Electromagnetic Probes II

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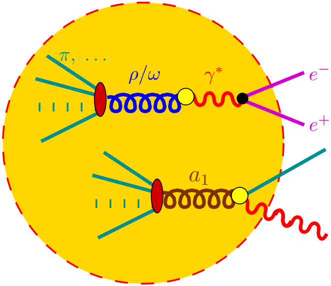
Outline

Why Electromagnetic Probes?

- $\gamma, \ell^{\pm}$: only e. m. interactions
- reflect whole "history" of collision
- chance to see chiral symm. rest. directly?



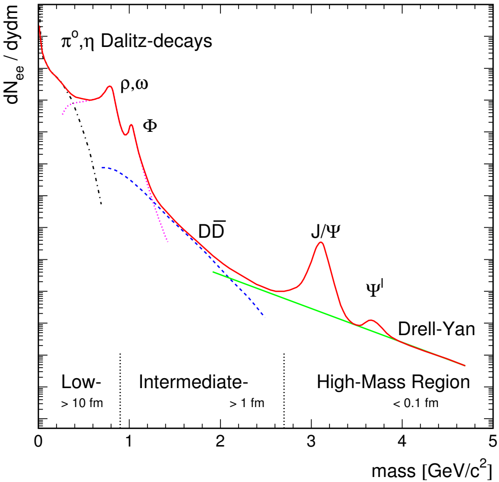


Fig. by A. Drees (from [RW00])

Theory of electromagnetic probes

The McLerran-Toimela formula

- derivation of dilepton-production rate [MT85, GK91]

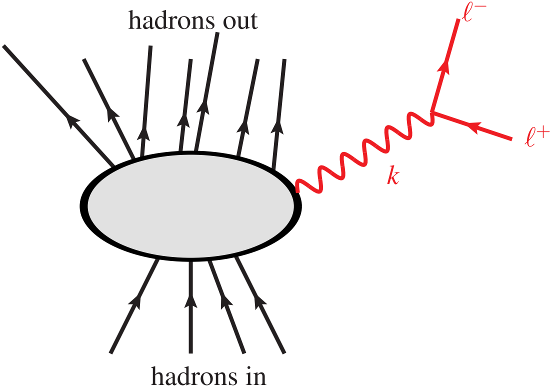
$$\frac{\mathrm{d}R_{\ell^+\ell^-}}{\mathrm{d}^4 q} = \frac{\mathrm{d}N_{\ell^+\ell^-}}{\mathrm{d}^4 x\, \mathrm{d}^4 q}$$

- radiation of dileptons from thermalized strongly interacting particles with total pair four-momentum k
- dileptons escape fireball without any final-state interactions
- calculation exact concerning strong interactions
- leading-order $\mathscr{O}(\alpha^2)$ in QED
- implies assumption that leptons don't suffer final-state interactions

$$\mathbf{H}_{\text{em}}^{(\text{int})} = e \int \mathrm{d}^3 \vec{x}\; \mathbf{J}_{\mu}(t, \vec{x}) A^{\mu}(t, \vec{x}), \quad A^{\mu}(t, \vec{x}) = \frac{\epsilon^{\mu}}{2\omega V} \exp(\mathrm{i} q \cdot x)$$

- $\mathbf{J}_{\mu}$: exact (wrt. strong interaction!)
  electromagnetic current operator of quarks or hadrons
  in the Heisenberg picture wrt. strong interactions
- $e = \sqrt{4\pi\alpha}$, $\alpha \simeq 1/137$ written out explicitly

The McLerran-Toimela formula



- Fermi's golden rule ⇒ transition-matrix element for process $|i\rangle \rightarrow \left|f'\right\rangle = \left|f\right\rangle + \left|\ell^+\ell^-(q)\right\rangle$
- QED Feynman rules

$$S_{f'i} = \left\langle f \left| \int \mathrm{d}^4 x \; \mathbf{J}_\mu(x) \right| i \right\rangle D_\gamma^{\mu\nu}(x, x') e \overline{u}_\ell(x') \gamma_\nu v_\ell(x')$$

The McLerran-Toimela formula

- Fourier transformation: energy-momentum conservation $\left|f'\right\rangle = \left|f, \ell^+\ell^-(q)\right\rangle$

$$S_{f'i} = T_{f'i}(2\pi)^4 \delta^{(4)}(P_f + q - P_i)$$

- Fermi's trick: Rate

$$R_{f'i} = \frac{|S_{f'i}|^2}{\tau V} = (2\pi)^4 \delta^{(4)}(P_f + q - P_i)|T_{f'i}|^2$$

- summing over $\left|f\right\rangle$ and polarizations of dilepton states
- averaging over initial hadron states: heat bath (grand canonical)

$$\boldsymbol{\rho} = \frac{1}{Z} \exp[-\beta(\mathbf{H}_{\text{QCD}} - \mu_B \mathbf{Q}_{\text{baryon}})]$$

The McLerran-Toimela formula

- result (derivation see [GK91], Appendices)

$$\frac{\mathrm{d}R_{\ell^+\ell^-}}{\mathrm{d}^4 q} = -\frac{\alpha^2}{3\pi^3} \frac{q^2 + 2m_\ell^2}{(q^2)^2} \sqrt{1 - \frac{4m_\ell^2}{q^2}} g_{\mu\nu} n_{\mathrm{B}}(q^0) \operatorname{Im} \Pi_{\mathrm{ret}}^{\mu\nu}(q)$$

