Heavy Probes in Heavy-Ion Collisions
Theory Part II

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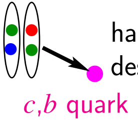
Outline

1. Heavy-quark transport in the sQGP
   - Open heavy-flavor observables in heavy-ion collisions
   - Transport equations
   - The Fokker-Planck equation
   - Realization as Langevin process
   - Langevin simulation for heavy-ion collisions

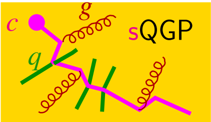
2. In-medium interactions of heavy quarks I
   - Elastic pQCD heavy-quark scattering
   - Non-perturbative interactions: effective resonance model

3. Non-photonic electrons at RHIC

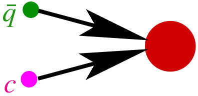
Heavy quarks in the sQGP



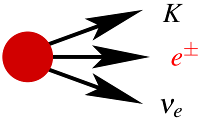
hard production of HQs
described by PDF's + pQCD (PYTHIA)



HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from
microscopic model for HQ interactions in the sQGP



Hadronization to D,B mesons via
quark coalescence + fragmentation



semileptonic decay ⇒
"non-photonic" electron observables

The relativistic Boltzmann equation

- describe heavy-quark scattering in the QGP by (semi-)classical transport equation
- $f_Q(t,\vec{x},\vec{p})$: phase-space distribution of heavy quarks
- equation of motion for HQ-fluid cell at time t at $(\vec{p},\vec{x})$:

$$\mathrm{d}f_Q = \mathrm{d}t \left(\frac{\partial}{\partial t} + \vec{v}\frac{\partial}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial}{\partial \vec{p}} \right) f_Q$$

  - change of phase-space distribution with time (non-equilibrium)
  - drift of HQ-fluid cell with velocity $\vec{v} = \vec{p}/E_{\vec{p}}$, $E_{\vec{p}} = \sqrt{m_Q^2 + \vec{p}^2}$
  - change of momentum with mean-field force, $\vec{F}$
- change must be due to collisions with surrounding medium

$$\mathrm{d}f_Q = C[f_Q] = \int \mathrm{d}^3\vec{k} [\underbrace{w(\vec{p}+\vec{k},\vec{k}) f_Q(t,\vec{x},\vec{p}+\vec{k})}_{\text{gain}} - \underbrace{w(\vec{p},\vec{k}) f_Q(t,\vec{x},\vec{p})}_{\text{loss}}]$$

  - $w(\vec{p},\vec{k})$: transition rate for collision of a heavy quark with momentum, $\vec{p}$ with a heat-bath particle with momentum transfer, $\vec{k}$

Transition rates

- relation to cross sections of microscopic scattering processes
- e.g., elastic scattering of heavy quark with light quarks

$$w(\vec{p},\vec{k}) = \gamma_q \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} f_q(\vec{q}) v_{\text{rel}}(\vec{p},\vec{q} \rightarrow \vec{p}-\vec{k},\vec{q}+\vec{k}) \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$

- $\gamma_q = 2 \times 3 = 6$: spin-color-degeneracy factor
- $v_{\text{rel}} := \sqrt{(p \cdot q)^2 - (m_Q m_q)^2}/(E_Q E_q)$; covariant relative velocity
- in terms of invariant matrix element

$$\begin{aligned}C[f_Q] =& \frac{1}{2E_Q} \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3 2E_q} \int \frac{\mathrm{d}^3\vec{p}\,'}{(2\pi)^3 2E_p'} \int \frac{\mathrm{d}^3\vec{q}\,'}{(2\pi)^3 2E_q'} \\ &\times \frac{1}{\gamma_Q} \sum_{c,s} \left|\mathcal{M}_{(\vec{p}\,',\vec{q}\,') \leftarrow (\vec{p},\vec{q})}\right|^2 \\ &\times (2\pi)^4 \delta^{(4)}(p+q-p'-q')[f_Q(\vec{p}\,')f_q(\vec{q}\,') - f_Q(\vec{p})f_q(\vec{q})]\end{aligned}$$

