

Creation of scalar and Dirac particles in the presence of a time varying electric field in an anisotropic Bianchi type I universe

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In this article we compute the density of scalar and Dirac particles created by a cosmological anisotropic Bianchi type I universe in the presence of a time varying electric field. We show that the particle distribution becomes thermal when one neglects the electric interaction.

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I. INTRODUCTION

During the last decades a great deal of effort has been made in understanding quantum processes in strong fields. Quantum field theory in the presence of strong fields is in general a theory associated with unstable vacua. The vacuum instability leads to many interesting features; among them particle creation is perhaps the most interesting nonperturbative phenomenon.

In order to analyze the mechanism of particle creation in cosmological backgrounds we have at our disposal different techniques such as the adiabatic approach [1,2], the Feynman path integral method [3], the Hamiltonian diagonalization technique [4,5], as well as the Green function approach [6]

The study of quantum processes in the presence of electromagnetic fields in curved backgrounds has been restricted almost entirely to constant electric fields. The density of particles created by an intense electric field was first calculated by Schwinger [7] and different authors have discussed this phenomenon in curved spaces. The technical difficulties associated with solving the Klein-Gordon or Dirac equations in time dependent electromagnetic fields have reduced the number of configurations studied to a few simple solvable models [4]. A further difficulty arises when one deals with cosmological backgrounds with singularities [8,9]. Here one cannot apply the standard adiabatic approach and other equivalent methods developed in the literature. It is worth mentioning that the presence of primordial electric fields could enhance the particle creation mechanism and also produce deviations from the thermal spectrum.

In order to discuss particle creation in an anisotropic universe, we consider the cosmological scenario associated with the metric

$$ds^2 = -dt^2 + t^2(dx^2 + dy^2) + dz^2. \quad (1)$$

The line element (1) presents a singularity at $t=0$. The scalar curvature is $R=2/t^2$, and consequently the adiabatic approach [1] cannot be applied in order to identify particle states. Using the Hamiltonian diagonalization method, Bukhbinder and co-workers [5,10,11] were able to compute the

density of particles created by the background associated with the metric (1), and in Ref. [12] this result was extended to include a time dependent electric field.

The purpose of the present article is to discuss the production of scalar and Dirac particles in the anisotropic cosmological background associated with the line element (1) in the presence of a time dependent homogeneous electric field. In order to compute the rate of particle creation we apply a quasiclassical approach that has been used successfully in different scenarios [13–16]. The idea behind the method is the following. First, we solve the relativistic Hamilton-Jacobi equation and, looking at its solutions, we identify positive and negative frequency modes. Second, we solve the Klein-Gordon and Dirac equations and, after comparing with the results obtained for the quasiclassical limit, we identify the positive and negative frequency states. The paper is structured as follows. In Sec. II we solve the relativistic Hamilton-Jacobi equation and compute the quasiclassical energy modes. In Sec. III we solve the Klein-Gordon and Dirac equations and obtain the density of scalar and Dirac particles created. The discussion of the results and final remarks are presented in Sec. IV.

II. COMPUTATION OF THE QUASICLASSICAL SOLUTIONS

The relativistic Hamilton-Jacobi equation can be written as

$$g^{\alpha\beta} \left(\frac{\partial S}{\partial x^\alpha} - eA_\alpha \right) \left(\frac{\partial S}{\partial x^\beta} - eA_\beta \right) + m^2 = 0 \quad (2)$$

where here and elsewhere we adopt the convention $c=1$ and $\hbar=1$.

The vector potential A^μ

$$A^\mu = \frac{C}{t} \delta_3^\mu \quad (3)$$

corresponds to a time dependent homogeneous electric field $C\hat{k}/t^2$. The invariants

$$\frac{1}{2} F^{\mu\nu} F_{\mu\nu} = -\frac{C^2}{t^4}, \quad F^{\mu\nu} F_{\mu\nu}^* = 0 \quad (4)$$

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indicate that there is no magnetic field. Since the line element (1) and the vector potential (3) do not depend on the space variables we can write the function S as follows:

$$S = \vec{k} \cdot \vec{r} + F(t). \quad (5)$$

Substituting Eq. (5) into the Hamilton-Jacobi equation one obtains

$$\frac{k_x^2 + k_y^2}{t^2} + \left(k_z - \frac{eC}{t}\right)^2 - \left(\frac{dF}{dt}\right)^2 + m^2 = 0. \quad (6)$$

