{r}, & r > r_0, \end{cases} \tag{1}$$

featuring "solid-body" gyration in the trunk $r \le r_0$ and differential gyration – out of them.

Let's present $P = P^0(r, z) + p(r, z, t)$, where $P^0(r, z)$ is the equilibrium pressure, *p*- pressure-perturbation by the vortex motion. The equations of axial-symmetric fluxion of viscous incompressible medium in cylindrical coordinates will be presented in a view [10]:

$$\frac{Wv_r}{t} + v_r \frac{\square v_r}{r} - \frac{v_j^2}{r} = - \frac{1}{r}\frac{p}{r} + n\frac{}{r}\left(\frac{v_r\square}{r} + \frac{v_r}{r}\right), \tag{2}$$

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[1] Email add: mabr@bionet.am

$$\frac{\partial v_\varphi}{\partial t}+v_r\left(\frac{\partial v_\varphi}{\partial r}+\frac{v_\varphi}{r}\right)=\nu\frac{\partial}{\partial r}\left(\frac{\partial v_\varphi}{\partial r}+\frac{v_\varphi}{r}\right), \tag{3}$$

$$\frac{\partial v_z}{\partial t}+v_z\frac{\partial v_z}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\frac{\partial^2 v_z}{\partial z^2}, \tag{4}$$

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}=0, \tag{5}$$

where ν - a kinematic viscosity coefficient, ρ - the homogeneous mass density of a protostar.

Note, that in the given equations because of axial symmetry the terms containing derivatives on φ and z (except pressure and v_z) are rejected, and also it is supposed $v_z=v_z(z, t)$.

Accepting $v_r = v_z = 0$ during the moment $t = 0$, from the equations (1) - (2) we will gain

$$\frac{\partial p}{\partial r}=\rho\frac{v_\varphi^2}{r}+2\rho\Omega v_\varphi, \quad \frac{\partial v_\varphi}{\partial t}=\nu\frac{\partial}{\partial r}\left(\frac{\partial v_\varphi}{\partial r}+\frac{v_\varphi}{r}\right) \tag{6}$$

Taking into account (1) the solution of first equation (6) will be presented in a view

$$p=\begin{cases} p_c+\rho(\omega_{in}^2+2\Omega\omega_{in})\dfrac{r^2}{2}, & r\le r_0,\\ \rho r_0^2\left[\dfrac{\omega_e^2 r_0^2}{2r^2}\left(1-\dfrac{r^2}{r_s^2}\right)+2\Omega\omega_e\ln\dfrac{r_s}{r}\right], & r>r_0, \end{cases} \tag{7}$$

Where it is accepted, that perturbation of pressure at the distance $r_s \gg r_0$ from a cylinder axis vanishes, p_c - a pressure drop on a vortex axis. From the requirement of a continuity of perturbation of pressure of surfaces of a trunk of a vortex it is gained:

$$p_c=-\rho\frac{r_0^2}{2}\left[\omega_{in}^2+2\Omega\omega_{in}+\omega_e^2+4\Omega\omega_e\ln\frac{r_s}{r_0}\right]. \tag{8}$$

Because of a pressure drop, through the inferior warrant of a vortex there will be a dilatational stream of the substance which velocity we will present in a view

$$v_z=\begin{cases} v_{z0}+\alpha z, & r\le r_0,\\ 0, & r>r_0, \end{cases} \tag{9}$$

where v_{z0} and α, in generally, can be time functions.

Taking into account (9) from the equation of continuity (5) it is found

$$v_r=-\frac{\alpha}{2}\begin{cases} r, & r\le r_0,\\ \dfrac{r_0^2}{r}, & r>r_0. \end{cases} \tag{10}$$

Hence, because of a pressure drop on axes of the vortex trunk, an *inhaust effect* creates, which causes a dilatational stream (9) and converging to a vortex axis radial stream of substance (10). The last transfers the angular moment and energy to field of a trunk of a vortex.

Irrespective of a function v_r from the equation (3) taking into account (1) follows, that in the field of a trunk ($r < r_0$) convective and Coriolis accelerations develop, and in exterior field ($r > r_0$) cancel each other. Considering expressions (1), (10) in the equation (2), we find

$$\frac{d\omega}{dt} = \begin{cases} \omega, & r \quad r_0, \\ , & r > r_0. \end{cases} \tag{11}$$

I.e. because of transport of the angular moment by a converging radial stream of substance (14), angular velocity of a trunk of a vortex grows in due course, and rates of change *ω* in the field of a vortex trunk ($r \le r_0$) and in its exterior field $r > r_0$ the different.