  - $\vec{p}$, $\vec{q}$ ($\vec{p}\,'$, $\vec{q}\,'$) initial (final) momenta of heavy and light quark
  - momentum transfer: $\vec{k} = \vec{q}\,' - \vec{q} = \vec{p} - \vec{p}\,'$
- sum over all ("relevant") scattering processes

The Fokker-Planck Equation

- heavy quarks $\leftrightarrow$ light quarks/gluons: momentum transfers small
- $w(\vec{p}+\vec{k},\vec{k})$: peaked around $\vec{k}=0$
- expansion of collision term around $\vec{k}=0$

$$\begin{split} w(\vec{p}+\vec{k},\vec{k}) f_Q(\vec{p}+\vec{k},\vec{k}) \simeq & w(\vec{p},\vec{k}) f_Q(\vec{p}) + \vec{k} \cdot \frac{\partial}{\partial \vec{p}} [w(\vec{p},\vec{k}) f_Q(\vec{p})] \\ & + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial \vec{p}_i \vec{p}_k} [w(\vec{p},\vec{k}) f_Q(\vec{p})] \end{split}$$

- collision term

$$C[f_Q] = \int \mathrm{d}^3 \vec{k} \left[k_i \frac{\partial}{\partial p_i} + \frac{1}{2} \frac{\partial^2}{\partial p_i \partial p_j} \right] [w(\vec{p},\vec{k}) f_Q(\vec{p})].$$

The Fokker-Planck Equation

- Boltzmann equation ⇒ simplifies to Fokker-Planck equation

$$\partial_t f_Q(t,\vec{x},\vec{p}) + \frac{\vec{p}}{E_{\vec{p}}} \cdot \frac{\partial}{\partial \vec{x}} f_Q(t,\vec{x},\vec{p}) = \frac{\partial}{\partial p_i} \left\{ A_i(\vec{p}) f_Q(t,\vec{x},\vec{p}) + \frac{\partial}{\partial p_j} [B_{ij}(\vec{p}) f_Q(t,\vec{p})] \right\}$$

- with drag and diffusion coefficients

$$A_i(\vec{p}) = \int \mathrm{d}^3\vec{k} k_i w(\vec{p},\vec{k}), \quad B_{ij}(\vec{p}) = \frac{1}{2} \int \mathrm{d}^3\vec{k} k_i k_j w(\vec{p},\vec{k})$$

- equilibrated light quarks and gluons: coefficients in heat-bath frame
- matter homogeneous and isotropic

$$A_i(\vec{p}) = A(p) p_i, \quad B_{ij}(\vec{p}) = B_0(p) P_{ij}^{\perp} + B_1(p) P_{ij}^{\parallel}$$

$$\text{with} \quad P_{ij}^{\parallel}(\vec{p}) = \frac{p_i p_j}{\vec{p}^2}, \quad P_{ij}^{\perp}(\vec{p}) = \delta_{ij} - \frac{p_i p_j}{\vec{p}^2}$$

Meaning of the Coefficients

- Simplified equation for momentum distribution, $F_Q(t, \vec{p})$
- Integrate Fokker-Planck equation over whole spatial volume:

$$F_Q(t,\vec{p}) = \int_V \mathrm{d}^3\vec{x} f_Q(t,\vec{x},\vec{p}),$$

$$\int_V \mathrm{d}^3\vec{x}\, \mathrm{div}_{\vec{x}} \left[\frac{\vec{p}}{E_{\vec{p}}} f(t,\vec{x},\vec{p})\right] = \int_{\partial V} \mathrm{d}\vec{S} \cdot \left[\frac{\vec{p}}{E_{\vec{p}}} f(t,\vec{x},\vec{p})\right] = 0 \Rightarrow$$

$$\frac{\partial}{\partial t} F_Q(t,\vec{p}) = \frac{\partial}{\partial p_i}\left\{A_i(\vec{p}) F_Q(t,\vec{p}) + \frac{\partial}{\partial p_j}[B_{ij}(\vec{p}) F_Q(t,\vec{p})]\right\}$$