- em. current-current correlator

$$\mathrm{i}\Pi_{\mathrm{ret}}^{\mu\nu}(q) := \int \mathrm{d}^4 x \; \exp(\mathrm{i} q \cdot x) \left\langle [\mathbf{J}^\mu(x), \mathbf{J}^\nu(0)] \right\rangle_{T,\mu_B} \Theta(x^0)$$

- written in (local) restframe of the medium
- in principle measureable: in linear response approximation Green's function for lepton current running through medium
- $q^2 = M^2 > 0$ invariant mass of dilepton
- probing medium with photons: same correlator for $q^2 = M^2 = 0$
- then correlator $\Leftrightarrow$ dielectric function $\epsilon(\omega)$ in electrodynamics!

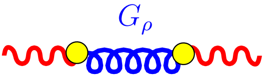
The McLerran-Toimela formula

- for real photons

$$\omega \frac{\mathrm{d}R}{\mathrm{d}^3 \vec{q}} = -\frac{\alpha g_{\mu\nu}}{2\pi^2} \operatorname{Im} \Pi^{\mu\nu}_{\text{ret}}(q) n_{\text{B}}(q^0), \quad q^0 = \omega = |\vec{q}|$$

- written in (local) restframe of the medium
- Phenomenological effective hadronic model: vector-meson dominance model
- em. current $\propto V^{\mu}$ (with $V \in \{\rho, \omega, \phi\}$)

$$\Sigma^{\gamma}_{\mu\nu} =$$



- Dilepton/photon rates: $\propto A_V = -2 \operatorname{Im} D_V^{(\text{ret})}$ (vector-meson spectral function!)
- measuring in-medium vector-meson spectral function!?!

Em. current-current correlator

Vector Mesons and electromagnetic Probes

- photon and dilepton thermal emission rates given by same electromagnetic-current-correlation function ($J_\mu = \sum_f Q_f \overline{\psi_f} \gamma_\mu \psi_f$)
- McLerran-Toimela formula

$$\Pi^{<}_{\mu\nu}(q) = \int \mathrm{d}^4 x \exp(\mathrm{i} q \cdot x) \left\langle J_\mu(0) J_\nu(x) \right\rangle_T = -2 n_{\mathrm{B}}(q_0) \operatorname{Im} \Pi^{(\mathrm{ret})}_{\mu\nu}(q)$$

$$q_0 \frac{\mathrm{d}N_\gamma}{\mathrm{d}^4 x \, \mathrm{d}^3 \vec{q}} = -\frac{\alpha_{\mathrm{em}}}{2\pi^2} g^{\mu\nu} \operatorname{Im} \Pi^{(\mathrm{ret})}_{\mu\nu}(q,u)\Big|_{q_0 = |\vec{q}|} n_{\mathrm{B}}(q \cdot u)$$

$$\frac{\mathrm{d}N_{\ell^+\ell^-}}{\mathrm{d}^4 x \, \mathrm{d}^4 q} = -g^{\mu\nu} \frac{\alpha^2}{3 q^2 \pi^3} \operatorname{Im} \Pi^{(\mathrm{ret})}_{\mu\nu}(q,u)\Big|_{q^2 = M^2_{\ell^+\ell^-}} n_{\mathrm{B}}(q \cdot u)$$

- manifestly Lorentz covariant (dependent on four-velocity of fluid cell, u)
- to lowest order in α: $4\pi\alpha \Pi_{\mu\nu} \simeq \Sigma^{(\gamma)}_{\mu\nu}$
- derivable from underlying thermodynamic potential, Ω!

Vector Mesons and chiral symmetry

- vector and axial-vector mesons $\longleftrightarrow$ respective current correlators

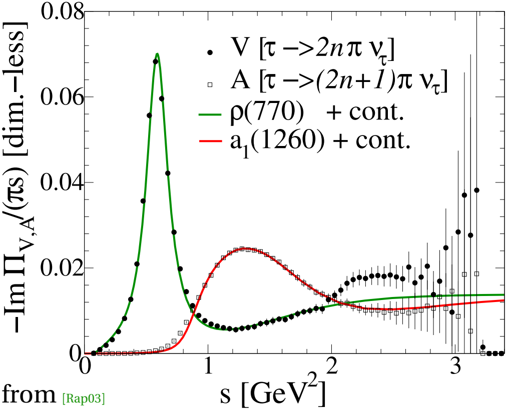
$$\Pi^{\mu\nu}_{V/A}(q) := \int \mathrm{d}^4 x \exp(\mathrm{i} q x) \left\langle J^{\nu}_{V/A}(0) J^{\mu}_{V/A}(x) \right\rangle_{\text{ret}}$$

- Ward-Takahashi Identities of χ symmetry $\Rightarrow$ Weinberg-sum rules

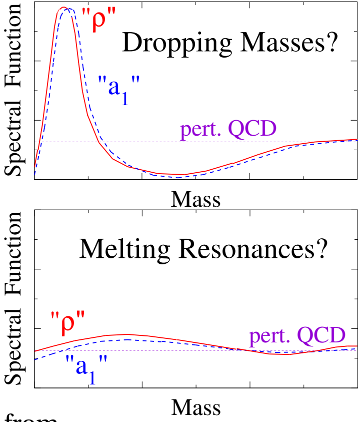
$$f_\pi^2 = -\int_0^\infty \frac{\mathrm{d}q_0^2}{\pi p_0^2} [\mathrm{Im}\,\Pi_V(q_0,0) - \mathrm{Im}\,\Pi_A(q_0,0)]$$

- spectral functions of vector (e.g. ρ) and axial vector (e.g. a_1) directly related to order parameter of chiral symmetry!

