Tutorial “General Relativity”

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Sheet No. 4

will be discussed on Dec/13/16

***1.* Ricci Theorem**

The affine connections (Christoffel symbols) are given as

$$\Gamma^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}{}_{\nu\mu} = \frac{1}{2} g^{\lambda\kappa} \left(g_{\nu\kappa|\mu} + g_{\kappa\mu|\nu} - g_{\mu\nu|\kappa} \right).$$

(a) Show through direct calculation that

$$g_{\mu\nu\|\kappa} = 0, \quad g^{\mu\nu}{}_{\|\kappa} = 0, \quad g^{\mu}{}_{\nu\|\kappa} = 0.$$

(b) Show the validity of product rule for the covariant derivative on the example $T^{\mu}{}_{\nu} = A^{\mu} B_{\nu}$, i.e.,

$$T^{\mu}{}_{\nu\|\rho} = A^{\mu}{}_{\|\rho} B_{\nu} + A^{\mu} B_{\nu\|\rho}.$$

(c) Why can one “naively” lower and raise indices in covariant derivatives, i.e., why is for, e.g., a tensor $T_{\mu\nu}$

$$T^{\mu}{}_{\nu\|\rho} = g^{\mu\sigma} T_{\sigma\|\rho}.$$

(d) Show that for the “covariant curl” for any vector field A_μ one can use the partial derivatives instead of the covariant ones:

$$F_{\mu\nu} = A_{\nu\|\mu} - A_{\mu\|\nu} = A_{\nu|\mu} - A_{\mu|\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

***2.* Ideal fluid**

The non-relativistic hydrodynamical equations describing mass-, and- momentum energy conservation, for an ideal fluid are given by

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0, \tag{1}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} + \frac{\vec{\nabla} P}{\rho} = 0, \tag{2}$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \cdot (\epsilon \vec{v}) + P \vec{\nabla} \cdot \vec{v} = 0. \tag{3}$$

(a) Express the energy density ϵ and the Pressure P of the ideal fluid as a function of the mass density density ρ and temperature T.

(b) Show via Eq. (3) that an isothermal ideal fluid, i.e., a fluid for which $T(t, \vec{x}) = T_0$, is also incompressible, meaning $\vec{\nabla} \cdot \vec{v} = 0$.