Accepting *α = const,* the equation (15) for dependence of angular velocity of a vortex on time gives:

$$\omega(t) = \begin{cases} \omega_0 e^{\alpha t} \quad \omega_{in}, & r \le r_0, \\ \omega_0 \equiv \omega_e, & r > r_0, \end{cases} \tag{12}$$

where ω_0 - angular velocity of a trunk during the initial moment of a vortex formation.

Hence, the exterior region of a vortex rotates with the constant angular velocity - ω_e, while the trunk's angular velocity is growing by exponential law in time - ω_{in}.

Velocities (1), (9), (10) identically vanish viscous terms in the equations (2) - (4), diagonal terms of a viscous stress tensor are distinct from zero that leads to the following constant power of a dissipation of a kinetic energy on unit length of a vortex:

$$\frac{dE_k}{dt} = -4\pi\nu\rho r_0^2\{\frac{3}{2}\alpha^2 + \omega_e^2\}. \tag{13}$$

So, in a viewed vortex motion the dissipation remains small, despite prompt growth of angular velocity of a trunk of a vortex.

Tangential springs of rotational and dilatational velocities on boundary of a vortex trunk are:

$$[v_\varphi] = V = \lvert\omega_{in}(t) - \omega_0\rvert = r_0\omega_0(e^{\alpha t} - 1), \quad [v_z]\,\omega\, u = v_{z0}(t) + \alpha z. \tag{14}$$

3. Vortex structure. Taking into account (9), (10) from Navier-Stokes equations we obtain distribution of the full pressure in the polar regions of protostar and isobaric funnel surface equations:

$$z^2(r,t) + 2(R - H + \frac{\dot v_{z0} + \alpha v_{z0} - \alpha^2(R-H)}{\Omega_0^2 + \alpha^2})z(r,t) - \frac{\omega_{in}^2 - \frac{1}{4}\alpha^2}{\Omega_0^2+\alpha^2}r^2 -$$
$$- \frac{\Omega_0^2(1-e^2)}{\Omega_0^2+\alpha^2}(2r_s^2 - r^2) + \frac{\omega_{in}^2 + 2\Omega\omega_{in} + \omega_e^2}{\Omega_0^2+\alpha^2}r_0^2 - \frac{2C(t)}{\rho(\Omega_0^2+\alpha^2)} = 0 \tag{15}$$

- in the field of a trunk $r \le r_0$, and

$$z^2 + 2(R-H)z - (1-e^2)(r_s^2 - r^2) + \frac{\omega_e^2 + \frac{1}{4}\alpha^2}{\Omega_0^2}(\frac{r_0^4}{r^2} - \frac{r_0^4}{r_s^2}) = 0 \tag{16}$$

- in the exterior field $r > r_0$, where $\Omega_0^2 = 4\pi G\rho/3$, H – the initial height of the cylindrical Rankin vortex region on the protostar pole.

Let's demand now a continuity of an isobaric surface on boundary of a trunk of a vortex $r = r_0$. For this purpose let's

$$\dot v_{z0} - \alpha(v_{z0} - \alpha R) = 0. \tag{17}$$

Solving this equation taking into account initial condition $v_{z0}(0) = 0$, we will gain dependence $v_{z0}(t)$ in a view:

$$v_{z0}(t) = \alpha R(1 - e^{-\alpha t}). \tag{18}$$

From a requirement of a continuity of an isobar on a surface $r = r_0$ we will gain $C(t)$, then we will gain a definitive view of the equation of an isobar in the polar region of the protostar:

$$z(r,t) = \begin{cases} R + [R^2 - \frac{\omega_{in}^2 - \frac{1}{4}\alpha^2}{\Omega_0^2 + \alpha^2}(r_0^2 - r^2) - \frac{\omega_0^2 + \frac{1}{4}\alpha^2}{\Omega_0^2} r_0^2]^{1/2}, & r \le r_0 \\ R + [R^2 - \frac{\omega_0^2 + \frac{1}{4}\alpha^2}{\Omega_0^2} r_0^2 (\frac{r_0^2}{r^2} - \frac{r_0^2}{r_s^2})]^{1/2}, & r > r_0. \end{cases} \quad (19)$$