Vector Mesons and chiral symmetry



from [Rap03]

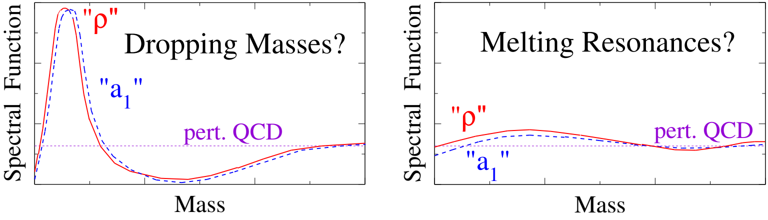


from [Rap05]

- at high enough temperatures and or densities: melting of $\langle \overline{q} q \rangle$
- $\Rightarrow$ spontaneous breaking of chiral symmetry supended
- $\Rightarrow$ chiral phase transition; chiral-symmetry restoration (χSR)
- which scenario is right? microscopic mechanisms behind χSR?

Scenarios for chiral symmetry restoration

- hadron spectrum must become degenerate between chiral partners



- models alone of little help (realization of χS not unique!)
  - "vector manifestation" $\rho_{\text{long}} = \chi$ partner of $\pi \Rightarrow$ dropping mass
  - "standard realization" $\rho = \chi$ partner of a_1,
    extreme broadening + little mass shifts
- theory "shopping list"
  - effective hadronic models (well constrained in vacuum!)
  - and concise evaluation in the medium!
  - models for fireball evolution
    (blast-wave parametrizations, hydro, transport, and transport-hydro hybrids)
  - must include partonic → phase transition → hadronic evolution
- precise $\ell^+\ell^-$ (γ) data from HICs mandatory!

QCD Sum Rules

- based on [LPM98]
- calculate current correlator, e.g., the vector part of the em. current

$$j_\mu = \frac{1}{2}(\overline{u}\gamma_\mu u - \overline{d}\gamma_\mu d)$$

- corresponds to the ρ meson!
- use pQCD to determine correlator

$$\Pi_{\mu\nu}(q) = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{k^2}\right)\Pi(q^2)$$

in deep spacelike region, $Q^2 = -q^2 \gg \Lambda_{\text{QCD}}$

- related to time-like region $\Rightarrow$ sum rule

$$\Pi(q^2) = \Pi(-Q^2) = \Pi(0) + c\,Q^2 + \frac{Q^4}{\pi}\int_0^\infty \mathrm{d}s \frac{\operatorname{Im}\Pi(s)}{s^2(s+Q^2-\mathrm{i}\epsilon)}$$

- dispersion relation: spectral function $\operatorname{Im}\Pi$!

QCD Sum Rules

- left-hand side of sum rule
- pQCD + chiral models for baryon-pion interactions [see, e.g., [DGH92]]

$$R(Q^2) := \frac{\Pi(k^2=-Q^2)}{Q^2} = -\frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right)\ln\left(\frac{Q^2}{\mu^2}\right)$$
$$+\frac{1}{Q^4} m_q \langle \overline{q} q \rangle + \frac{1}{24 Q^4}\left\langle \frac{\alpha_s}{\pi} F^a_{\mu\nu} F^{a\mu\nu} \right\rangle - \frac{112}{81 Q^6} \kappa \langle \overline{q} q \rangle^2$$

- additional cold-nuclear matter contributions

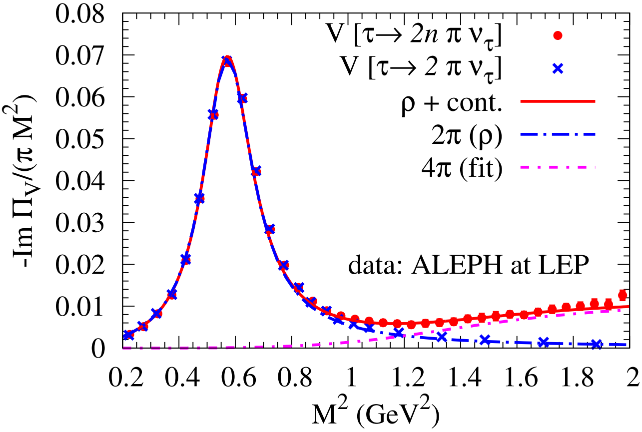
$$\Delta R(Q^2) = \frac{m_N}{4Q^4} A_2 \rho_N - \frac{5 m_N^3}{12 Q^6} A_4 \rho_N$$

- $A_{2,4}$ from parton-distribution functions
- also condensates medium-modified (in low-density approximation)

$$\langle \overline{q} q \rangle = \langle \overline{q} q \rangle_{\text{vac}} + \frac{\sigma_N}{2 m_q} \rho_N,$$
$$\left\langle \frac{\alpha_s}{\pi} F^a_{\mu\nu} F^{a\mu\nu} \right\rangle = \left\langle \frac{\alpha_s}{\pi} F^a_{\mu\nu} F^{a\mu\nu} \right\rangle_{\text{vac}} - \frac{8}{9} m_N^{(0)} \rho_N$$

