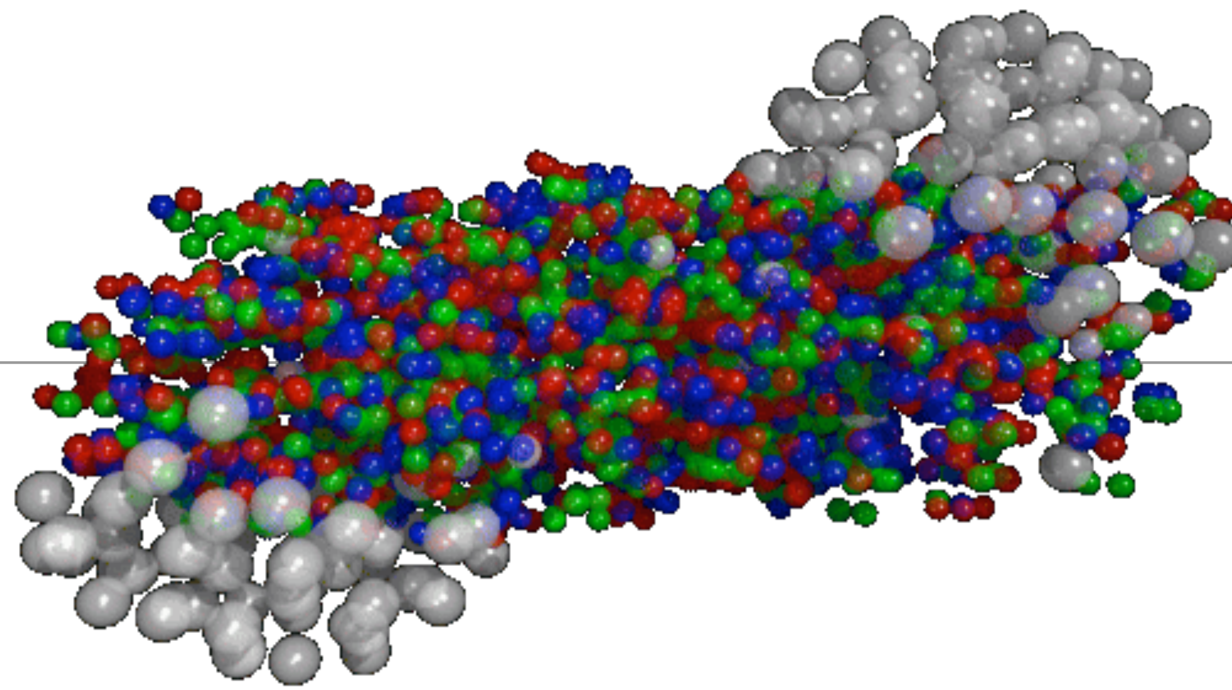

WHAT ARE YOU DOING, RAT?
I'M PORING OVER PHYSICS BOOKS IN AN ATTEMPT TO UNIFY GENERAL RELATIVITY AND QUANTUM MECHANICS INTO ONE UNIFIED THEORY THAT GOVERNS OUR ENTIRE EXISTENCE.
WHAT HAVE YOU GOT SO FAR?
THIS.
Beer is good.
I DIDN'T KNOW IT WAS THAT SIMPLE.

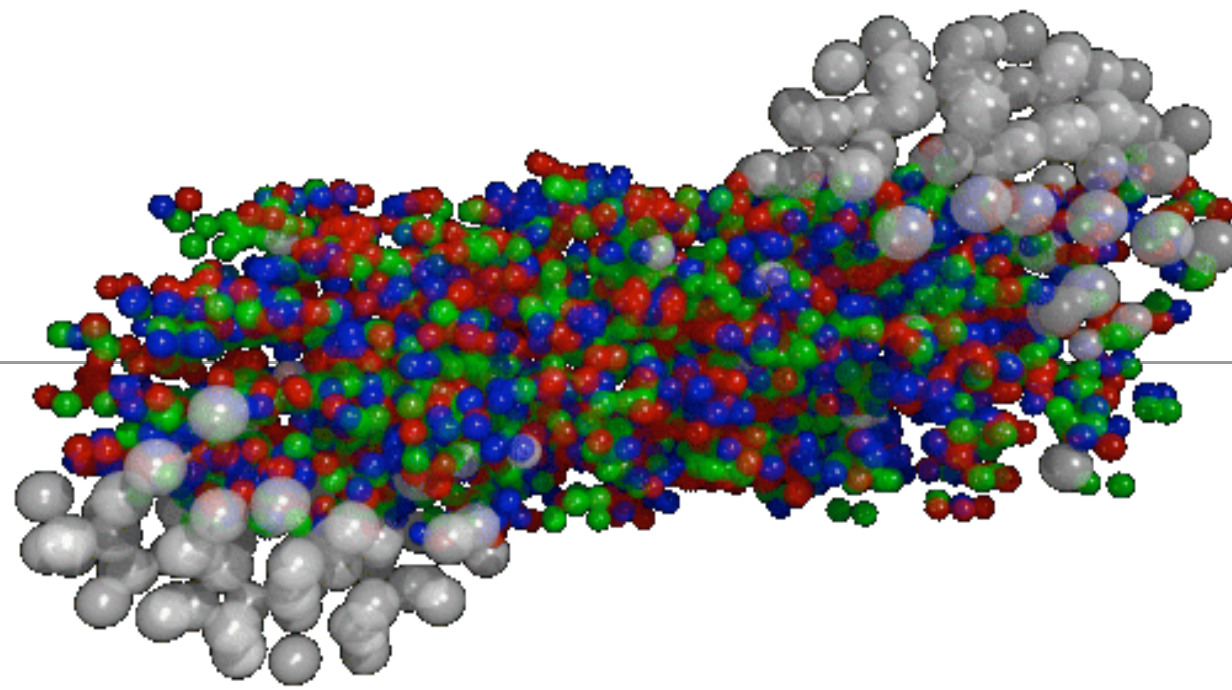
Dileptons in heavy ion collisions



Sascha Vogel

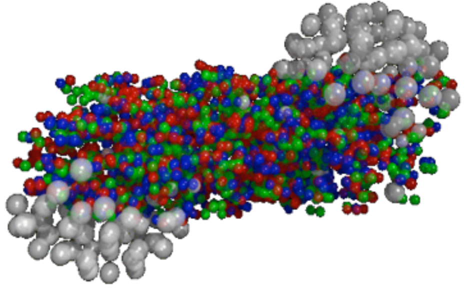
H-QM Lecture Week Strasbourg
April 1-5

Everything you need to survive a dilepton meeting



Sascha Vogel

H-QM Lecture Week Strasbourg
April 1-5

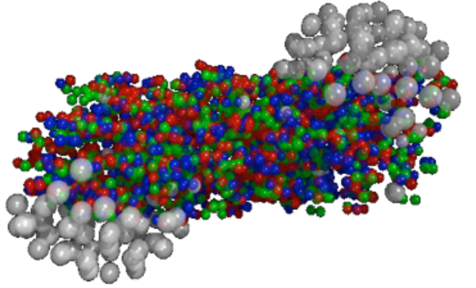


Outline

- A brief reminder on chiral symmetry

  why are we doing this?

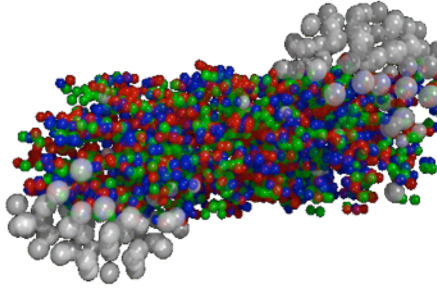
- Quarks to dileptons
- Pions to dileptons
- Resonance decays to dileptons
- Other processes

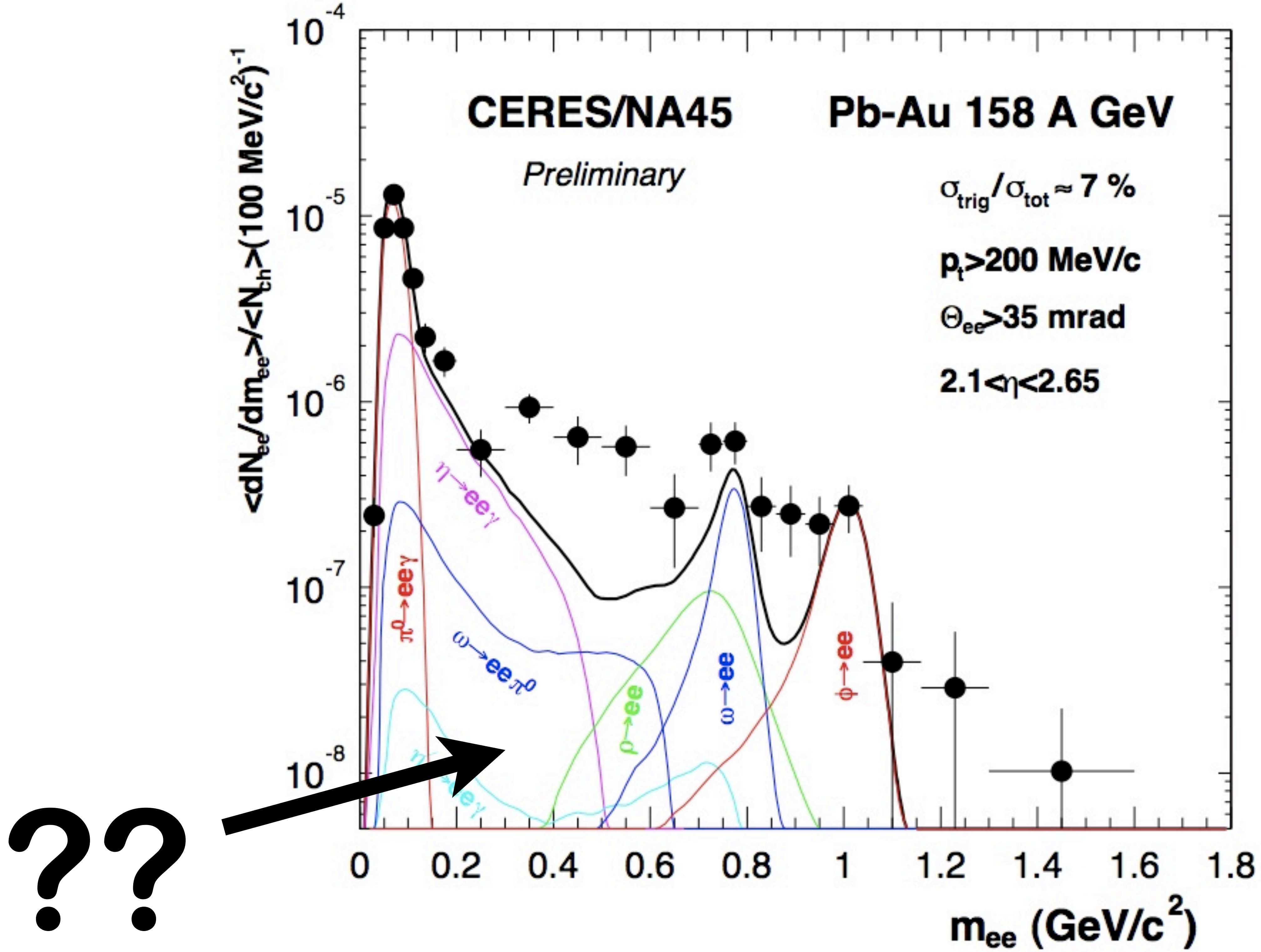


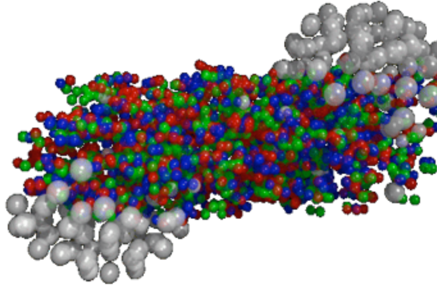
Outline

- Formulas
  - More formulas
- Equations
  - More equations
  - Even more equations
- Physics?

