Speeding up entanglement degradation

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We present a method for tracking time-dependent entanglement between different modes of a quantum system as measured by observers in different states of relative (nonuniform) motion. By describing states on a given spacelike hypersurface, observers or detectors in different states of motion can detect different modes of excitation in a quantum field at any desired instant and thereby track various measures of entanglement as function of time. We illustrate our method for a scalar field, showing how entanglement degrades as a function of time if one observer begins in a state of inertial motion but ends in a state of uniform acceleration while the other remains inertial.

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I. INTRODUCTION

Entanglement is a key resource in quantum computational tasks [1] such as teleportation[2], communication, quantum control [3], and quantum simulations [4]. It is a property of multipartite quantum states that arises from the superposition principle and the tensor product structure of Hilbert space. It can be quantified uniquely for nonrelativistic bipartite pure states by the von Neumann entropy, and several measures such as entanglement cost, distillable entanglement, and logarithmic negativity have been proposed for mixed states [5].

Understanding entanglement in a relativistic setting is important both for providing a more complete framework for theoretical considerations, and for practical situations such as the implementation of quantum computational tasks performed by observers in arbitrary relative motion. For observers in uniform relative motion the total amount of entanglement is the same in all inertial frames [6], although different inertial observers may see these correlations distributed differently among various degrees of freedom. However for observers in relative uniform acceleration a communication horizon appears, limiting access to information about the whole of space-time, resulting in a degradation of entanglement as demonstrated for scalars [7,8] and fermions [9], and restricting the fidelity of processes such as teleportation [10] and other communication protocols [11].

The most general situation is that of observers in different states of nonuniform motion. This situation is relevant even for inertial observers in curved space-time, who will undergo relative nonuniform acceleration due to the geodesic deviation equation, and therefore disagree on the degree of entanglement in a given bipartite quantum state. While it is expected such entanglement will be time dependent, there is presently no approach for determining how the entanglement changes throughout the course of the motion.

We address this issue by constructing a method for determining such time-dependent entanglement measures. By considering the description of the field modes on a given spacelike hypersurface we are able to compute the Bogoliubov transformations between states of instantaneous positive and negative frequency as determined by observers in different states of nonuniform motion. We illustrate our approach by considering specifically the entanglement between two modes of a noninteracting scalar field when one of the observers describing the state begins in a state of inertial motion and ends in a state of uniform acceleration. From the perspective of the inertial observer (Alice) the state is a maximally entangled pure state. However the nonuniformly accelerated (NUA) observer (Vic) finds that the entanglement degrades as a function of time, approaching at late times the constant value measured by an observer (Rob) undergoing the same asymptotic uniform acceleration. The paper is structured as follows: In Sec II we solve the Klein-Gordon equation in the NUA coordinates and compute the Bogoliubov coefficients. In Sec. III, we quantify the degree of entanglement with the help of the logarithmic negativity N and mutual information I. Finally, we summarize our results in the conclusions

II. SOLUTION OF THE KLEIN-GORDON EQUATION

For the inertial observer Minkowski coordinates (t,x) are the most suitable for describing the field, whereas for a uniformly accelerated (UA) observer Rindler coordinates (τ,ξ) are appropriate. As these coordinates cover only a quadrant of Minkowski space, the UA observer remains constrained to a particular Rindler quadrant and has no access to the other Rindler sector. Both the inertial and UA observers can define a vacuum state and a Fock space for a scalar field obeying the Klein-Gordon equation, with solutions in each related by Bogoliubov transformations. A given mode seen by an inertial observer corresponds for the UA observer to a two-mode squeezed state, associated with the field observed in the two distinct Rindler regions [12]. The UA observer can neither access nor influence field modes in the causally disconnected region and so information is lost about the quantum state, resulting in detection of a thermal state, a phenomenon known as the Unruh effect [13].