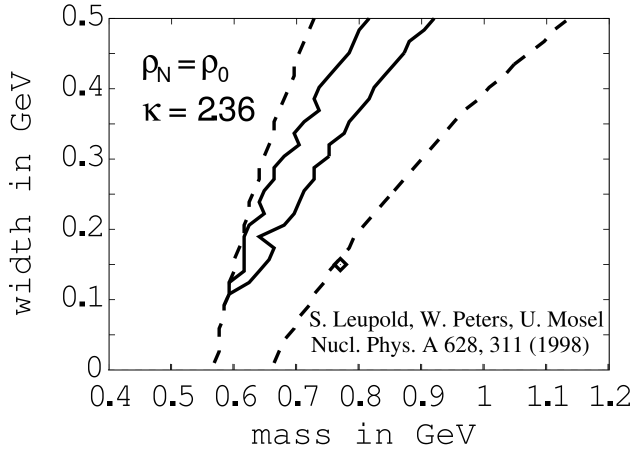
QCD Sum Rules

- right-hand side of sum rule
- use hadronic models to fit measured vector-current correlator
- e.g., ALEPH/OPAL data of $\tau \rightarrow \nu + 2n\pi$



QCD Sum Rules

- typical result from [LPM98]



- possible medium effects on ρ meson
  - dropping mass, unchanged/small width
  - unchanged mass, broadened spectrum (large width)

Weinberg Sum Rules

- vector and axial-vector mesons $\longleftrightarrow$ respective current correlators

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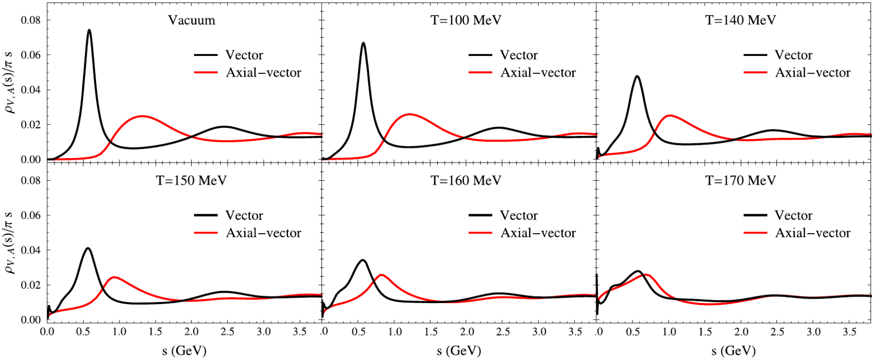
Weinberg Chiral Sum Rules

- Chiral-Sum-Rule analysis by Hohler and Rapp [HR14]
- using detailed in-medium models for vector-meson spectral functions
- construct axial-vector-meson spectral functions



- compatible with chiral-symmetry restoration

Hadronic models

Effective hadronic models: chiral-symmetry constraints

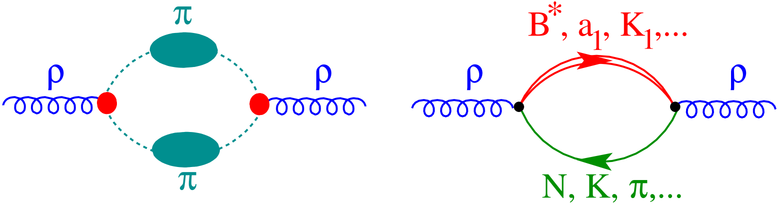
- different realizations of chiral symmetry
- equivalent only on shell (“low-energy theorems”)
- model-independent conclusions only in low-temperature/density limit (chiral perturbation theory) or from lattice-QCD calculations
- QCD sum rules: allow dropping-mass or melting-resonance scenario
- use phenomenological hadronic many-body theory (HMBT) to assess medium modifications of vector mesons
  - build models with hadrons as effective degrees of freedom
  - based on (chiral) symmetries
  - constrained by data on cross sections, branching ratios,... in the vacuum
  - in-medium properties assessed by many-body (thermal) field theory

Realistic hadronic models for light vector mesons

- CERES data: pion-ρ model too simplistic
- many approaches to more realistic models
  - gauged linear σ-model + vector-meson dominance [Pis95, UBW02]
    gauge-symmetry breaking $\Rightarrow$ pions still in physical spectrum!
  - massive Yang-Mills model; gauged non-linear chiral model with explicitly broken gauge symmetry [Mei88, LSY95]
  - hidden local symmetry: Higgs-like chiral model [BKU+85, HY03]
    allows for vector manifestation or usual manifestation (with a_1)
- here: phenomenological model by Rapp, Wambach, et al [RW99a]

Hadronic many-body theory

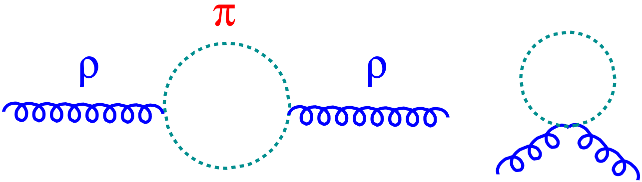
- Phenomenological HMBT [RW99a] for vector mesons
- $\pi\pi$ interactions and baryonic excitations