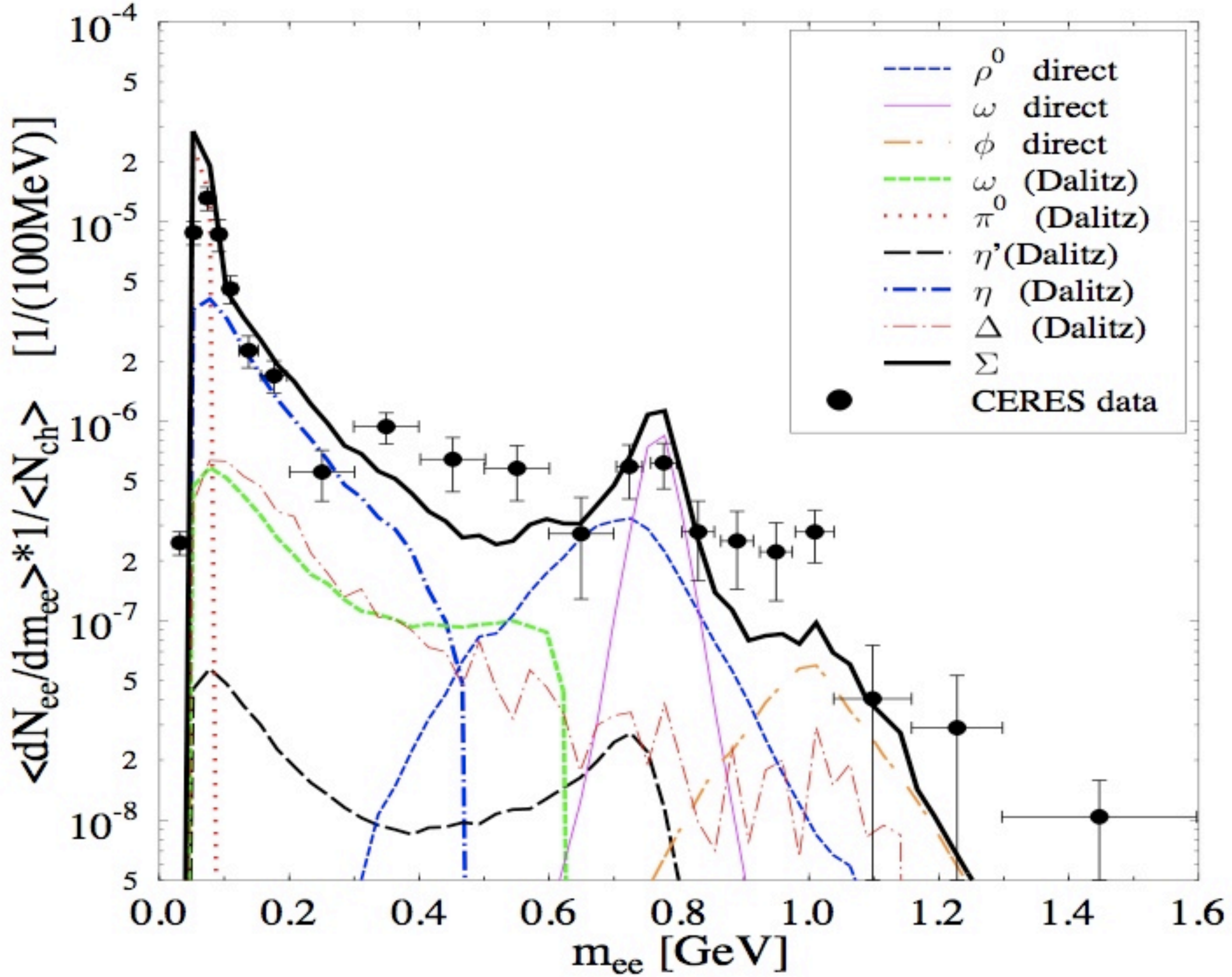
Dilepton spectra

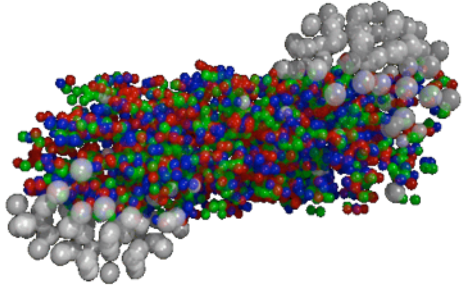




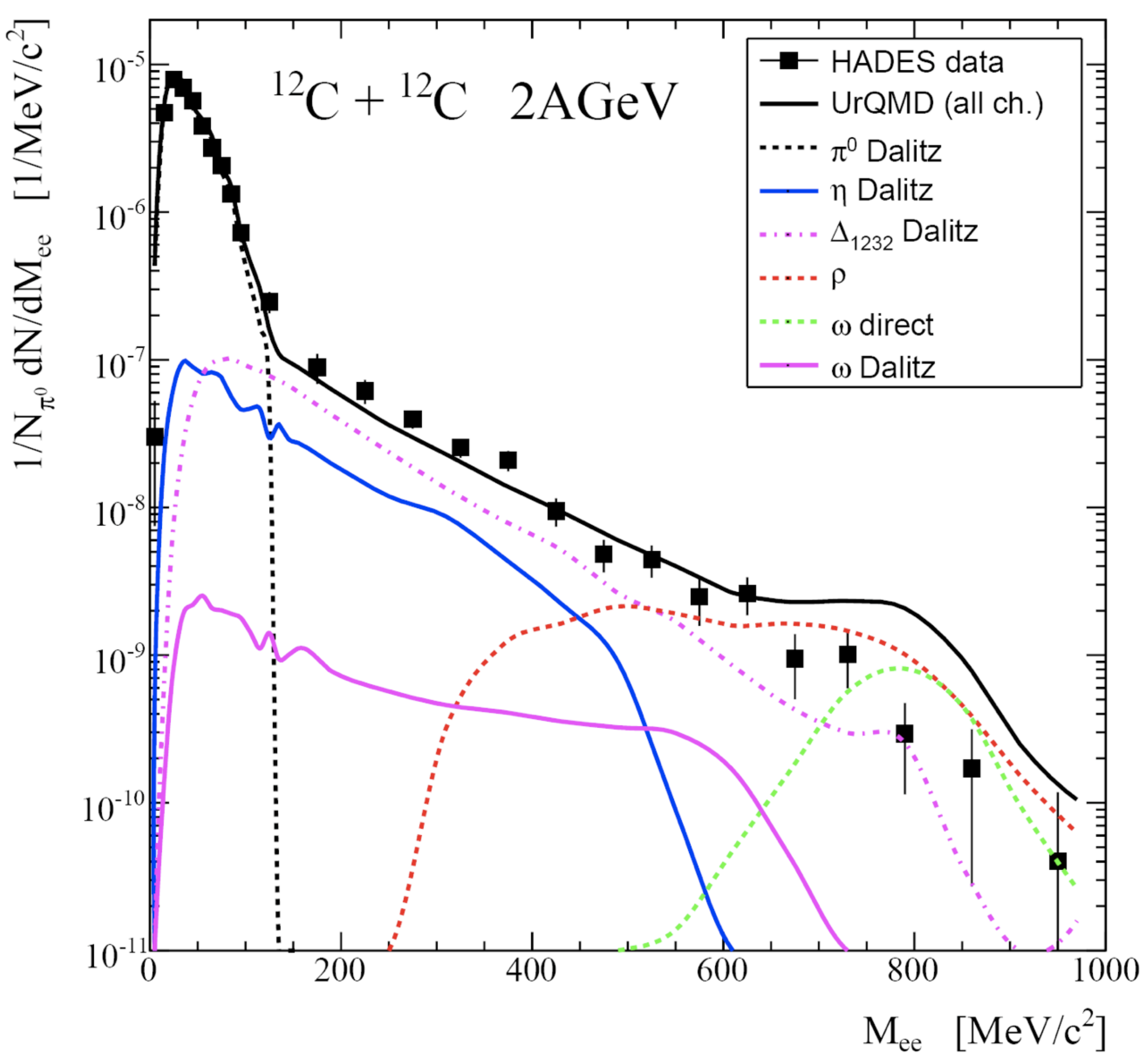


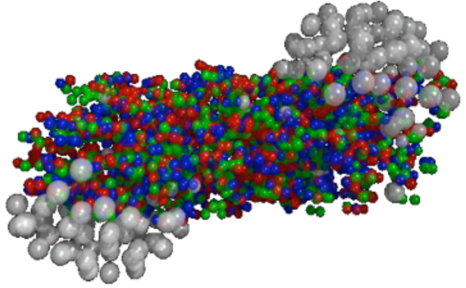
Dilepton spectra



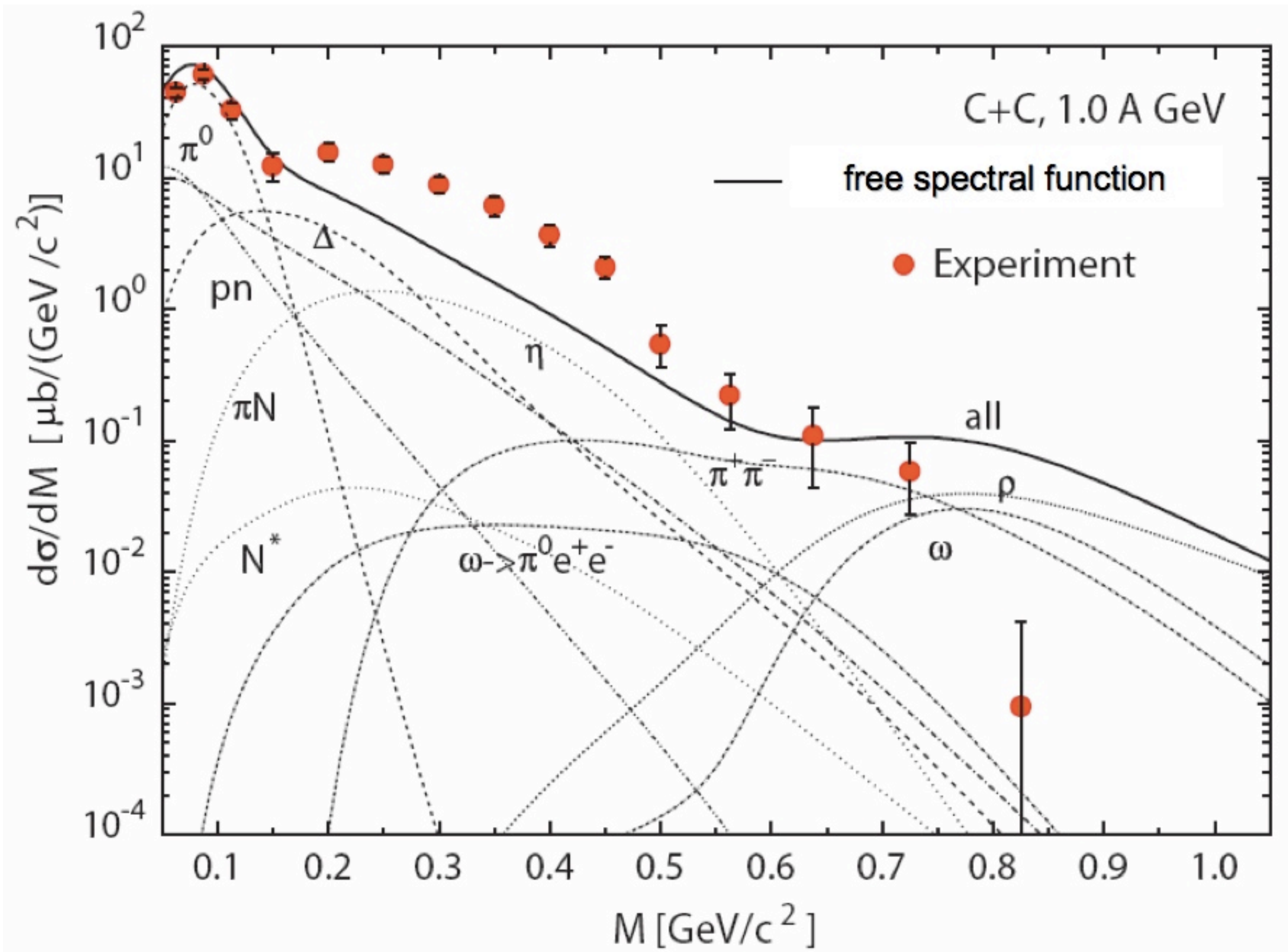


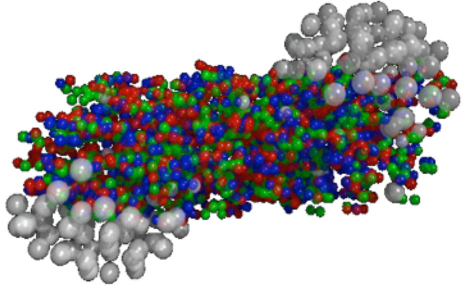
Dilepton spectra





Dilepton spectra





Chiral symmetry

Consider the Lagrangian of massless fermions and the following transformations:

$$\mathcal{L} = i\bar{\psi}_j \partial\!\!\!/ \psi_j$$

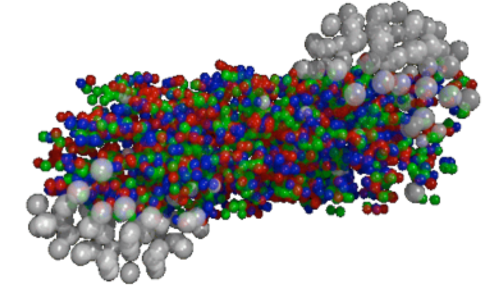
$$\Lambda_V : \;\; \psi \longrightarrow e^{-i\frac{\vec{\tau}}{2}\vec{\Theta}}\psi \simeq (1 - i\frac{\vec{\tau}}{2}\vec{\Theta})\psi$$

$$\bar{\psi} \longrightarrow e^{+i\frac{\vec{\tau}}{2}\vec{\Theta}}\bar{\psi} \simeq (1 + i\frac{\vec{\tau}}{2}\vec{\Theta})\bar{\psi}$$

$$\begin{aligned} i\bar{\psi}\partial\!\!\!/\psi \quad &\longrightarrow \quad i\bar{\psi}\partial\!\!\!/\psi - i\vec{\Theta}\left(\bar{\psi} i\partial\!\!\!/ \frac{\vec{\tau}}{2}\psi - \bar{\psi}\frac{\vec{\tau}}{2} i\partial\!\!\!/\psi\right) \\ &= \quad i\bar{\psi}\partial\!\!\!/\psi \end{aligned}$$

$$V_\mu^a = \bar{\psi}\,\gamma_\mu \frac{\tau^a}{2}\,\psi$$

The Lagrangian is invariant under this transformation.



Chiral symmetry

$$\mathcal{L} = i\bar{\psi}_j \partial\!\!\!/ \psi_j$$

$$\Lambda_A : \qquad \psi \longrightarrow e^{-i\gamma_5 \frac{\vec{\tau}}{2}\vec{\Theta}}\psi = (1 - i\gamma_5 \frac{\vec{\tau}}{2}\vec{\Theta})\psi$$

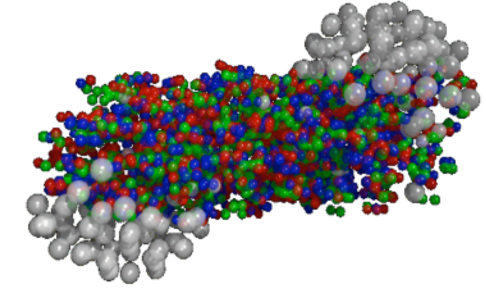
$$\bar{\psi} \longrightarrow e^{-i\gamma_5 \frac{\vec{\tau}}{2}\vec{\Theta}}\bar{\psi} \simeq (1 - i\gamma_5 \frac{\vec{\tau}}{2}\vec{\Theta})\bar{\psi}$$

$$i\bar{\psi}\partial\!\!\!/\psi \longrightarrow i\bar{\psi}\partial\!\!\!/\psi - i\vec{\Theta}\left(\bar{\psi}\, i\partial_\mu\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}\,\psi + \bar{\psi}\,\gamma_5\frac{\vec{\tau}}{2} i\partial_\mu\gamma^\mu\,\psi\right)$$

$$= i\bar{\psi}\partial\!\!\!/\psi$$

$$A_\mu^a = \bar{\psi}\gamma_\mu\gamma_5\frac{\tau}{2}\psi$$

The Lagrangian is invariant under this transformation.
Both transformations together are called
"Chiral Symmetry"



Chiral symmetry

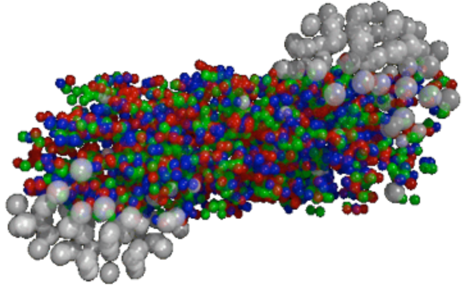
What happens if we introduce a mass term?

$$\delta\mathcal{L} = -m\,(\bar{\psi}\psi)$$

$$\Lambda_V : \; m\,(\bar{\psi}\psi) \longrightarrow m(\bar{\psi}\psi)$$

$$\Lambda_A : \; m\,(\bar{\psi}\psi) \longrightarrow m\bar{\psi}\psi - 2im\vec{\Theta}\left(\bar{\psi}\frac{\vec{\tau}}{2}\gamma_5\psi\right)$$

The mass term breaks the symmetry of the axial part, so it's not a good symmetry anymore. Yet, the quark masses are small compared to the relevant energy scales of QCD (Λ_{QCD} ~ 200 MeV), so one speaks of an approximate symmetry.



Chiral symmetry

Let us consider a combination of quark fields with the correct quantum numbers of some mesons:

$$\vec{\pi} \equiv i\bar{\psi}\vec{\tau}\gamma_5\psi \qquad \sigma \equiv \bar{\psi}\psi$$

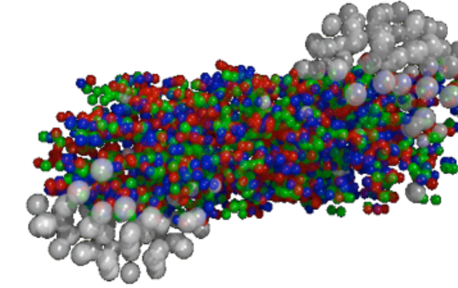
$$\vec{\rho}_\mu \equiv \bar{\psi}\vec{\tau}\gamma_\mu\psi \qquad \vec{a_1}_\mu \equiv \bar{\psi}\vec{\tau}\gamma_\mu\gamma_5\psi$$

Vector transformations:

$$\begin{aligned} \pi_i: \quad i\bar{\psi}\tau_i\gamma_5\psi \quad &\longrightarrow \quad i\bar{\psi}\tau_i\gamma_5\psi + \Theta_j\left(\bar{\psi}\tau_i\gamma_5\frac{\tau_j}{2}\psi - \bar{\psi}\frac{\tau_j}{2}\tau_i\gamma_5\psi\right) \\ &= \quad i\bar{\psi}\tau_i\gamma_5\psi + i\Theta_j\epsilon_{ijk}\,\bar{\psi}\gamma_5\tau_k\psi \end{aligned}$$

$$\vec{\pi} \longrightarrow \vec{\pi} + \vec{\Theta} \times \vec{\pi}$$

$$\vec{\rho_\mu} \longrightarrow \vec{\rho_\mu} + \vec{\Theta} \times \vec{\rho_\mu}$$



Chiral symmetry

Axial transformations:

$$
\begin{aligned}
\pi_i : \quad i\bar{\psi}\tau_i\gamma_5\psi \quad &\longrightarrow \quad i\bar{\psi}\tau_i\gamma_5\psi + \Theta_j\left(\bar{\psi}\tau_i\gamma_5\gamma_5\frac{\tau_j}{2}\psi + \bar{\psi}\gamma_5\frac{\tau_j}{2}\tau_i\gamma_5\psi\right) \\
&= \quad i\bar{\psi}\tau_i\gamma_5\psi + \Theta_i\bar{\psi}\psi
\end{aligned}
$$

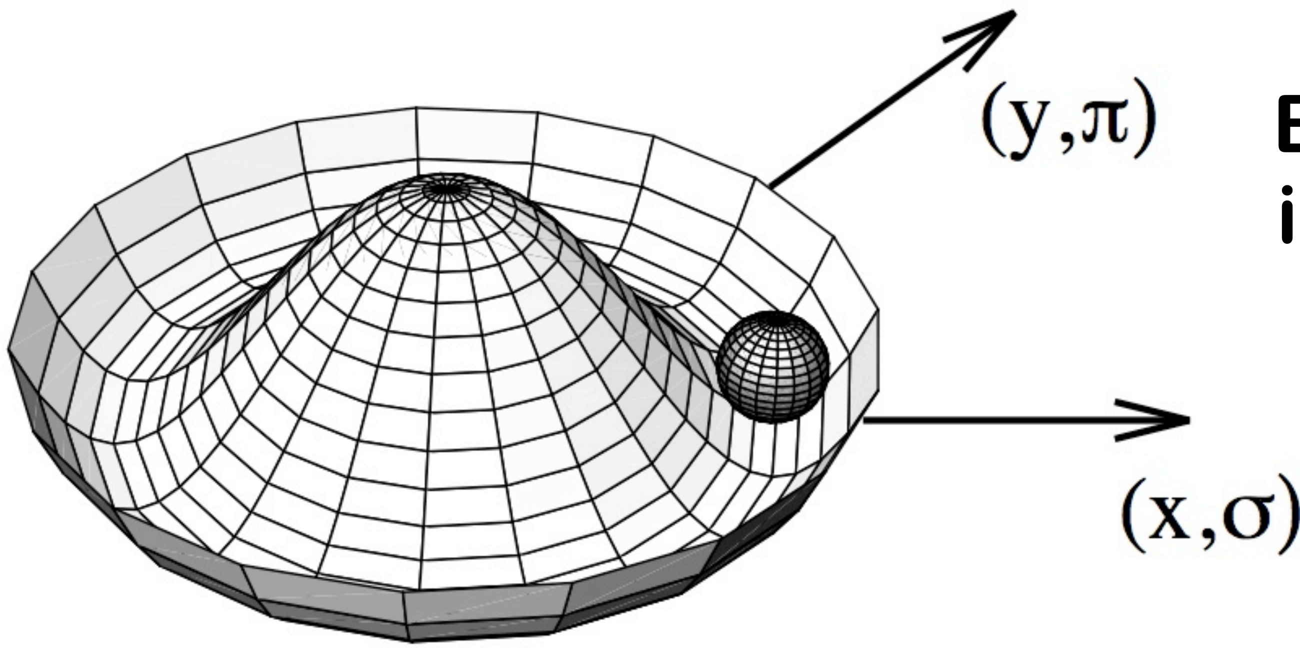
$$\vec{\pi} \longrightarrow \vec{\pi} + \vec{\Theta}\sigma$$

$$\vec{\rho}_\mu \longrightarrow \vec{\rho}_\mu + \vec{\Theta} \times \vec{a_1}_\mu$$

Since chiral symmetry is supposed to be a symmetry of the Lagrangian and the ρ state can be rotated into the a_1 state, both should have the same eigenvalues, i.e. the same masses.

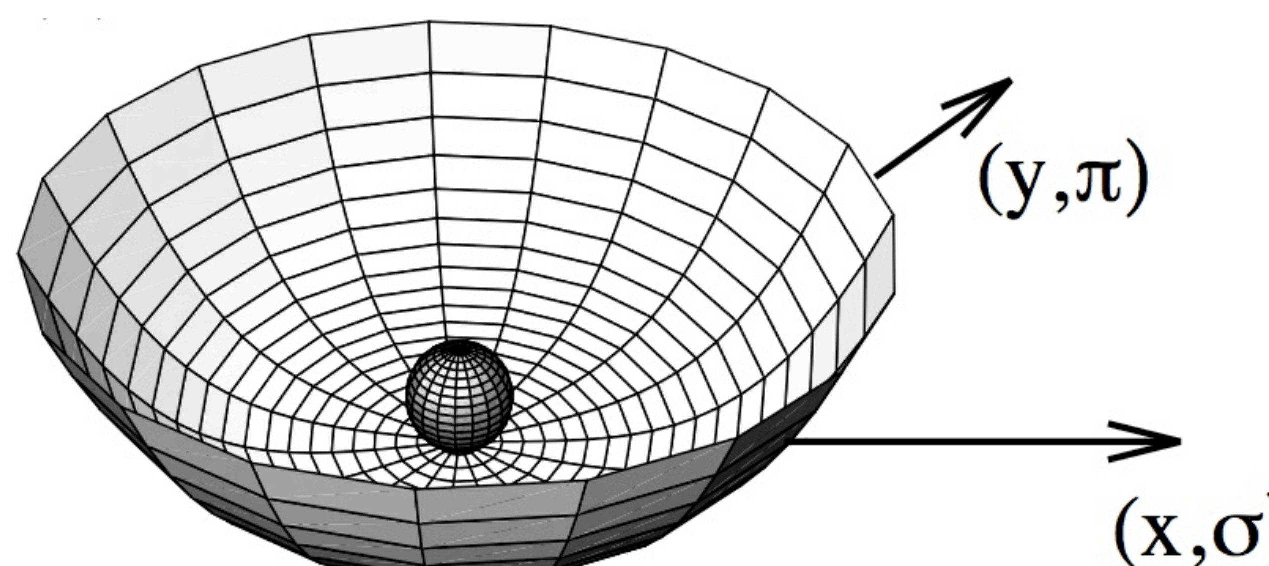
Yet m(ρ) = 770 MeV and m(a_1) = 1260 MeV.
The symmetry is broken.

Mexican hat

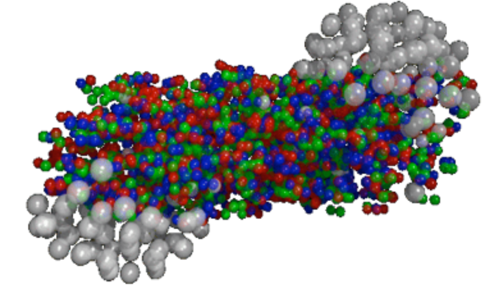


Broken Symmetry (excitations in x-direction cost energy, not in y-direction though)

Restored symmetry, neither movement into x-, nor in y-direction cost energy



V. Koch, Int.J.Mod.Phys.E6:203-250,1997

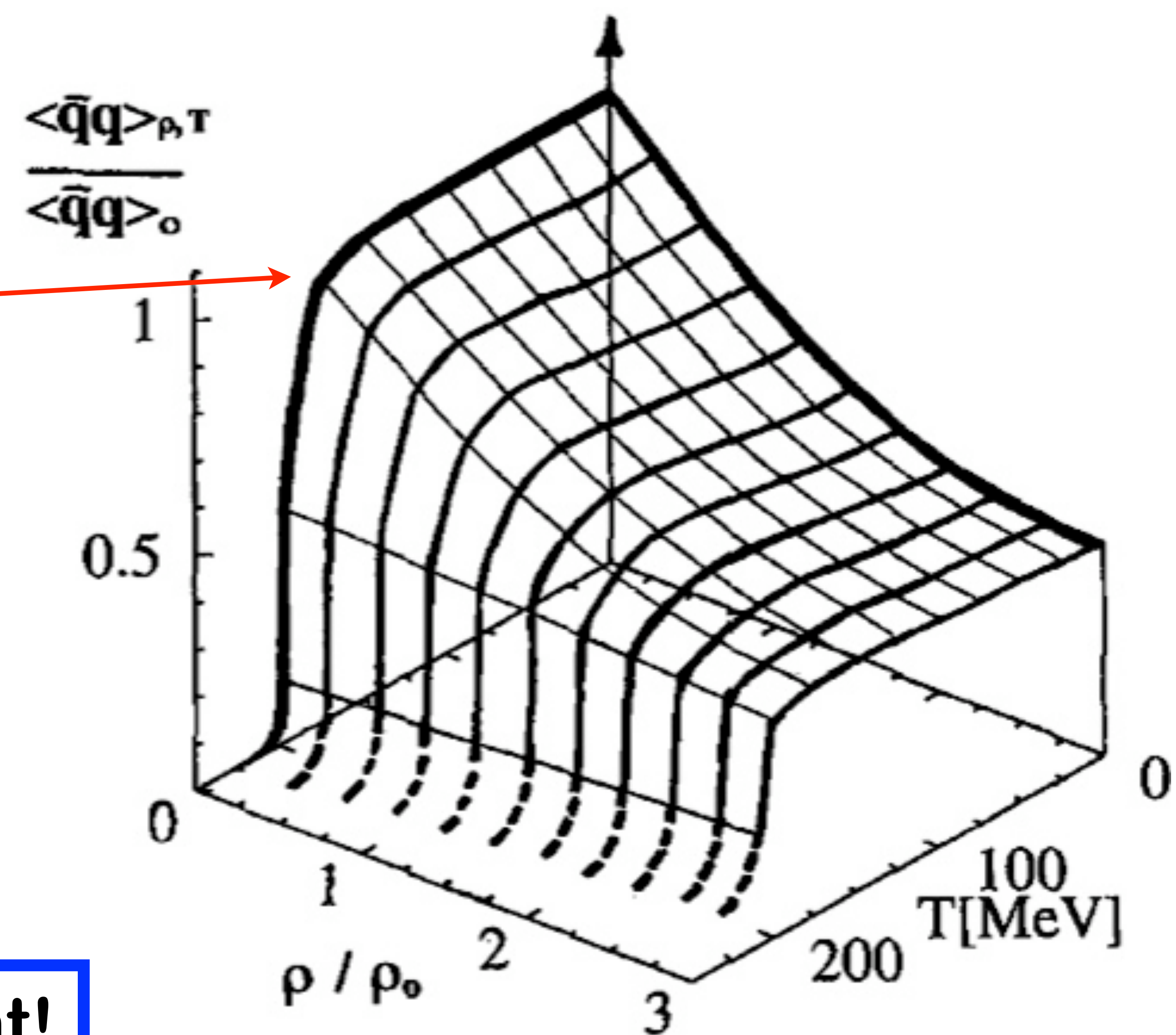


Chiral symmetry

Restoration of chiral symmetry has effects on particle masses.

$$m^* \sim m(1 - \alpha \frac{< \bar{q}q >_0}{< \bar{q}q >_{\rho,T}})$$

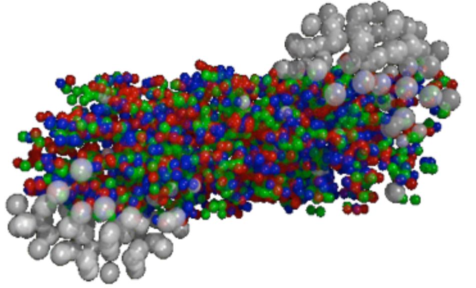
Critical temperature of ~170MeV (~10^{10} K)



Properties of particles, especially masses, change with the temperature and the density of the medium they are put in!

