Umkehrung der Kurvendiskussion

1. a) $\boxed{f(x) = ax^2 + bx + c}$

$$\begin{array}{l|l|}
\mathrm{I} & 4a + 2b + c = 1 \\
\mathrm{II} & 9a + 3b + c = 2 \\
\mathrm{III} & 16a + 4b + c = 5
\end{array}$$

$$\left|\begin{array}{l} 3 = 12a + 6b + 3c \\ -4 = -18a - 6b - 2c \end{array}\right| \begin{array}{l} \mathrm{I} \cdot (3) \\ \mathrm{II} \cdot (-2) \end{array}$$

IV $\quad |-1 = -6a + c|$

$$\left|\begin{array}{l} -2 = -8a - 4b - 2c \\ 5 = 16a + 4b + c \end{array}\right| \begin{array}{l} \mathrm{II} \cdot (-1) \\ \mathrm{III} \end{array}$$

$|3 = 8a - c|$ $\quad \mathrm{V}$

$\mathrm{IV} + \mathrm{V}$

$$\left|\begin{array}{l} -1 = -6a + c \\ 3 = 8a - c \end{array}\right|$$

$2 = 2a \Rightarrow a = 1$

↳ $\boxed{a = 1}$ in V $\quad 3 = 8 \cdot (1) - c \Rightarrow c = 5$

in I nun um b zu bestimmen

$4 \cdot (1) + 2b + 5 = 1$

↳ $b = -4$

1a) $\boxed{f(x) = x^2 - 4x + 5}$

1b) $\boxed{f(x) = x^2 - 2x - 6}$

1c) $\boxed{f(x) = \frac{3}{4}x^2 + 3x - 1}$

2. $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f(0) = 1 \longrightarrow d = 1$

$f'(0) = -1 \longrightarrow c = -1$

$f(-1) = 4$

$f''(-1) = 0$

$$\left| \begin{array}{l} 4 = -a + b + 1 + 1 \\ 0 = -6a + 2b \end{array} \right| \Rightarrow \left| \begin{array}{l} -\cancel{2}^{4} = +2a - 2b \\ 0 = -6a + 2b \end{array} \right|$$

$-\cancel{2}^{4} = -4a$

$\boxed{a = 1}$ $(a=1)$

$0 = -6 \cdot 1 + 2b$

$\boxed{b = 3}$

$$\boxed{f(x) = x^3 + 3x^2 - x + 1}$$

---

3. $f(x) = ax^4 + bx^2$

$f'(x) = 4ax^3 + 2bx$

$f''(x) = 12ax^2 + 2b$

$f(1) = -5/2$

$f(-1) = -5/2$ (Symmetrie)

$f''(1) = 0$

$$\left| \begin{array}{l} -\frac{5}{2} = a + b \\ 0 = 12a + 2b \end{array} \right| \cdot(-2)$$

$$\left| \begin{array}{l} 5 = -2a - 2b \\ 0 = 12a + 2b \end{array} \right|$$

$5 = 10a$

$\boxed{a = \frac{1}{2}}$

$5 = -2 \cdot \frac{1}{2} - 2b$

$2b = -6$

$\boxed{b = -3}$

$$\boxed{f(x) = \frac{1}{2}x^4 - 3x^2}$$

4

$$\boxed{f(x)=ax^2+bx+c}$$

$f(0)=-4 \rightarrow -4=c$

$f'(1)=5 \rightarrow 5=2a+b \rightarrow b=3$

$f''(x)=2 \rightarrow 2=2a \Rightarrow \boxed{a=1}$

$$\boxed{f(x)=x^2+3x-4}$$

5

$$\boxed{f(x)=ax^3+bx}$$

$f(1)=-1 \rightarrow -1=a+b$ } SAME

$f(-1)=1 \rightarrow 1=-a-b$

$f'(2)=0 \rightarrow 0=12a+b$

$\rightarrow 1=11a \Rightarrow a=\frac{1}{11}$

$-1=\frac{1}{11}+b \Rightarrow b=-\frac{11}{11}-\frac{1}{11}=-\frac{12}{11}$

$$\boxed{f(x)=\frac{1}{11}x^3-\frac{12}{11}x}$$

6

$$\boxed{f(x)=ax^4+bx^2+c}$$

$f'(x)=4ax^3+2bx$

$f''(x)=12ax^2+2b$

$f(2)=0 \overset{①}{\Rightarrow} 16a+4b+c=0$

$f'(2)=2 \overset{②}{\Rightarrow} 32a+4b=2$

$f''(-1)=0 \overset{③}{\Rightarrow} 12a+2b=0$

② $-16a-2b=-1$

③ $12a+2b=0$

$-1=-4a$

$\boxed{a=\frac{1}{4}}$ in ③ $\rightarrow 2b=-3$ $\boxed{b=-3/2}$ in ① $\rightarrow 4-6=-c \Rightarrow c=2$

