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Resonant states in an attractive one-dimensional cusp potential

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Abstract

We solve the two-component Dirac equation in the presence of a spatially one-dimensional symmetric attractive cusp potential. The components of the spinor solution are expressed in terms of Whittaker functions. We compute the bound states solutions and show that, as the potential amplitude increases, the lowest energy state sinks into the Dirac sea becoming a resonance. We characterize and compute the lifetime of the resonant state with the help of the phase shift and the Breit–Wigner relation. We discuss the limit when the cusp potential reduces to a delta point interaction.

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1. Introduction

Spontaneous particle production in the presence of strong electric fields is perhaps one of the most interesting phenomena associated with the charged quantum vacuum [1, 2]. The study of supercritical effects and resonant particle production by strong Coulomb-like potentials dates back to the pioneering works of Pieper and Greiner [3] and Gershtein and Zeldovich [4] where it was shown that spontaneous positron production was possible when two heavy bare nuclei with total charge larger than some critical value Z_c collided with each other. The critical Z_c is the value for which the $1S$ state of the hydrogenlike atom with total charge Z has energy $E = -m$. The idea behind supercriticality lies in spontaneous positron emission induced by the presence of very strong attractive vector potentials. The presence of a strong electric field induces the energy level of an unoccupied bound state to sink into the negative energy continuum, i.e., an electron of the Dirac sea is trapped by the potential, leaving a positron that escapes to infinity. The electric field responsible for supercritical effects should be stronger than $2m_ec^2$, which is the value of the gap between the negative and positive energy continua. Such strong electric fields could be produced in heavy-ion collisions [1, 5, 6].

In the last years the one-dimensional Dirac equation in an external potential V has been studied in connection to the Levinson theorem in relativistic quantum mechanics [7–9]. The interest in this problem can be found in their applications in semiconductor physics, field theory and one-dimensional quantum electrodynamics (QED) among others. One-dimensional Dirac electrons can be considered as three-dimensional problems when V depends only on one space variable [7]. The relation between supercritical states and the Levinson theorem in the one-dimensional Dirac has been established by Ma *et al* [9], and Calogeracos and Dombey [8].

Recently the relation between transmission resonances and supercriticality effects have been studied in a one-dimensional barrier [10] and in a Woods–Saxon potential [11, 12], which is a smoothed form of a square well. Here, the Dirac equation presents half-bound states with the same asymptotic behaviour of those obtained with the potential barrier. Supercritical states with $E = -m$ have also been studied in a one-dimensional cusp potential [13, 14] and in a class of short-range potentials [15].

In the present paper, we discuss the problem of computing the phenomenon of resonant particle creation in a one-dimensional symmetric cusp potential of the form

$$eU(x) = -V\exp\left(-|x|/a\right), \tag{1}$$

where $V > 0$, $a > 0$ and e is the charge of the particle.

The potential (1) is an asymptotically vanishing potential for large values of the space variable x. The parameter V shows the depth of the well. The constant a determines the shape of the potential. The expression (1) can be regarded as a screened one-dimensional Coulomb potential [16].

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It is well known that a vectorial delta potential is strong enough to pull the bound state into the negative energy continuum $E = -m$ [17, 18], nevertheless this supercritical state does not evolve to a real resonant state. It is of interest to investigate the resonant behaviour of the lowest energy states in a potential exhibiting a delta point interaction as asymptotic limit. The cusp potential (1) reduces to an attractive vectorial delta interaction of strength g_v in the limit $2Va \to g_v$, as $a \to 0$. The attractive potential (1) does not possess compact support and, in contrast to the delta potential case, it can exhibit more than one bound state.

It is the purpose of the present paper to show that the cusp potential (1) is strong enough to sink the lowest electron bound state into the negative continuum to create a resonant state. In order to calculate the process of embedding of the bound state into the negative energy continuum we construct the Jost solutions and their corresponding Jost functions [19, 20]. We estimate, with the help of the phase shift and the Wigner time-delay [21, 22], the mean life of the resonance and how it depends on the shape and strength of the potential.

This paper is structured as follows. In section 2, we solve the 1+1 Dirac equation in the potential (1) and derive the equation governing bound states, energy resonances and supercritical states. In section 3, we compute, with the help of the Jost functions, the transmission and reflection amplitudes and the phase shifts. We derive the condition for energy resonances and estimate the mean life of the antiparticle state. Finally, in section 4 we summarize our results.

