Electromagnetic Probes I

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

ITP Institute for Theoretical Physics

CRC–TR 211 Strong-interaction matter under extreme conditions

Outline

Plan of the lectures

Plan of the Lectures

- Lecture I: Fundamentals
  - Motivation: Heavy-ion collisions and electromagnetic probes
  - symmetries and conservation laws in (quantum) field theory
  - the Standard Model in a nutshell
  - QCD, chiral symmetry, and effective models for hadrons
- Lecture II: Dileptons in heavy-ion collisions I
  - dilepton radiation from a transparent medium (McLerran-Toimela formula)
  - QCD and chiral sum rules
  - hadronic many-body theory
  - bulk-medium description: Boltzmann equation and hydrodynamics

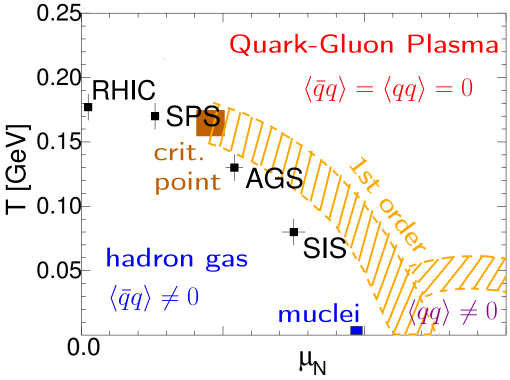
Plan of the Lectures

- Lecture III: Dileptons in heavy-ion collisions II
  - hard-thermal-loop approved dilepton rates (emission from QGP)
  - hadronic many-body theories (emission from hadron gas)
  - dileptons at SPS and RHIC
  - access to the phase diagram?
- Website:
  https://th.physik.uni-frankfurt.de/~hees/hgs-hire-lectweek18/index.html

Phase diagram, HICs, dileptons

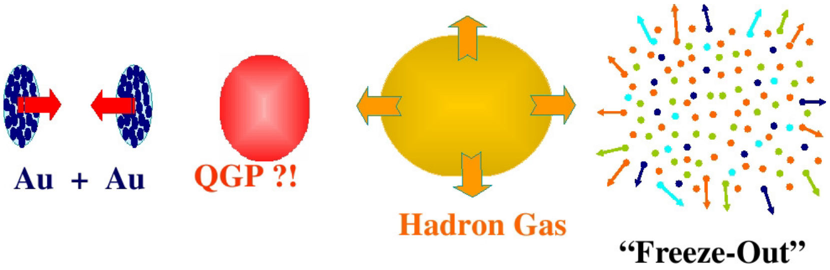
The phase diagram of strongly interacting matter

- hot and dense matter: quarks and gluons inside hadrons compressed
- highly energetic scatterings ⇒ deconfinement/chiral phase transition
- quarks and gluons relevant d.o.f. ⇒ Quark-Gluon Plasma
- still strongly couples: fast thermalization!
- in the vacuum quark condensate $\langle \bar{q} q \rangle \neq 0$
- at high T, μ_{B}: chiral phase transition $\langle \bar{q} q \rangle = 0$



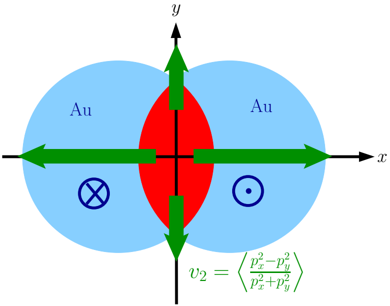
Ultra-relativistic heavy-ion collisions

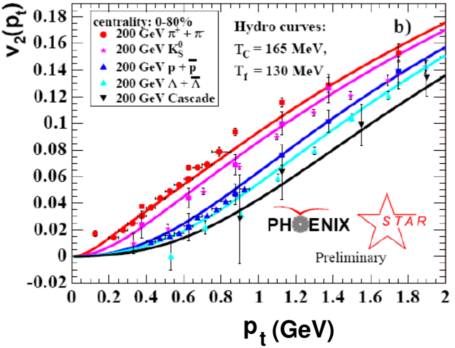
- highly energetic collisions of (heavy) nuclei
- many collisions of partons inside the nucleons
- creation of many particles ⇒ hot and dense fireball
- generation of the Quark-Gluon Plasma (QGP)?
- properties of QGP and/or compressed baryonic matter?
- signatures of 1st-order phase transition (critical end point)?