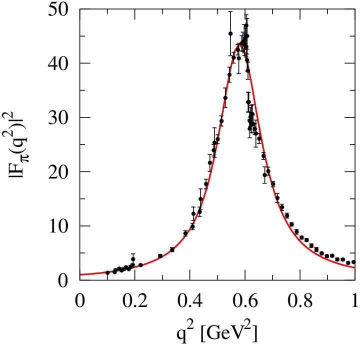


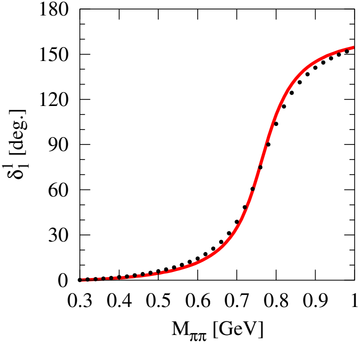
- Baryon (resonances) important, even at RHIC with low **net** baryon density $n_B - n_{\bar{B}}$
- reason: $n_B + n_{\bar{B}}$ relevant (CP inv. of strong interactions)

The meson sector (vacuum)

- most important for ρ-meson: pions

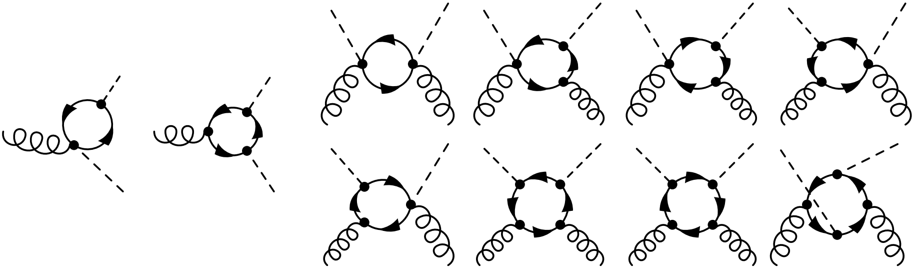




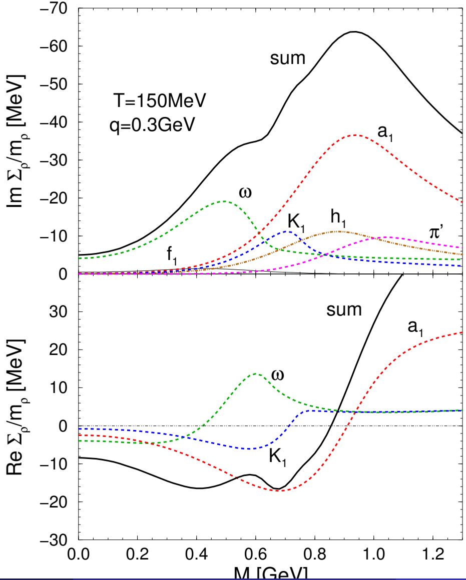


The meson sector (matter)

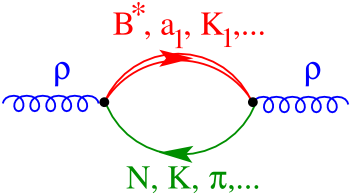
- Pions dressed with N-hole-, Δ-hole bubbles
- Ward-Takahashi ⇒ vertex corrections mandatory!



The meson sector (contributions from higher resonances)

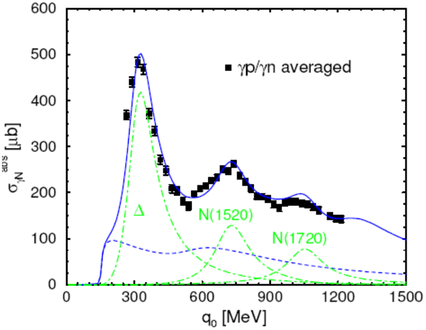


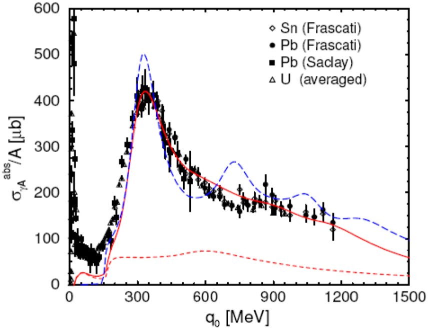
The baryon sector (vacuum)



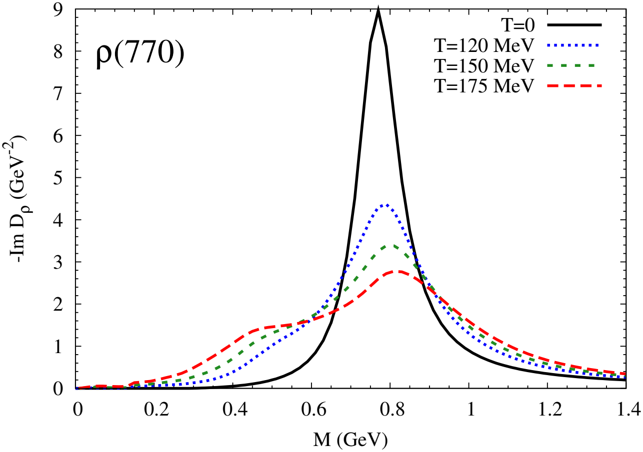
- $P = 1$-baryons: p-wave coupling to ρ:
  N(939), Δ(1232), N(1720), Δ(1905)
- $P = -1$-baryons: s-wave coupling to ρ:
  N(1520), Δ(1620), Δ(1700)