2. Solution of the Dirac equation and energy resonances

In two-dimensions, the Dirac equation in the presence of the spatially dependent electric field (1) can be written as [13]

$$\left\{\gamma^0(\partial_t - \mathrm{i}V\exp\left(-|x|/a\right)) + \gamma^1\partial_x + m\right\}\Psi = 0, \tag{2}$$

where the γ^μ matrices satisfy the commutation relation $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$ and the metric $\eta^{\mu\nu}$ has the signature $(-, +)$. Here and along the paper, we adopt the natural units $\hbar = c = 1$.

Since we are working in 1+1 dimensions, it is possible to choose the following representation of the Dirac matrices

$$\gamma^0 = \mathrm{i}\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^1 = \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3}$$

Taking into account that the potential (1) does not depend on time, we can write the spinor Ψ as follows

$$\Psi = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix} \exp\left(-\mathrm{i}Et\right). \tag{4}$$

Substituting the gamma γ^μ matrices (3) into the Dirac equation (2), we obtain the following system of coupled differential equations

$$\left(\frac{\mathrm{d}}{\mathrm{d}x} + \mathrm{i}(V\exp\left(-|x|/a\right) + E)\right)\Psi_1 + m\Psi_2 = 0, \tag{5}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}x} - \mathrm{i}(V\exp\left(-|x|/a\right) + E)\right)\Psi_2 + m\Psi_1 = 0. \tag{6}$$

The spinor solution of the system (5) and (6), exhibiting a regular behaviour as $x \to +\infty$, can be expressed in terms of the Whittaker function $M_{k,\mu}(y)$ [23] as follows

$$\Psi_{\mathrm{right}}(y) = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = b_1 \begin{pmatrix} \frac{1/2+\mu+k}{ma} y^{-1/2} M_{k+1,\mu}(y) \\ y^{-1/2} M_{k,\mu}(y) \end{pmatrix}, \tag{7}$$

where b_1 is a constant, $y = -2\mathrm{i}aV\exp(-x/a)$ and

$$k = \mathrm{i}Ea - 1/2, \quad \mu = \mathrm{i}a\sqrt{E^2 - m^2}. \tag{8}$$

For negative values of x, we have that the solution of the system (5) and (6) showing a regular behaviour as $x \to -\infty$ can be expressed in terms of the Whittaker functions $M_{k,\mu}(y)$ as

$$\Psi_{\mathrm{left}}(\bar{y}) = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = c_1 \begin{pmatrix} \bar{y}^{-1/2} M_{k,\mu}(\bar{y}) \\ -\frac{\frac{1}{2}+\mu+k}{ma} \bar{y}^{-1/2} M_{k+1,\mu}(\bar{y}) \end{pmatrix}, \tag{9}$$

where c_1 is a constant and $\bar{y} = -2a\mathrm{i}V\exp(x/a)$

Imposing the continuity of the spinor solution Ψ at $x = 0$, and demanding the existence of non-trivial values for c_1 and b_1, we obtain that resonances and bound levels satisfy the energy condition

$$\left(\frac{1}{2} + \mu + k\right)^2 M^2_{k+1,\mu}(-2\mathrm{i}eaV) + m^2a^2 M^2_{k,\mu}(-2\mathrm{i}eaV) = 0. \tag{10}$$

Given the values of the strength V and the shape parameter a, the expression (10) exhibits solutions with E real, showing that the cusp potential (1) is able to bind particles, a property that, for instance, is absent in the pure one-dimensional Coulomb potential. The number of energy levels in the cusp potential decreases as the shape parameter decreases.