Hydrodynamical behavior

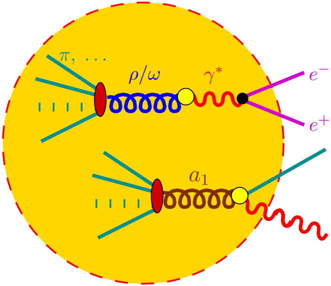
- particle spectra compatible with collective motion of an ideal fluid ⇒ small viscosity
- Medium in local thermal equilibrium (after short formation time $\lesssim 1\,\text{fm}/c$)





Why Electromagnetic Probes?

- $\gamma, \ell^{\pm}$: only e. m. interactions
- reflect whole “history” of collision
- chance to see chiral symm. rest. directly?



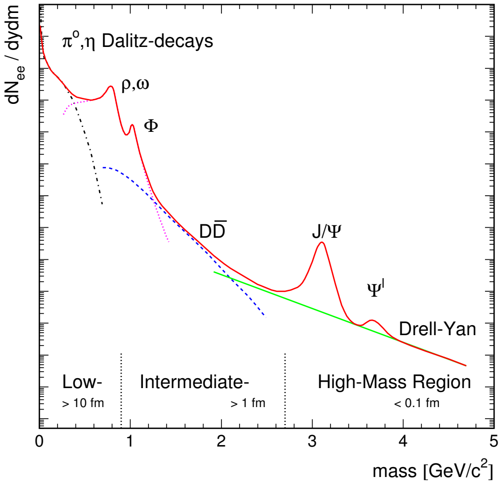
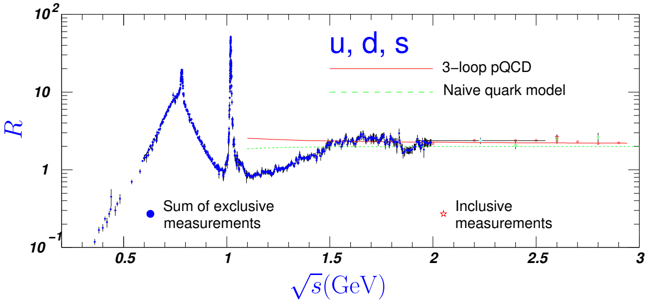


Fig. by A. Drees (from [RW00])

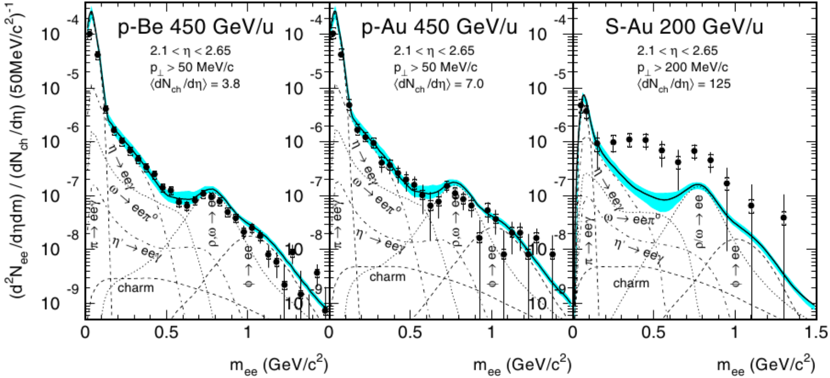
Vacuum Baseline: $e^+e^- \rightarrow$ hadrons



$$R := \frac{\sigma_{\mathrm{e^+e^-\rightarrow hadrons}}}{\sigma_{e^+e^-\rightarrow\mu^+\mu^-}}$$

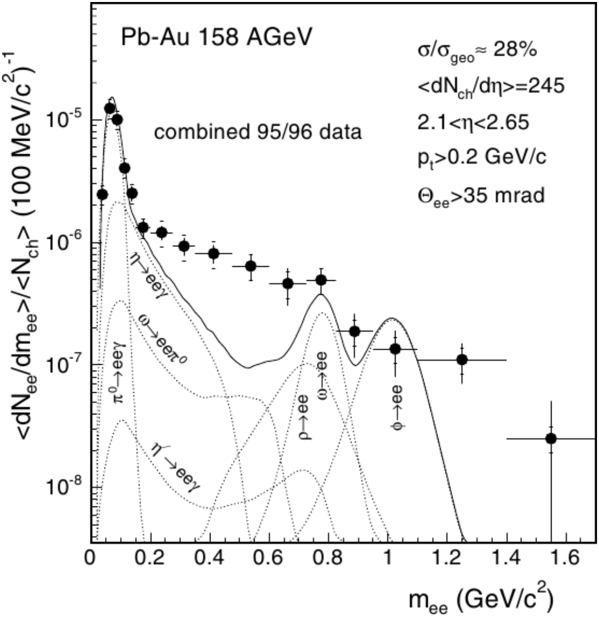
- probes all hadrons with quantum numbers of γ^*
- $R_{\mathrm{QM}} = N_c \sum_{f=u,d,s} Q_f^2 = 3 \times [(2/3)^2 + (-1/3)^2 + (-1/3)^2] = 2$
- Our aim $\mathrm{pp} \rightarrow \ell^+\ell^-$, $\mathrm{pA} \rightarrow \ell^+\ell^-$, $\mathrm{AA} \rightarrow \ell^+\ell^-$ ($\ell = \mathrm{e}, \mu$)