Photoabsorption on nucleons and nuclei





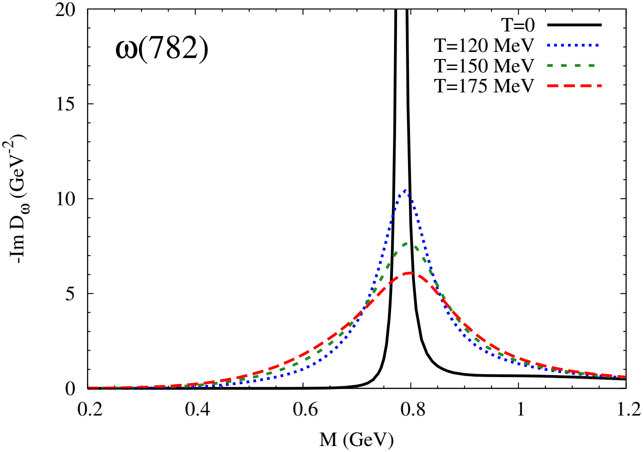
In-medium spectral functions and baryon effects



[R. Rapp, J. Wambach 99]

- baryon effects important
  - large contribution to broadening of the peak
  - responsible for most of the strength at small M

In-medium spectral functions and baryon effects



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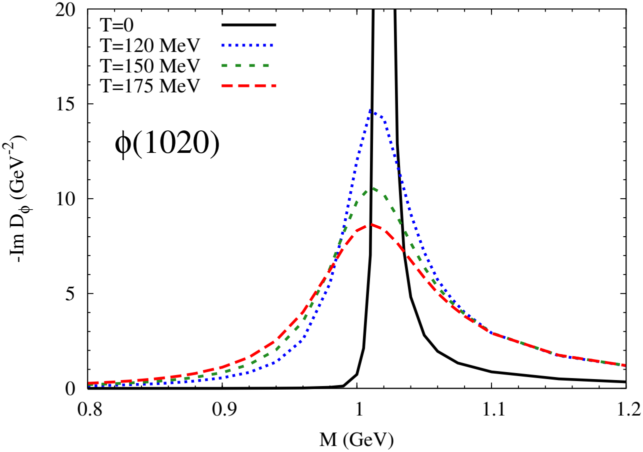

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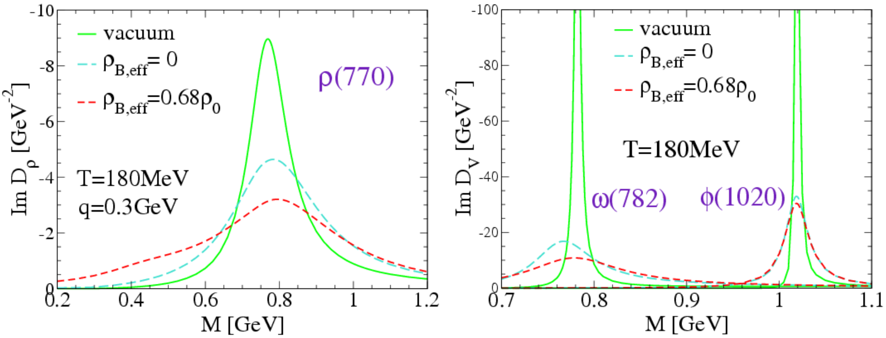

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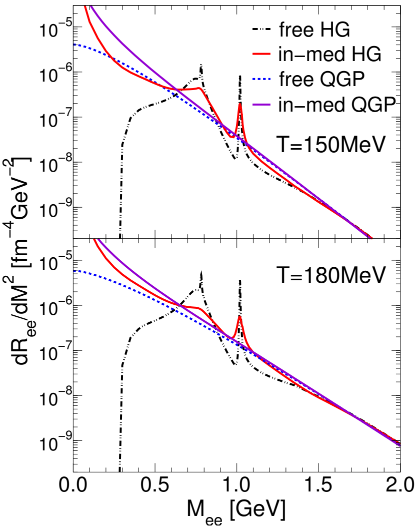
Dilepton rates: Hadron gas ⟷ QGP



- in-medium hadron gas matches with QGP
- similar results also for γ rates
- "quark-hadron duality"?
- hidden local symm.+baryons?

[BKU+85, HY03, HS06, HSW08]

Transport theory and hydrodynamics

Phase-space distribution

- classical many-body system of relativistic particles
- all particles are on their mass shell: $E = E_p := \sqrt{\vec{p}^2 + m^2}$
- Boltzmann equation [dvv80, CK02, Hee15]:
  dynamical equation for phase-space distribution function $f(t, \vec{x}, \vec{p})$
- relativistic covariance of phase-space distribution
  - $f(t, \vec{x}, \vec{p})$ defined as Lorentz scalar quantity
  - particle number N: $\mathrm{d}N = \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{p}\, f(t, \vec{x}, \vec{p})$
  - particle-number four-vector current $(N^\mu) = (n, \vec{N})$

$$N^\mu = \int_{\mathbb{R}^3} \mathrm{d}^3\vec{p}\, \frac{p^\mu}{E_p} f(t, \vec{x}, \vec{p})$$