Which is accessible by experiment!

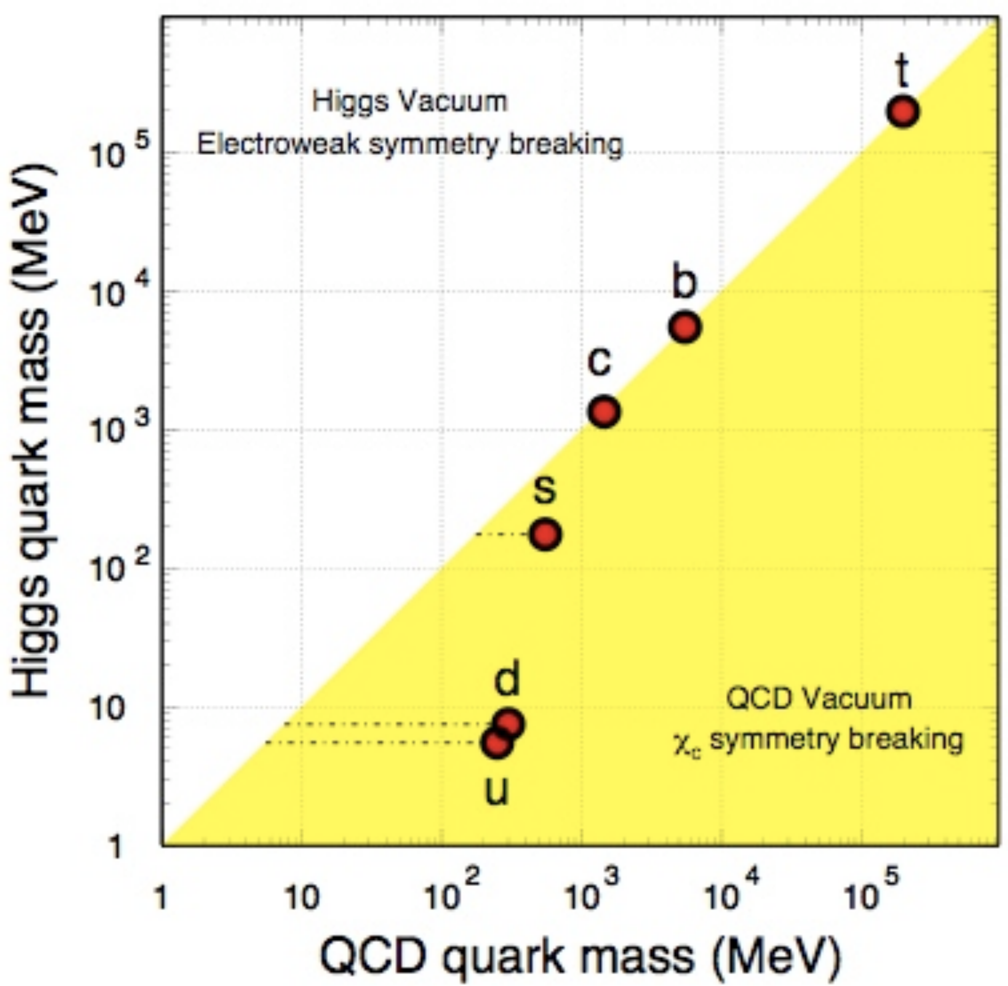
W. Weise, Nucl.Phys.A690(2001) 98



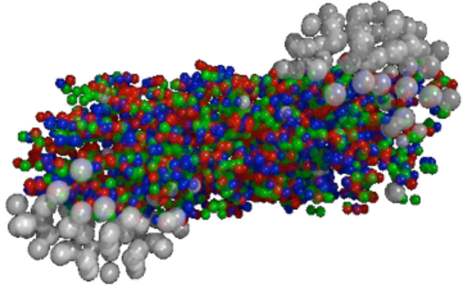
Chiral symmetry

Chiral symmetry is broken in nature, which gives particles their "real" (QCD) mass.

~99% of the mass of the light quarks originates from chiral symmetry breaking.



Chiral symmetry

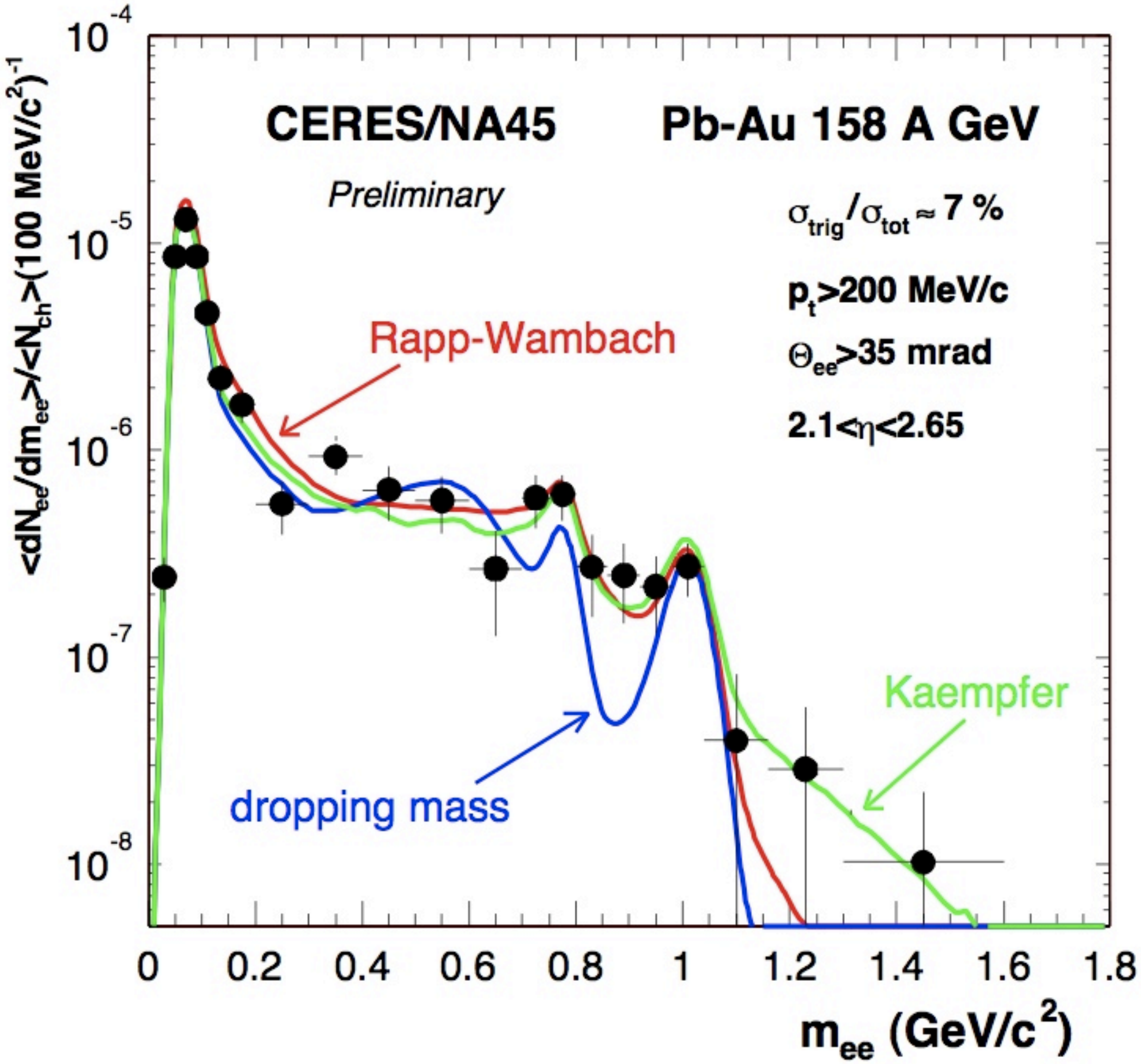


Chiral symmetry...

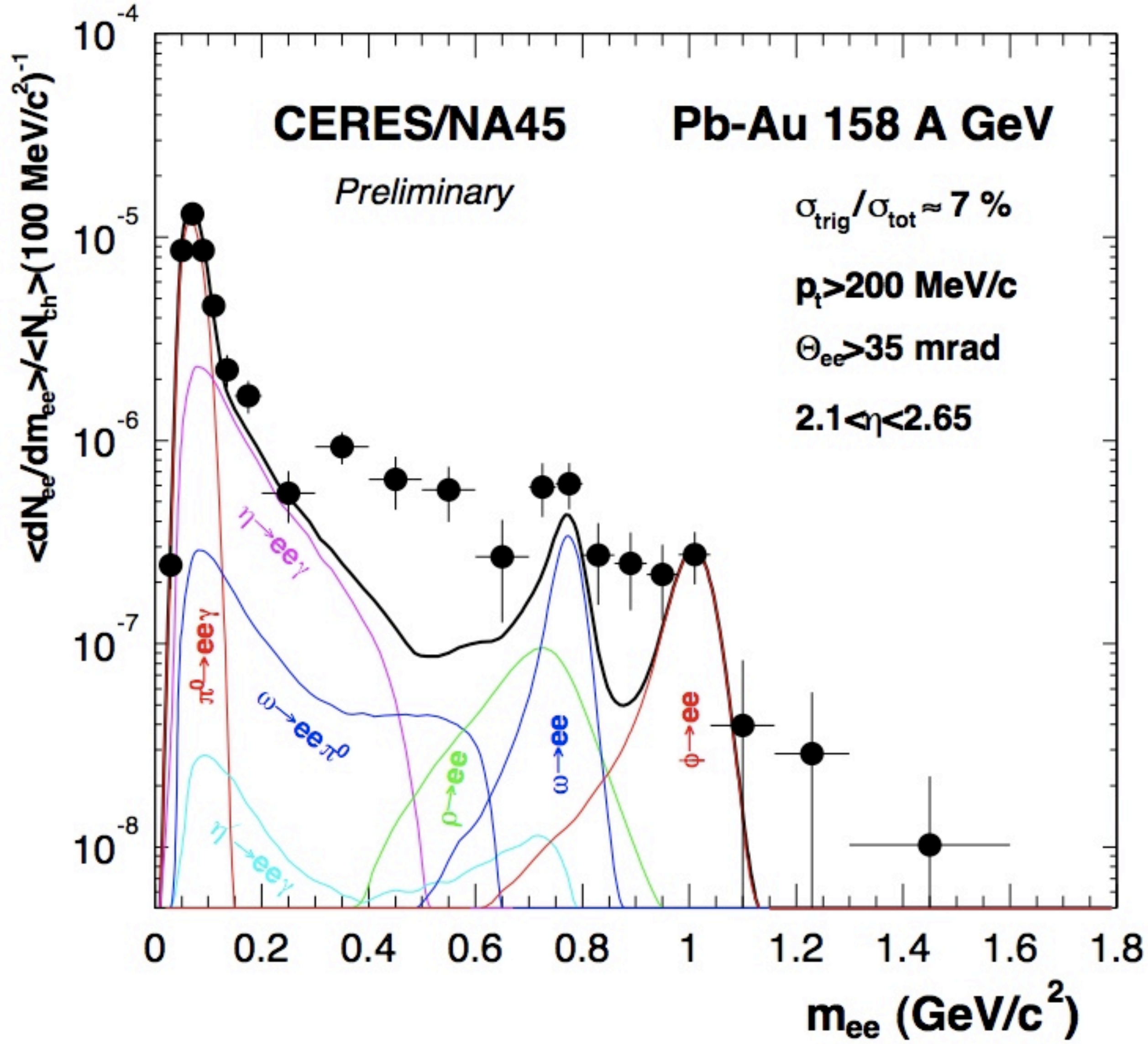
- is a symmetry of the QCD Lagrangian
- is broken in nature
- is related to the mass of particles
- is expected to be restored at high temperatures and/or densities

CERES

Sergey Yurevich, PhD thesis

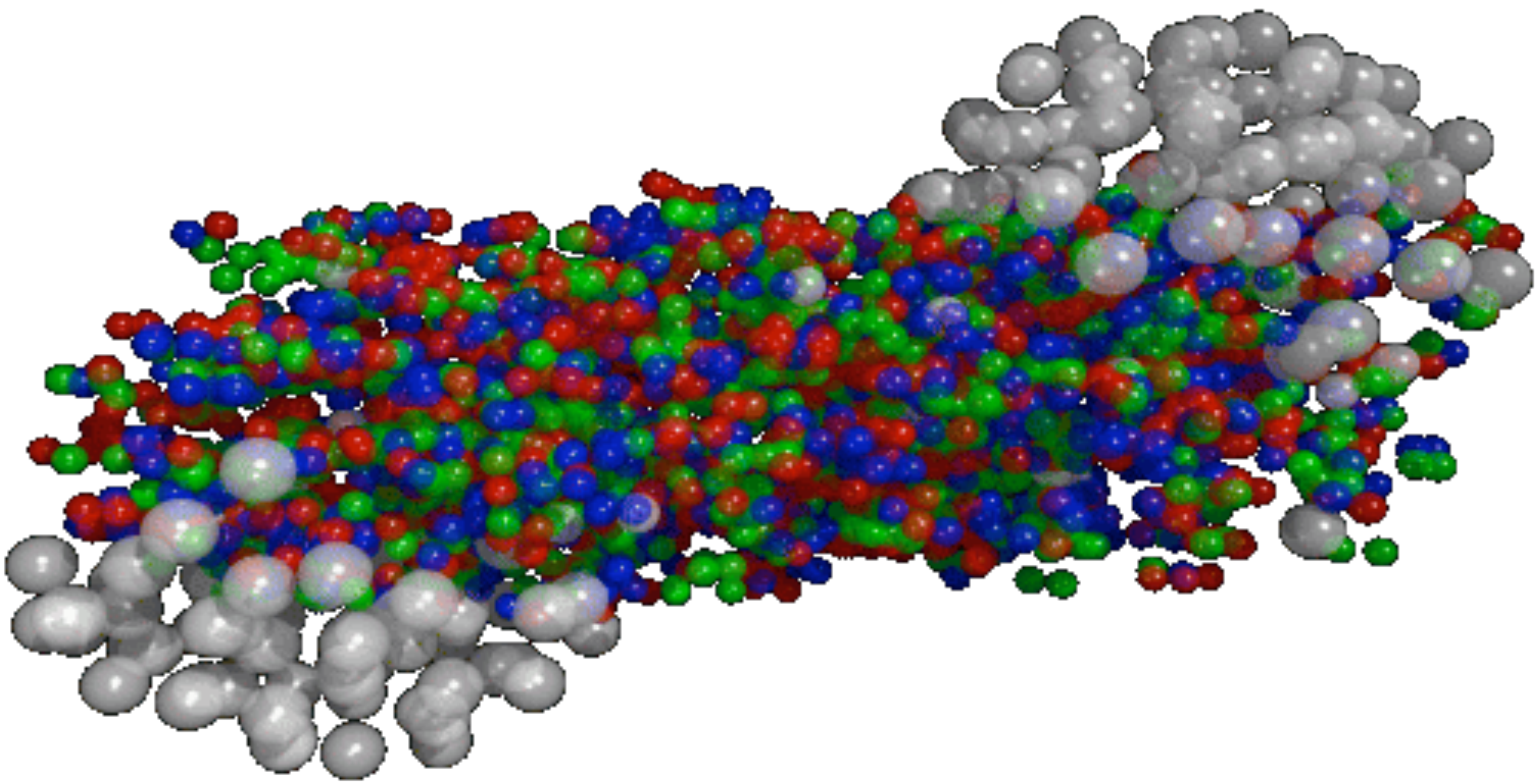


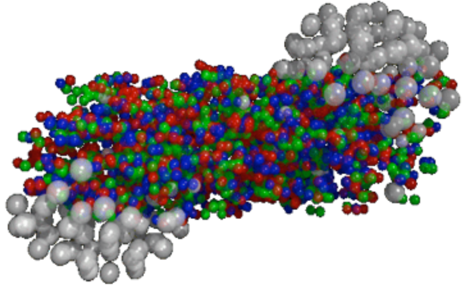
Data explainable with in-medium spectral functions assuming chiral symmetry restoration.

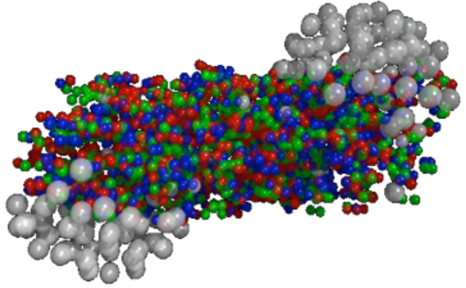


Data not explainable with free (i.e. vacuum) spectral functions.