As the potential depth increases for large values of V, the energy eigenvalues of the bound states decrease. When the lowest energy bound state reaches the value $E = -m$, this level merges with the negative energy continuum and the potential becomes supercritical. For $E = -m$, we have that $\mu = 0$ and, using equation (10), we obtain the condition for supercriticality

$$M_{k,0}(-2\mathrm{i}eaV_c)M_{k,0}(-2\mathrm{i}eaV_c) - M_{k+1,0}(-2\mathrm{i}eaV_c)M_{k+1,0}(-2\mathrm{i}eaV_c) = 0. \tag{11}$$

As soon as the potential strength V becomes larger than the supercritical value V_c, the lowest bound state dives into the energy continuum becoming a resonance. Figure 1 shows the dependence of the supercritical potential V_c on the shape a. As $a \to 0$ we have that $-2aV_c$ approaches π, showing that the Dirac equation in the presence of a vectorial delta interaction of strength g_v becomes supercritical as $g_v = \pi$ [17].

Figure 2 shows, for a given value of the shape parameter a, the dependence of the resonant energy of the lowest bound state on the potential strength V. The curve of the imaginary part as a function of the real part of the resonant energy shows a concavity that decreases as the potential V increases.

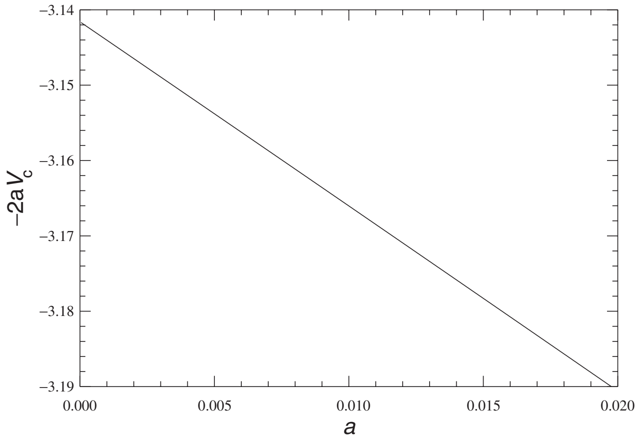

Figure 1. Plot of the supercritical potential strength $V\ (E=-m)$ against the shape a.

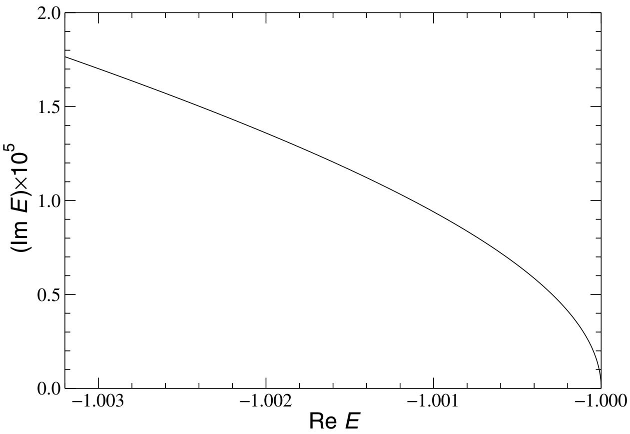

Figure 2. Resonant energy spectrum for $a=1.57$ with $3.145533 < V < 3.15$.

3. Transmission amplitude and phase shifts

In order to study resonant states associated with Dirac particles in a cusp potential, we proceed to discuss the scattering process with the help of the Jost functions [19, 22]. Resonances can be identified as poles of the scattering matrix S [19, 21, 22]. Since the components of the scattering matrix as well as the reflection and transmission coefficients R and T can be expressed in terms of the reflection and transmission amplitudes, we proceed to compute r and t with the help of the incoming $\phi^{\pm}(x)$ and outgoing $\psi^{\pm}$ Jost solutions, where plus and minus indicates the asymptotic right and left solutions.

Since the cusp potential (1) decays faster than $1/x^2$ for large values of x, there are solutions of the Dirac equation (2) that behave asymptotically as free travelling plane wave spinors. Those solutions exhibiting this asymptotic behavior are called Jost solutions [22]. The one-dimensional Dirac equation possesses two outgoing $\psi^{\pm}$ and two incoming $\varphi^{\pm}$ Jost solutions [20, 24].