- most simple case in non-relativistic limit $A(\vec{p}) = A = \text{const}$, $B_0(\vec{p}) = B_1(\vec{p}) = B = \text{const}$

$$F_Q(t,\vec{p}) = \left\{\frac{A}{2\pi D}[1 - \exp(-2\gamma t)]\right\}^{-3/2} \times \exp\left[-\frac{A}{2B}\frac{[\vec{p} - \vec{p}_0 \exp(-At)]^2}{1 - \exp(-2\gamma t)}\right]$$

Meaning of the Coefficients

- solution: Gaussian with

$$\langle \vec{p}(t) \rangle = \vec{p}_0 \exp(-At), \quad \Delta \vec{p}^2(t) = \langle \vec{p}^2 \rangle - \langle \vec{p} \rangle^2 = \frac{3B}{A}[1-\exp(-2At)].$$

  - A: friction/drag coefficient $\Rightarrow$ dissipation
  - $1/A$: relaxation time to reach equilibrium
  - B: momentum-diffusion coefficient
  - measures size of momentum fluctuations (result of random uncorrelated collisions of heavy quarks with medim)
  - $\Rightarrow$ effective description of collisions: white-noise-random force
- equilibrium limit $(t \to \infty)$

$$F_Q(t,\vec{p}) \underset{t\to\infty}{\cong} \left(\frac{2\pi B}{A}\right)^{3/2} \exp\left(-\frac{A\vec{p}^2}{2B}\right)$$

- has to be Maxwell-Boltzmann distribution $\Rightarrow$

$$B = m_Q A T$$

  - T: given temperature of the QGP
  - Einstein's dissipation-flucutation relation (1905)

Realization as Langevin process

- Langevin process: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

$$\mathrm{d}\vec{x} = \frac{\vec{p}}{E_p}\mathrm{d}t,$$
$$\mathrm{d}\vec{p} = -A\,\vec{p}\,\mathrm{d}t + \hat{C}\vec{w}\sqrt{\mathrm{d}t}$$

- $\vec{w}(t)$: Gaussian-distributed random variable

$$\langle \vec{w}(t)\rangle = 0, \quad \left\langle w_j(t) w_k(t')\right\rangle = \delta(t-t')$$

- $\hat{C} = \hat{C}^t$: covariance matrix of random force
- stochastic process depends on choice of momentum argument of $\hat{C}$

$$\hat{C} \rightarrow \hat{C}(t, \vec{x}, \vec{p} + \xi \mathrm{d}\vec{p}), \quad \xi \in [0,1]$$

- usual values of ξ
  - $\xi = 0$: pre-point Ito realization
  - $\xi = 1/2$: Stratonovich realization
  - $\xi = 1$: post-point Ito (Hänggi-Klimontovich) realization

Langevin $\leftrightarrow$ Fokker-Planck

- heavy-quark phase-space distribution

$$f_Q(t,\vec{x},\vec{p}) = \left\langle \delta^{(3)}[\vec{x}-\vec{x}\,'(t)]\delta^{(3)}[\vec{p}-\vec{p}\,'(t)] \right\rangle \tag{1}$$

- $[\vec{x}\,'(t),\vec{p}\,'(t)]$: trajectories according to stochastic Langevin process

$$\begin{aligned} \mathrm{d}\vec{x} &= \frac{\vec{p}}{E_p}\mathrm{d}t, \\ \mathrm{d}\vec{p} &= -A\,\vec{p}\,\mathrm{d}t + \hat{C}\vec{w}\,\sqrt{\mathrm{d}t} \end{aligned} \tag{2}$$

- perform timestep of Eq. (1) using (2)

$$\frac{\partial f_Q}{\partial t} + \frac{p_j}{E}\frac{\partial f_Q}{\partial x_j} = \frac{\partial}{\partial p_j}\left[\left(A p_j - \xi C_{lk}\frac{\partial C_{jk}}{\partial p_l}\right) f_Q\right] + \frac{1}{2}\frac{\partial^2}{\partial p_j \partial p_k}(C_{jl}C_{kl}f_Q)$$

$$\Rightarrow \quad C_{jk} = \sqrt{2B_0}P_{jk}^{\perp} + \sqrt{2B_1}P_{jk}^{\parallel}$$

- Form of Fokker-Planck equation ok, but how to chose ξ?