The solution of Eq. (6) has the following asymptotic behavior:

$$\lim_{t \rightarrow 0} F(t) = \pm \sqrt{k_x^2 + k_y^2 + e^2 C^2} \log t, \quad (7)$$

$$\Phi = e^{iS} \rightarrow C_0 t^{\pm i \sqrt{k_x^2 + k_y^2 + e^2 C^2}} \quad (8)$$

as $t \rightarrow 0$, and

$$\lim_{t \rightarrow +\infty} F(t) = \pm \sqrt{k_z^2 + m^2} \mp \frac{k_z e C}{\sqrt{k_z^2 + m^2}} \log t, \quad (9)$$

$$\Phi = e^{iS} \rightarrow C_1 e^{\pm i \sqrt{k_z^2 + m^2} t \mp i k_z e C / \sqrt{k_z^2 + m^2}} \quad (10)$$

as $t \rightarrow \infty$. The positive or negative frequency mode is selected depending on the sign of the operator $i \partial_t$. Positive and negative frequency modes will have, respectively, positive and negative eigenvalues; thus in Eqs. (7)–(10) the upper signs are associated with negative frequency modes and the lower signs correspond to positive frequency states. After making this identification we proceed to analyze the solutions of the Klein-Gordon and Dirac equations in the cosmological background (1).

III. PARTICLE PRODUCTION

A. Scalar particles

The covariant Klein-Gordon equation has the form

$$g^{\alpha\beta} (\nabla_\alpha - ieA_\alpha) (\nabla_\beta - ieA_\beta) \Phi - (m^2 + \xi R) \Phi = 0 \quad (11)$$

where ∇_α is the covariant derivative, R is the scalar curvature and ξ is a dimensionless coupling constant which takes the value $\xi=0$ in the minimal coupling case and $\xi=1/6$ when a conformal coupling is considered. The scalar wave function Φ is normalized according to the inner product [17]

$$\langle \Phi_1, \Phi_2 \rangle = \int_\sigma (\Phi_2 \partial_\mu \Phi_1^* - \Phi_1^* \partial_\mu \Phi_2) \sqrt{-g} ds^\mu \quad (12)$$

where σ is an arbitrary spacelike hypersurface. Since the metric (1) is stationary we choose a hypersurface orthogonal to the timelike vector $t^\mu = \delta_0^\mu$. Since Eq. (11) commutes with each of the components of the linear momentum $\vec{p} = -i\nabla$, the wave function Φ can be written as

$$\Phi = t^{-1} \Delta(t) e^{i(k_x x + k_y y + k_z z)}. \quad (13)$$

Substituting Eq. (13) into Eq. (11) we reduce the problem of solving Eq. (11) to that of finding a solution of the following second order differential equation:

$$\left(\frac{d^2}{dt^2} + \frac{k_x^2 + k_y^2 + e^2 C^2 + 2\xi}{t^2} + k_z^2 - \frac{2k_z e C}{t} + m^2 \right) \Delta(t) = 0. \quad (14)$$

The solution can be expressed in terms of Whittaker functions [18,19]

$$\Delta = \mathfrak{C}_1 M_{k, \bar{\mu}}^-(\rho) + \mathfrak{C}_2 W_{k, \bar{\mu}}^-(\rho) \quad (15)$$

where \mathfrak{C}_1 and \mathfrak{C}_2 are arbitrary constants, and ρ, k and $\bar{\mu}$ are given by the expressions

$$\rho = -2i \sqrt{k_z^2 + m^2} t, \quad k = -\frac{ik_z e C}{\sqrt{k_z^2 + m^2}},$$

$$\bar{\mu} = i \sqrt{-\frac{1}{4} + k_x^2 + k_y^2 + e^2 C^2 + 2\xi}, \quad (16)$$