Since the Jost solutions $\psi_{\pm}(E,x)$ and $\varphi_{\pm}(E,x)$, are linearly independent, we can write regular solutions $\Psi^{(-)}(E,x)$, $\Psi^{(+)}(E,x)$ of the Dirac equation as [20]

$$\begin{aligned}\Psi^{(\pm)}(E,x) &= -\frac{1}{W(\psi_{\pm},\varphi_{\pm})(E)}\\ &\quad\times\left[W\left(\Psi^{(\pm)}(E,x),\psi_{\pm}(E,x)\right)\varphi_{\pm}(E,x)\right.\\ &= -W\left(\Psi^{(\pm)}(E,x),\varphi_{\pm}(E,x)\right)\left.\psi_{\pm}(E,x)\right]\\ &= -\frac{1}{W(\psi_{\pm},\varphi_{\pm})(E)}\Big[f_{+}^{(\pm)}(E)\,\varphi_{\pm}(E,x)\\ &\quad - f_{-}^{(\pm)}(E)\,\psi_{\pm}(E,x)\Big],\end{aligned} \tag{12}$$

where the Wronskian $W(\psi(x),\varphi(x))$ of the two two-component solutions $\psi(x)$ and $\varphi(x)$ of the Dirac equation $\psi(x)=(\psi_1(x),\psi_2(x))^t$ and $\varphi(x)=(\varphi_1(x),\varphi_2(x))^t$ is defined as [25, 26]

$$\begin{aligned}W(\psi(x),\varphi(x)) &\equiv \det\begin{pmatrix}\psi_1(x) & \varphi_1(x)\\ \psi_2(x) & \varphi_2(x)\end{pmatrix}\\ &= \psi_1(x)\varphi_2(x)-\psi_2(x)\varphi_1(x).\end{aligned} \tag{13}$$

It is easy to see that if $W(\psi,\varphi)\neq 0$ we have that $\psi(x)$ and $\varphi(x)$ are linearly independent, $f_{-}^{(\pm)}(E)$, $f_{+}^{(\pm)}(E)$ are in equation (12), analogous to the non-relativistic case, the Jost functions [22].

We make use of the Jost solutions in order to describe a scattering process. The solution ψ_+ should be a linear combination of the solutions ψ_- and φ_-, that is,

$$t^{\pm}(E)\,\psi_{\pm}(E,x)=\varphi_{\mp}(E,x)+r^{\mp}(E)\,\psi_{\mp}(E,x). \tag{14}$$

Since the cusp potential (1) is even, using the invariance under parity [1] and the representation (3) for the gamma matrices we have that

$$\psi_{\pm}(E,x)=-\mathrm{i}\sigma_y\psi_{\mp}(E,-x),\ \ \varphi_{\pm}(E,x)=-\mathrm{i}\sigma_y\psi_{\mp}(E,-x). \tag{15}$$

The parity conditions (15) imply that the left and right transmission and reflection amplitudes are identical. The transmission amplitude $t(E)=t^{\pm}(E)$ can be expressed as

$$t(E)\equiv t^{\pm}(E)=-\frac{W(\psi_+,\varphi_+)(E)}{W(\psi_-,\psi_+)(E)}, \tag{16}$$

analogously, we have that the reflection amplitude $r(E)=r^{\mp}(E)$ reads

$$r(E)\equiv r^{\mp}(E)=\frac{W(\psi_+,\varphi_-)(E)}{W(\psi_-,\psi_+)(E)}, \tag{17}$$

The incoming Jost solution φ_- for the one-dimensional Dirac equation (2) takes the form

$$\begin{aligned}\varphi_-(\bar{y}) &= \frac{m}{(-2\mathrm{i}aeV)^{\mu}\sqrt{2E\left(E-\sqrt{E^2-m^2}\right)}}\\ &\quad\times\begin{pmatrix}\frac{\frac{1}{2}+k-\mu}{ma}\,\bar{y}^{-1/2}M_{k,\mu}(\bar{y})\\ \bar{y}^{-1/2}M_{k+1,\mu}(\bar{y})\end{pmatrix},\end{aligned} \tag{18}$$

a spinor that exhibits the following asymptotic behaviour

$$\varphi_-(x) \to \frac{m}{\sqrt{2E(E-\sqrt{E^2-m^2})}} \times \begin{pmatrix} \frac{\frac{1}{2}+k-\mu}{ma} \\ 1 \end{pmatrix} \exp\left(\mathrm{i}x\sqrt{E^2-m^2}\right). \tag{19}$$