The CERES findings: Dilepton enhancement



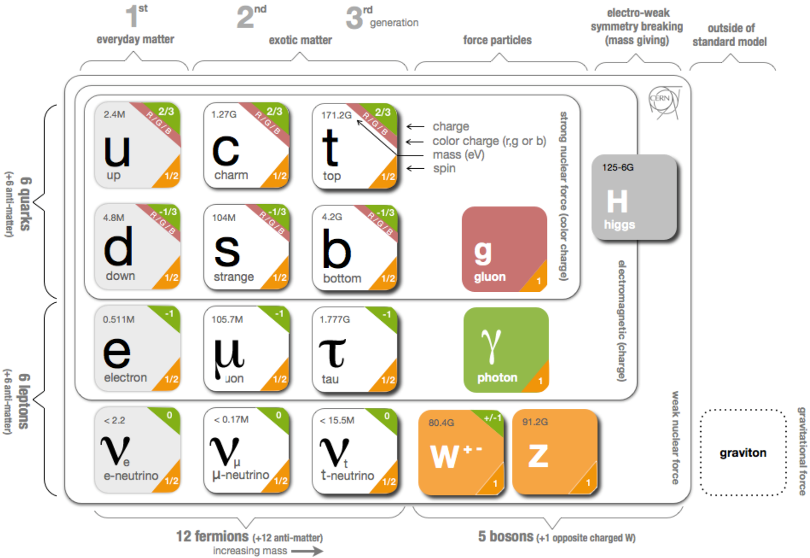
- pp (pBe): “elementary reactions”; baseline (mandatory to understand first!)
- pA: “cold nuclear matter effects”; next step (important as baseline for other observables like “J/ψ suppression”)
- AA: “medium effects”; hope to learn something about in-medium properties of vector mesons, fundamental QCD properties

The CERES findings: Dilepton enhancement



QCD and chiral symmetry

The standard model in a nutshell: particles and forces



[graphics from http://www.isgtw.org/spotlight/go-particle-quest-first-cern-hackfest]

Quantum Electrodynamics (QED)

Literature: [Nac90, DGH92, B+12], conventions as in [Nac90]

- electrons+positrons: massive spin-1/2 Dirac field $\psi \in \mathbb{C}^4$
- describes electron (charge $q_{\text{e}} = -1$) and antielectron (=positron)
- photon: massless vector field A_μ
- antisymmmetric field-strength tensor $\rightarrow (\vec{E}, \vec{B})$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \begin{pmatrix} 0 & E^1 & E^2 & E^3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix}$$

- Lagrangian ($e > 0$: em. coupling constant $e^2/(4\pi) = \alpha_{\text{em}} \simeq 1/137$

$$\mathscr{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\psi}[\mathrm{i}(\not{\partial} + \mathrm{i} q_{\text{e}} e \not{A}) - m]\psi, \quad q_e = -1$$

- Dirac matrices: $\gamma^\mu \in \mathbb{C}^{4\times 4}$, $\mu \in \{0,1,2,3\}$,
  $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$, $\overline{\psi} = \psi^\dagger \gamma^0$
- "Feynman slash" $\not{A} = A_\mu \gamma^\mu$, $\not{\partial} = \gamma^\mu \partial_\mu = \gamma^\mu \frac{\partial}{\partial x^\mu}$

Symmetries of QED

- as a classical field theory: Least-action principle ⇒ equations of motion
- action (Lorentz invariant!)

$$S[A,\psi] = \int d^4x \, \mathscr{L}$$

- symmetries lead to conservation laws (Noether's Theorem)
- space-time symmetries
  - time translations: energy conservation
  - space translations: momentum conservation
  - rotations: angular-momentum conservation
- intrinsic symmetry: invariant under change of phase factor $\psi \rightarrow \exp(-\mathrm{i} q_{\mathrm{e}} e \alpha)\psi$, $\alpha \in \mathbb{R}$ ⇒ electric-charge conservation

$$j_{\mathrm{em}}^{(\mathrm{e})\mu} = q_{\mathrm{e}} e \overline{\psi} \gamma^{\mu} \psi, \quad \partial_{\mu} j_{\mathrm{em}}^{(\mathrm{e})\mu} = 0$$