For an NUA observer consider the coordinates (T,X)

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$$w(t+x) = 2\sinh[w(T+X)]w(t-x) = -e^{-w(T-X)}, \quad (1)$$

yielding the metric

$$ds^2 = [\exp(-2wT) + \exp(2wX)][dT^2 - dX^2], \quad (2)$$

which covers $t-x<0$, referred to as region (I). The remaining half-plane $t-x>0$ [region (II)] can be covered using coordinates (1) after making the substitutions $X\rightarrow -X$, $T\rightarrow -T$, and $w\rightarrow -w$. This observer's acceleration

$$a(T,X_0) = [e^{-2wT} + e^{2wX_0}]^{-3/2} w \exp(2wX_0) \quad (3)$$

along $X=X_0$ is nonuniform, with $a(T\rightarrow -\infty, X_0)=0$ and $a(T\rightarrow +\infty, X_0)=w\exp(-wX_0)$. This observer, restricted to the region $t-x<0$ can causally influence events in $t-x>0$ but cannot be causally influenced by them. Consequently one expects a loss of information about the state of a quantum field, with an associated degradation of entanglement.

For both the Rindler and NUA observers entropy no longer quantifies entanglement because an entangled pure state seen by inertial observers appears mixed from their frames. However for the NUA observer this mixed state changes with time, making its description somewhat problematic. Furthermore, since $\partial/\partial T$ is not a Killing vector there is no clear way to define a vacuum state.

Our approach is to describe the field modes on a given spacelike hypersurface at time T_0. On any such hypersurface instantaneous positive and negative frequencies (with their respective annihilation/creation operators) can be defined and a solution to the Klein-Gordon equation in (T,X) coordinates can be related to one in (t,x) coordinates by a Bogoliubov transformation at $T=T_0$. It is then possible to bound the entanglement of the mixed state using logarithmic negativity, which is a full entanglement monotone that bounds distillable entanglement from above [14]. This quantity will depend on T_0, and so one can track the change in entanglement as T_0 is varied. Similar considerations apply for mutual information [15], which can be used to quantify the state's total correlations (classical plus quantum). We find that two modes maximally entangled in an inertial frame become less entangled as a function of T_0, with the entanglement approaching a fixed value as $T_0 \rightarrow \infty$.

The Klein-Gordon equation in (T,X) coordinates is

$$[\partial_T^2 - \partial_X^2 + m^2(e^{-2wT} + e^{2wX})]\Phi(X,T) = 0. \quad (4)$$

Separating variables via $\Phi(X,T)=F(T)G(X)$ yields the second-order differential equations

$$\frac{d^2F(T)}{dT^2} + [m^2\exp(-2wT) + K^2]F(T) = 0, \quad (5)$$

$$\frac{d^2G(X)}{dX^2} - [m^2\exp(2wX) - K^2]G(X) = 0, \quad (6)$$

where K^2 is a constant of separation. Writing $\nu=K/w$, the solutions

$$F(T) = A_\nu J_{\mathrm{I}\nu}\left(\frac{m}{w}e^{-wT}\right) + B_\nu J_{-\mathrm{I}\nu}\left(\frac{m}{w}e^{-wT}\right), \quad (7)$$

$$G(X) = C_\nu K^1_{\mathrm{I}\nu}\left(\frac{m}{w}e^{wX}\right) + D_\nu \mathrm{I}^2_{\mathrm{I}\nu}\left(\frac{m}{w}e^{wX}\right) \quad (8)$$

can be expressed as linear combinations of Bessel functions $J_\mu(Z)$ and McDonald functions $K^1_\mu(Z)$ and $\mathrm{I}^2_\mu(Z)$.