We can see that the solution (18) behaves asymptotically as a travelling to the right plane wave solution to the free Dirac equation. The outgoing Jost solution ψ_- can be written as

$$\psi_-(\bar{y}) = \frac{m}{(-2\mathrm{i}aeV)^{-\mu}\sqrt{2E(E+\sqrt{E^2-m^2})}} \times \begin{pmatrix} \frac{\frac{1}{2}+k+\mu}{ma}\bar{y}^{-1/2}M_{k,-\mu}(\bar{y}) \\ \bar{y}^{-1/2}M_{k+1,-\mu}(\bar{y}) \end{pmatrix}, \tag{20}$$

such a spinor exhibits the following asymptotic behaviour as $x \to -\infty$

$$\psi_-(x) \to \frac{m}{\sqrt{2E(E+\sqrt{E^2-m^2})}} \times \begin{pmatrix} \frac{\frac{1}{2}+k+\mu}{ma} \\ 1 \end{pmatrix} \exp\left(-\mathrm{i}x\sqrt{E^2-m^2}\right). \tag{21}$$

This asymptotic form shows that the Jost solution (20) asymptotically behaves like a travelling left solution wave to the free Dirac equation. The right outgoing Jost function ψ_+ is

$$\psi_+(y) = \frac{m}{(-2\mathrm{i}aeV)^{-\mu}\sqrt{2E(E-\sqrt{E^2-m^2})}} \times \begin{pmatrix} \frac{\frac{1}{2}+k-\mu}{ma}y^{-1/2}M_{k+1,-\mu}(y) \\ y^{-1/2}M_{k,-\mu}(y) \end{pmatrix}, \tag{22}$$

such a spinor exhibits the following asymptotic behaviour.

$$\psi_+(x) \to \frac{m}{\sqrt{2E(E-\sqrt{E^2-m^2})}} \times \begin{pmatrix} \frac{\frac{1}{2}+k-\mu}{ma} \\ 1 \end{pmatrix} \exp\left(\mathrm{i}x\sqrt{E^2-m^2}\right). \tag{23}$$

Finally, we have that the incoming Jost solution φ_+ takes the form

$$\varphi_+(y) = \frac{m}{(-2\mathrm{i}aeV)^{\mu}\sqrt{2E(E+\sqrt{E^2-m^2})}} \times \begin{pmatrix} \frac{\frac{1}{2}+k+\mu}{ma}y^{-1/2}M_{k+1,\mu}(y) \\ y^{-1/2}M_{k,\mu}(y) \end{pmatrix}, \tag{24}$$

whose asymptotic behaviour is that of an incoming wave from the right.

$$\varphi_+(x) \to \frac{m}{\sqrt{2E(E+\sqrt{E^2-m^2})}} \begin{pmatrix} \frac{\frac{1}{2}+\mu+k}{ma} \\ 1 \end{pmatrix} \exp\left(-\mathrm{i}x\sqrt{E^2-m^2}\right). \tag{25}$$

With the help of the Jost functions and using equation (16), we have that the transmission amplitude t for the Dirac equation (2) takes the form

$$t(E) = \frac{ma}{(1/2+k+\mu)} \times \begin{bmatrix} (1/2+k+\mu/ma)M_{k+1,\mu}(-2\mathrm{i}aeV)M_{k,-\mu}(-2\mathrm{i}aeV) \\ -(1/2+k-\mu/ma)M_{k+1,-\mu}(-2\mathrm{i}aeV)M_{k,\mu}(-2\mathrm{i}aeV) \end{bmatrix} \times \begin{bmatrix} (-2\mathrm{i}aeV)^{2\mu}(M^2_{k,-\mu}(-2\mathrm{i}aeV) \\ +(((1/2+k-\mu)^2/m^2a^2)M^2_{k+1,-\mu}(-2\mathrm{i}aeV)) \end{bmatrix}^{-1}. \tag{26}$$

Analogously, we have that the reflection amplitude $r(E)$ is

$$r(E) = \mathrm{i}\frac{ma}{1/2+k+\mu} \times \begin{bmatrix} M_{k+1,\mu}(-2\mathrm{i}aeV)M_{k+1,-\mu}(-2\mathrm{i}aeV) \\ -M_{k,-\mu}(-2\mathrm{i}aeV)M_{k,\mu}(-2\mathrm{i}aeV) \end{bmatrix} \times \begin{bmatrix} (-2\mathrm{i}aeV)^{2\mu}(M^2_{k,-\mu}(-2\mathrm{i}aeV) \\ +(((1/2+k-\mu)^2/m^2a^2)M^2_{k+1,-\mu}(-2\mathrm{i}aeV)) \end{bmatrix}^{-1}. \tag{27}$$