Langevin $\leftrightarrow$ Fokker-Planck

- Choice of ξ: $f_Q \to$ Maxwell-Boltzmann distribution for $t \to \infty$:

$$f_Q^{\text{eq}}(\vec{p}) \propto \exp(-\sqrt{\vec{p}^2 + m_Q^2}/T)$$

- Langevin process with $B_0 = B_1 = D(E) \Rightarrow C_{jk} = \sqrt{2D(E)}\delta_{jk}$
- MB distribution solution of stationary FP equation $\Rightarrow$

$$A(E)ET - D(E) + (1-\xi)TD'(E) \stackrel{!}{=} 0$$

- simples choice: $\xi = 1$ (post-point Ito realization)
- then simple Einstein dissipation-fluctuation relation

$$D = TEA$$

- for models for FP coefficients: relation not well satisfied for B_1
- $\Rightarrow$ use $\xi = 1$ and $B_1 = TEA$
- numerical check: Langevin simulation has right equilibrium limit

Langevin simulation for heavy-ion collisions

- need to simulate heavy-quark diffusion in sQGP
- "bulk" (light quarks + gluons) described by thermal fireball model
- flowing medium in local thermal equilibrium
- FP coefficients and Langevin process in restframe of the heat bath
- way out: boost to local heat-bath frame with flow velocity $v(t, \vec{x}$
- do time step to "update" momenta
- boost back to "lab frame"
- defines HQ distribution as "freezeout at constant lab time"
- NB: leads to covariant equilibrium distribution

$$\mathrm{d}N_Q = \frac{\gamma_Q}{(2\pi)^3} \mathrm{d}^3\vec{x}^{(\mathsf{hb})} \frac{\mathrm{d}^3\vec{p}}{p_0} p \cdot u(x) \exp\left(-\frac{p \cdot u(x)}{T(x)}\right)$$

  - $u(t, \vec{x}) = [1, \vec{v}(t, \vec{x})]/\sqrt{1 - \vec{v}^2(t, \vec{x})}$: velocity-flow field (4-vector)
  - $T(x)$: temperature field (4-scalar)

Fire-ball model

- Elliptic fire-ball parameterization
  fitted to hydrodynamical flow pattern [Kolb '00]

$$V(t) = \pi(z_0 + v_z t)a(t)b(t), \quad a, b\text{: semi-axes of ellipse,}$$
$$v_{a,b} = v_\infty[1 - \exp(-\alpha t)] \mp \Delta v[1 - \exp(-\beta t)]$$

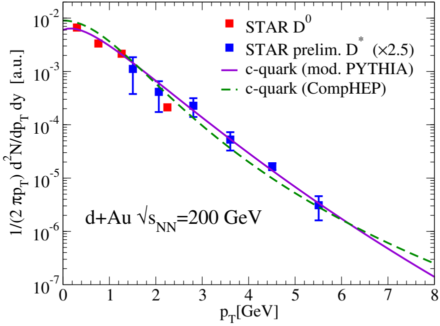
- Isentropic expansion: $S = \text{const}$ (fixed from N_{ch})
- QGP Equation of state:

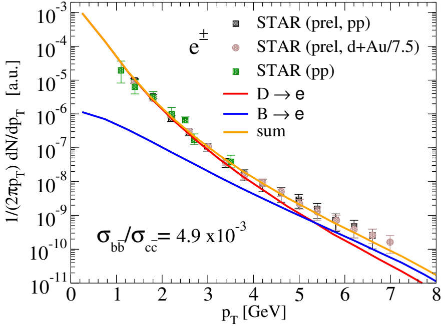
$$s = \frac{S}{V(t)} = \frac{4\pi^2}{90} T^3 (16 + 10.5 n_f^*), \quad n_f^* = 2.5$$

- obtain $T(t) \Rightarrow A(t,p)$, $B_0(t,p)$ and $B_1 = TEA$
- for semicentral collisions ($b = 7$ fm): $T_0 = 340$ MeV,
  QGP lifetime $\simeq 5$ fm/c.
- simulate FP equation as relativistic Langevin process