Looking at the asymptotic behavior of $M_{k, \mu}(z)$ as $z \rightarrow 0$,

$$M_{k, \mu}(z) \sim e^{-z/2} z^{1/2 + \mu}, \quad (17)$$

we find that, according to Eq. (8), and using the fact that all the coefficients in Eq. (14) are real, the positive and negative frequency solutions (15) at $t=0$ are

$$\begin{aligned} \Delta_0^+ &= \mathfrak{C}_0^+ M_{k, \bar{\mu}}^-(\rho), \\ \Delta_0^- &= [\mathfrak{C}_0^+ M_{k, \bar{\mu}}^-(\rho)]^* \\ &= \mathfrak{C}_0^+ (-1)^{-\bar{\mu} + 1/2} M_{k, -\bar{\mu}}^-(\rho) \end{aligned} \quad (18)$$

where \mathfrak{C}_0^+ is a normalization constant. Analogously, looking at the behavior of $W_{k, \mu}(z)$ as $|z| \rightarrow \infty$,

$$W_{k, \mu}(z) \sim e^{-z/2} z^k, \quad (19)$$

we see that the corresponding positive and negative frequency modes as $|\rho| \rightarrow \infty$ are

$$\Delta_\infty^+ = \mathfrak{C}_\infty^+ W_{k, \bar{\mu}}^-(\rho), \quad \Delta_\infty^- = \mathfrak{C}_\infty^- W_{-k, \bar{\mu}}^-(\rho) \quad (20)$$

where \mathfrak{C}_∞^+ and \mathfrak{C}_∞^- are normalization constants. The positive frequency mode at $|\rho| \rightarrow \infty$ can be expressed in terms of the positive Δ_0^+ and negative Δ_0^- frequency modes via the Bogoliubov transformation

$$\Delta_\infty^+ = \alpha \Delta_0^+ + \beta \Delta_0^- \quad (21)$$

Since the Whittaker function $W_{k, \mu}(z)$ can be expressed in terms of $M_{k, \mu}(z)$ as follows:

$$W_{k,\mu}(z) = \frac{\Gamma(-2\mu)}{\Gamma\left(\frac{1}{2} - \mu - k\right)} M_{k,\mu}(z) + \frac{\Gamma(2\mu)}{\Gamma\left(\frac{1}{2} + \mu - k\right)} M_{k,-\mu}(z), \quad (22)$$

we obtain that α and β are given by the expressions

$$\alpha = \frac{\mathfrak{C}_\infty^+}{\mathfrak{C}_0^+} \frac{\Gamma(-2\bar{\mu})}{\Gamma\left(\frac{1}{2} - \bar{\mu} - k\right)},$$

$$\beta = -i \frac{\mathfrak{C}_\infty^+}{\mathfrak{C}_0^+} \frac{\Gamma(2\bar{\mu})}{\Gamma\left(\frac{1}{2} + \bar{\mu} - k\right)} \exp(-\pi|\bar{\mu}|); \quad (23)$$

then

$$\left| \frac{\beta}{\alpha} \right|^2 = \left| \frac{\Gamma\left(\frac{1}{2} - \bar{\mu} - k\right)}{\Gamma\left(\frac{1}{2} + \bar{\mu} - k\right)} \right|^2 \exp(-2\pi|\bar{\mu}|). \quad (24)$$

The computation of the density of particles created is straightforward from Eq. (24) and the normalization condition of the wave function which, for scalar particles, means that the Bogoliubov coefficients satisfy the relation

$$|\alpha|^2 - |\beta|^2 = 1. \quad (25)$$

Taking into account that [18]

$$\left| \Gamma\left(\frac{1}{2} + iy\right) \right|^2 = \frac{\pi}{\cosh \pi y}, \quad (26)$$

we find that the density of particles created is

$$n = |\beta|^2 = \frac{\exp[\pi(|\bar{\mu}| + |k|)] \cosh[\pi(|\bar{\mu}| + |k|)] \exp(-2\pi|\bar{\mu}|)}{\sinh(2\pi|\bar{\mu}|)}. \quad (27)$$

It is worth mentioning that, thanks to the relations (25) and (24), we did not have to compute the normalization constants \mathfrak{C}_∞^+ , \mathfrak{C}_∞^- , and \mathfrak{C}_0^+ . When the electric field is switched off, the parameter k is zero and the density of particles created (27) reduces to a thermal distribution [5]:

$$n = \frac{1}{\exp 2\pi \sqrt{k_\perp^2 + 2\xi - \frac{1}{4}} - 1}. \quad (28)$$

B. Dirac particles

In order to compute the density of spin 1/2 particles created, we proceed to solve the Dirac equation in the cosmological background (1) in the presence of the electric field (3).