It is not difficult to see that the components $t(E)$ and $r(E)$ of the scattering matrix S have the same poles, they are the roots of the Jost function $f_+(E)$

$$f_+(E) = W(\psi_-, \psi_+)(E) = M^2_{k,-\mu}(-2\mathrm{i}aeV) + \frac{(1/2+k-\mu)^2}{m^2a^2}M^2_{k+1,-\mu}(-2\mathrm{i}aeV). \tag{28}$$

It is worth noticing that the equation $f_+(E) = 0$ coincides with the resonant energy condition given by equation (10).

When the transmission coefficient reaches the unit value, that is for $E = \mathrm{Re}E_{\mathrm{res}}$, where E_{res} is the resonance energy, the phase shift $\delta_{\mathrm{res}}(E)$ reaches the value $\pi/2$. In the vicinity of the resonant value, provided that the Breit–Wigner formula [22] is valid, the resonance energy is a simple zero of the Jost function $f_+^{(\pm)}(E)$. In the neighbourhood of $E = \mathrm{Re}E_{\mathrm{res}}$ we have

$$f_+^{(\pm)}(E) \approx \left(\frac{\mathrm{d}f_+^{(\pm)}}{\mathrm{d}E}\right)_{E=E_{\mathrm{res}}}(E - E_{\mathrm{res}}). \tag{29}$$

If $\mathrm{Im}E_{\mathrm{res}}$ is sufficiently close to the real axis for the approximation (29) to be applicable in a real neighbourhood of $E = \mathrm{Re}E_{\mathrm{res}}$, we will have

$$\delta(E) = -\arg\left(f_+(E)\right)_{E=E_{\mathrm{res}}} \approx \delta_{\mathrm{bg}}(E) + \delta_{\mathrm{res}}(E), \tag{30}$$

hence

$$\delta_{\mathrm{res}}(E) \approx \delta(E) - \delta_{\mathrm{bg}}(E), \tag{31}$$

where $\delta_{\mathrm{bg}} = -\arg\left(\frac{\mathrm{d}f_+}{\mathrm{d}E}\right)_{E=E_{\mathrm{res}}}$ is the background phase shift. Figure 3 shows the dependence of the phase shift δ on the real part of the energy E. For the resonance value $\mathrm{Re}E = -1.0032$, we have that $\delta = 2.62769$ and $\delta_{\mathrm{bg}} = 1.05706$, obtaining in this way $\delta_{\mathrm{res}} = 1.57063 \approx \pi/2$.

It is worth mentioning that as the lowest-bound energy state reaches the supercritical value $E = -m$, the phase shift δ takes the value $\delta = \pi/2$, result that is in agreement with the phase shift for the one-dimensional Dirac equation in the presence of a symmetric potential well [1, 8].

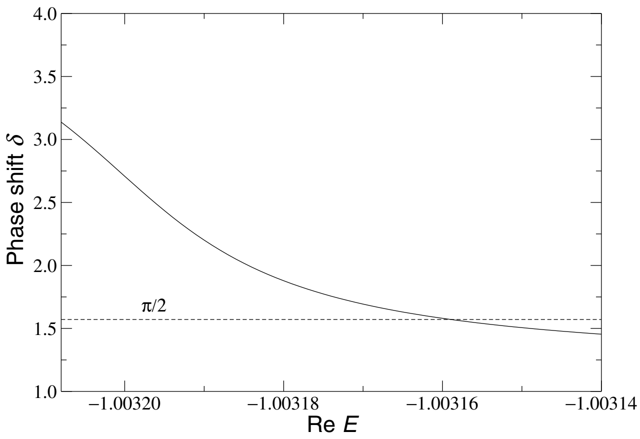

Figure 3. Phase shift δ as function of the real part of the energy E for $V = 3.15$, $a = 1.57$. Notice that the phase shift δ of the resonance value $\Re E = -1.0032$ is larger than $\pi/2$.