As $T\rightarrow\infty$ the solutions to Eq. (5) asymptote to $F \sim e^{\mp \mathrm{I}KT}$, similar to those for a Rindler observer with time coordinate $\tau=T$, whereas for $T\rightarrow -\infty$ the solutions asymptote to $F\sim \exp(\mp \mathrm{I}\frac{m}{w}e^{-wT})$, similar to those for an inertial observer with time coordinate $wt=-e^{-wT}$. Requiring the solutions to be nonsingular in X we obtain from Eqs. (7) and (8) the positive frequency solution with the former asymptotic behavior is

$$\Phi^{\mathrm{R}+}_\nu(X,T) = \left(\frac{\nu}{\pi w}\right)^{1/2} J_{\mathrm{I}\nu}(\tilde{T})K^1_{\mathrm{I}\nu}(\tilde{X}), \quad (9)$$

where $\tilde{T}=\frac{m}{w}e^{-wT}$ and $\tilde{X}=\frac{m}{w}e^{wX}$. The solution with the latter asymptotic behavior is

$$\Phi^{\mathrm{I}+}_\nu(X,T) = \frac{\sqrt{1-e^{-2\pi\nu}}}{2}\left(\frac{\nu}{\pi w}\right)^{1/2} H^1_{\mathrm{I}\nu}(\tilde{T})K^1_{\mathrm{I}\nu}(\tilde{X}), \quad (10)$$

as can be determined from the asymptotic behavior of the Bessel functions. Solutions (9) and (10) are orthogonal

$$\langle \Phi^{\mathrm{R}+}_\mu(X,T), \Phi^{\mathrm{I}+}_\nu(X,T)\rangle = (we^{\pi\nu}\sqrt{e^{2\pi\nu}-1})^{-1}\delta(\mu-\nu) \quad (11)$$

under the inner product

$$\langle \Phi, \Psi\rangle = -\mathrm{I}\int_S dS^\alpha \Phi \overleftrightarrow{\partial}_\alpha \Psi^*, \quad (12)$$

where dS^α is the measure orthogonal to the Cauchy surface S with timelike unit normal n^α.

We proceed in a way analogous to that applied in discussing particle production in expanding universes [16]. Consider now the hypersurface $T=T_0$. We consider a subspace of solutions of Eq. (4), possessing instantaneous positive frequency at the hypersurface T_0. A general solution at time T

$$\Phi^+_\nu(X,T) = [c_{\nu+}(T_0)J_{\mathrm{I}\nu}(\tilde{T}) + c_{\nu-}(T_0)J_{-\mathrm{I}\nu}(\tilde{T})]K_{\mathrm{I}\nu}(\tilde{X}) \quad (13)$$

will be a positive frequency solution at $T=T_0$ provided

$$c_{\nu\pm}(T) = \frac{\mp \mathrm{I}\nu\pi W^{1/2}}{2K\sinh(\pi\nu)}[\dot{J}_{\mp \mathrm{I}\nu}(\tilde{T})W^{-1} + \mathrm{I}J_{\mp \mathrm{I}\nu}(\tilde{T})], \quad (14)$$

where $W=\sqrt{m^2e^{-2wT}+K^2}$ and the overdot denotes a derivative with respect to T.

A solution for the NUA observer can be expressed in terms of positive and negative frequency solutions

$$\Phi^{\mathrm{M}\pm}_k(x,t) = \frac{1}{2}(\pi\epsilon)^{-1/2}\exp[\mp \mathrm{I}(\epsilon t - kx)] \quad (15)$$

of the inertial observer via the Bogoliubov transformations

$$\alpha_{\nu,k} = \langle \Phi^+_\nu(X,T), \Phi^{\mathrm{M}+}_k(x,t)\rangle, \quad (16)$$

$$\beta_{\nu,k} = -\langle \Phi_\nu^+(X,T), \Phi_k^{\mathrm{M}-}(x,t)\rangle, \tag{17}$$

where $\epsilon^2 = k^2 + m^2$. For positive frequency solution (13), Eqs. (16) and (17) give at $T = T_0$

$$\alpha_{\nu,k} = -\mathcal{C}c_{\nu+}(T_0)A_{\nu,k} + \mathcal{C}c_{\nu-}(T_0)B^*_{\nu,k}, \tag{18}$$

$$\beta_{\nu,k} = \mathcal{C}c_{\nu+}(T_0)B_{\nu,k} + \mathcal{C}c_{\nu-}(T_0)A^*_{\nu,k}, \tag{19}$$

where

$$A_{\nu,k} = -\left(\frac{\nu}{\epsilon w}\right)^{1/2} \frac{e^{\pi\nu/2}}{2\Gamma(1+\mathrm{I}\nu)\sinh(\pi\nu)}\left(\frac{\epsilon - k}{2w}\right)^{\mathrm{I}\nu}, \tag{20}$$

$B_{\nu,k} = -A_{\nu,k}e^{-\pi\nu}$, and $\mathcal{C} = \sqrt{\frac{\pi w}{\nu}}$.