Initial conditions

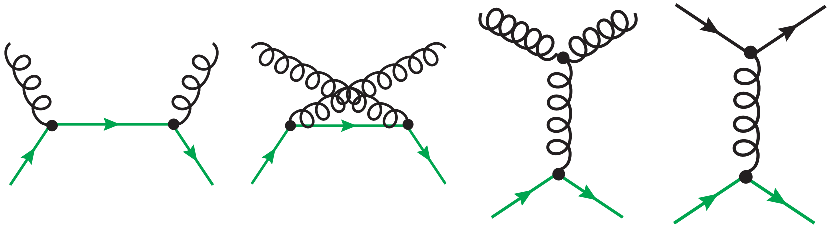
- need initial p_T-spectra of charm and bottom quarks
  - (modified) PYTHIA to describe exp. D meson spectra, assuming δ-function fragmentation
  - exp. non-photonic single-$e^{\pm}$ spectra: Fix bottom/charm ratio





Elastic pQCD processes

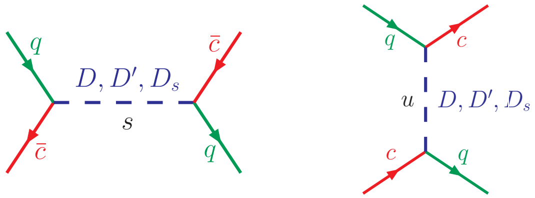
- Lowest-order matrix elements [Combridge 79]



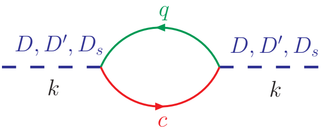
- Debye-screening mass for t-channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$
- not sufficient to understand RHIC data on "non-photonic" electrons
  [Moore, Teaney 2005]

Non-perturbative interactions: Resonance Scattering

- General idea: Survival of D- and B-meson like resonances above T_c
- model based on chiral symmetry (light quarks) HQ-effective theory
- elastic heavy-light-(anti-)quark scattering

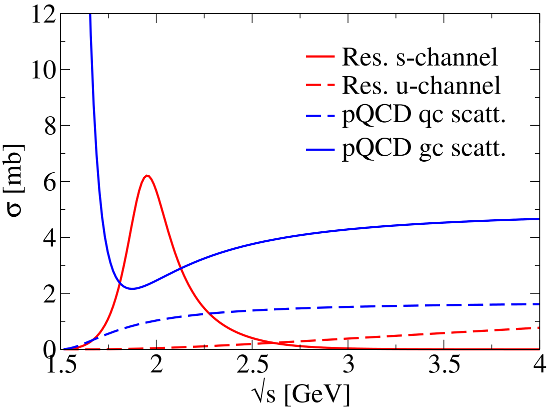


- D- and B-meson like resonances in sQGP



- parameters
  - $m_D = 2$ GeV, $\Gamma_D = 0.4 \ldots 0.75$ GeV
  - $m_B = 5$ GeV, $\Gamma_B = 0.4 \ldots 0.75$ GeV

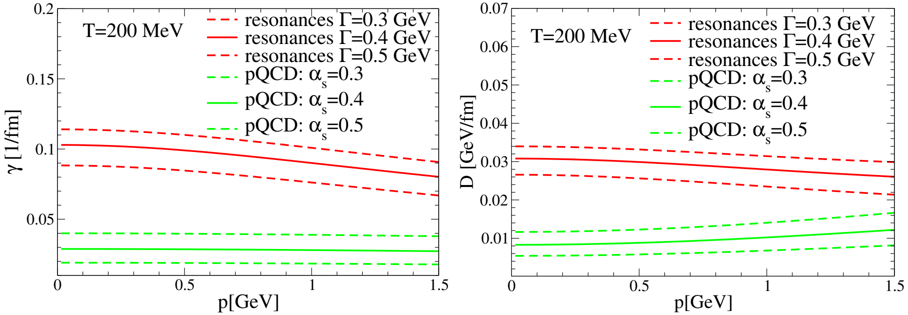
Cross sections



- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked ↔ resonance isotropic
- resonance scattering more effective for friction and diffusion