The covariant Dirac equation in curved space in the presence of electromagnetic fields can be written as

$$\{\gamma^\mu(\partial_\mu - \Gamma_\mu - ieA_\mu) + m\}\Psi = 0 \quad (29)$$

where the curved gamma γ^μ matrices satisfy the anticommutation relation $\{\gamma^\mu, \gamma^\nu\}_+ = 2g^{\mu\nu}$ and the spinor connections Γ_μ are

$$\Gamma_\mu = \frac{1}{4} g_{\lambda\alpha} \left[\left(\frac{\partial b_\nu^\beta}{\partial x^\mu} \right) a_\beta^\alpha - \Gamma_{\nu\mu}^\alpha \right] s^{\lambda\nu} \quad (30)$$

where

$$s^{\lambda\nu} = \frac{1}{2} (\gamma^\lambda \gamma^\nu - \gamma^\nu \gamma^\lambda). \quad (31)$$

The matrices $b_\alpha^\beta, a_\beta^\alpha$ establish the connection between the curved γ^μ and Minkowski $\tilde{\gamma}^\mu$ Dirac matrices as follows:

$$\gamma_\mu = b_\mu^\alpha \tilde{\gamma}_\alpha, \quad \gamma^\mu = a_\beta^\mu \tilde{\gamma}^\beta \quad (32)$$

and

$$\tilde{\gamma}^\lambda \tilde{\gamma}^\nu + \tilde{\gamma}^\nu \tilde{\gamma}^\lambda = 2\eta^{\lambda\nu}. \quad (33)$$

Since the line element (1) is diagonal, we choose to work in the diagonal tetrad

$$\gamma^\mu = \sqrt{|g^{\mu\mu}|} \tilde{\gamma}^\mu, \quad \text{no sum.} \quad (34)$$

Substituting Eq. (34) into Eq. (30) we obtain

$$\Gamma_1 = \frac{1}{2} \tilde{\gamma}^0 \tilde{\gamma}^1, \quad \Gamma_2 = \frac{1}{2} \tilde{\gamma}^0 \tilde{\gamma}^2, \quad \Gamma_3 = 0, \quad \Gamma_0 = 0, \quad (35)$$

and the Dirac equation (29) takes the form

$$\left\{ \tilde{\gamma}^0 \frac{\partial}{\partial t} + \frac{1}{t} \left(\tilde{\gamma}^1 \frac{\partial}{\partial x} + \tilde{\gamma}^2 \frac{\partial}{\partial y} \right) + \tilde{\gamma}^3 \left(\frac{\partial}{\partial z} - \frac{ieC}{t} \right) + m \right\} \Psi_0 = 0 \quad (36)$$

where $\Psi_0 = t\Psi$. The factor t was introduced in order to cancel the contribution due to the spinor connections (35). Equation (36) can be written as a sum of two first order commuting differential operators as follows [20,21]:

$$(\hat{K}_1 + \hat{K}_2)\Phi = 0, \quad (37)$$

$$\hat{K}_2\Phi = k\Phi = -\hat{K}_1\Phi, \quad (38)$$

where the spinor Φ is related to Ψ_0 via the expression

$$\tilde{\gamma}^3 \tilde{\gamma}^0 \Psi_0 = \Phi, \quad (39)$$

and k is a separation constant. The operators \hat{K}_1 and \hat{K}_2 read

$$\hat{K}_1 = t \left[\gamma^3 \frac{\partial}{\partial t} + \gamma^0 \left(\frac{\partial}{\partial z} - i \frac{eC}{t} \right) + m \gamma^3 \gamma^0 \right], \quad (40)$$

$$\hat{K}_2 = \left(\tilde{\gamma}^1 \frac{\partial}{\partial x} + \tilde{\gamma}^2 \frac{\partial}{\partial y} \right) \tilde{\gamma}^3 \tilde{\gamma}^0. \quad (41)$$