The isobar featured by formulas (19) represents a funnel going deep in due course. The outer part of a vortex funnel remains stationary. The isobar funnel (19) develops in fields of a vortex trunk going deep in due course under the exponential law. The bottom of a funnel is located on a vortex axis $r = 0$. The co-ordinate of a bottom of a funnel becomes deeper exponentially:

$$z(0,t) \sim -\frac{\omega_0^2}{\Omega_0^2 + \alpha^2} \frac{r_0^2}{2R} e^{2\alpha t}. \quad (20)$$

On the fig. 1 the illustration of approximate process of an isobaric surface of a protostar in the field of the vortex motion during the different moments with an identical interval of time is given.

5. Instability of a tangential shear of velocity on trunk boundary and vortex saturation.

As is known [10], surfaces with tangential shear of velocity are unstable to the superficial perturbations. This instability forms around the trunk boundary a turbulized transition stratum of a thickness $2\zeta(t)$. The effective coefficient of turbulent viscosity can be estimated by formula $\nu^*(t) \sim \frac{1}{2}\alpha v_0 |\zeta(t)| t$, where $\zeta(t)$ is the solution of equation $ln\frac{\zeta(t)}{\zeta_0} \approx \frac{\alpha \omega_0 r_0 t^2}{2\zeta(t)}$. The coefficient of turbulent viscosity grows in due course very promptly ($\sim t^3$), and can reach great values ($v^* \gg v$).

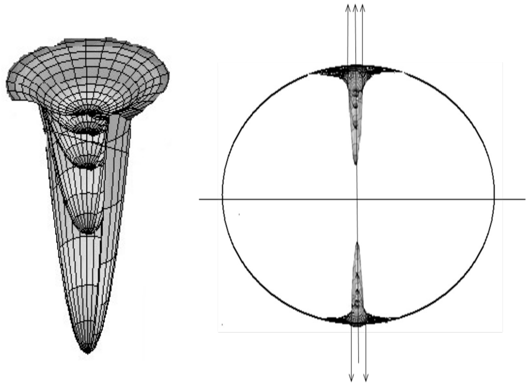

Fig.1. *The approximate evolution of an isobaric funnel in the region of a vortex through equal time steps (scale lengths are not maintained).*

Saturation of turbulent perturbations occurs, when growth of a kinetic energy of ground waves for a time unit, as a result of instability of a tangential share of velocity, becomes on an order of magnitude equal to power of a dissipation of turbulent energy in volume unity.

The acceleration of gyration of a vortex trunk will stop, when the tangential velocity jump $V(t)$ will reach to values, close to sound velocity c_s. The characteristic time t_s of vortex saturation is

$$t_s = \frac{1}{\alpha} \ln \frac{c_s}{\omega_0 r_0}. \qquad (21)$$

For this time the bottom of an isobaric funnel moves deep into on distance

$$z_s \quad \frac{c_s^2}{2R(\Omega_0 + \alpha^2)}, \qquad (22)$$

and $v_{z0}(t_s)$ accrues to value $v_{z0}(t_s) = \alpha R(1 - \frac{\omega_0 r_0}{c_s}) : \alpha R$.

So, the velocity of a vertical stream of substance (stream) on a protostar surface will be equal

$$v_j \quad v_{z0}(t_s) + \alpha(z_s + H). \qquad (23)$$

The mass loss of a protostar for a year will make $dM/dt = \pi r_0^2 \rho v_j$.

5. Summary

So, occurrence of the Rankin's vortex in the polar stratum of a protostar causes dilatational and converging to trunk streams of substance which provide exponentially growth of angular velocity of gyration of a trunk and a pressure drop on its axis. Process of sedate growth of angular velocity of gyration and a pressure drop stops and the vortex motion transfers in a saturation state when rotational velocity on a trunk reaches the sound velocity. It occurs in time (21) after vortex occurrence, during which the vortex motion sweeps more and more deep stratums of a protostar, moving on distance (22). Dilatational velocity of a stream along a vortex trunk thus accrues to value (23), causing the mass outflow through a surface of a protostar in the form of a stream with velocity.

The same mechanism can be used for the collimation and acceleration of jets after the stream eruption from the pole of a protostar [9].

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