$$\boxed{f(x)=\frac{1}{4}x^4-\frac{3}{2}x^2+2}$$

7

$$f(x) = a(x+3)\cdot x(x-2)$$
$$= ax(x^2+x-6) = ax^3+ax^2-6ax$$

1 $f(-2) = 16 \rightarrow 16 = -8a + 4a + 12a$

$\boxed{a=2}$

$$\boxed{f(x) = 2x(x+3)(x-2) = 2x^3+2x^2-12x}$$

2 oder $f(-2) = 16 \Rightarrow 16 = a(1)(-2)(-4)$

$\boxed{a=2}$

8

$f(x) = ax^3+bx^2+cx+d$

$f(0) = 0 \Rightarrow d = 0$

$f'(x) = 3ax^2+2bx+c$

$f''(x) = 6ax+2b$

Wendetangente

$y = mx+b$

$m = \frac{4-0}{-2-4} = -\frac{2}{3}$

(Steigung ist dann auch in $(-2/4) = -2/3$)

$f(-2) = 4 \rightarrow -8a+4b-2c = 4$

$f''(-2) = 0 \rightarrow -12a+2b = 0$

$f'(-2) = -2/3 \rightarrow 12a-4b+c = -2/3$

$-4 = -2b$

$\boxed{b=2}$

$-\frac{2}{3}+8-4 = c = \frac{10}{3}$

~~[illegible]~~

$$\begin{vmatrix} -4a+2b-c = 2 \\ 12a-4b+c = -2/3 \end{vmatrix}$$

$8a-2b = 4/3$

$-12a+2b = 0$

$-4a = 4/3$

$\boxed{a = +\frac{1}{3}}$

$$\boxed{f(x) = \frac{1}{3}x^3+2x^2+\frac{10}{3}x}$$

(9) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f(0) = 0 \Rightarrow d = 0$

$f'(0) = 0 \Rightarrow c = 0$ (bewiesen!)

~~f(-3)=0~~ $\Rightarrow f(-3) = 0 \Rightarrow -27a + 9b = 0$

~~f'(-3)=6~~ $\Rightarrow f'(-3) = 6 \Rightarrow 27a - 6b = 6$

$3b = 6$ $\boxed{b = \frac{1}{2}}$

$27a = 6 + 3$

$\boxed{a = \frac{1}{3}}$

$$\boxed{f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2}$$

---

(10) $f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 6ax + 2b$

$f(0) = -6$

$f(3) = 0$

$f''(3) = 0$

$f'(3) = 5$

(1) $\rightarrow d = -6$ → schon einsetzen

(2) $\rightarrow 27a + 9b + 3c = 6$

(3) $\rightarrow 18a + 2b = 0$

(4) $\rightarrow 27a + 6b + c = 5$

(2) $-9a - 3b - c = -2$

(4) $27a + 6b + c = 5$

$18a + 3b = 3$

(3) $-18a - 2b = 0$

$\boxed{b = 3}$

(3) $18a = -6$

$\boxed{a = -\frac{1}{3}}$

in (4) $-(-9 + 18 - 5) = c$

$\boxed{c = -4}$

$$\boxed{f(x) = -\frac{1}{3}x^3 + 3x^2 - 4x - 6}$$

11

$f(x) = ax^4 + bx^2 + c$ $\quad f(2) = 0$

$f'(x) = 4ax^3 + 2bx$ $\quad f'(2) = 4$

$f''(x) = 12ax^2 + 2b$ $\quad f''(2) = 0$

$\boxed{f(x) = \frac{1}{40}x^4 - \frac{3}{5}x^2 + 2}$

$$\left| \begin{array}{l} 16a + 4b + c = 0 \\ 48a + 2b = 0 \\ 32a + 8b = 4 \end{array} \right|$$

$48a + 2b = 0$

$-\ 8a - 2b = \cancel{4} - 1$

$40a = \cancel{4} - 1$

$\boxed{a = \frac{1}{40}}$

$-\left(\frac{\overset{24}{\cancel{48}}}{2 \cdot 40}\right) = \boxed{b = -\frac{3}{5}}$

$-\left(\frac{16}{40} - \frac{12}{5}\right) = c = -\left(-\frac{80}{40}\right) = 2$