III. QUANTIFICATION OF THE ENTANGLEMENT

Consider now two modes, k and s, of a free scalar field in Minkowski space-time that are maximally entangled from an inertial perspective

$$\frac{1}{\sqrt{2}}(|0_s\rangle^{\mathcal{M}}|0_k\rangle^{\mathcal{M}} + |1_s\rangle^{\mathcal{M}}|1_k\rangle^{\mathcal{M}}), \tag{21}$$

where $|0_j\rangle^{\mathcal{M}}$ and $|1_j\rangle^{\mathcal{M}}$ are the vacuum and single-particle excitation states of the mode j with respect to an inertial observer in Minkowski space. We assume that Alice can only detect mode s with her detector and that Vic can only detect mode k with his. The Minkowski vacuum state $|0\rangle^{\mathcal{M}} = \Pi_j|0_j\rangle^{\mathcal{M}}$ is defined as the absence of any particle excitation in any of the modes. It can be expressed in terms of the vacuum $|0\rangle^{\mathcal{A}}$ with respect to the accelerated observer as [17]

$$|0\rangle^{\mathcal{M}} = \frac{1}{{}^{\mathcal{M}}\langle 0|0\rangle^{\mathcal{A}}}\exp\left(-\frac{1}{2}\Sigma_{\nu,k}\tilde{a}_\nu V_{\nu\lambda}\tilde{a}^\dagger_\lambda\right)|0\rangle^{\mathcal{A}}. \tag{22}$$

Using $(\alpha^*_{\nu,k})_{\mathrm{I}} = (\alpha_{\nu,k})_{\mathrm{II}}$ and $(\beta^*_{\nu,k})_{\mathrm{I}} = (\beta_{\nu,k})_{\mathrm{II}}$, where subindices I and II indicate, respectively, regions (I) and (II), we get

$$V_{\nu,\lambda} = \sum_l \beta^*_{\nu l}\alpha^{-1}_{l\lambda} = -\delta(\nu - \lambda)q, \tag{23}$$

where q is given by

$$q = \frac{e^{-\pi\nu}c^*_{\nu+}(T_0) - c^*_{\nu-}(T_0)}{[c_{\nu+}(T_0) + e^{-\pi\nu}c_{\nu-}(T_0)]}. \tag{24}$$

We then rewrite Eq. (22) in terms of a product of two-mode squeezed states of the NUA vacuum defined on the hypersurface $T = T_0$,

$$|0\rangle^{M} = \sqrt{1 - |q|^2}\sum_{n=0}^{\infty} q^n |n_k\rangle_{\mathrm{I}}|n_k\rangle_{\mathrm{II}}, \tag{25}$$

where $|n_k\rangle_{\mathrm{I}}$ and $|n_k\rangle_{\mathrm{II}}$ refer to the mode decomposition in regions (I) and (II), respectively. Each Minkowski mode j has a mode expansion given by Eq. (25). We assume that all modes except for mode s for Alice and k for Vic are in the vacuum. Tracing over all of these other modes yields a pure state since each set of solutions in regions (I) and (II) are orthogonal and so different modes j and j' do not mix on the hypersurface $T = T_0$.