  - flow-velocity of fluid cell (“Eckart frame”)

$$\vec{v}_{\text{Eck}}(x) = \frac{\vec{N}(x)}{N^0(x)}, \quad u^\mu_{\text{Eck}} = \frac{N^\mu}{\sqrt{N_\mu N^\mu}} = \frac{N^\mu}{n_0}$$

  - n_0: particle density in local fluid (Eckart) restframe

Relativistic Boltzmann equation

- particles moving along trajectories $(\vec{x}(t), \vec{p}(t))$
- for infinitesimal time step $\mathrm{d}t$

$$\mathrm{d}N(t+\mathrm{d}t) = f(t+\mathrm{d}t, \vec{x}+\mathrm{d}t\,\vec{v}, \vec{p}+\mathrm{d}t\,\vec{F})\mathrm{d}^6\xi(t+\mathrm{d}t), \quad \mathrm{d}^6\xi = \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{p}$$

- Jacobian for phase-space volume

$$\mathrm{d}^6\xi(t+\mathrm{d}t) = \mathrm{d}^6\xi(t)\det\left(\frac{\partial(\vec{x}+\mathrm{d}t\,\vec{v}, \vec{p}+\mathrm{d}t\,\vec{F})}{\partial(\vec{x},\vec{p})}\right) = \mathrm{d}^6\xi(t)\left(1+\mathrm{d}t\,\vec{\nabla}_p\cdot\vec{F}\right) + \mathcal{O}(\mathrm{d}t^2)$$

- total change of $\mathrm{d}N$

$$\mathrm{d}N(t+\mathrm{d}t) - \mathrm{d}N(t) = \mathrm{d}^6\xi(t)\mathrm{d}t\left[\frac{\partial f}{\partial t} + \frac{\vec{p}}{E}\cdot\frac{\partial \vec{f}}{\partial \vec{x}} + \frac{\partial(\vec{F}f)}{\partial \vec{p}}\right]$$

Relativistic Boltzmann equation

- covariance: $\mathrm{d}\tau = \mathrm{d}t\sqrt{1-\vec{v}^2}$ proper time, $\vec{v}=\vec{p}/E_p$, $\sqrt{1-\vec{v}^2}=m/E_p$

$$\mathrm{d}t\left[\frac{\partial f}{\partial t}+\frac{\vec{p}}{E}\cdot\frac{\partial \vec{f}}{\partial \vec{x}}\right]=\mathrm{d}\tau\frac{p^\mu}{m}\frac{\partial f}{\partial x^\mu} \Rightarrow \text{covariant!}$$

- covariant equation of motion for point particle

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau}=K^\mu, \quad p_\mu p^\mu = m^2 = \text{const} \Rightarrow$$

$$K^0=\frac{\vec{p}}{E_p}\cdot\vec{K} \Rightarrow \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}=\vec{F}=\vec{K}\frac{m}{E_p}$$

$$\frac{E_p}{m}\vec{\nabla}_p(\vec{F}f)=\frac{\partial}{\partial p^\mu}(K^\mu f) \Rightarrow \text{covariant!}$$

$$\frac{\mathrm{d}N(t+\mathrm{d}t)-\mathrm{d}N(t)}{\mathrm{d}^6\xi}=\mathrm{d}t\left[\frac{\partial f}{\partial t}+\frac{\vec{p}}{E}\cdot\frac{\partial \vec{f}}{\partial \vec{x}}+\frac{\partial(\vec{F}f)}{\partial \vec{p}}\right]=\mathrm{d}\tau\left[\frac{p^\mu}{m}\frac{\partial f}{\partial x^\mu}+\frac{\partial(K^\mu f)}{\partial p^\mu}\right]$$

Relativistic Boltzmann equation

- change of particle number due to collisions
  - short-range interactions: collisions at one point (local) in space
  - invariant cross section

$$\mathrm{d}N_{\text{coll}}(p_1', p_2' \leftarrow p_1, p_2) = \mathrm{d}^4 x \frac{\mathrm{d}^3 \vec{p}_1}{E_1} \frac{\mathrm{d}^3 \vec{p}_2}{E_2} \frac{\mathrm{d}^3 \vec{p}\,'_1}{E_1'} \frac{\mathrm{d}^3 \vec{p}\,'_2}{E_2'} f_1 f_2 W(p_1', p_2' \leftarrow p_1, p_2),$$

$$\mathrm{d}\sigma = \frac{W(p_1', p_2' \leftarrow p_1, p_2) \mathrm{d}^4 x \frac{\mathrm{d}^3 \vec{p}_1}{E_1} \frac{\mathrm{d}^3 \vec{p}_2}{m} \frac{\mathrm{d}^3 \vec{p}\,'_1}{E_1'} \frac{\mathrm{d}^3 \vec{p}\,'_2}{E_2'} f_1 f_2}{\mathrm{d}^4 x \mathrm{d}^3 \vec{p}_1 v_{\text{rel}} f_1 \mathrm{d}^3 \vec{p}_2 f_2},$$

$$\mathrm{d}\sigma = \frac{\mathrm{d}^3 \vec{p}\,'_1}{E_1'} \frac{\mathrm{d}^3 \vec{p}\,'_2}{E_2'} \frac{W(p_1', p_2' \leftarrow p_1, p_2)}{I}, \quad I = \sqrt{(p_1 \cdot p_2)^2 - m^4}$$