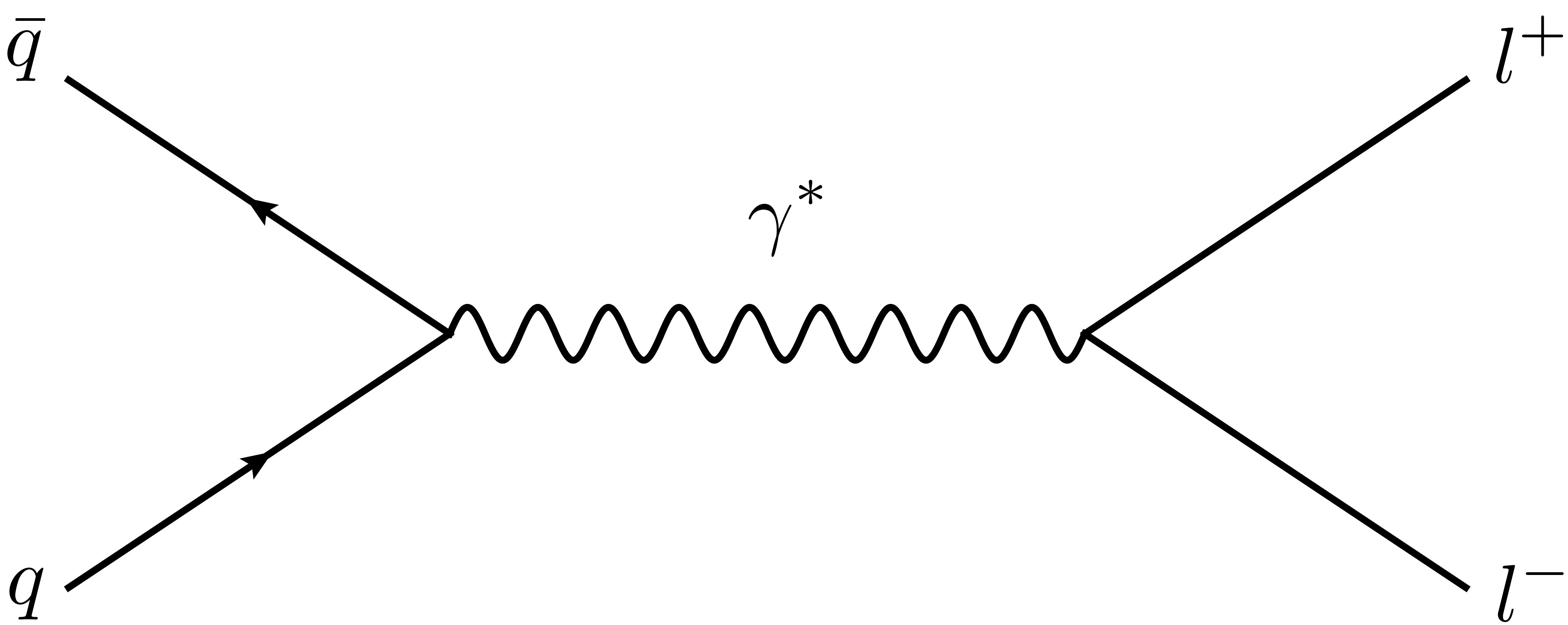
Understanding the vacuum

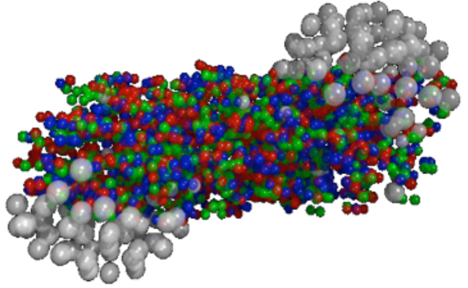






Understanding the vacuum

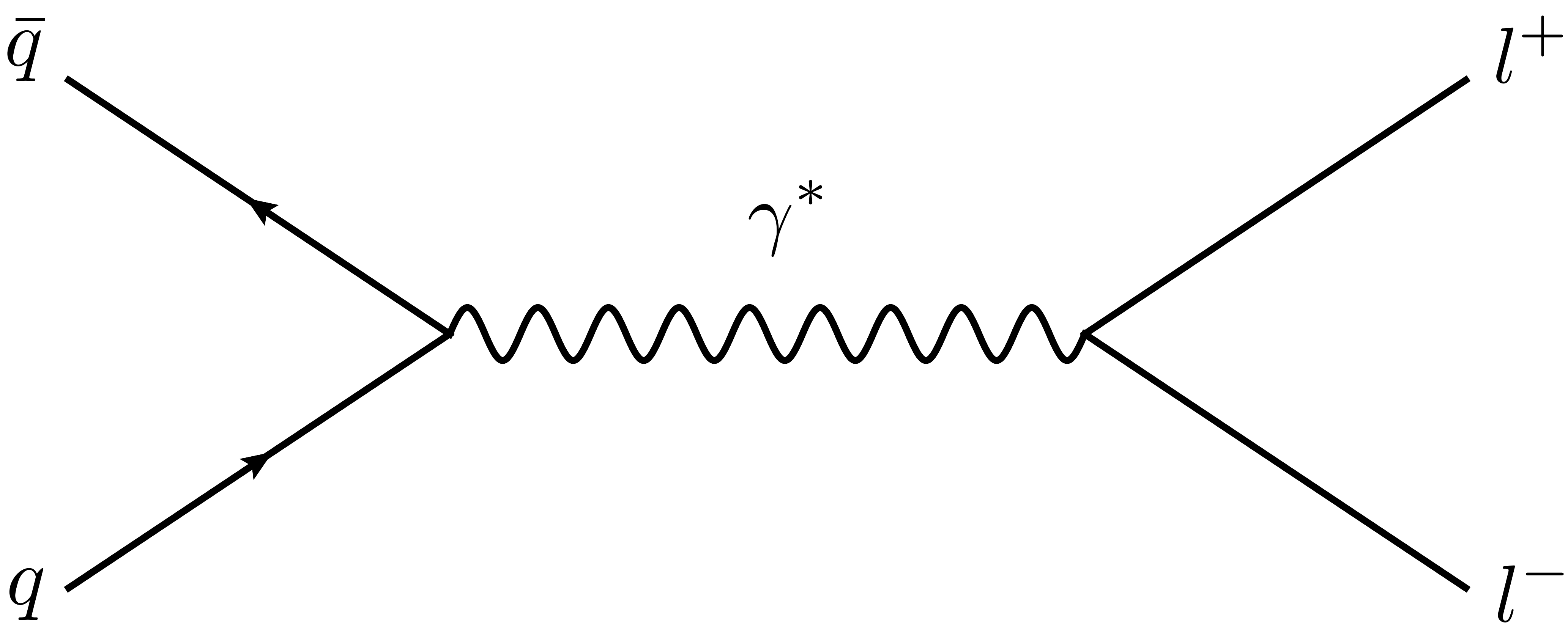


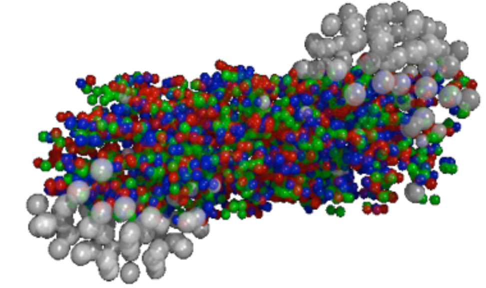


Understanding the vacuum

We need to understand the vacuum and kinematics before tackling 'in medium' physics!

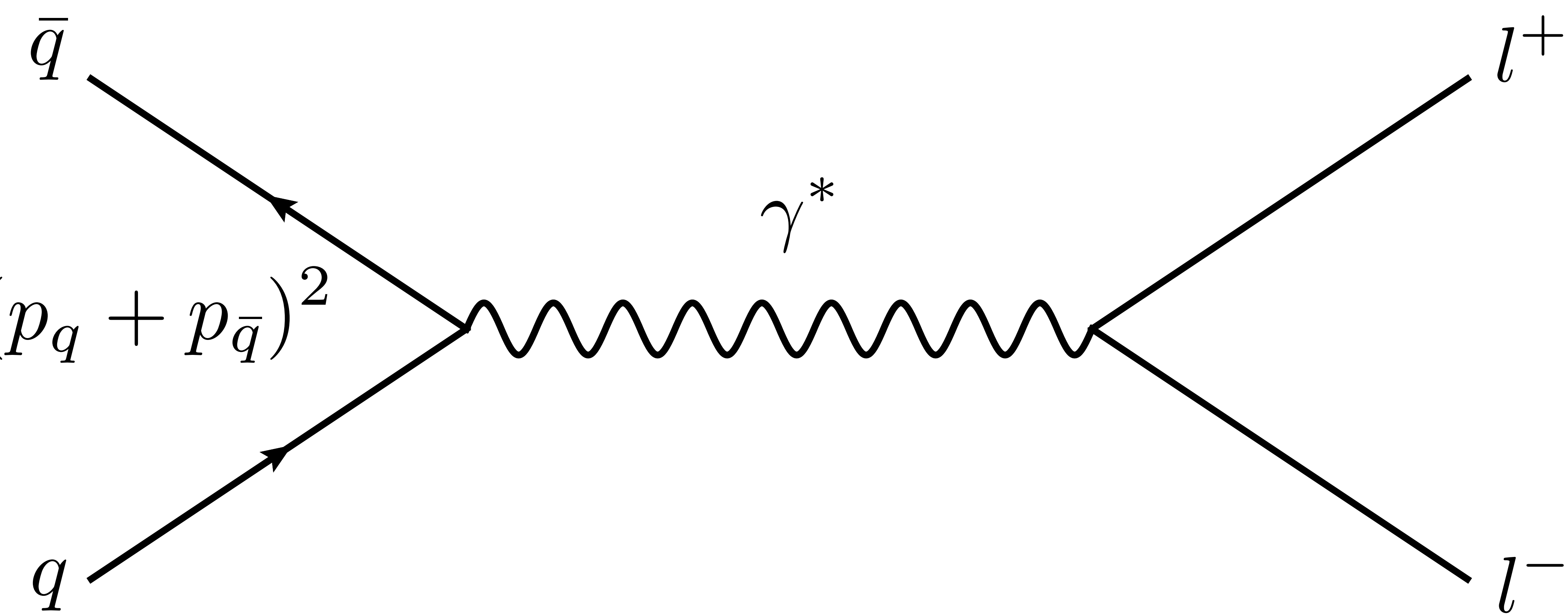
$q\bar{q} \to l^+ l^-$





Mandelstam variables

$$u = (p_q + p_{\bar{l}})^2$$

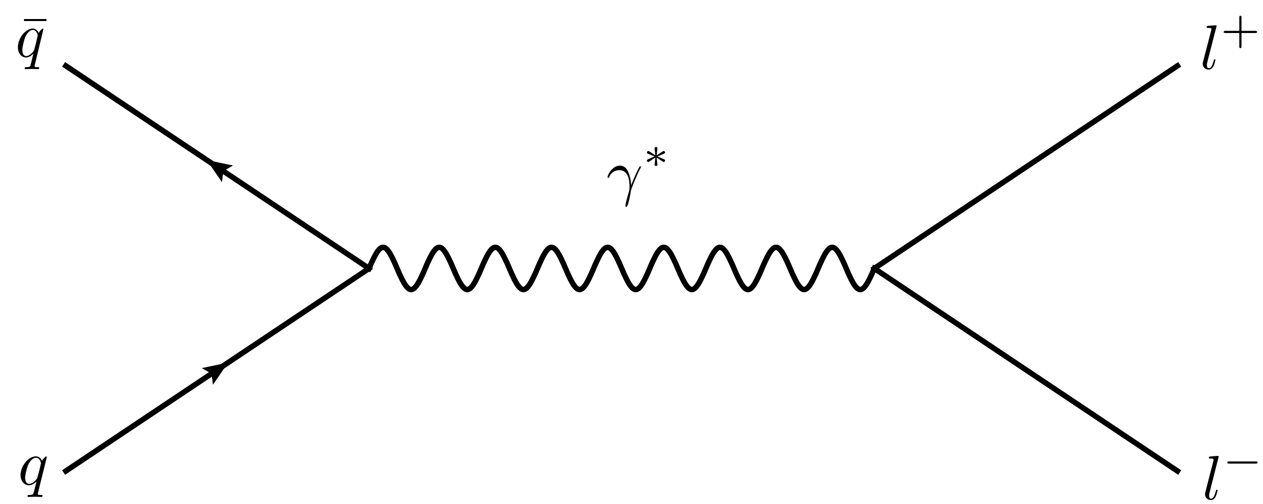


$$s = (p_q + p_{\bar{q}})^2$$

$$t = (p_q + p_l)^2$$

$q\bar{q} \to l^+ l^-$

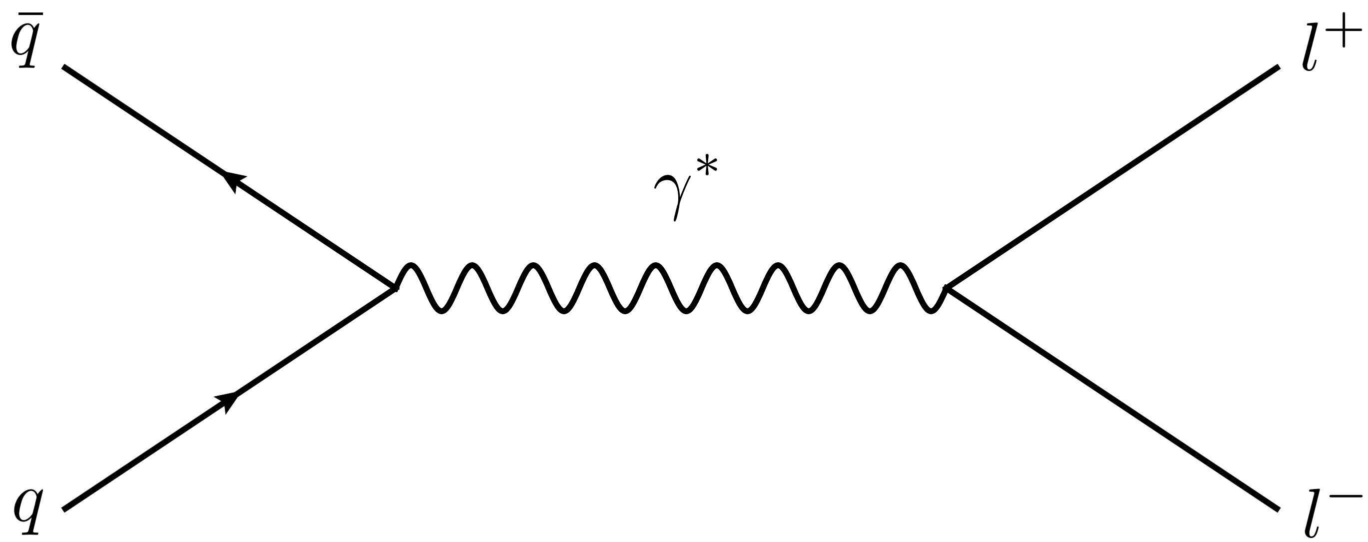
$$\sigma(q\bar{q} \to l^+ l^-) = \left(\frac{e_q}{e}\right) \sigma(M)$$



$$M = (p_q + p_{\bar{q}})^2 = (p_l + p_{\bar{l}})^2 = s$$

calculation done in quark CMS

Kinematics

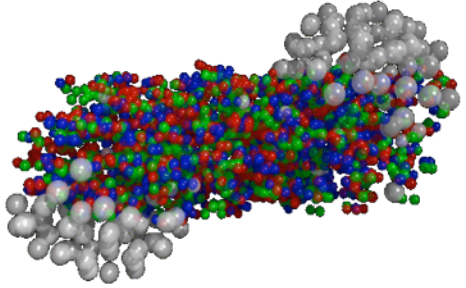


$$s = (p_q + p_{\bar{q}})^2 = (p_l + p_{\bar{l}})^2$$

$$p_q^0 = p_{\bar{q}}^0 = p_l^0 = p_{\bar{l}}^0 = \frac{\sqrt{s}}{2}$$

$$|\vec{p_q}| = |\vec{p_{\bar{q}}}| = \frac{1}{2}\sqrt{s - 4m_q^2}$$

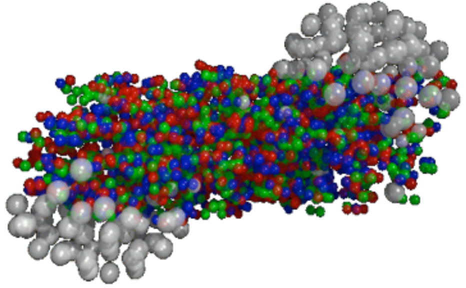
$$|\vec{p_l}| = |\vec{p_{\bar{l}}}| = \frac{1}{2}\sqrt{s - 4m_l^2}$$



Differential cross section

Starting point:

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4 (p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$



Differential cross section

Starting point:

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4(p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

incident fermions

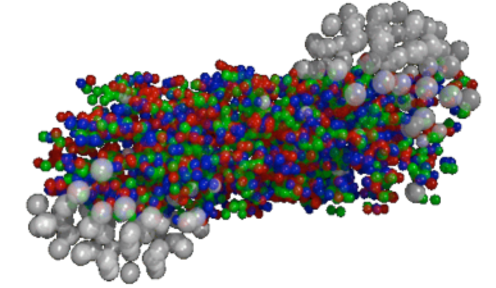
Differential cross section

Starting point:

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4(p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

incident fermions

Differential cross section



Starting point:

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4 (p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

incident fermions

relative velocity factor

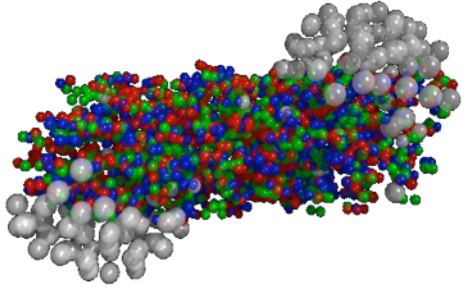
Differential cross section

Starting point:

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4 (p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

incident fermions

relative velocity factor



Differential cross section

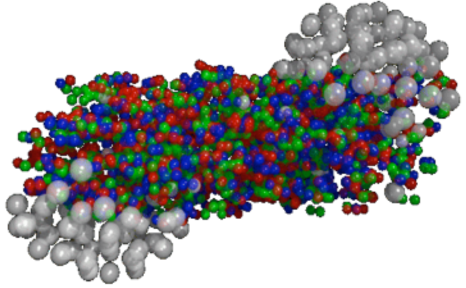
Starting point:

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4(p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

incident fermions

relative velocity factor

transition matrix element
(initial state i to final state f)



Differential cross section

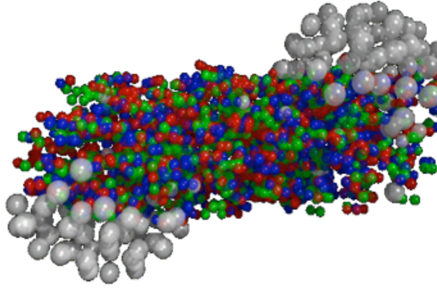
Starting point:

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4(p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

incident fermions

relative velocity factor

transition matrix element
(initial state i to final state f)



Differential cross section

Starting point:

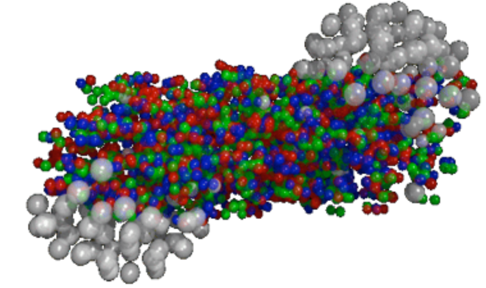
$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{(2\pi)^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4(p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

incident fermions

relative velocity factor

phase space factors

transition matrix element
(initial state i to final state f)



Differential cross section

Starting point:

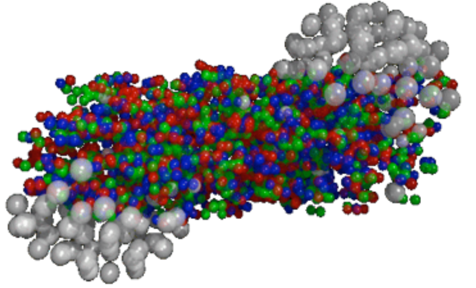
$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4 (p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

incident fermions

relative velocity factor

phase space factors

transition matrix element
(initial state i to final state f)



Differential cross section

Starting point:

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4 (p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

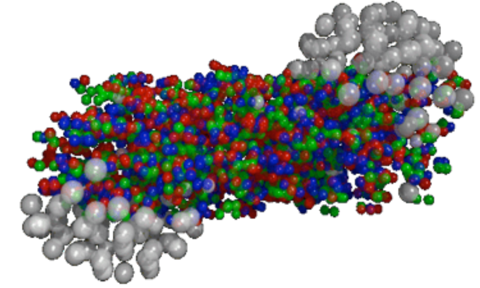
energy-momentum conservation

incident fermions

relative velocity factor

phase space factors

transition matrix element
(initial state i to final state f)



Differential cross section

Starting point:

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4 (p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