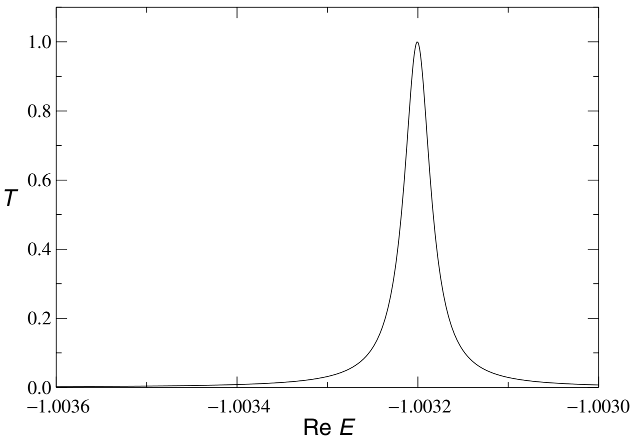

Figure 4. Transmission coefficient for $V = 3.15$, $a = 1.57$. A maximum corresponding to a resonance is reached for $\Re E = -1.0032$.

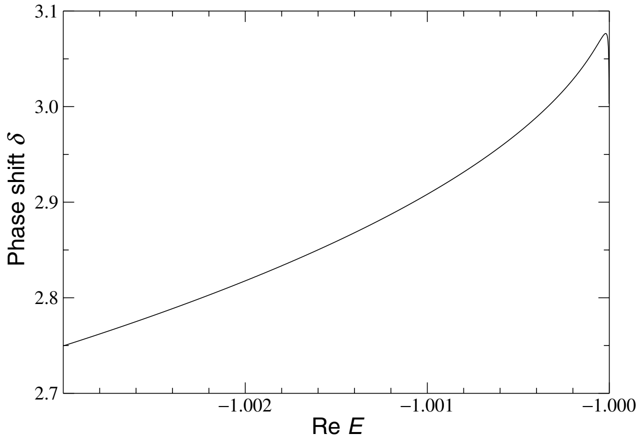

Figure 5. Phase shift δ as a function of the real part of the resonant energy for $3.145533 < V < 3.15$ with $a = 1.57$.

The Wigner time τ permits one to estimate how long the bound state lives before it degenerates into the negative continuum. From figure 4 and the solution to equation (10), we can estimate the mean life of the resonant state in terms of the half-width Γ of the Lorentzian curve. For $\mathrm{Re}E = -1.0032$, we have $\mathrm{Im}E = 1.765 \times 10^{-5}$, corresponding to a Wigner time of $\tau = \frac{\mathrm{d}\delta}{\mathrm{d}E} \approx \frac{2}{\Gamma} = 5.667 \times 10^5$ expressed in natural units. Resonant states make the phase shift surpass the value $\pi/2$ with a Wigner time delay $\tau > 0$. Figure 5 shows the behaviour of the phase shift δ as a function of the real part of the resonant energy, it can be observed that the phase shift increases surpassing the value $\pi/2$ and after reaching a maximum, it monotonically decreases, showing in this way a behaviour that has been observed in square well case [15].

4. Concluding remarks

The relation (10) shows that the one-dimensional cusp potential (1) supports supercritical states and is strong enough to produce resonant states. This result is non-trivial and interesting in view of the fact that the Dirac equation in a one-dimensional vectorial Dirac delta interaction does not exhibit resonances [17]. The potential (1) does not exhibit a square barrier limit, and for very small values of a and constant value of $2Va$ it can be regarded as a delta potential [16], therefore the relation (11) also holds for a delta potential. Equation (1) represents a local interaction whose support does not vanish anywhere and exhibits supercritical resonant states.

We have computed the phase shift associated with the scattering of a Dirac particle by the potential (1), and have shown that resonant states make the phase shift surpass the value $\pi/2$ with a Wigner time-delay $\tau > 0$. The asymptotic behaviour of the cusp potential has permitted us to derive the transmission and reflection amplitudes $t(E)$ and $r(E)$ with the help of the Jost solutions. This approach can be applied in the study of resonant states in the presence of potentials not allowing exact solutions of the Dirac equation in terms of special functions, this problem will be discussed in a forthcoming publication.

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