Tutorial "General Relativity"

Winter term 2016/2017

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Sheet No. 1

will be discussed on Nov/01/16

1. Decay of the muon

Muons have been discovered while studying cosmic radiation at Caltech in the thirties of the last century. The muon is an unstable subatomic particle with a mean life time of $\tau \sim 2.2\mu$s (measured in its rest frame). Their decay via the weak interaction is described by

$$N(t) = N_0 \exp\left(-\frac{t}{\tau}\right),$$

where $N(t)$ is the number of muons after the time t, and N_0 is the initial number at $t = 0$. They travel nearly with the speed of light, $v = 0.998c$.

a.) What distance can a muon manage in its proper time[1]?

b.) Why does an observer on Earth measure a mean lifetime of around 34.8μs. What distance would a muon travel in this time?

c.) Suppose, that in 9 kilometers above sea level 10^8 muons were produced. How many of them reach the Earth's surface (non-relativistically)? Why does an observer detect nearly 42% of them nonetheless?

2. Addition of velocities

Given a particle in frame Σ, which is moving at $\vec{u} = \frac{3}{4}c\vec{e}_1$ to the right and another observer in frame Σ', which is moving with $\vec{v} = -\frac{3}{4}c\vec{e}_1$ (i.e., to the left with respect to Σ). Why doesn't the observer in Σ' measure a total speed of $\frac{3}{2}c$ of the particle? What speed does he measure?

3. Arrow

An arrow of length 1 m has been shot. While passing your view, you measure a length of 86.6 cm. At what speed v travels the arrow?

4. Speed of a particle

If a particle's kinetic energy is n times its rest energy, what is its speed?

5. Lorentz invariance

Which of the following quantities is Lorentz-invariant (and which manifestly Lorentz covariant)?

a.) $\vec{x}^2$ b.) $x_\mu x^\mu$ c.) $x^\mu x^\nu$ d.) $\eta_{\mu\nu}$ e.) $\mathrm{d}s^2$ f.) $(\mathrm{d}x^0)^2$ g.) γ

6. General rotation-free Lorentz boost

Find the rotation-free Lorentz-boost matrix $\Lambda^\mu{}_\nu$, $x'^\mu = \Lambda^\mu{}_\nu x^\nu$ between two inertial frames Σ and Σ', where Σ' moves with the speed $\vec{v}$ wrt. to Σ ($|\vec{v}| < c$ in arbitrary direction).

[1] Eigenzeit