- here even local gauge symmetry:

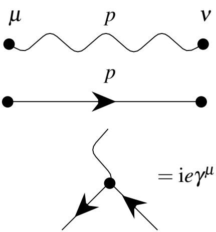
$$\psi \rightarrow \exp[-\mathrm{i} q_{\mathrm{e}} e \chi(x)]\psi, \quad A_{\mu} \rightarrow A_{\mu} + q_{\mathrm{e}} \partial_{\mu} \chi$$

- local symmetry ⇔ gauge boson

Quantization

- fields ⇒ operators
- physical quantities S-matrix elements: $|T_{fi}|^2$ transition probabilities for scattering from asymptotic free initial to asymptotic free final state
- local, microcausal quantum field theory with stable ground state
  - spin-statistics relation:
    half-integer spin ⇔ fermions, integer spin ⇔ bosons
- can only evaluate in perturbation theory ⇒ Feynman rules

Internal lines: Propagators

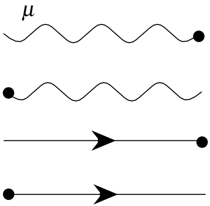


$= \mathrm{i} G_\gamma^{\mu\nu}(p)$

$= \mathrm{i} G_e(p)$

$= \mathrm{i} e \gamma^\mu$

External lines: Initial and final states



ϵ^μ

$(\epsilon^\mu)^*$

e^+ in final state or e^- in initial state

e^+ in initial state or e^- in final state

- $G_\gamma^{\mu\nu} = -\eta_{\mu\nu}/(p^2 + \mathrm{i}0^+)$, $G_\mathrm{e} = (\not{p} - m)/(p^2 - m^2 + \mathrm{i}0^+)$

Quantum Chromodynamics: QCD

- Theory for strong interactions: QCD

$$\mathscr{L}_{\text{QCD}} = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \overline{\psi}(\mathrm{i}\not{D} - \hat{M})\psi$$

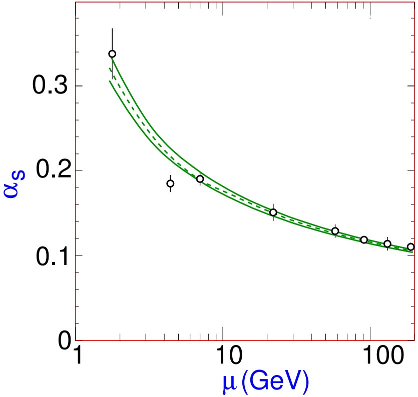
- non-Abelian gauge group $\mathrm{SU}(3)_{\text{color}}$
  - each quark: color triplet
  - covariant derivative: $\mathrm{D}_\mu = \partial_\mu + \mathrm{i}g\,\hat{T}^a A^a$ $(a \in \{1,\ldots,8\})$
  - field-strength tensor $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\,f^{abc} A_\mu^b A_\nu^c$
  - group structure constants: $[\hat{T}^a, \hat{T}^b] = \mathrm{i} f^{abc}\,\hat{T}^c, \quad \hat{T}^a = (\hat{T}^a)^\dagger \in \mathbb{C}^{3\times 3}$
- Particle content:
  - ψ: Quarks with flavor $(u, d; c, s; t, b)$ (mass eigenstates!)
  - $\hat{M} = \mathrm{diag}(m_u, m_d, m_s, \ldots) =$ current quark masses
  - A_μ^a: gluons, gauge bosons of $\mathrm{SU}(3)_{\text{color}}$
- Symmetries
  - fundamental building block: local $\mathrm{SU}(3)_{\text{color}}$ symmetry
  - in light-quark sector: approximate chiral symmetry ($\hat{M} \to 0$)
  - dilation symmetry (scale invariance for $\hat{M} \to 0$)

“Accidental” Symmetries of QCD

- chiral symmetry
  - light-quark sector: $m_q \ll m_{\text{had}} \Rightarrow$ approximate chiral symmetry
  - strong interaction: $\langle \overline{q} q \rangle_{\text{vac}} \neq 0$ (quark condensate)
  - chiral symmetry spontaneously broken (order parameter = quark condensate)
  - pions (pseudo-)Nambu-Goldstone modes: $m_\pi^2 f_\pi^2 = -m_q \langle \overline{q} q \rangle_{\text{vac}}$
- scaling symmetry
  - classical field theory: in limit of massless quarks no scale
  - symmetry under scale transformations
  - Noether: trace of energy-momentum tensor $T^{\mu}{}_{\mu} = 0$
  - quantization: need to introduce renormalization scale
  - scale invariance broken explicitly by quantization (anomaly)
  - $\langle T^{\mu}{}_{\mu} \rangle \neq 0$; $\propto$ gluon condensate + $m_{\text{q}} \times$ quark condensate
  - $\langle \text{nucleon} | T^{\mu}{}_{\mu} | \text{nucleon} \rangle = m_{\text{nucleon}}^2$ [Rob17]