  - important: v_{rel} is velocity of particle 1 in rest frame of particle 2
  - from relativistic covariance (or unitarity of S-matrix!) $\Rightarrow$ detailed-balance relation

$$W(p_1', p_2' \leftarrow p_1, p_2) = W(p_1, p_2 \leftarrow p_1', p_2')$$

- Boltzmann equation (manifestly covariant form)

$$p^\mu \frac{\partial f}{\partial x^\mu} + m \frac{\partial (K^\mu f)}{\partial p^\mu} = \frac{1}{2} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}_2}{E_2} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}\,'_1}{E_1'} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3 \vec{p}\,'_2}{E_2'} W(p_1', p_2' \leftarrow p, p_2)(f_1' f_2' - f f_2)$$

- collision integral: "gain minus loss"

Entropy

- input from quantum mechancis: particle in a cubic box (periodic boundary cond.)

$$\vec{p} = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3$$

- $\Delta^6 \xi_j = L^3 \Delta^3 \vec{p}$ (“microscopically large, macroscopically small”)
- contains G_j single-particle states (g: degeneracy due to spin, isospin, ...)

$$G_j = g \frac{\Delta^6 \xi_j}{(2\pi)^3}$$

- statistical weight for N_j particles in $\Delta^6 \xi_j$:
- factor $1/N_j!$: indistinguishability of particles

$$\Delta\Gamma_j = \frac{1}{N_j!} G_j^{N_j}$$

- entropy a la Boltzmann

$$\begin{aligned} S &= \sum_j \ln \Delta\Gamma_j \simeq \sum_j [N_j \ln G_j - N_j(\ln N_j - 1)] \\ &= -\int \mathrm{d}^3\vec{x}\, \mathrm{d}^3\vec{p}\, f(x,p)\{\ln[(2\pi)^3 f(x,p)/g] - 1\} \end{aligned}$$

The Boltzmann H theorem

- H = greek Eta: Boltzmann's notation for entropy
- covariant description of entropy: entropy four-flow

$$S^\mu(x) = -\int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}}{E} p^\mu f(x,p)\{\ln[(2\pi)^3 f(x,p)/g] - 1\}$$

- Boltzmann equation + symmetries of $W(p_1' p_2' \leftarrow p_1 p_2)$

$$\frac{\partial S^\mu}{\partial x^\mu} := \zeta = +\frac{1}{4} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}}{E} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}_2}{E_2} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}\,'_1}{E_1'} \int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}\,'_2}{E_2'} f f_2 \times \left[\frac{f_1' f_2'}{f f_2} - \ln\left(\frac{f_1' f_2'}{f f_2}\right) - 1\right] W(p_1' p_2' \leftarrow p, p_1) \geq 0$$

- (on average) entropy can never decrease with time!
- equilibrium $\Leftrightarrow$ S maximal!
- bracket must vanish $\Rightarrow$ Maxwell-Boltzmann distribution

$$f_{\text{eq}}(x,p) = \frac{g}{(2\pi)^3} \exp\left[-\beta(x)\Big(u(x)\cdot p - \mu(x)\Big)\right], \quad p^0 = E = \sqrt{m^2 + \vec{p}^2}$$

- $\beta = 1/T$: inverse temperature, u: fluid four-velocity, μ: chemical potential
- temperature, chemical potential are Lorentz scalars!

Hydrodynamics

- in the limit of very small mean-free path: system in local thermal equilibrium
- switch to ideal hydrodynamics description
- forget about "particles" ⇒ fluid description
- equations of motion for $\vec{v}(t,\vec{x})$: conservation laws

$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0$$

- N^μ: net-baryon number, $T^{\mu\nu}$: energy-momentum tensor
- ideal hydrodynamics

$$N^\mu = n u^\mu, \quad T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P \eta^{\mu\nu}$$
$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0$$

- n: proper net-baryon density, ϵ: proper energy density, P: pressure
- 5 equations of motion, 6 unknowns: $\vec{v}$, n, ϵ, P
- need also equation of state $\epsilon = \epsilon(P)$
- hadron-resonance gas EoS (low energies)
  lQCD based cross-over phase transition (high energies)

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Flash Talks

Flash Talks

1. Summarize the basic steps to derive the McLerran-Toimela formula (slides 5-7, private notes to be distributed)
2. What are the basic ideas behind QCD and chiral sum rules (slides 15-17, [LPM98, Wei67])
3. What's the main mechanism behind in-medium modifications of the ρ meson? (slides 24-32, [RW99b])
4. Explain (intuitively) what's behind the Boltzmann equation (slides 35-37, [Hee15])

Quiz

Quiz

1. Which important “theoretical quantity” can be assessed by observing electromagnetic probes in HICs (and elementary reactions)?
2. What is chiral-symmetry restoration (χSR) and in which ways could it be realized in nature?
3. What have em. probes in heavy-ion collisions to do with chiral symmetry?
4. What can we learn from QCD and chiral sum rules about χSR?
5. what’s basic assumption of the vector-meson dominance (VMD) model?
6. What tell effective hadronic models about the medium modification of light vector mesons and the related χSR?
7. why are baryon-vector-meson interactions important even at high collision energies, where $\mu_B \simeq 0$ (nearly 0 net-baryon density)?