Since events in region (II) cannot causally influence those in region (I) we rewrite Eq. (21) using Eq. (25), tracing over states in region (II). This yields a mixed state between Alice (A) and Vic (V),

$$\rho_{AV} = \frac{1 - |q|^2}{2}\sum_n q^{2n}\rho_n,$$

$$\begin{aligned}\rho_n = {} & |0n\rangle\langle 0n| + \sqrt{1 - |q|^2}\sqrt{n+1}|0n\rangle\langle 1n+1| \\ & + \sqrt{1 - |q|^2}\sqrt{n+1}|1n+1\rangle\langle 0n| + (1 - |q|^2)(n+1)|1n+1\rangle \\ & \times\langle 1n+1|,\end{aligned} \tag{26}$$

whose elements are functions of T_0 via the parameter q, where $|nm\rangle = |n_s\rangle^{\mathcal{M}}|m_k\rangle_{\mathrm{I}}$.

From here the calculation proceeds in a manner similar to that for the Rindler case [7]. If at least one eigenvalue of the partial transpose of ρ is negative, then the density matrix is entangled; but a state with positive partial transpose can still be entangled. This criterion is sufficient for determining entanglement [18], whose type in this case is called bound or nondistillable entanglement [14]. We obtain the partial transpose by interchanging Alice's qubits, obtaining

$$\lambda^n_{\pm} = \frac{1}{4}|q|^{2n}(1 - |q|^2)\left[\left(\frac{n|q|^2}{\sqrt{1 - |q|^2}} + |q|^2\right) \pm \sqrt{Z_n}\right]$$

for the eigenvalues in the $(n, n+1)$ block, where

$$Z_n = \left(\frac{n|q|^2}{\sqrt{1 - |q|^2}} + |q|^2\right)^2 + 4(1 - |q|^2). \tag{27}$$

One eigenvalue is always negative since $|q| < 1$ for any value of T_0 and so the state is always entangled. Summing over all the negative eigenvalues yields the logarithmic negativity [14], defined as $N(\rho) = \log_2\|\rho^T\|_1$, where $\|\rho^T\|_1$ is the trace-norm of the density matrix ρ. The quantity $N(\rho)$ bounds the distillable entanglement (the amount of "almost-pure state entanglement" that can be asymptotically distilled) contained in ρ. We find

$$N(\rho_{AV}) = \log_2\left(\frac{1 - |q|^2}{2} + \sum_{n=0}^{\infty}\frac{|q|^{2n}}{4}(1 - |q|^2)\sqrt{Z_n}\right). \tag{28}$$

Figure 1 shows that logarithmic negativity decreases with increasing time, its slope more pronounced close to $T_0 = 0$ where the change in acceleration is maximal. For vanishing acceleration ($q \to 0$ as $T \to -\infty$), $N(\rho_{AV}) \to 1$, and as $T_0 \to +\infty$, it approaches the values obtained for the UA case [7], with different values of $K = \nu w$ yielding different asymptotic values of $N(\rho_{AV})$.

Hence entanglement degradation increases as a function of time. This behavior could in principle be empirically determined by Alice from an ensemble of experiments with different NUA observers each with the same value of w, making measurements for different choices of T_0 that are classically communicated to Alice.

To estimate the total amount of correlation in the state we compute the mutual information, defined as $\mathrm{I}(\rho_{AV}) = S(\rho_A) + S(\rho_V) - S(\rho_{AV})$, where $S(\rho) = -\mathrm{Tr}[\rho\log_2(\rho)]$ is the entropy

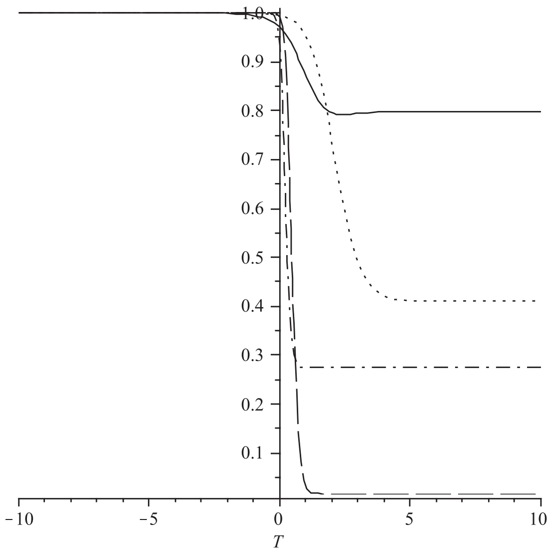

FIG. 1. Logarithmic negativity plotted against T_0, with $\nu = K/w$, for various values of K and w. (a) The dot line corresponds to $K=0.1$ and $w=1$, (b) the solid line corresponds to $K=0.3$ and $w=1$, (c) the dash line corresponds to $K=0.1$ and $w=5$, and (d) the dash-dot line corresponds to $K=0.3$ and $w=5$