Features of QCD

- asymptotically free: at large momentum transfers $\alpha_s = 4\pi g_s^2 \longrightarrow 0$
- running from renormalization group (due to self-interactions of gluons!): Nobel prize 2004 for Gross, Wilczek, Politzer



- quarks and gluons confined in hadrons
- theoretically not fully understood (nonperturbative phenomenon!)
- need of effective hadronic models at low energies: (Chiral) symmetry!

Chiral Symmetry of (massless) QCD

- Consider only light u, d quarks
- iso-spin 1/2 doublet: $\psi = \begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$
- NB: ψ has three “indices”: Dirac spinor, color, flavor iso-spin!
- γ matrices: $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}\mathbb{1}$, $\gamma_5 := \mathrm{i}\gamma_0\gamma_1\gamma_2\gamma_3$, $\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$, $\gamma_5^\dagger = \gamma_5$, $\gamma_5^2 = \mathbb{1}$
- Diracology of left and right-handed components

$$\psi_L = \frac{\mathbb{1} - \gamma_5}{2}\psi = P_L\psi, \quad \psi_R = \frac{\mathbb{1} + \gamma_5}{2}\psi = P_R\psi,$$
$$P_{L/R}^2 = P_{L/R}, \quad P_R P_L = P_L P_R = 0, \quad P_{L/R}\gamma_5 = \gamma_5 P_{L/R} = \mp P_{L/R}$$
$$P_{L/R}\gamma_\mu = \gamma_\mu P_{R/L}, \quad \overline{P_L\psi} = \overline{\psi}P_R, \quad \overline{P_R\psi} = \overline{\psi}P_L$$
$$\overline{\psi}\gamma_\mu\psi = \overline{\psi_L}\gamma_\mu\psi_L + \overline{\psi_R}\gamma_\mu\psi_R, \quad \overline{\psi}\psi = \overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L$$

- $\overline{\psi} := \psi^\dagger\gamma_0$, $\overline{\gamma_5\psi} = \psi^\dagger\gamma_5^\dagger\gamma_0 = -\overline{\psi}\gamma_5$
- in the massless limit ($m_u = m_d = 0$)

$$\mathscr{L}_{u,d} = \overline{\psi}\mathrm{i}\not{D}\psi = \overline{\psi_L}\mathrm{i}\not{D}\psi_L + \overline{\psi_R}\mathrm{i}\not{D}\psi_R$$

Chiral Symmetry of (massless) QCD

- in the massless limit ($m_u = m_d = 0$)
- a lot of global chiral symmetries:
  - change of independent phases for left and right components:

$$\psi_L(x) \to \exp(-\mathrm{i}\phi_L)\psi_L(x), \quad \psi_R(x) \to \exp(-\mathrm{i}\phi_R)\psi_R(x)$$

  - symmetry group $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$
  - independent "iso-spin rotations"

$$\psi_L(x) \to \exp(-\mathrm{i}\vec{\alpha}_L \cdot \vec{T})\psi_L(x), \quad \psi_R(x) \to \exp(-\mathrm{i}\vec{\alpha}_R \cdot \vec{T})\psi_R(x)$$

  - $\vec{T} = \vec{\tau}/2$, $\vec{\tau}$: Pauli matrices; symmetry group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$
- alternative notation scalar-pseudoscalar phases/iso-spin rotations

$$\psi \to \exp(-\mathrm{i}\phi_s)\psi, \quad \psi \to \exp(-\mathrm{i}\gamma_5\phi_a)\psi$$
$$\psi \to \exp(-\mathrm{i}\vec{\alpha}_V \cdot \vec{T})\psi, \quad \psi \to \exp(-\mathrm{i}\gamma_5\vec{\alpha}_A \cdot \vec{T})\psi$$

- $\mathrm{U}(1)_s$ and $\mathrm{SU}(2)_V$ are subgroups that are symmetries even if $m_u = m_d \neq 0 \Rightarrow$ Heisenberg's iso-spin symmetry!