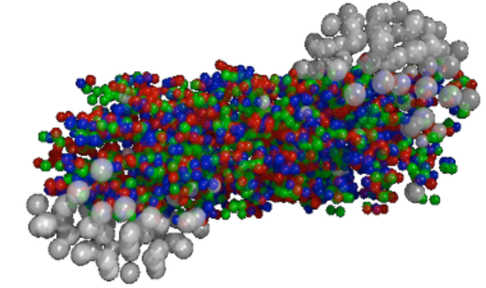
incident fermions

energy-
momentum
conservation

relative velocity factor

phase space factors

transition matrix element
(initial state i to final state f)



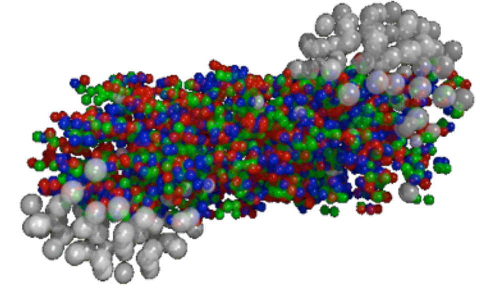
Integrating everything...

$$d\sigma = \frac{2m_q 2m_{\bar{q}}}{4[(p_q p_{\bar{q}})^2 - m_q^2 2m_{\bar{q}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4(p_q + p_{\bar{q}} - p_l + p_{\bar{l}})$$

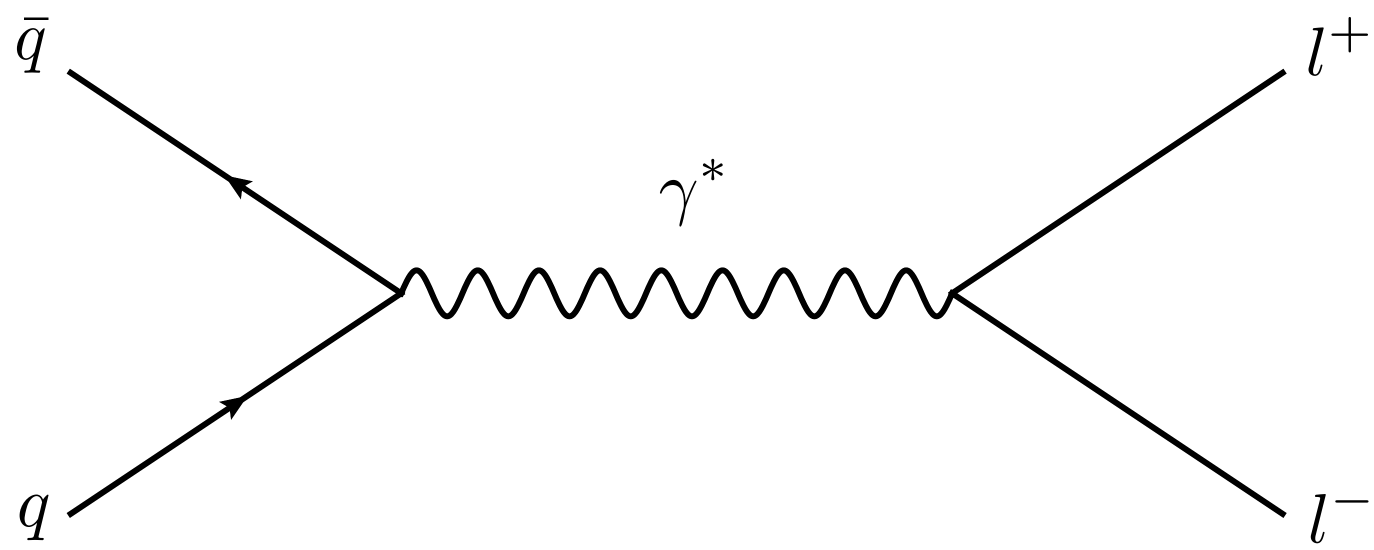
(plenty of integrations, using the delta function)

$$d\sigma = \frac{m_q^2 2m_{\bar{l}}^2}{(2\pi)^2} \frac{dt d\phi_l}{s(s - 4m_q^2)} |T_{fi}|^2$$

The big elephant in the room...



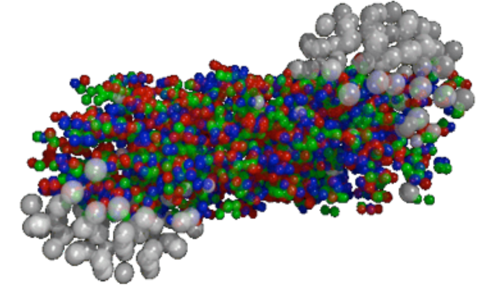
Feynman graphs



$$T_{fi} = \bar{u}(p_l, \varepsilon_l)(ie\gamma^\nu)v(p_{\bar{l}}, \varepsilon_{\bar{l}})\frac{-ig_{\mu\nu}}{(p_q + p_{\bar{q}})^2}\bar{v}(p_{\bar{q}}, \varepsilon_{\bar{q}})(ie\gamma^\mu)v(p_q, \varepsilon_q)$$

(page 301-307, Wong)

$$\frac{1}{4}\sum_{\varepsilon_q, \varepsilon_{\bar{q}}, \varepsilon_l, \varepsilon_{\bar{l}}}^{2} |T_{fi}|^2 = \frac{e^2 e_q^2}{4m_q^2 m_l^2 s^2}\left[sm_l^2 + sm_q^2 + \frac{1}{2}(t - m_q^2 - m_l^2)^2 + \frac{1}{2}(u - m_q^2 - m_l^2)^2\right]$$



Cross section

$$d\sigma = \frac{e^2 e_q^2}{8\pi} \frac{dt}{s^3(s-4m_q^2)} [2s(m_l^2 - m_q^2) + (t - m_q^2 - m_l^2)^2 - (u - m_q^2 - m_l^2)^2]$$

(integrating)

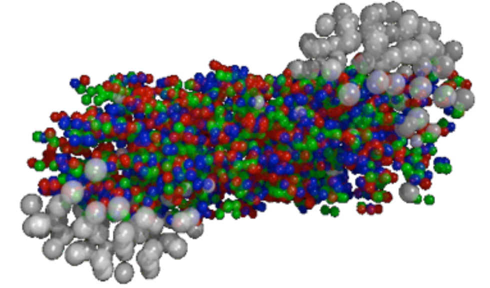
$$\sigma = \frac{4\pi}{3} \left(\frac{e_q}{e}\right)^2 \frac{\alpha^2}{s} \left(1 - \frac{4m_q^2}{s}\right)^{-1/2} \sqrt{1 - \frac{4m_l^2}{s}} \left(1 + 2\frac{m_q^2 + m_l^2}{s} + 4\frac{m_q^2 m_l^2}{s^2}\right)$$

Cross section

$$d\sigma = \frac{e^2 e_q^2}{8\pi} \frac{dt}{s^3(s-4m_q^2)} [2s(m_l^2 - m_q^2) + (t - m_q^2 - m_l^2)^2 - (u - m_q^2 - m_l^2)^2]$$

(integrating)

$$\sigma = \frac{4\pi}{3} \left(\frac{e_q}{e}\right)^2 \frac{\alpha^2}{s} \left(1 - \frac{4m_q^2}{s}\right)^{-1/2} \sqrt{1 - \frac{4m_l^2}{s}} \left(1 + 2\frac{m_q^2 + m_l^2}{s} + 4\frac{m_q^2 m_l^2}{s^2}\right)$$

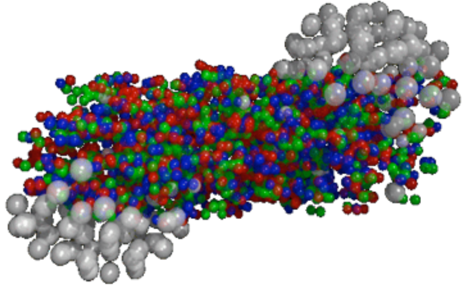


Cross section

$$d\sigma = \frac{e^2 e_q^2}{8\pi} \frac{dt}{s^3(s-4m_q^2)}[2s(m_l^2 - m_q^2) + (t - m_q^2 - m_l^2)^2 - (u - m_q^2 - m_l^2)^2]$$

(integrating)

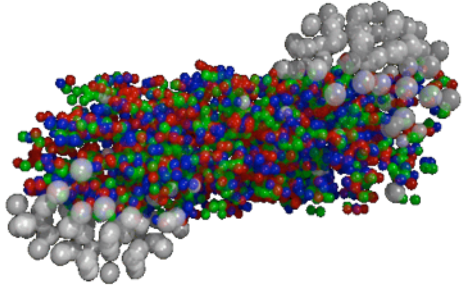
$$\sigma = \frac{4\pi}{3}\left(\frac{e_q}{e}\right)^2 \frac{\alpha^2}{s}\left(1 - \frac{4m_q^2}{s}\right)^{-1/2}\sqrt{1-\frac{4m_l^2}{s}}\left(1 + 2\frac{m_q^2 + m_l^2}{s} + 4\frac{m_q^2 m_l^2}{s^2}\right)$$

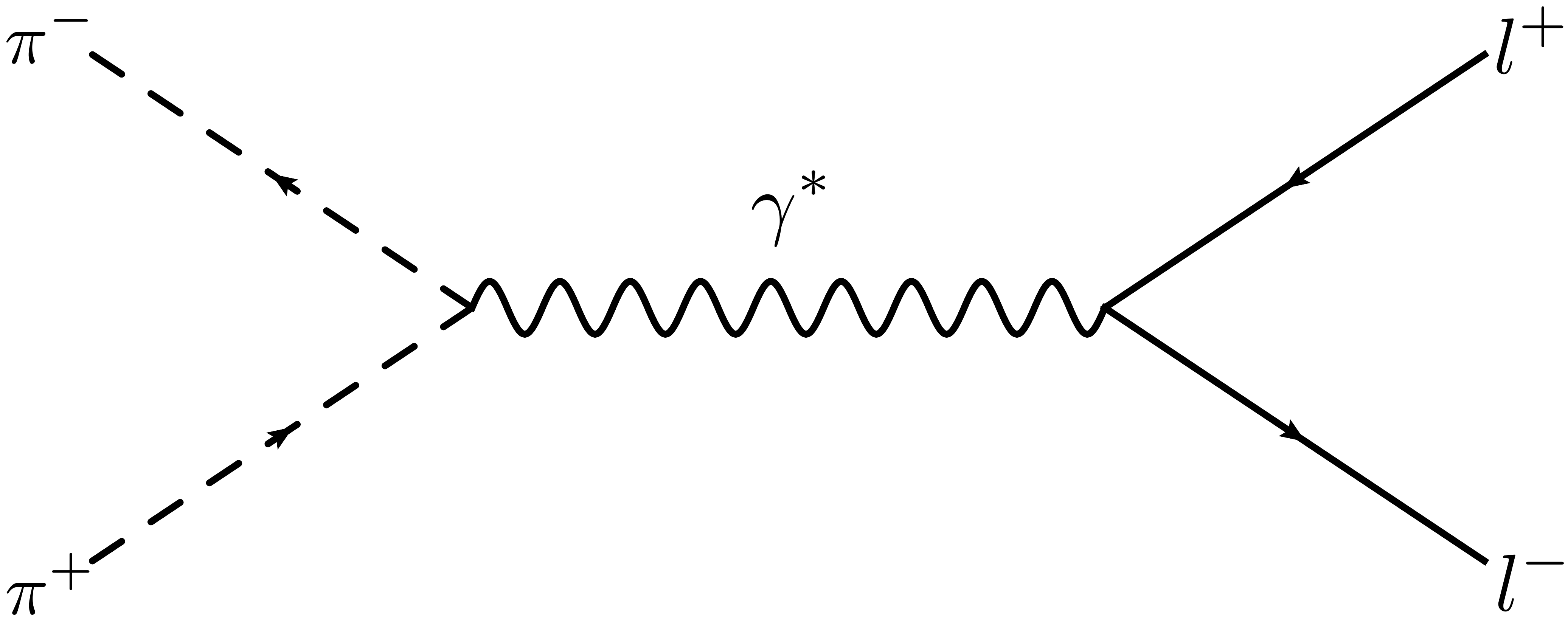


Cross section

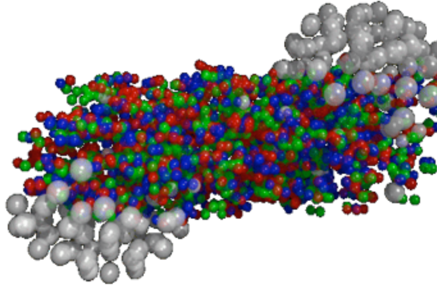
$$\sigma(M) = \frac{4\pi}{3}^{2}\,\frac{\alpha^2}{M^2}\left(1-\frac{4m_q^2}{M^2}\right)^{-1/2}\sqrt{1-\frac{4m_l^2}{M^2}}\left(1+2\frac{m_q^2+m_l^2}{M^2}+4\frac{m_q^2 m_l^2}{M^4}\right)$$

$\pi\pi \to l^+ l^-$





$\pi\pi \to l^+ l^-$

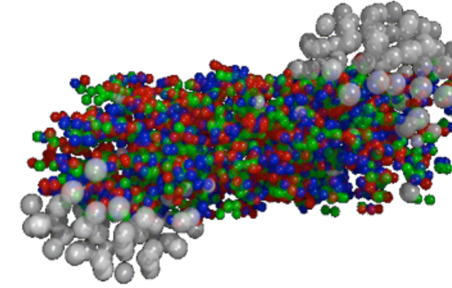


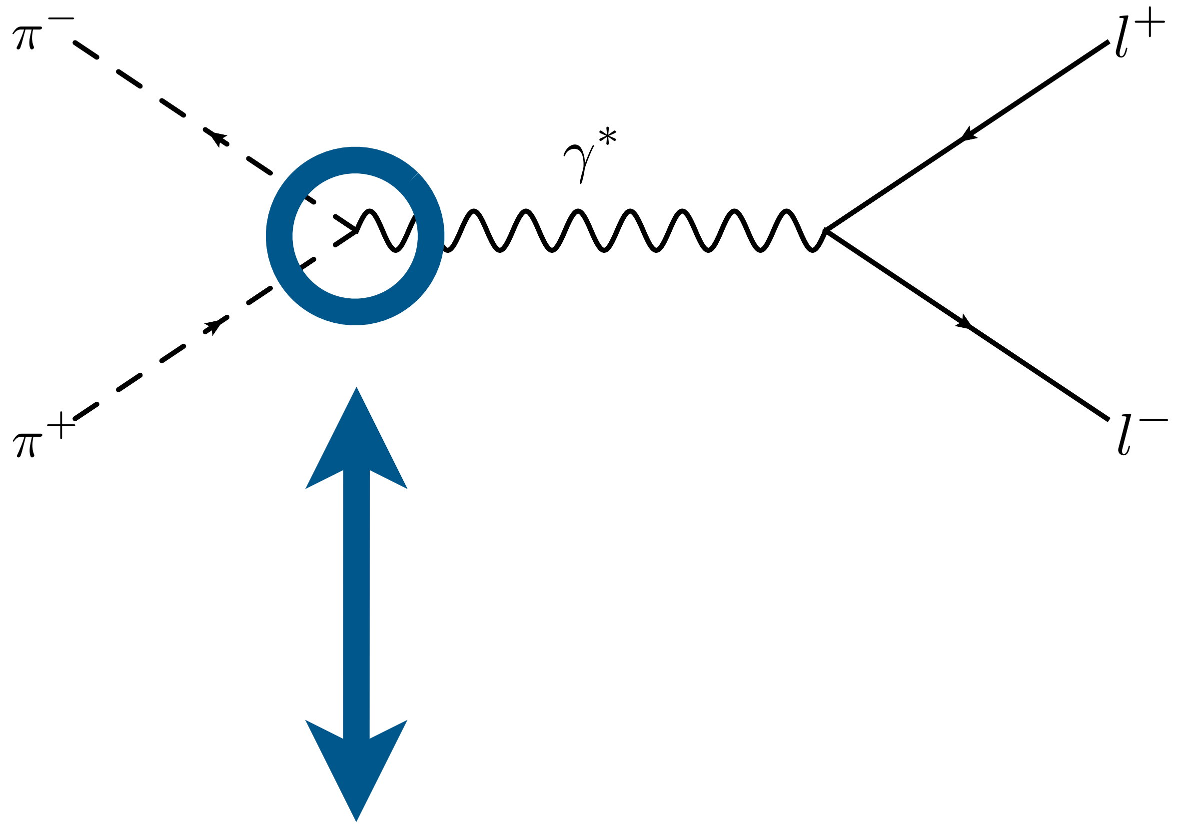
$$d\sigma = \frac{1}{4[(p_\pi p_{\bar{\pi}})^2 - m_\pi^2 2m_{\bar{\pi}}^2]^{1/2}} |T_{fi}|^2 \frac{d^3 p_l}{2\pi^3} \frac{m_l}{E_l} \frac{d^3 p_{\bar{l}}}{(2\pi)^3} \frac{m_{\bar{l}}}{E_{\bar{l}}} (2\pi)^4 \delta^4(p_\pi + p_{\bar{\pi}} - p_l + p_{\bar{l}})$$

(the usual deal...)

$$d\sigma = \frac{m_l^2}{2(2\pi)^2} \frac{dt d\phi_l}{s(s - 4m_q^2)} |T_{fi}|^2$$

$\pi\pi \to l^+ l^-$

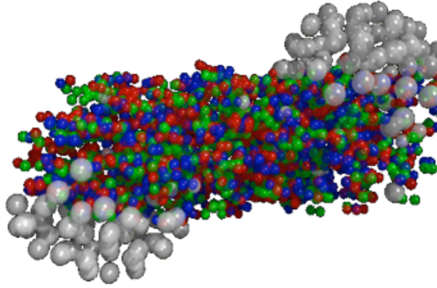




$$\int \frac{d^4k}{(2\pi)^4}(-ie)(p^\mu_\pi - p^\mu_{\bar{\pi}})(2\pi)^4\delta^4(p^\mu_\pi + p^\mu_{\bar{\pi}} - k)$$

$$T_{fi} = \bar{u}(p_l, \varepsilon_l)(ie\gamma^\nu)v(p_{\bar{l}}, \varepsilon_{\bar{l}})\frac{-ig_{\mu\nu}}{(p_\pi + p_{\bar{\pi}})^2}[-ie(p^\mu_\pi - p^\mu_{\bar{\pi}})]$$

$\pi\pi \to l^+ l^-$



$$\sigma(M) = \frac{4\pi}{3}\frac{\alpha^2}{M^2}\sqrt{1-\frac{4m_\pi^2}{M^2}}\sqrt{1-\frac{4m_l^2}{M^2}}\left(1+2\frac{m_l^2}{M^2}\right)$$

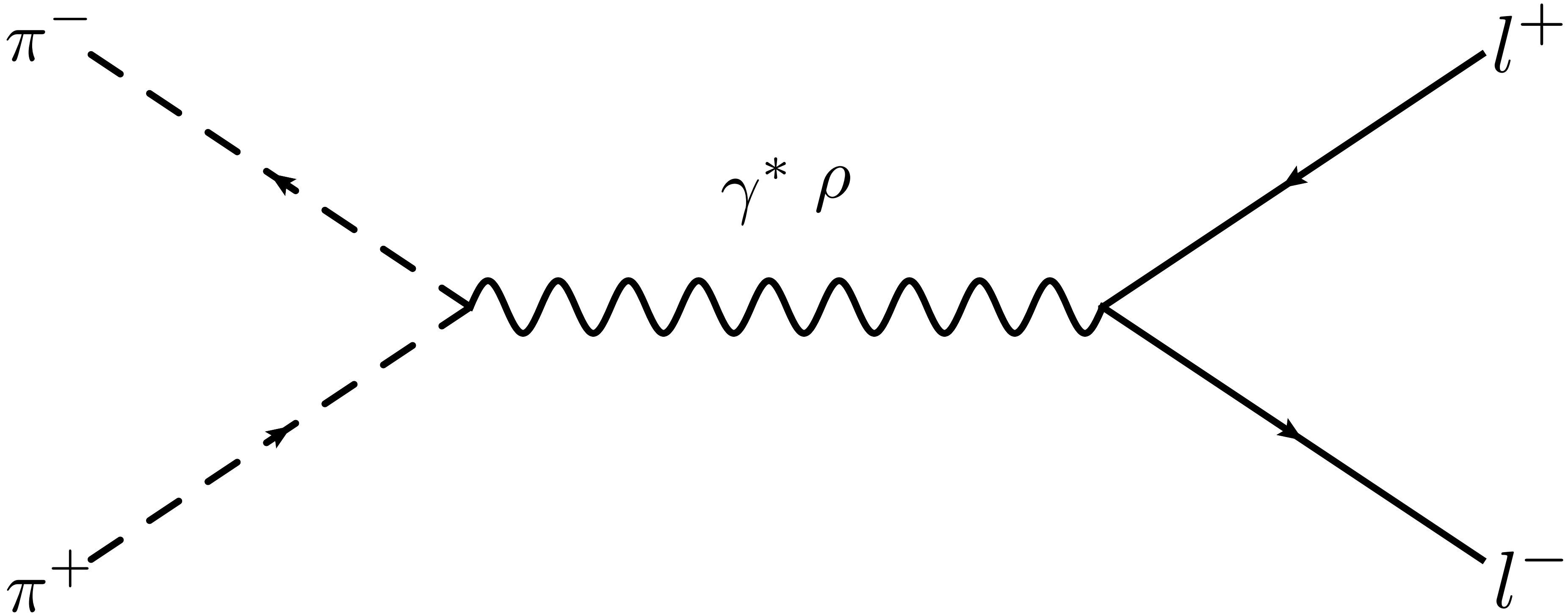
Vector Meson Dominance

So far:

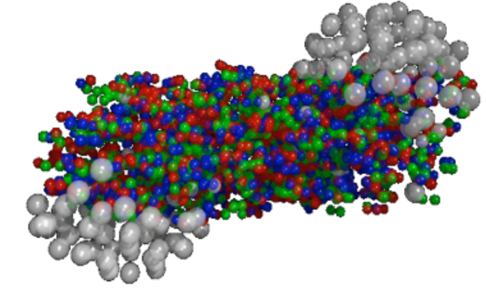
Photon propagator!

$$\frac{1}{(p_\pi + p_{\bar{\pi}})^2} = \frac{1}{s}$$

Vector Meson Dominance



$$\frac{1}{(p_\pi + p_{\bar{\pi}})^2} = \frac{1}{s} \quad \longrightarrow \quad \frac{1}{(p_\pi + p_{\bar{\pi}})^2 - (m_\rho + i\Gamma_\rho/2)^2}$$

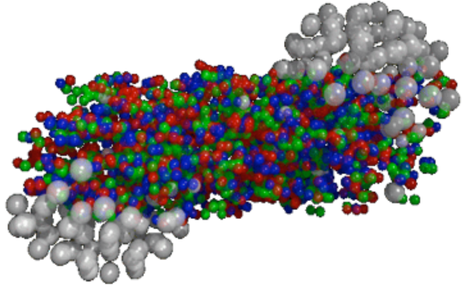


Form factor scaling

Solution: Scale with the respective absolute values!

$$|F_\pi(m_\pi)|^2 = \frac{\left|\frac{1}{(p_\pi+p_{\bar\pi})^2-(m_\rho+i\Gamma_\rho/2)^2}\right|^2}{\left|\frac{1}{(p_\pi+p_{\bar\pi})}\right|^2}$$

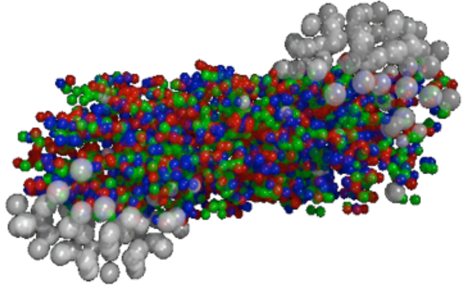
$$|F_\pi(m_\pi)|^2 \sim \frac{m_\rho^4}{(M^2-m_\rho^2)^2+\Gamma_\rho^2 m_\rho^2}$$



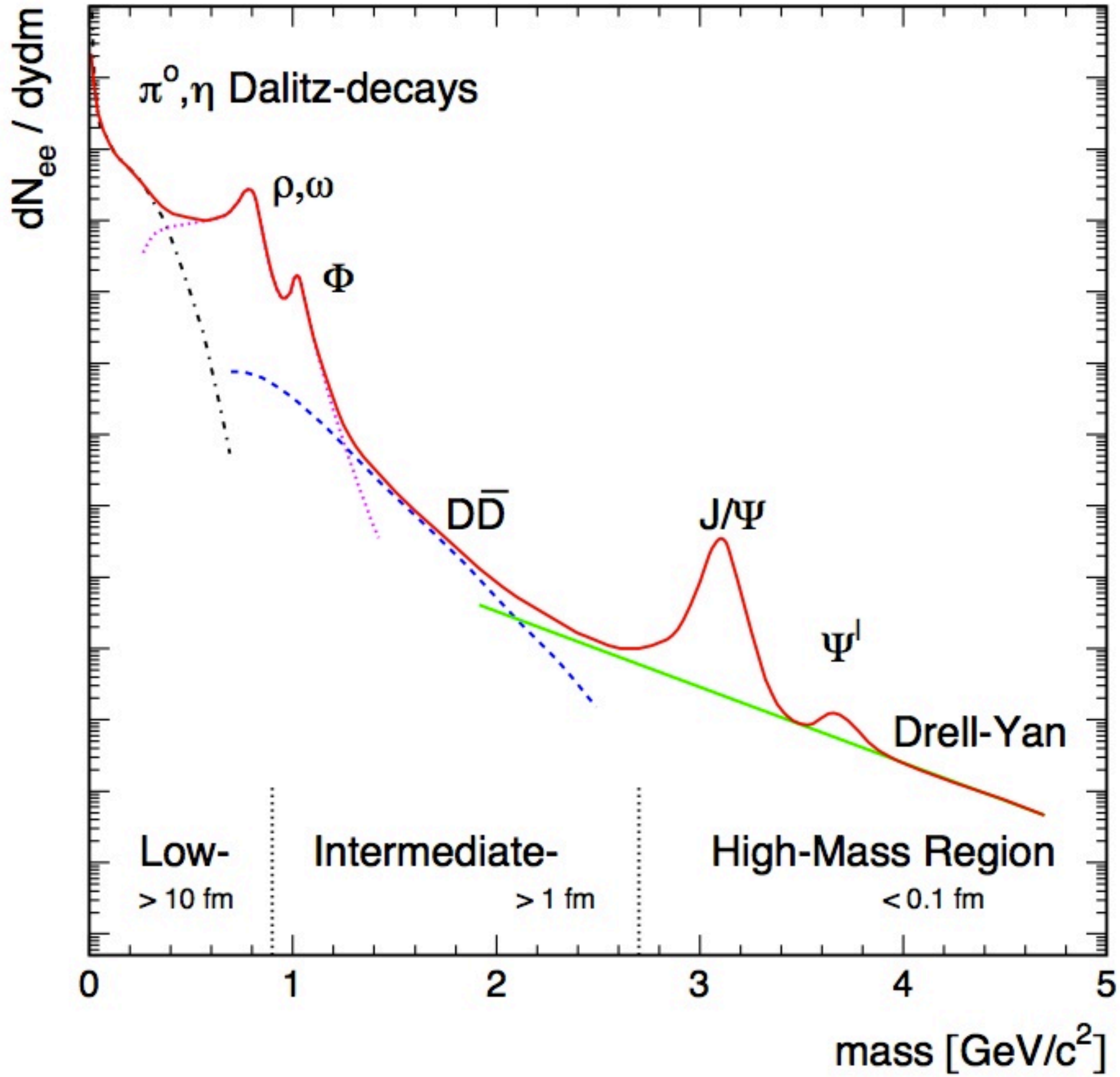
Form factor scaling

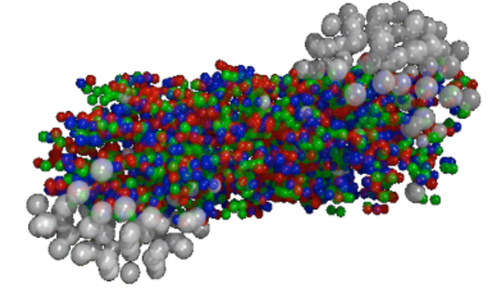
$$\sigma(M) = \frac{4\pi}{3}\frac{\alpha^2}{M^2}\sqrt{1-\frac{4m_\pi^2}{M^2}}\sqrt{1-\frac{4m_l^2}{M^2}}\left(1+2\frac{m_l^2}{M^2}\right)|F_\pi(m_\pi)|^2$$

$$\sigma(M) = \frac{16\pi}{(M^2-4m_\pi^2)}\frac{|F_\pi(m_\pi)|^2}{m_\rho^4}m_\rho\Gamma(\rho\to\pi\pi)m_\rho\Gamma(\rho\to e^+e^-)$$



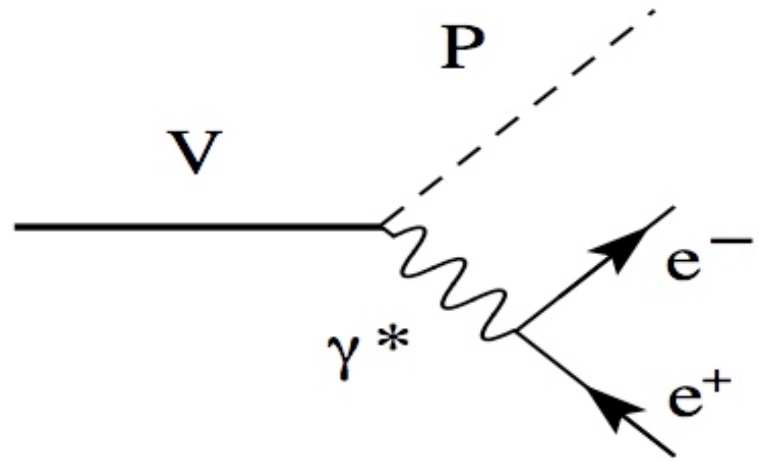
What about decays?



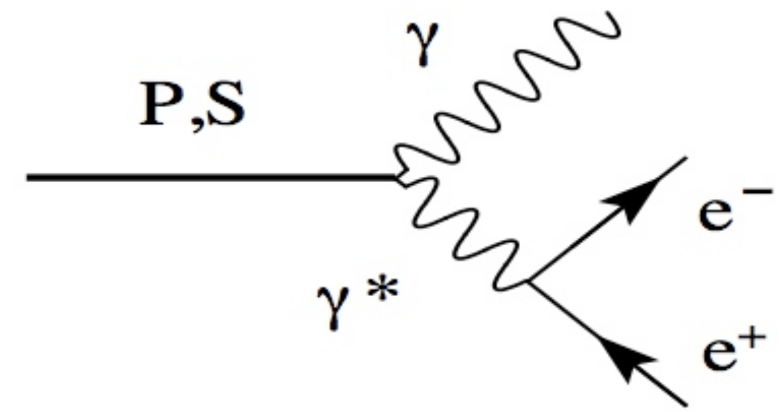


Dilepton sources

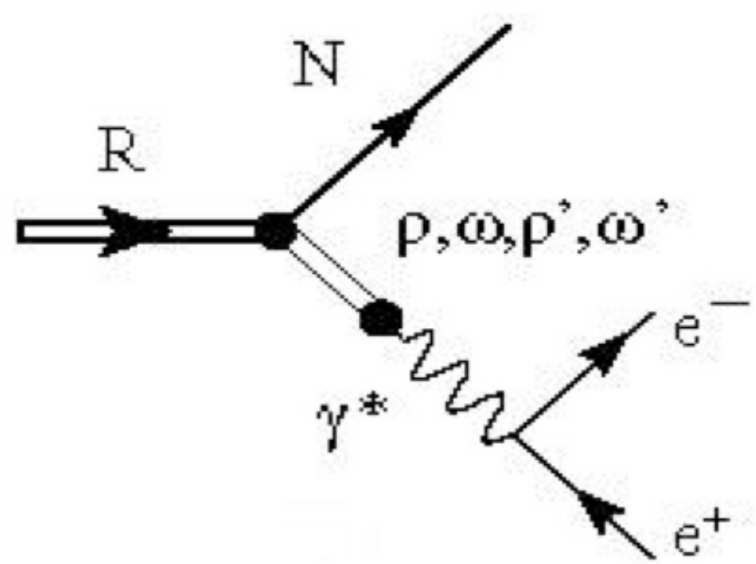
- Dalitz decays
  - π^0, η, η’, ω, Δ …
- Direct decays
  - ρ, ω, ϕ …



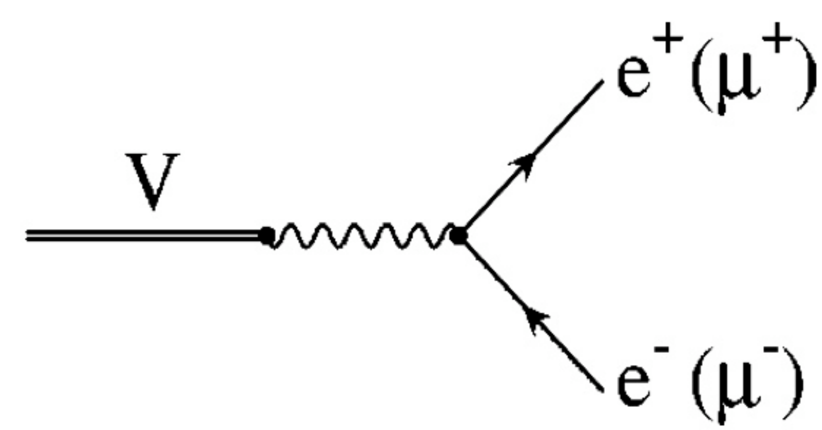
Dalitz decay (vector meson)



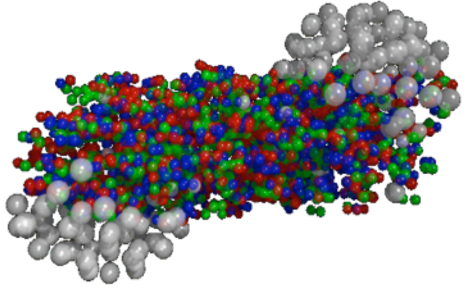
(pseudoscalar meson)



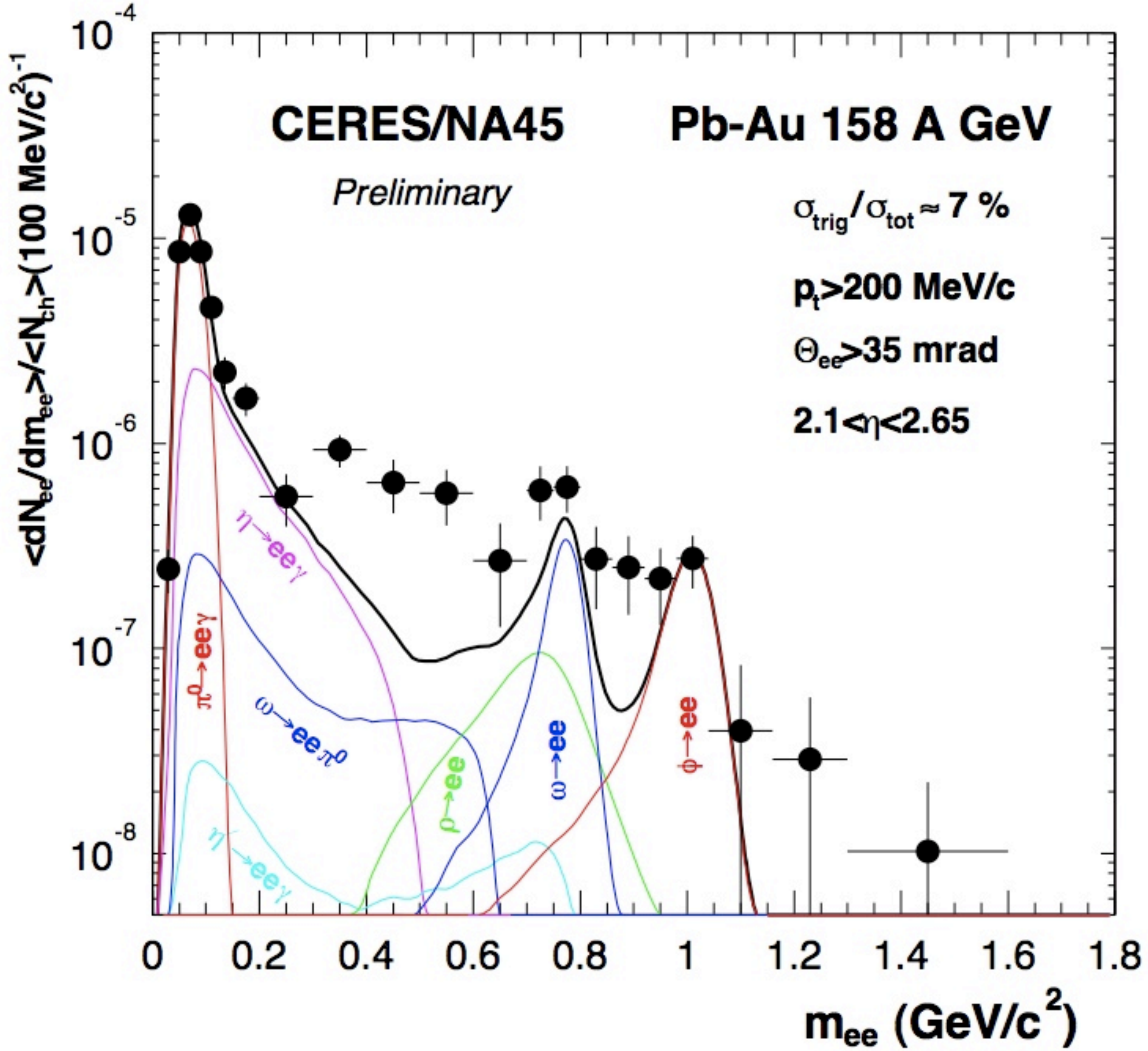
Dalitz decay (Δ)