Transport coefficients: pQCD vs. resonance scattering

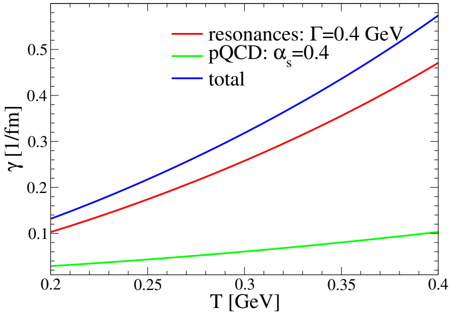
- three-momentum dependence

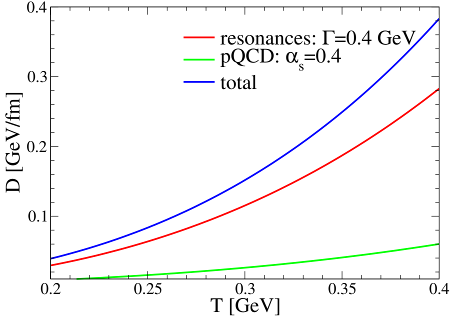


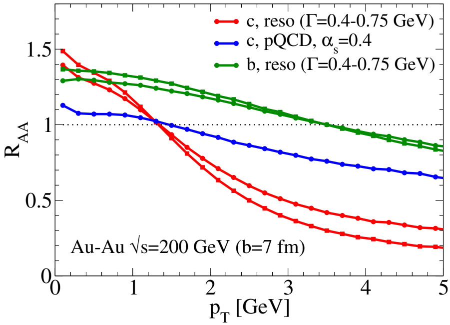
- resonance contributions factor $\sim 2 \ldots 3$ higher than pQCD!

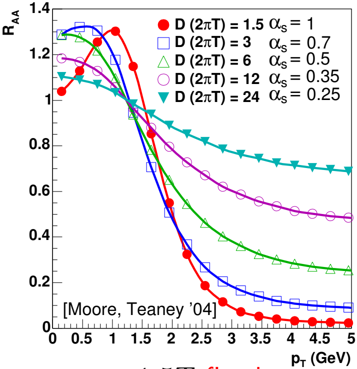
Transport coefficients: pQCD vs. resonance scattering

- Temperature dependence





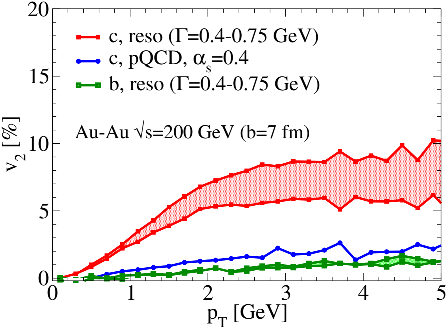


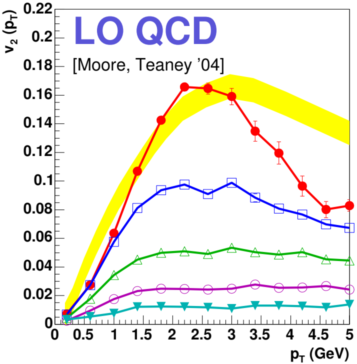


- $\mu_D = gT$, $\alpha_s = g^2/(4\pi) = 0.4$
- resonances $\Rightarrow$ c-quark thermalization without upscaling of cross sections
- Fireball parametrization consistent with hydro

- $\mu_D = 1.5T$ fixed
- spatial diff. coefficient: $D = D_s = \frac{T}{mA}$
- $2\pi TD \simeq \frac{3}{2\alpha_s^2}$

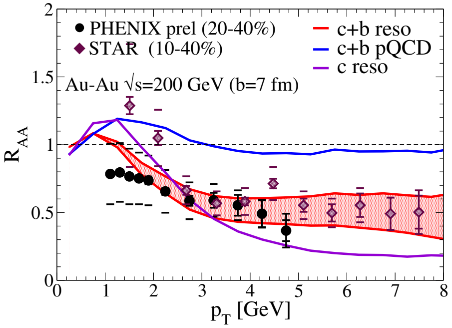
Spectra and elliptic flow for heavy quarks