Currents: relation to mesons

- based on [Koc97, Sch03, Din11]
- Noether: each global symmetry leads to a conserved quantity
- from chiral symmetries

$$j_s^\mu = \overline{\psi}\gamma^\mu\psi, \quad j_a^\mu = \overline{\psi}\gamma^\mu\gamma_5\psi$$
$$\vec{j}_V^\mu = \overline{\psi}\gamma^\mu\vec{T}\psi, \quad \vec{j}_A^\mu = \overline{\psi}\gamma^\mu\gamma_5\vec{T}\psi$$

- Link to mesons: Build Lorentz-invariant objects with corresponding quantum numbers
  - σ: $\overline{\psi}\psi$ (scalar and iso-scalar)
  - π's: $\mathrm{i}\overline{\psi}\vec{T}\gamma_5\psi$ (pseudoscalar and iso-vector)
  - ρ's: $\overline{\psi}\gamma_\mu\vec{T}\psi$ (vector and iso-vector)
  - a_1's: $\overline{\psi}\gamma_\mu\gamma_5\vec{T}\psi$ (axialvector and iso-axialvector)
- in nature: σ and π's; ρ's and a_1's do not have same mass!
- reason: QCD ground state not symmetric under pseudoscalar and pseudovector trafos since $\langle \mathrm{vac} | \overline{\psi}\psi | \mathrm{vac} \rangle \neq 0$

Spontaneous symmetry breaking

- spontaneously broken symmetry: ground state not symmetric
- vacuum necessarily degenerate
- vacuum invariant under scalar and vector transformations: $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ broken to $\mathrm{U}(1)_s$; $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ broken to $\mathrm{SU}(2)_V$
- for each broken symmetry massless scalar Goldstone boson
- there are three pions which are very light compared to other hadrons (finite masses due to explicit breaking through m_u, m_d!)
- but no pseudoscalar isoscalar light particle! ($m_{\eta'} \simeq 958$ MeV)
- reason: $\mathrm{U}(1)_a$ anomaly
- in $\mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}$ (also taking the s quark as light):
  8 instead of 9 nearly massless Goldstone bosons (3π, 4 K, 1 η)
    - axialscalar symmetry does not survive quantization!
    - good for explanation of correct decay rate for $\pi_0 \rightarrow \gamma\gamma$
    - axialscalar current not conserved $\partial_\mu j_a^\mu = 3/8\alpha_s \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a$
- explicit breaking due to quark masses
    - can be treated perturbatively $\Rightarrow$ chiral perturbation theory
    - axial-vector current only approximately conserved $\Rightarrow$ PCAC
    - a lot of low-energy properties of hadrons derivable

Chiral transformations

- σ meson and pions (chiral partners)
- meson $= \bar{q}$-q bound state
- infinitesimal chiral transformations for quarks ($\vec{T} = \vec{\tau}/2$) in $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{SU}(2)_{\mathrm{R}}$ model

$$\psi \rightarrow (1 - \mathrm{i}\delta\vec{\alpha}_V \cdot \vec{\tau}/2)\psi \quad \text{(iso-vector transformation)}$$
$$\psi \rightarrow (1 - \mathrm{i}\gamma_5\delta\vec{\alpha}_A \cdot \vec{\tau}/2)\psi \quad \text{(axial-vector transformation)}$$

- transformation properties under χ transformations as $\sigma \sim \overline{\psi}\psi$, $\vec{\pi} \sim \mathrm{i}\overline{\psi}\vec{\tau}\gamma_5\psi$

$$\sigma \rightarrow \sigma - \delta\vec{\alpha}_A \cdot \vec{\pi}, \quad \vec{\pi} \rightarrow \vec{\pi} + \delta\vec{\alpha}_V \times \vec{\pi} + \delta\vec{\alpha}_A\sigma$$

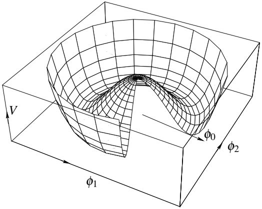
- $\sigma^2 + \vec{\pi}^2$ invariant $\Rightarrow$ χ symmetry realized as SO(4) rotations

The minimal linear σ model

- chiral symmetry realized by SO(4): meson fields $\phi \in \mathbb{R}^4$
- describes σ and pions $(\pi^{\pm}, \pi^0)$

$$\mathscr{L}_{\chi\text{limit}} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{\lambda}{4}(\phi^2 - f_\pi^2)^2$$