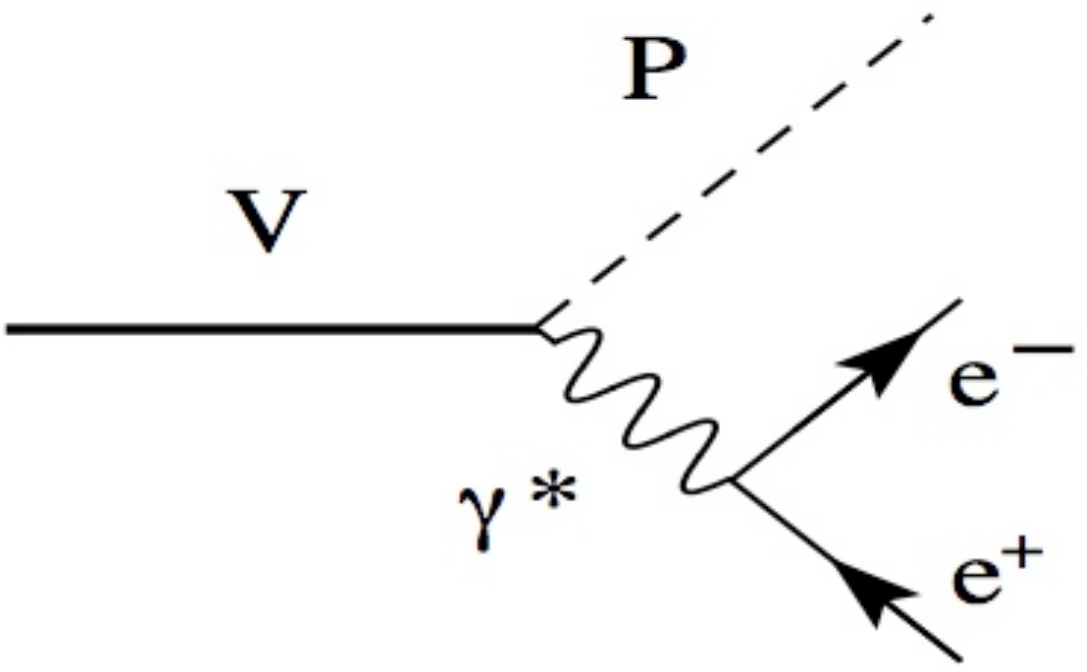
Direct decay

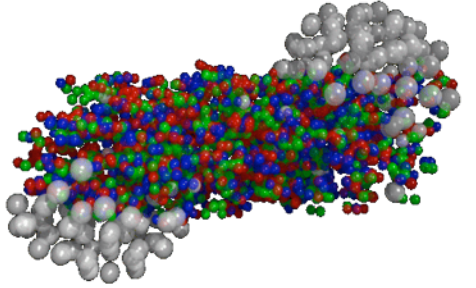


Dilepton sources

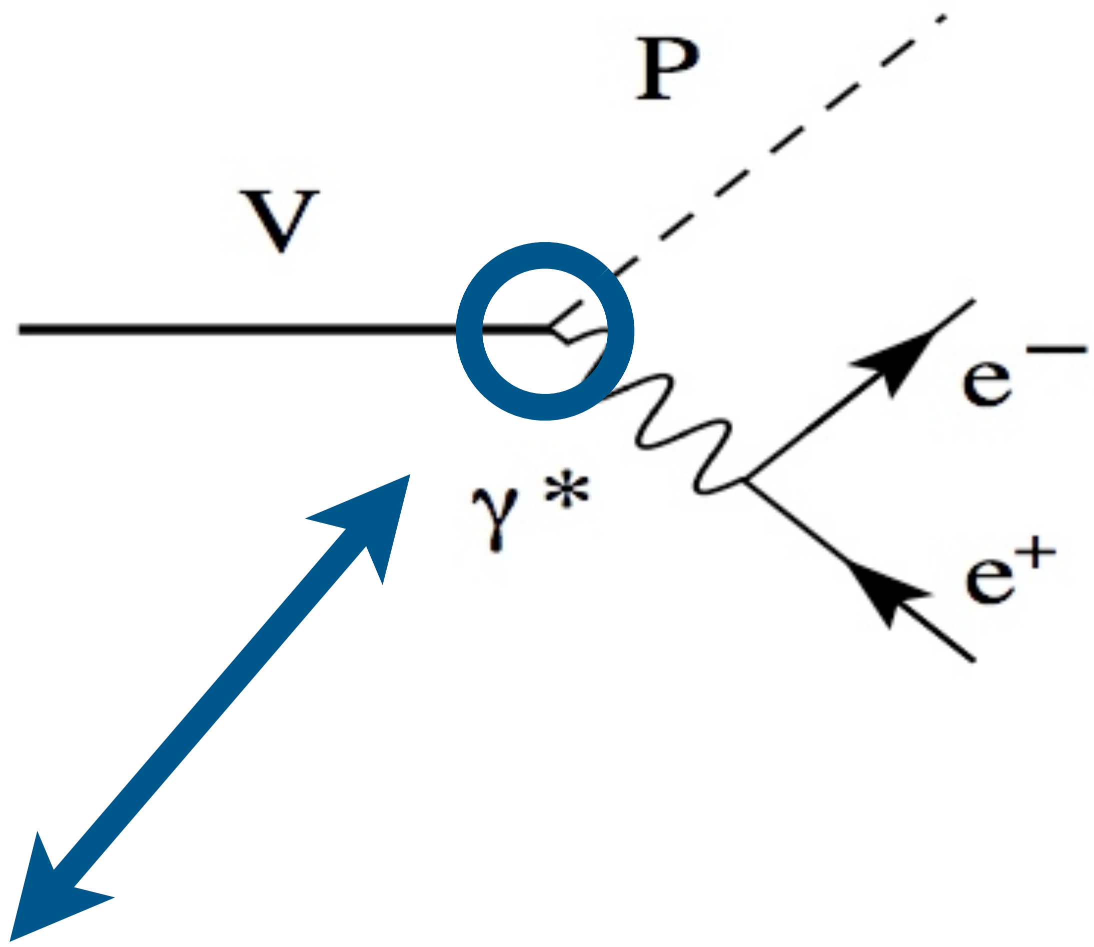


Feynman graphs



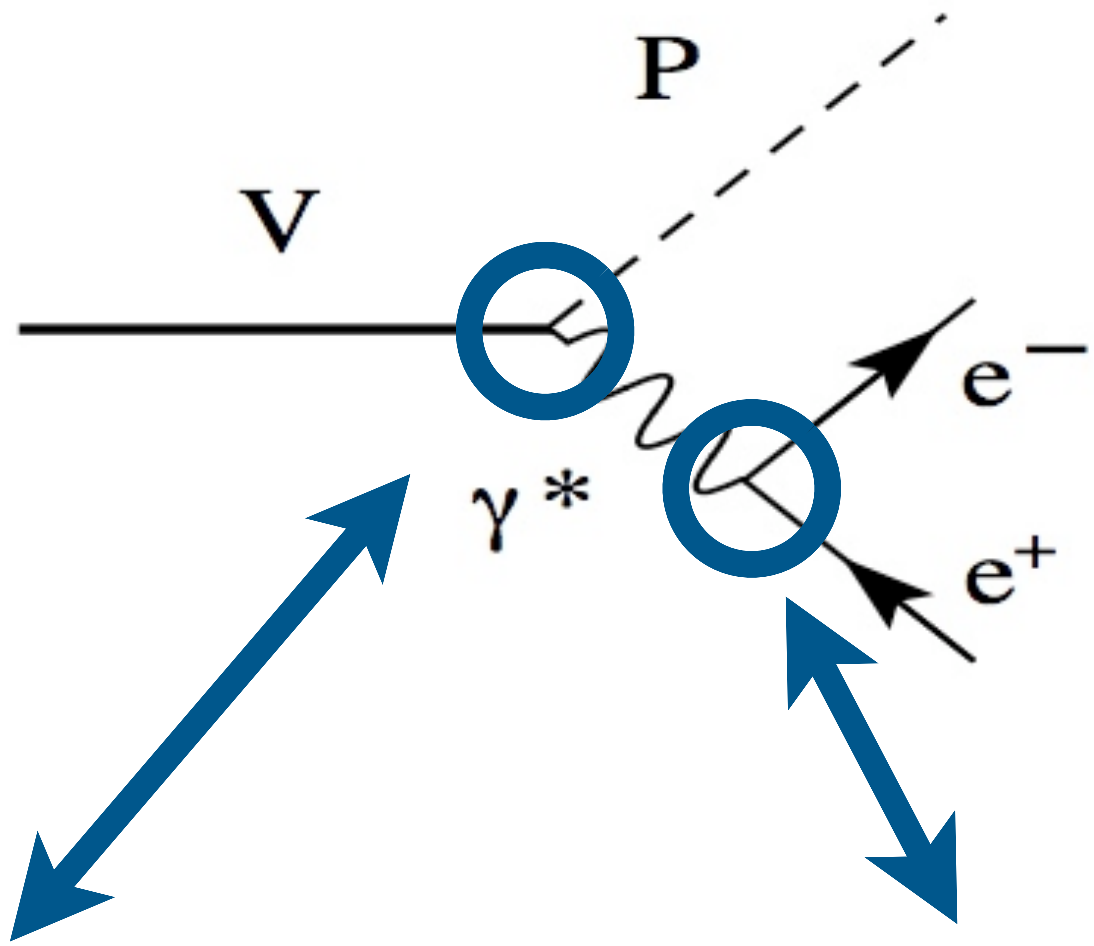


Feynman graphs



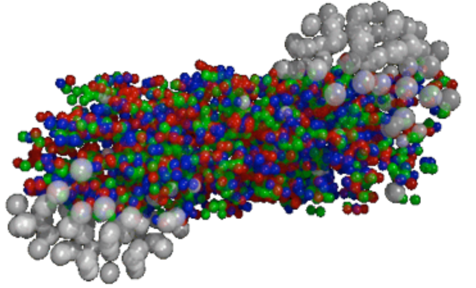
$$V \to P\gamma^*$$

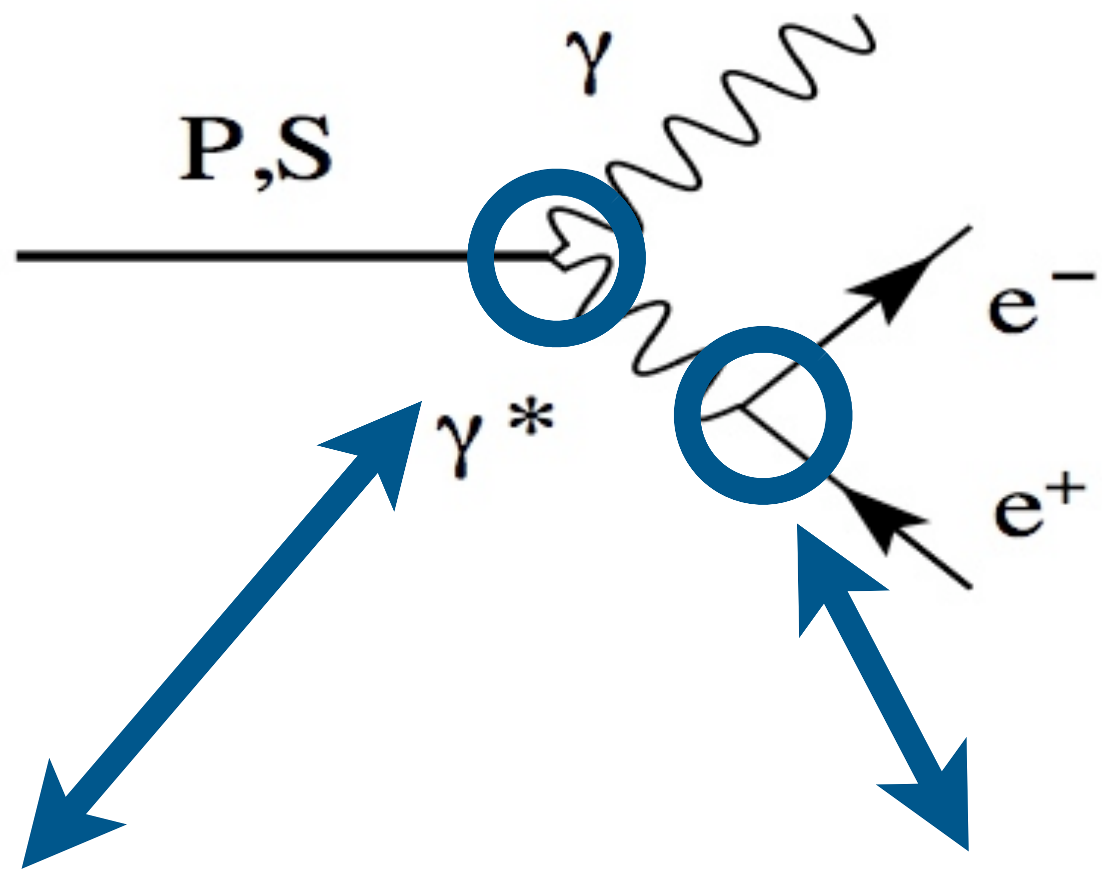
Feynman graphs



$V \to P\gamma^*$ $\gamma^* \to e^+e^-$

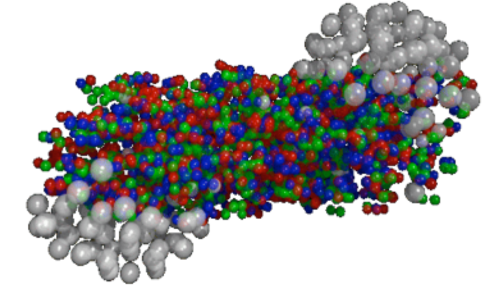
Feynman graphs





$$P, S \to \gamma\gamma^*$$

$$\gamma^* \to e^+e^-$$



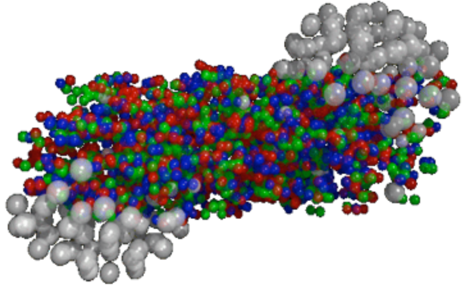
Decay rate

$$\Gamma = \frac{\text{Number of decays per unit time}}{\text{Numbers of particles present}}$$

$$d\Gamma = \frac{1}{2M}\left(\prod_f \int \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)|\mathcal{M}(M \to \{p_f\})|^2 (2\pi)^4 \delta^4\left(P - \sum p_f\right)$$

Phase space:

$$\int d\Pi_n = \frac{1}{2M}\left(\prod_f \int \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)(2\pi)^4 \delta^4\left(P - \sum p_f\right)$$



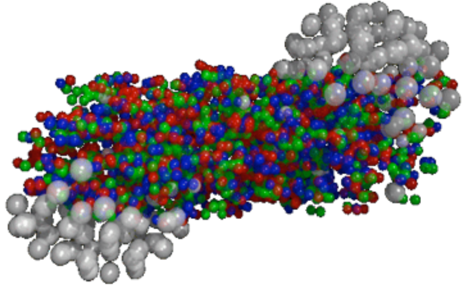
Phase space

2 body phase space

$$\int d\Pi_2 = \int d\Omega \frac{1}{16\pi^2} \frac{|\vec{p}|}{\sqrt{s}}$$

2 body decay rate

$$d\Gamma = \frac{1}{2M} |\mathcal{M}(M \to \{p_f\})|^2 d\Pi_2$$



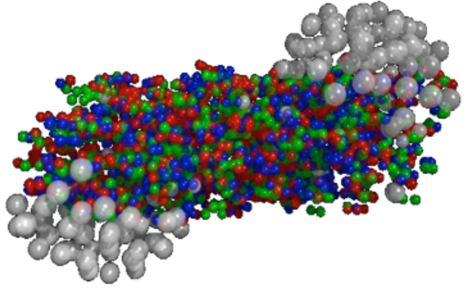
Phase space

2 body phase space

$$\int d\Pi_2 = \int d\Omega \frac{1}{16\pi^2} \frac{|\vec{p}|}{\sqrt{s}}$$

2 body decay rate

$$d\Gamma = \frac{1}{2M} |\mathcal{M}(M \to \{p_f\})|^2 d\Pi_2$$



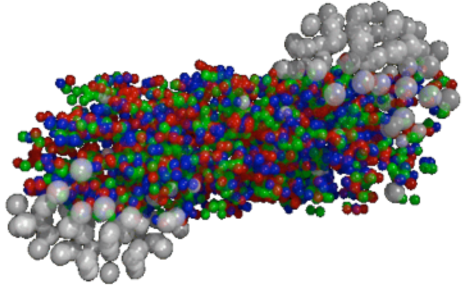
Phase space

2 body phase space

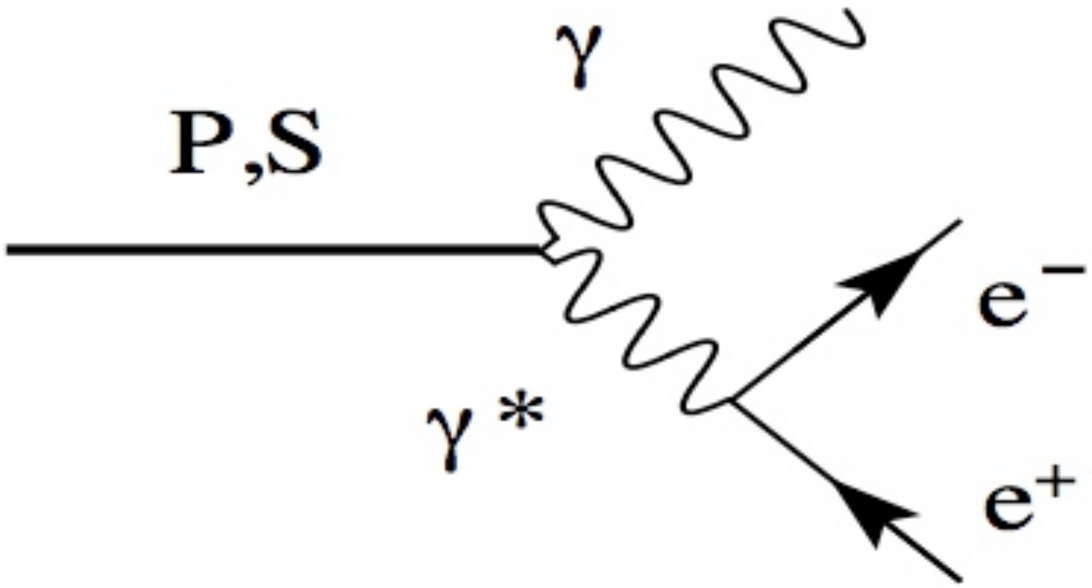
$$\int d\Pi_2 = \int d\Omega \frac{1}{16\pi^2} \frac{|\vec{p}|}{\sqrt{s}}$$

2 body decay rate

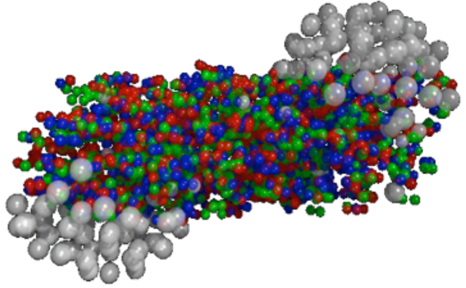
$$d\Gamma = \frac{1}{2M} |\mathcal{M}(M \to \{p_f\})|^2 d\Pi_2$$



Pion decay

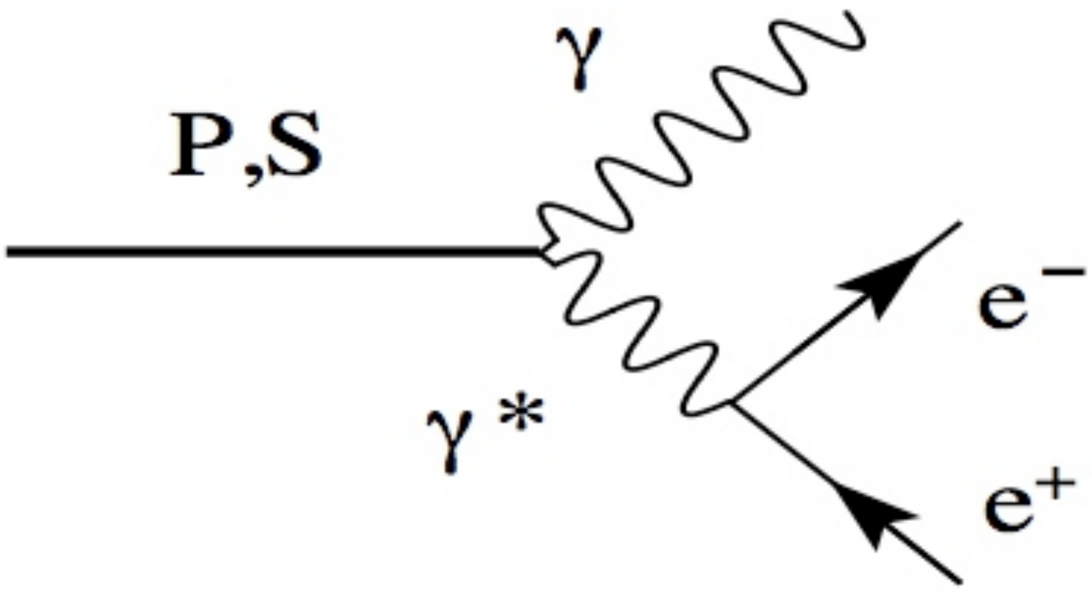


$$P, S \to \gamma\gamma^* \qquad \gamma^* \to e^+ e^-$$

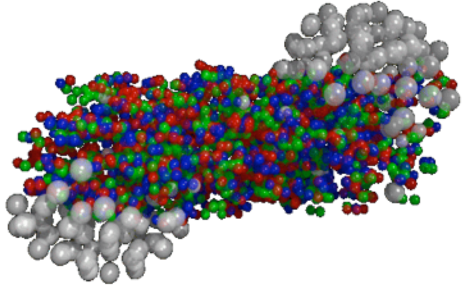


Second vertex

$$\gamma^* \to e^+ e^-$$



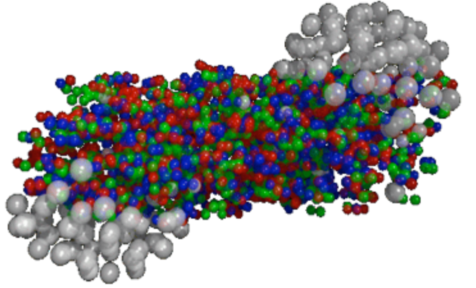
$$i\mathcal{M}(\gamma^* \to e^+ e^-) = -ie\varepsilon_\mu(k)\bar{u}(p)\gamma^\mu v(p')$$