Since Eq. (36) commutes with $-i\nabla$, the spinor Φ can be written as $\Phi = \Phi_0 \exp[i(k_x x + k_y y + k_z z)]$. Choosing to work in the following representation of the Dirac matrices:

$$\begin{aligned}\tilde{\gamma}^0 &= \begin{pmatrix} -i\sigma^1 & 0 \\ 0 & i\sigma^1 \end{pmatrix}, & \tilde{\gamma}^1 &= \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \\ \tilde{\gamma}^2 &= \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}, & \tilde{\gamma}^3 &= \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix},\end{aligned}\quad (42)$$

we see that the equation $\hat{K}_2 \Phi = k \Phi$ helps us to determine the relation between the components of the bispinor Φ

$$\Phi_0 = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \Phi_1 \\ \frac{k_x \sigma^2}{ik_y - k} \Phi_1 \end{pmatrix}\quad (43)$$

where

$$k = i\sqrt{k_x^2 + k_y^2}.\quad (44)$$

Using the representation (42) we find that Eq. (40) reduces to the system of equations

$$\left\{ \sigma^3 \frac{d}{dt} + \sigma^1 \left(k_z - \frac{eC}{t} \right) + m\sigma^2 + \frac{k}{t} \right\} \Phi_1 = 0,\quad (45)$$

$$\left\{ -\sigma^3 \frac{d}{dt} + -\sigma^1 \left(k_z - \frac{eC}{t} \right) + m\sigma^2 + \frac{k}{t} \right\} \Phi_2 = 0.\quad (46)$$

Taking into account the structure of the bispinor (43), it is not difficult to see that Eq. (46) is equivalent to Eq. (45) and consequently we have reduced the problem of solving Eq. (36) to that of finding the solution of Eq. (45).

In order to solve Eq. (45) we proceed as follows. Since the structure of Eq. (45) resembles the one obtained after solving the hydrogen atom in spherical coordinates [22,23] and we are interested in identifying positive and negative frequency modes, it would be desirable to have solutions of Eq. (45), as in the scalar case, in terms of single Whittaker functions. For this purpose, we apply to the spinor Φ_1 the linear transformation T :

$$T\Phi_1 = F = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}\quad (47)$$

with

$$T = \begin{pmatrix} \sqrt{im+k_z} & -\sqrt{im-k_z} \\ \sqrt{im+k_z} & \sqrt{-im-k_z} \end{pmatrix}.\quad (48)$$

The components of the spinor F (47) satisfy the system of equations

$$\rho \frac{df_1}{d\rho} - f_1 \left(\frac{-Cek_z}{\sqrt{-k_z^2 - m^2}} + \frac{\rho}{2} \right) - f_2 \left(\frac{IeCm}{\sqrt{-k_z^2 - m^2}} - k \right) = 0,\quad (49)$$

$$\rho \frac{df_2}{d\rho} + f_2 \left(\frac{-Cek_z}{\sqrt{-k_z^2 - m^2}} + \frac{\rho}{2} \right) + f_1 \left(\frac{IeCm}{\sqrt{-k_z^2 - m^2}} + k \right) = 0,\quad (50)$$

whose solutions can be expressed in terms of the Whittaker functions $W_{k,\mu}(z)$ and $M_{k,\mu}(z)$ [18,19].

In order to construct the positive and negative energy frequency modes, we use the asymptotic behavior of the hypergeometric functions (17) and (19) and compare the solutions of Eq. (49) and (50) with Eqs. (8) and (10), obtained after solving the Hamilton-Jacobi equation. Using this procedure, we identify the positive frequency mode as $t \rightarrow +\infty$ as

$$F_{+\infty}^+ = \begin{pmatrix} \mathfrak{C}_\infty^+ \rho^{-1/2} W_{-1/2+eCk_z/\sqrt{-k_z^2-m^2}, \sqrt{k^2-e^2C^2}}(\rho) \\ \frac{1}{k-ICem/\sqrt{-k_z^2-m^2}} \mathfrak{C}_\infty^+ \rho^{-1/2} W_{1/2+eCk_z/\sqrt{-k_z^2-m^2}, \sqrt{k^2-e^2C^2}}(\rho) \end{pmatrix}\quad (51)$$

with

$$\mu = \sqrt{k^2 - e^2 C^2} = i\sqrt{k_\perp^2 + e^2 C^2} = i\tilde{\mu},\quad (52)$$