- spontaneous symmetry breaking: mexican-hat potential



- doesn't cost energy to excite field in direction of the rim
  $\Rightarrow$ massless Nambu-Goldstone bosons (pions)
- vacuum expectation value $\langle \phi^0 \rangle = f_\pi \neq 0$
- symmetry spontaneously broken from SO(4) to $\mathrm{SO}(3)_{\mathrm{V}}$
- particle spectrum: 4 field-degrees of freedom $\Rightarrow$ vacuum inv. 3-dim SO(3)
  $\Rightarrow$ 3 massless pions $\Rightarrow 4 - 3 = 1$ massive σ

Pion decay and PCAC

- weak decay $\pi^+ \to \mu^+ + \nu_\mu$
- weak interaction $\propto J_V^\mu - J_A^\mu$
- pion pseudoscalar $\Rightarrow$ decay must be due to axial current $\Rightarrow$

$$\langle 0 | J_A^{a\mu}(x) | \pi_b(p) \rangle = \mathrm{i} p^\mu \delta_{ab} f_\pi \exp(-\mathrm{i} p \cdot x)$$

- decay rate from Fermi model $\Rightarrow$ $f_\pi \simeq 93$ MeV

$$\langle 0 | \partial_\mu J_A^{a\mu}(x) | \pi_b(p) \rangle = -f_\pi p^2 \delta^{ab} \exp(-\mathrm{i} p \cdot x) = -f_\pi m_\pi^2 \delta^{ab} \exp(-\mathrm{i} p \cdot x)$$

- exact chiral symmetry $\Rightarrow m_\pi = 0$ (Goldstone theorem!) $\Rightarrow \partial_\mu J_A^{a\mu} = 0$ Noether
- $m_\pi \neq 0$ but "small" $\Rightarrow$ "partial conservation of axial current" (PCAC)
- in effective model

$$J_{A,\pi}^{a\mu} = f_\pi \partial_\mu \phi^a, \quad a \in \{1,2,3\}$$

Explicit symmetry breaking

- explicit breaking due to finite quark masses
- symmetry-breaking term in QCD: $\mathscr{L}_{\chi\text{SB}} = -m\bar{\psi}\psi$
- $m = (m_u + m_d)/2$; from $\bar{\psi}\psi \sim \sigma \Rightarrow$

$$\mathscr{L}_{\chi\text{SB}} = -\epsilon\sigma$$

- now write for σ-pion potential

$$V(\sigma, \vec{\pi}) = \frac{\lambda}{4}\left[(\sigma^2 + \vec{\pi}^2) - v_0^2\right]^2 - \epsilon\sigma$$

- tilted in σ-direction $\Rightarrow$ "selects" direction of vacuum expectation value
- minimum at $f_\pi \Rightarrow$

$$v_0 = f_\pi - \frac{\epsilon}{2\lambda f_\pi^2}, \quad m_\sigma^2 = 2\lambda f_\pi^2 + \frac{\epsilon}{f_\pi}, \quad m_\pi^2 = \frac{\epsilon}{f_\pi}$$

- Noether with symmetry breaking + PCAC (consistency!):

$$\partial_\mu J_A^{a\mu} = -\epsilon\pi^a \overset{\text{PCAC}}{=} -f_\pi m_\pi^2 \pi^a \Rightarrow \epsilon = f_\pi m_\pi^2$$

- χSB in QCD as in effective model $\Rightarrow$ Gell-Mann-Oaks-Renner Relation

$$\langle 0|\epsilon\sigma|0\rangle = f_\pi \epsilon = m_\pi^2 f_\pi^2 = -m\langle 0|\bar{\psi}\psi|0\rangle$$

Adding nucleons

- axial current of nucleon

$$\vec{J}^{\mu}_{A,\text{nucl}} = g_a \overline{\Psi} \gamma^{\mu} \gamma_5 \frac{\vec{\tau}}{2} \Psi$$

- from neutron-β decay $\Rightarrow g_a = 1.25$
- total axial current $\vec{J}^{\mu}_{A} = \vec{J}^{\mu}_{A,\pi} + \vec{J}^{\mu}_{A,\text{nucl}}$ should fulfill PCAC $\partial_{\mu} \vec{J}^{\mu}_{A} = -f_{\pi} m_{\pi}^2 \vec{\pi} \Rightarrow$

$$(\Box + m_{\pi}^2) \pi^a = -g_a \mathrm{i} \frac{M}{f_{\pi}} \overline{\Psi} \gamma_5 \vec{\tau} \Psi$$

- Goldberger-Treiman relation

$$g_{\pi NN} = g_a \frac{M}{f_{\pi}} \simeq 12.6 \quad \text{vs.} \quad g_{\pi NN}^{\text{exp}} = 13.4$$