12

Nullstellen von $f(x) = -\frac{1}{2}x^3 + 2x$: $f(x) = 0$

$\left.\begin{array}{l} x_1 = 0 \\ x_{2,3} = \pm 2 \end{array}\right\}$ $g(x) = a(x+2)(x-2)x$

stehen im Ursprung senkrecht, heißt $f'(x) = -\frac{1}{g'(x)}$

$f'(x) \to f'(0) = -\frac{3}{2}x^2 + 2 = 2$

Steigung von $g(x)$ in $(0/0) \Rightarrow -2^{-1}$ $(m = 2)$ $(m = -\frac{1}{2})$

$g(x) = ax^3 - 4ax \to g'(x) = 3ax^2 - 4a$

$g'(x) = -\frac{1}{2}$ $(x = 0)$ $g'(0) = -\frac{1}{2}$

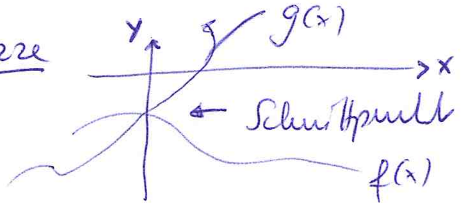
$-4a = -\frac{1}{2}$

$\boxed{a = 1/8}$

$$\boxed{g(x) = \frac{1}{8}x^3 - \frac{1}{2}x}$$

13 • if $f(x) = g(x)$ bei $(0/-3)$ $\exists$ Schnittpunkt

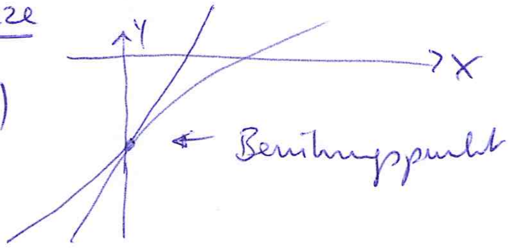
Skizze



• if $f'(x) = g'(x)$ in $(0/-3)$ besitzen beide Fnct. dort eine Tangente (mit identischer Steigung).

Skizze

(z.B.)



check: $f(0) = 3 = g(0)$ gegeben ✓

$f'(x) = \frac{3}{5}x^2 - 4x + 5 \rightarrow f'(0) = 5$

$g'(x) = -2x + 5 \rightarrow g'(x) = 5$ ebenso gegeben □

14 Alle Extrema: $\boxed{\left(\underbrace{\frac{2}{3}t}_{x} \Big/ \underbrace{\frac{2}{9}t^3}_{y}\right)}$

$\frac{2}{3}t = x \Rightarrow x = \frac{3}{2}t$ ↳ in $y = f(x)$ einsetzen: $\frac{2}{9} \cdot \left(\frac{3}{2}\right)^3 x^3$

$\boxed{f(x) = \frac{3}{4}x^3}$ Ortskurve aller Extrema

15 (SAME AS 14) $\boxed{\left(\sqrt{\frac{a}{3}} \Big/ -\frac{2}{3}a\sqrt{\frac{a}{3}}\right)}$

$\downarrow$ x $\qquad \downarrow$ y

$a = 3x^2$ in y einsetzen $f(x) = y = -\frac{2}{\cancel{3}} \cdot \cancel{3}x^2\sqrt{\frac{\cancel{3}x^2}{\cancel{3}}} = -2x^3$

$\boxed{f(x) = -2x^3}$

16

$f(x) = 2x^2$

$g(x) = -t\,x^2 + 4$

sollen im Schnittpt. senkrecht aufeinander stehen.

$$\boxed{f'(x) = -\frac{1}{g'(x)}}$$

Schnittpunkt

$(2+t)\,x^2 = 4$

$$\boxed{x^2 = \frac{4}{2+t}}$$

einsetzen und umstellen

$f'(x) = 4x$

$g'(x) = -2tx$

$4x = -\frac{1}{-2tx}$

$-8tx^2 = -1 \quad |\cdot(-1)$

$8t\,\frac{4}{2+t} = 1 \quad |\cdot(2+t)$

$32t = 2+t \quad |-t$

$31t = 2 \quad |\cdot\frac{1}{31}$

$t = \frac{2}{31}$

Für $t = \frac{2}{31}$ stehen beide senkrecht aufeinander.

17

$f(x) = 2x^2 - 5x + 1$

parallele Gerade hat $m = \frac{4}{-4} = -1$ (Punkte einsetzen in $m = \frac{\Delta y}{\Delta x}$)

$f'(x) = 4x - 5 = -1$

$x = 1 \qquad f(1) = 2 - 5 + 1 = -2 = y$

$y = mx + b$

$-2 = (-1)(1) + b \Rightarrow b = -1$

$$\boxed{y = -x - 1}$$