# Tutorial "General Relativity"

Winter term 2014/2015

Lecturer: Prof. Dr. C. Greiner
Tutor: Andreas Zacchi

## Sheet No. 2

will be discussed at: 18.11.2014

***1.* Line Element**
Consider a fictitious two-dimensional line element given by

$$ds^2 = x^2 dx^2 + 2dxdy - dy^2$$

Write down $g_{ab}$, $g^{ab}$ and then raise and lower indices on $V_a = (1, -1)^T$ and $W^a = (0, 1)^T$.

***2.* Coordinate Transformations**
In a coordinate transformation, the components of the transformation matrix $\Lambda_a^b$ are formed by taking the partial derivative of one coordinate with respect to the other

$$\Lambda_a^b = \frac{\partial x^b}{\partial x'^a}$$

whereas basis vectors transform as

$$e'_a = \Lambda_a^b e_b$$

Plane polar coordinates are related to cartesian coordinates by

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$

Describe the transformation matrix that maps cartesian coordinates to polar coordinates, and write down the polar coordinate basis vectors in terms of the basis vectors of cartesian coordinates.

***3.* Coordinate Transformations and Metrics**
Under a coordinate transformation $\xi^A = \xi^A(x^\mu)$, the Minkowski metric $\eta_{AB}$ transforms to a new metric $g_{\mu\nu}$ in such a way that proper distances are invariant. In other words, the line element $ds^2 = \eta_{AB} d\xi^A d\xi^B$ is invariant, i.e. $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$.

(a) Show, that this implies that $g_{\mu\nu}$ is related to $\eta_{AB}$ by

$$g_{\mu\nu}(x) = \frac{\partial \xi^A}{\partial x^\mu}\frac{\partial \xi^B}{\partial x^\nu}\eta_{AB}$$

(b) Show, that the inverse metric $g^{\mu\nu}$, i.e. $g^{\mu\nu} g_{\nu\lambda} = \delta^\mu_\lambda$ is given by

$$g^{\mu\nu}(x) = \eta_{AB}\frac{\partial x^\mu}{\partial \xi^A}\frac{\partial x^\nu}{\partial \xi^B}$$

**4. Rotating frame**

A rotating frame can be described with

$$\begin{aligned} x &= x' \cos \omega t' - y' \sin \omega t' \\ y &= x' \sin \omega t' + y' \cos \omega t' \\ z &= z' \\ t &= t' \end{aligned}$$

The invariant line element (ct,x,y,z) reads $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$

(a) Calculate the metric in the rotating frame.

(b) The affine connections for the primed coordinates are given as

$$\Gamma^{\kappa}_{\lambda\mu} = \frac{1}{2} g^{\kappa\nu} \left( \frac{\partial g_{\mu\nu}}{\partial x'^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x'^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x'^{\nu}} \right) \tag{1}$$

Calculate the non-vanishing affine connections.

(c) Evaluate the geodesic equation to determine $g_{\mu\nu}$ of a rotating frame. Use your results from (b) to derive the Centrifugal- and the Coriolis Force in case $v << c$.