- extension of linear σ model

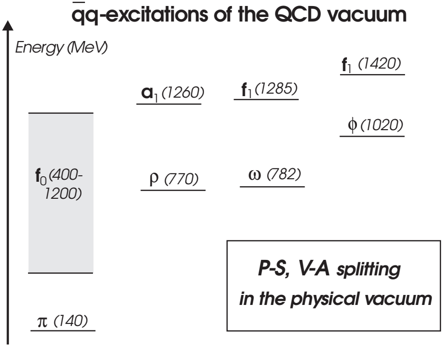
$$\mathscr{L}_{\text{nucl}} = \overline{\Psi} \mathrm{i} \not{\partial} \Psi - g_{\pi NN} \left[\mathrm{i} \overline{\Psi} \gamma_5 \vec{\tau} \Psi \cdot \vec{\pi} + \overline{\Psi} \Psi \sigma \right] + \mu \overline{\Psi} \Psi$$

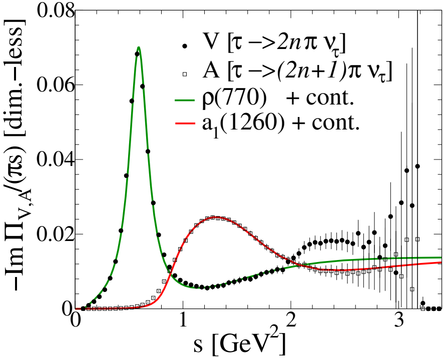
- GT relation: $\langle 0 | \sigma | 0 \rangle = f_{\pi} \Rightarrow M = g_A M - \mu$

$$\mu = (g_A - 1) M \simeq \frac{M}{4}$$

Hadron phenomenology and chiral symmetry

- QCD in light-quark sector (u, d, (s)): chiral symmetry
- in vacuum: Spontaneous breaking of chiral symmetry because $\langle \overline{q} q \rangle \neq 0$
- $\Rightarrow$ mass splitting of chiral partners





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Flash Talks

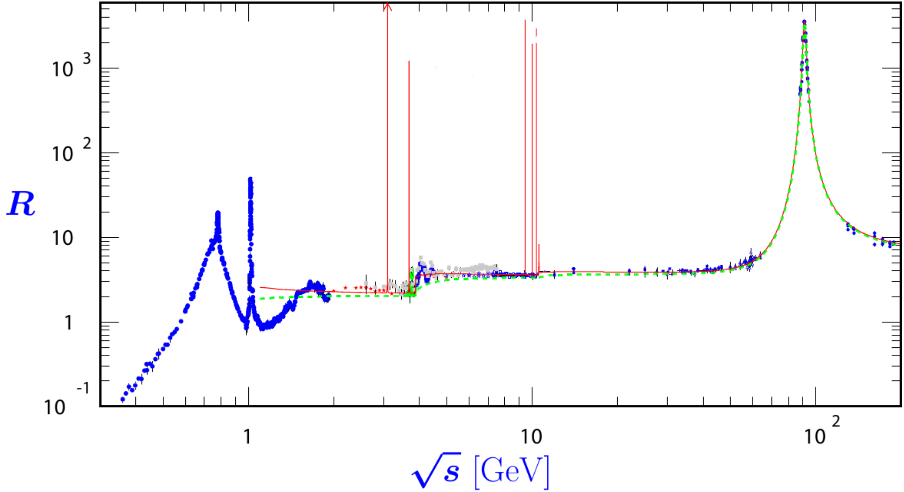
Flash Talks

1. Lagrangian of QED: Abelian Gauge symmetry $\longleftrightarrow$ charge conservation
   [Nac90] (Sect. 7.1)
2. Lagrangian of QCD: Non-Abelian Gauge symmetry $\longleftrightarrow$ gluon self-interaction
   [Nac90] (Sect. 19.1)
3. Lagrangian of minimal linear-σ model $\longleftrightarrow$ Nambu-Goldstone modes
   [Koc97] (Sects. 2.3.1 + 3.1)
4. Explicit breaking of χ symmetry $\longleftrightarrow$ pion mass and
   Gell-Mann-Oakes-Renner relation
   [Koc97] (Sect. 3.2)

Quiz

Quiz

1. What are the peaks in the following figure of $R_{e^+e^- \rightarrow \text{hadrons}}$?
2. Can you explain the horizontal lines (values: 2, 3.333, 3.667)?



Quiz

4. Why is chiral symmetry (intuitively) only true for massless quarks?
5. What's the main consequence of spontaneous symmetry breaking?
6. Why can one measure the vector and axial-vector current-current correlators from $\tau \rightarrow$ even/odd number of pions $+ \nu$?