\mathfrak{C}_∞^+ is a normalization constant according to the inner product

$$\langle \Phi_1, \Phi_2 \rangle = \int_\sigma \bar{\Phi}_1 \gamma^\mu \Phi_2 \sqrt{-g} d\sigma_\mu\quad (53)$$

where σ is an arbitrary spacelike hypersurface. Analogously, we have that as $t \rightarrow 0$ the positive mode is

$$F_0^+ = \begin{pmatrix} \mathfrak{C}_0^+ \rho^{-1/2} M_{-1/2+eCk_z/\sqrt{-k_z^2-m^2}, \mu}(\rho) \\ -\frac{eCk_z/\sqrt{-k_z^2-m^2} + \mu}{k-ICem/\sqrt{-k_z^2-m^2}} \mathfrak{C}_0^+ \rho^{-1/2} M_{1/2+eCk_z/\sqrt{-k_z^2-m^2}, \mu}(\rho) \end{pmatrix}\quad (54)$$

where \mathfrak{C}_0^+ is also a normalization constant. Using the asymptotic behavior of the Whittaker function $M_{k,\mu}(z)$ (17) and the normalization condition (53), we obtain the result for that the negative frequency mode F_0^- as $t \rightarrow 0$:

$$F_0^- = \begin{pmatrix} e^{\pi\tilde{\mu}} \mathfrak{C}_0^+ \rho^{-1/2} M_{-1/2 + eCk_z/\sqrt{-k_z^2 - m^2}, -\mu}(\rho) \\ -\frac{eCk_z/\sqrt{-k_z^2 - m^2} - \mu}{k - ICem/\sqrt{-k_z^2 - m^2}} e^{\pi\tilde{\mu}} \mathfrak{C}_0^+ \rho^{-1/2} M_{1/2 + eCk_z/\sqrt{-k_z^2 - m^2}, -\mu}(\rho) \end{pmatrix}. \quad (55)$$

The density of particles created can be calculated with the help of the Bogoliubov transformation

$$F_\infty^+ = \alpha F_0^+ + \beta F_0^-, \quad (56)$$

$$F_{+\infty}^+ = \frac{\Gamma(-2i\tilde{\mu})}{\Gamma(\frac{1}{2} - i\tilde{\mu} - \mathfrak{k})} F_0^+ + \frac{\Gamma(2i\tilde{\mu})}{\Gamma(\frac{1}{2} + i\tilde{\mu} - \mathfrak{k})} e^{-\pi\tilde{\mu}} F_0^-, \quad (57)$$

where $\mathfrak{k} = -1/2 + Ck_z/\sqrt{-k_z^2 - m^2}$. Using the property of the Gamma function [18]

$$|\Gamma(iy)|^2 = \frac{\pi}{y \sinh \pi y} \quad (58)$$

we obtain

$$\left| \frac{\beta}{\alpha} \right|^2 = e^{-2\pi\tilde{\mu}} \frac{\sinh \pi(\tilde{\mu} + eCk_z/\sqrt{k_z^2 + m^2})}{\sinh \pi(\tilde{\mu} - eCk_z/\sqrt{k_z^2 + m^2})} \frac{(\tilde{\mu} - eCk_z/\sqrt{k_z^2 + m^2})}{(\tilde{\mu} + eCk_z/\sqrt{k_z^2 + m^2})}. \quad (59)$$

From Eq. (59) and taking into account the normalization condition for Dirac particles

$$|\alpha|^2 + |\beta|^2 = 1, \quad (60)$$

we find that the density of Dirac particles created is

$$|\beta|^2 = \frac{\exp(-2\pi\tilde{\mu})(\tilde{\mu} - eCk_z/\sqrt{k_z^2 + m^2}) \sinh \pi(\tilde{\mu} + eCk_z/\sqrt{k_z^2 + m^2})}{\exp(-2\pi\tilde{\mu})(\tilde{\mu} - eCk_z/\sqrt{k_z^2 + m^2}) \sinh \pi(\tilde{\mu} + eCk_z/\sqrt{k_z^2 + m^2}) + (\tilde{\mu} + eCk_z/\sqrt{k_z^2 + m^2}) \sinh \pi(\tilde{\mu} - eCk_z/\sqrt{k_z^2 + m^2})}. \quad (61)$$