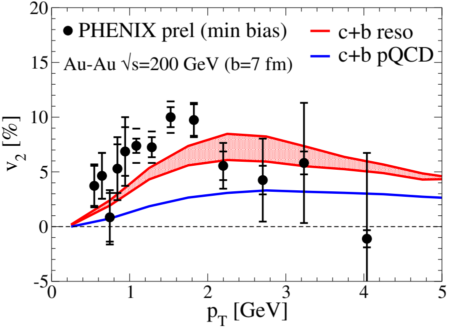




Observables: p_T-spectra (R_{AA}), v_2

- Hadronization: Coalescence with light quarks + fragmentation $\Leftrightarrow c\bar{c}$, $b\bar{b}$ conserved
- single electrons from decay of D- and B-mesons

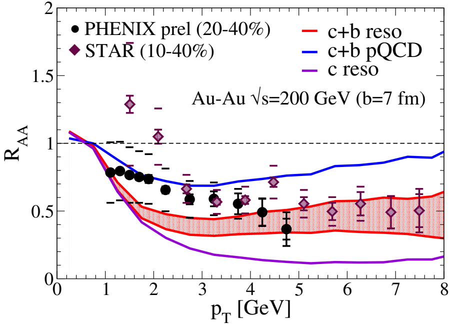


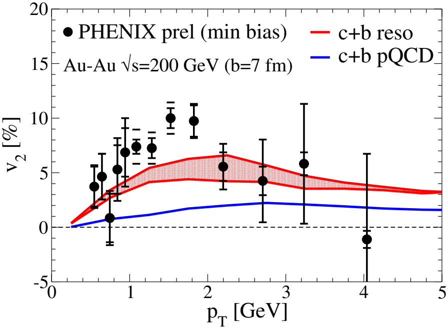


- Without further adjustments: data quite well described [HvH, V. Greco, R. Rapp, Phys. Rev. C **73**, 034913 (2006)]

Observables: p_T-spectra (R_{AA}), v_2

- Hadronization: Fragmentation only
- single electrons from decay of D- and B-mesons



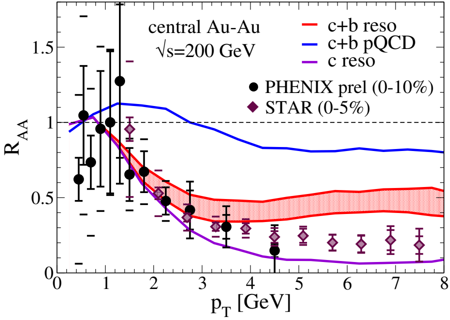


- coalescence brings up both, R_{AA} and v_2
- due to additional momentum kick from light quarks