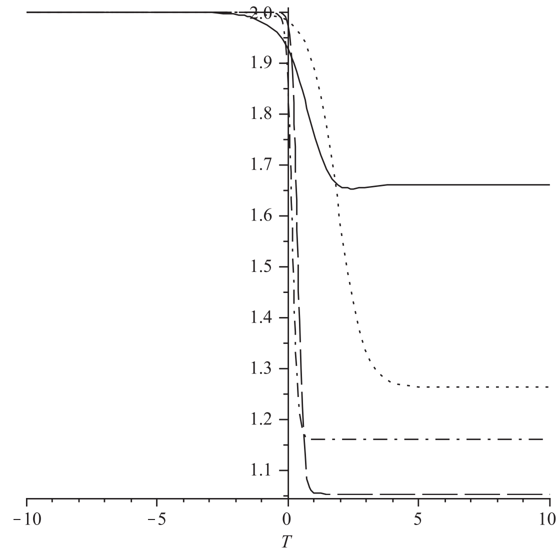

FIG. 2. Mutual information plotted against T_0, with $\nu = K/w$, for various values of K and w. (a) The dot line corresponds to $K=0.1$ and $w=1$, (b) the solid line corresponds to $K=0.3$ and $w=1$, (c) the dash line corresponds to $K=0.1$ and $w=5$, and (d) the dash-dot line corresponds to $K=0.3$ and $w=5$.

of the density matrix ρ. We obtain Alice's density matrix $\rho_A = \frac{1}{2}\mathrm{I}$ by tracing over Vic's states and Vic's density matrix by tracing over Alice's states. The resultant entropies can be straightforwardly calculated along with the entropy of the joint state, yielding

$$\mathrm{I} = 1 - \frac{1}{2}\log_2(|q|^2) - \frac{1-|q|^2}{2}\sum_{n=0}^{\infty}|q|^{2n}\mathcal{D}_n,$$

$$\mathcal{D}_n = \left[1 + \frac{n(1-|q|^2)}{|q|^2}\right]\log_2\left[1 + \frac{n(1-|q|^2)}{|q|^2}\right] - \{1 + (n+1)(1-|q|^2)\log_2[1 + (n+1)(1-|q|^2)]\},$$

for the mutual information, which we plot in Fig. 2 as a function of T_0. For large negative values of T_0 the acceleration vanishes and the mutual information is 2 as expected. As the acceleration increases, the mutual information decreases reaching, as $T_0 \to +\infty$, a constant value larger than unity.

The logarithmic negativity as well as the mutual information approach for large values of T a constant asymptotic value that depends on $\nu = K/w$. Figure 1 and Fig. 2 show that, as expected from the Rindler case [7], for a given value of K, the asymptote of N and I decrease as w increases. We also see that for a given vale of w, the asymptotic values of N and I increase monotonically with K.

IV. CONCLUDING REMARKS

Our method for tracking the time dependence of measures of quantum information via a sequence of measurements on hypersurfaces where positive or negative frequencies can be defined is quite general and can be applied to different kinds of motions and fields beyond the example we consider here. For a situation in which both observers begin freely falling into a black hole with one observer increasing acceleration to avoid this fate, the distillable entanglement degrades to a finite value. The entanglement degradation is due to the increase in entanglement with the modes in the region causally undetectable by the NUA observer. In curved space-time, we expect in general that entanglement is a time-dependent as well as an observer-dependent concept.

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