In the absence of an electric field, the density of particles created (61) becomes thermal:

$$n = \frac{1}{\exp 2\pi|k_\perp| + 1}. \quad (62)$$

IV. DISCUSSION OF THE RESULTS

From the expressions (27) and (61) we observe that, in the presence of the electric field (3), the distribution of particles created is not thermal and it strongly depends on the strength eC . For very strong electric fields we have that Eq. (27) reduces to

$$n \sim \exp\left(-2\pi\sqrt{k_\perp^2 + e^2 C^2 + 2\xi} - \frac{1}{4} + \frac{2\pi e C k_z}{\sqrt{k_z^2 + m^2}}\right), \quad (63)$$

an analogous result can be obtained for spin 1/2 particles. Expression (63) corresponds to a thermal distribution with an effective mass and a chemical potential proportional to eC . Equation (63) shows that the presence of strong electric fields contributes significantly to the creation of particles. The results obtained in this article show that the quasiclassical approach permits one to compute positive and negative frequency modes even when spacetime and electromagnetic sources are not static, and encourage us to study quantum effects in more realistic anisotropic scenarios.

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- [1] L. Parker, Phys. Rev. Lett. **21**, 562 (1968); *Asymptotic Structure of Space-Time* (Plenum, New York, 1977), p. 107.
- [2] S.A. Fulling, *Aspects of Quantum Field Theory in Curved Space-Time* (Cambridge University Press, Cambridge, England, 1991).
- [3] D.M. Chitre and J.B. Hartle, Phys. Rev. D **16**, 251 (1977).
- [4] A.A. Grib, S.G. Mamaev, and V.M. Mostepanenko, *Quantum Vacuum Effects in Strong Fields* (Energoatomizdat, Moscow, 1988).
- [5] I.L. Bukhbinder, Izv. Vyssh. Uchebn. Zaved. Fiz. **7**, 3 (1980).
- [6] S. Gavrilov, D.M. Gitman, and S.D. Odintsov, Int. J. Mod. Phys. A **12**, 4837 (1997).
- [7] J. Schwinger, Phys. Rev. **82**, 664 (1951).
- [8] M.V. Fischetti, J.B. Hartle, and B.L. Hu, Phys. Rev. D **20**, 1757 (1979).
- [9] V. Sahni, Class. Quantum Grav. **1**, 579 (1984).
- [10] I.L. Bukhbinder and D.M. Gitman, Izv. Vyssh. Uchebn. Zaved. Fiz. **3**, 900 (1979).
- [11] I.L. Bukhbinder and D.M. Gitman, Izv. Vyssh. Uchebn. Zaved. Fiz. **4**, 55 (1979).
- [12] I.L. Bukhbinder and S.V. Odintsov, Izv. Vyssh. Uchebn. Zaved. Fiz. **5**, 93 (1983).
- [13] K.H. Lotze, Class. Quantum Grav. **2**, 351 (1985).
- [14] K.H. Lotze, Class. Quantum Grav. **2**, 363 (1985).
- [15] V.M. Villalba, Phys. Rev. D **60**, 127501 (1999).
- [16] V.M. Villalba, Phys. Rev. D **52**, 3742 (1995).
- [17] N.D. Birrel and P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, England, 1982).
- [18] N.N. Lebedev, *Special Functions and Their Applications* (Dover, New York, 1972).
- [19] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. Stegun (Dover, New York, 1974).
- [20] G.V. Shishkin and V.M. Villalba, J. Math. Phys. **30**, 2132 (1989).
- [21] V.M. Villalba, Mod. Phys. Lett. A **8**, 2351 (1993).
- [22] A.I. Akhiezer and V.B. Berestetskii, *Quantum Electrodynamics* (Interscience, New York, 1965).
- [23] V.M. Villalba, Rev. Mex. Fís. **42**, 1 (1996).