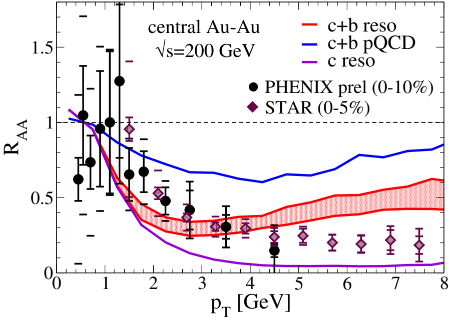
Observables: p_T-spectra (R_{AA}), v_2

- Central Collisions
- single electrons from decay of D- and B-mesons

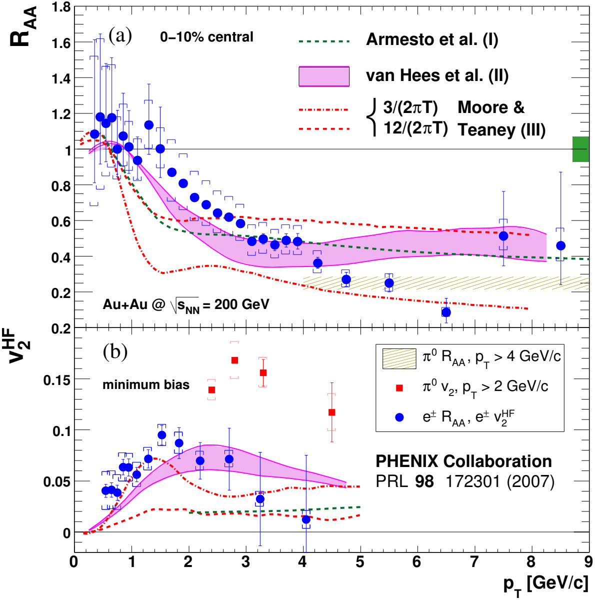
Coalescence+Fragmentation



Fragmentation only



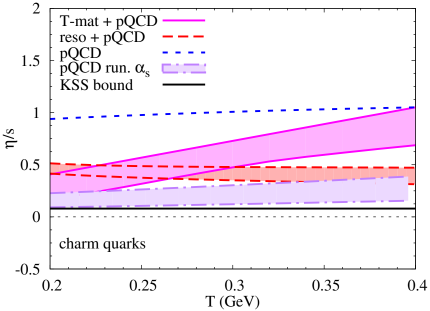
Comparison to newer data

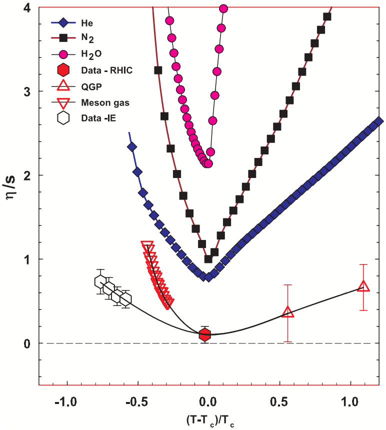


Transport properties of the sQGP

- spatial diffusion coefficient: Fokker-Planck $\Rightarrow D_s = \frac{T}{mA} = \frac{T^2}{D}$
- coupling strength in plasma: viscosity/entropy density, η/s

$$\frac{\eta}{s} \simeq \frac{1}{2} T D_s \qquad \text{(AdS/CFT)}, \qquad \frac{\eta}{s} \simeq \frac{1}{5} T D_s \qquad \text{(wQGP)}$$





[Lacey, Taranenko (2006)]

Instead of a summary: Questions

- How relate (semi-)classical transport models the behavior of many-body systems to microscopic constituents?
- Why can for heavy quarks the transport equations be approximated by a Fokker-Planck equation?
- How are medium properties characterized within the Fokker-Planck equation?
- What is the microscopic picture arising from the Langevin equation?
- What can we learn within this theoretical picture from heavy-quark observables in heavy-ion collisions?
- Which properties of the sQGP can be extracted from that model?

Summary

- Boltzmann Transport Equations
  - can be derived from classical mechanics or quantum-many-body theory
  - (semi-)classical statistical description of interacting many-body systems
  - equations for single-particle phase-space distribution
  - collision term: transition probabilities from microscopic cross sections
  - many-body systems ⇔ microscopic properties of constituents
- Fokker-Planck Equations
  - heavy particles immersed in medium of light particles
  - momentum transfer in single collision small ⇒
    integro-differential Boltzmann equation ⇒ partial differential equation
  - HQ-medium interactions ⇒ friction/drag coefficient + diffusion coefficients
  - related by Einstein dissipation-fluctuation relation

Summary

- Langevin Equations
  - stochastic differential equation equivalent to Fokker-Planck equation
  - drag/friction force + random forces = uncorrelated Gaussian noise
  - depends on realization of stochastic process
  - right process ⇒ equilibrium limit = relativistic MB distribution
  - application to flowing sQGP
- Heavy-quark interactions in the sQGP I
  - elastic scattering with light quarks and gluons: pQCD + screening
  - resonance scattering with light (anti-)quarks
- Non-photonic single electron observables
  - $R_{AA}(p_T)$ and $v_2(p_T)$ of electrons from D- and B-meson decays
  - Langevin simulation → coalescence+fragmentation hadronization → semi-leptonic decay
  - pQCD (with realistic α_s) too weak
  - with resonance-scattering interactions good description of data