Feynman graphs

[calculating, calculating...]

$$|\mathcal{M}(\gamma^* \to e^+ e^-)|^2 = \frac{1}{3} e^2 (4M_{\gamma^*}^2 + 8m_e^2)$$

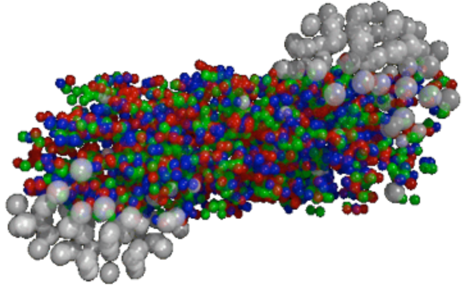


Phase space

$$p = \sqrt{\frac{M_{\gamma^*}^2}{4} - m_e^2}$$

$$\sqrt{s} = M_{\gamma^*}$$

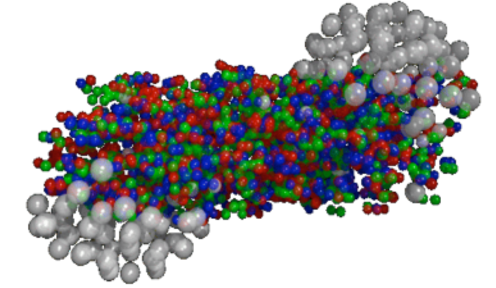
$$\int d\Pi_2 = \int d\Omega \frac{1}{32\pi^2}\sqrt{1 - \frac{4m_e^2}{M_{\gamma^*}}}$$



Feynman graphs

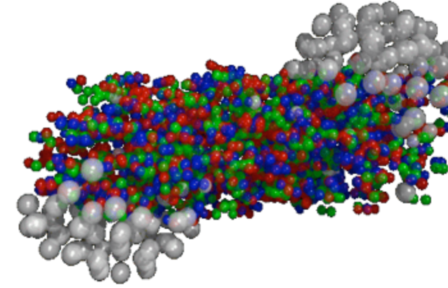
[plugging it all in...]

$$\Gamma(\gamma^* \to e^+ e^-) = \frac{\alpha}{3} \frac{1}{M_{\gamma^*}} (M_{\gamma^*}^2 + 2m_e^2) \frac{4m_e^2}{M_{\gamma^*}}$$



Full process

$$d\Gamma(A \to Be^+e^-) = d\Gamma(A \to B\gamma^*) M\Gamma(\gamma^* \to e^+e^-) \frac{dM^2}{\pi M^4}$$



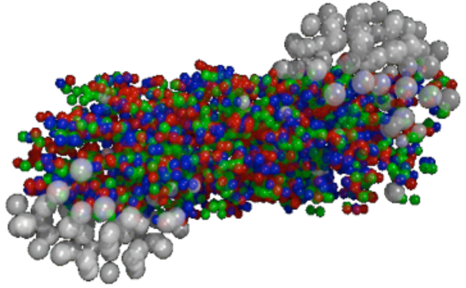
Feynman graphs

Final results:

$$\frac{d\Gamma(\pi^0 \to \gamma e^+ e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} = 2\left(1 - \frac{M_{\gamma^*}}{m^2_{\pi^0}}\right)^3 |F_{\pi^0 \to \gamma\gamma}(M_{\gamma^*})^2|^2 M_{\gamma^*} \Gamma\gamma^* \to e^+ e^-) \frac{dM_{\gamma^*}}{\pi M^4_{\gamma^*}}$$

Final mass spectrum:

$$\frac{dN_{\pi^0 \to \gamma e^+ e^-}}{dM} = \frac{2\alpha}{3\pi M}\sqrt{1 - \frac{4m_e^2}{M^2_{\gamma^*}}}\left(1 + \frac{2m_e^2}{M^2_{\gamma^*}}\right)\left(1 - \frac{M^2_{\gamma^*}}{m^2_{\pi^0}}\right)^3 |F_{\pi^0 \to \gamma\gamma}(M_{\gamma^*})^2|^2 \frac{\Gamma_{\pi^0 \to 2\gamma}}{\Gamma_{tot}} \langle N_{\pi^0} \rangle$$



Dilepton sources

Pseudoscalar mesons:

$$\frac{dN_{A\to\gamma e^+e^-}}{dM} = \frac{4\alpha}{3\pi M}\sqrt{1-\frac{4m_e^2}{M^2}}\left(1+\frac{2m_e^2}{M^2}\right)\left(1-\frac{M^2}{m_A^2}\right)^3 \times \; |F_{AB}(M^2)|^2\frac{\Gamma_{A\to 2\gamma}}{\Gamma_{tot}}\langle N_A\rangle$$

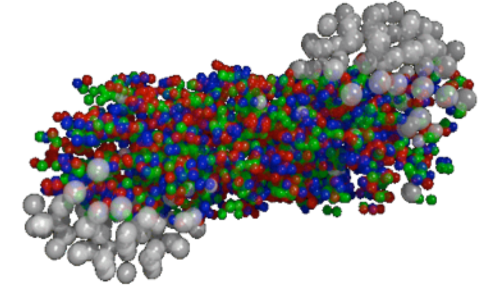
Vector mesons:

$$\frac{dN_{A\to Be^+e^-}}{dM} = \frac{2\alpha}{3\pi M}\sqrt{1-\frac{4m_e^2}{M^2}}\left(1+\frac{2m_e^2}{M^2}\right)|F_{AB}(M^2)|^2\frac{\Gamma_{A\to 2\gamma}}{\Gamma_{tot}}\langle N_A\rangle \times \left(\left(1+\frac{M^2}{m_A^2-m_B^2}\right)^2-\left(\frac{2m_AM}{m_A^2-m_B^2}\right)^2\right)^{3/2}$$

Direct decays:

$$BR(V\to e^+e^-) = \frac{\Gamma_{V\to e^+e^-}(M)}{\Gamma_{tot}}$$

$$\Gamma_{V\to e^+e^-}(M) = \frac{\Gamma_{V\to e^+e^-}(m_V)}{m_V}\frac{m_V^4}{M^3}\sqrt{1-\frac{4m_e^2}{M^2}}\left(1+2\frac{m_e^2}{M^2}\right)$$



Dilepton sources

Δ baryon:

$$\frac{dN_{e^+e^-}}{dM} = \int \frac{dN_{\Delta \to N e^+ e^-}}{dM}(M_\Delta)\frac{dN_\Delta}{dM_\Delta}dM_\Delta = \int \frac{2\alpha}{3\pi M}\frac{\Gamma(M_\Delta, M)}{\Gamma^{tot}_{\Delta 0}}\frac{dN_\Delta}{dM_\Delta}dM_\Delta$$

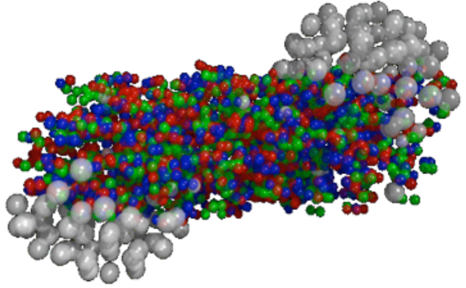
$$\Gamma(M_\Delta, M) = \frac{\lambda^{1/2}(M^2, m_N^2, M_\Delta^2)}{16\pi M_\Delta^2} m_N [2\mathcal{M}_t(M, M_\Delta) + \mathcal{M}_l(M, M_\Delta)]$$

$$\lambda(m_A^2, m_1^2, m_2^2) = (m_A^2 - (m_1 + m_2)^2)(m_A^2 - (m_1 - m_2)^2)$$

$$\mathcal{M}_l = (efg)^2 \frac{m_\Delta^2}{9 m_N} M^2 4(m_\Delta - m_N - q_0)$$

$$\mathcal{M}_t = (efg)^2 \frac{m_\Delta^2}{9 m_N}[q_0^2(5m_\Delta - 3(q_0 + m_N)) - M^2(m_\Delta + m_N + q_0)]$$

L.G. Landsberg, Phys.Rept.128:301-376,1985
P. Koch, Z. Phys. C57:283-304, 1993

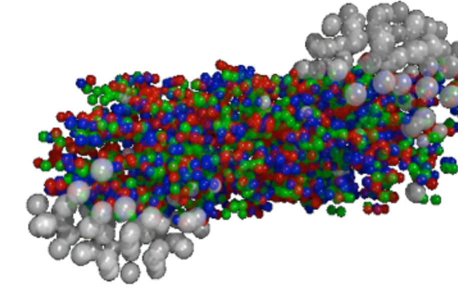


Form factors

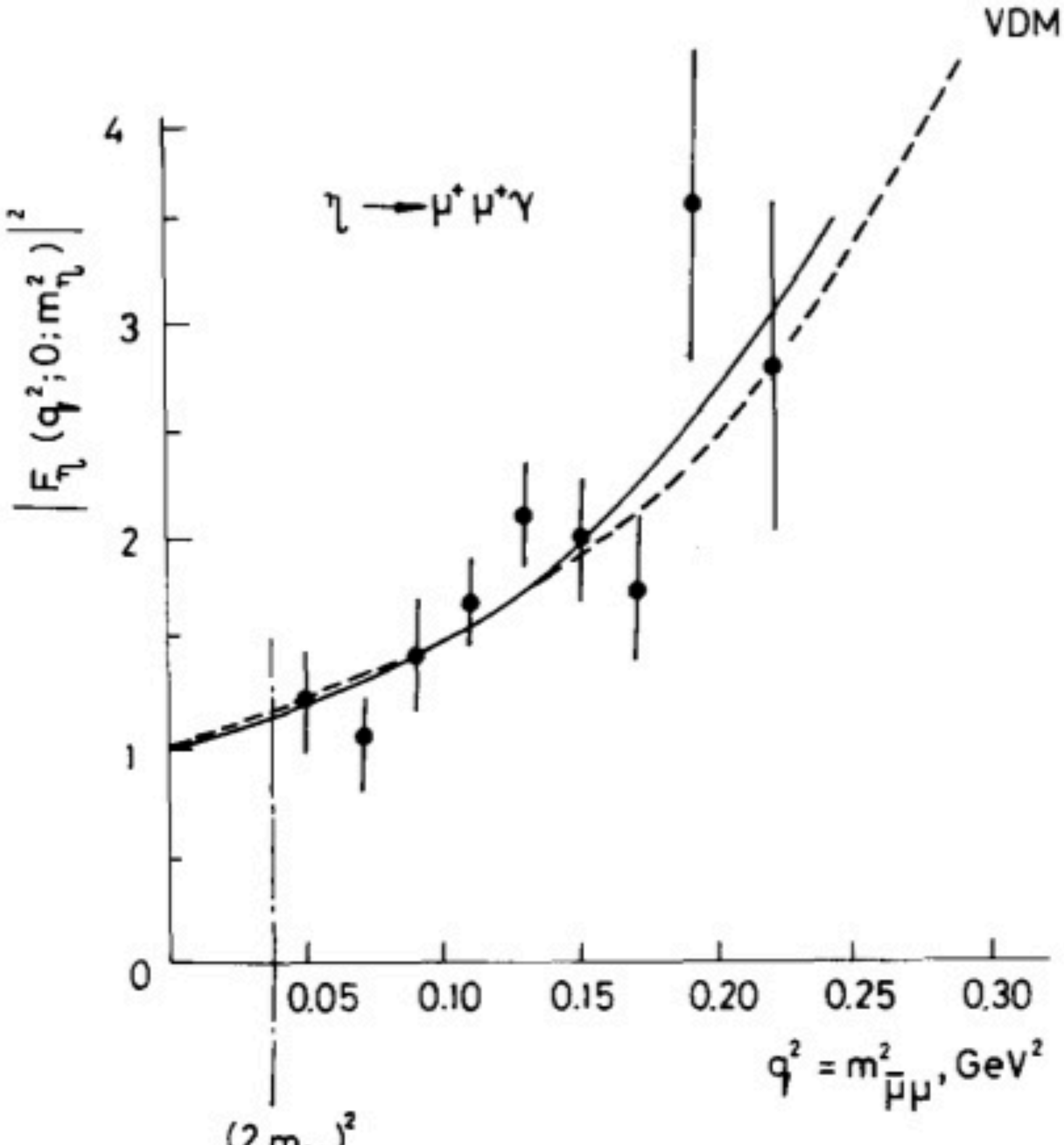
Hadrons are no point particles!

⇒ Form Factors!

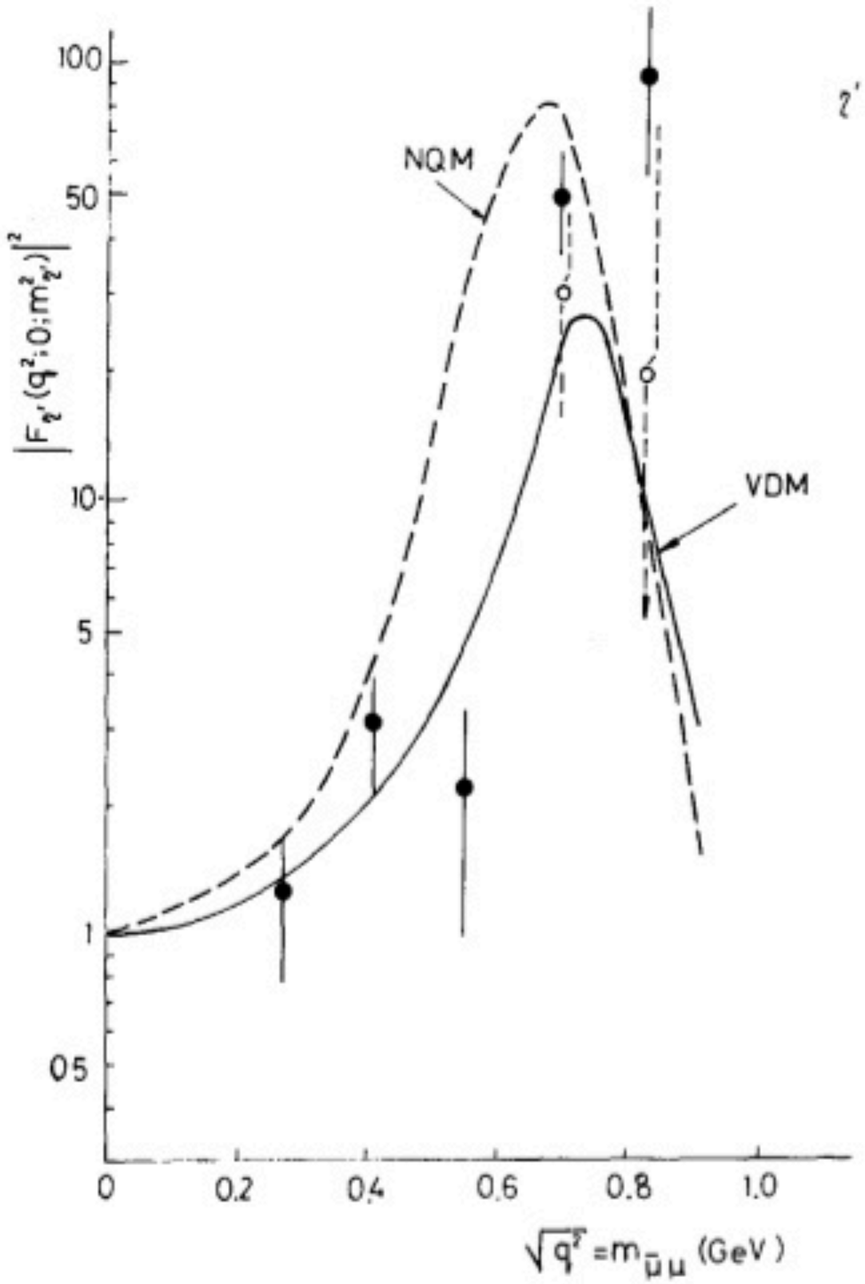
Important: no first principle calculations!
Huge model dependence.
Need to be adjusted experimentally!

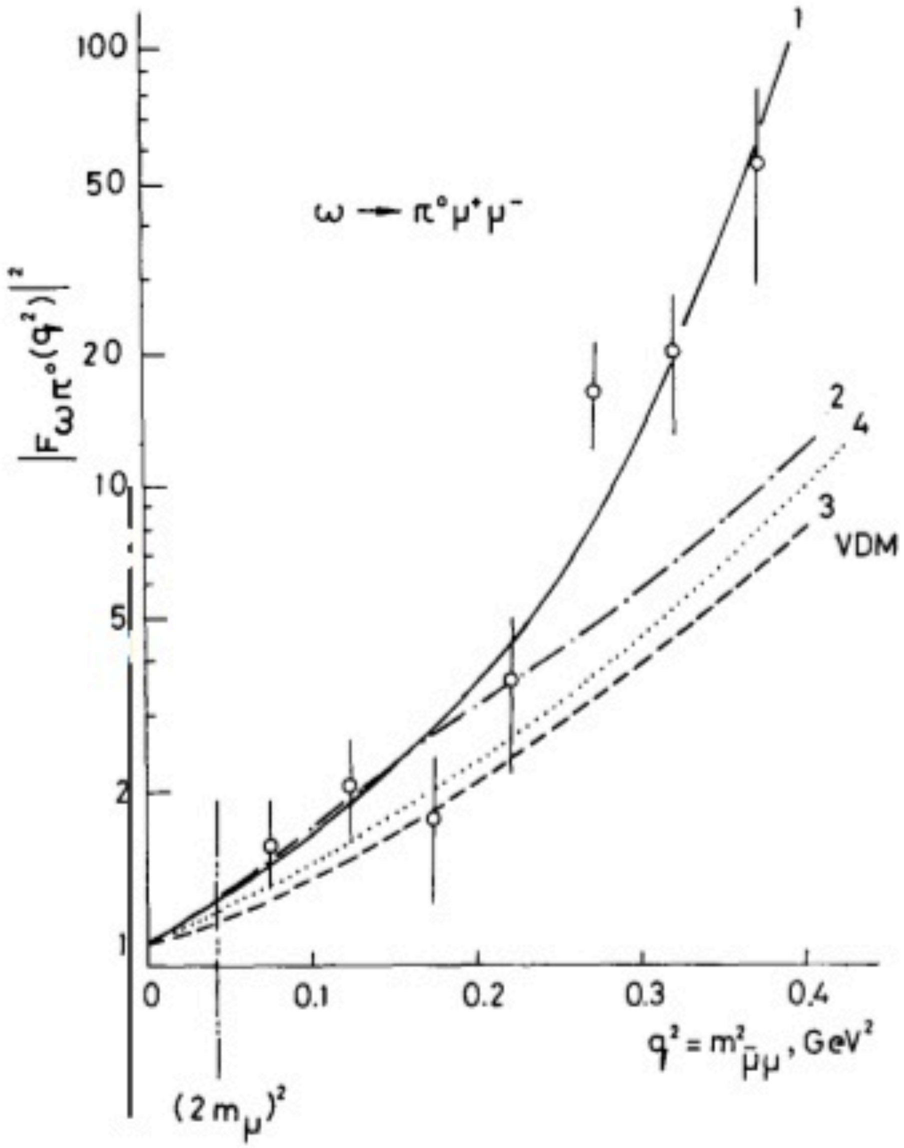


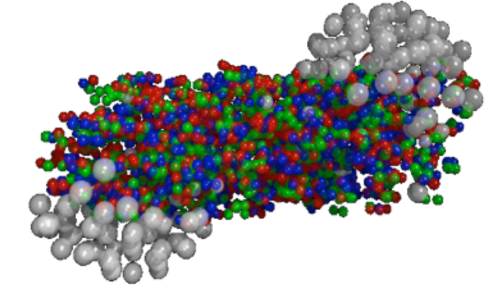
Form factors



Form factors







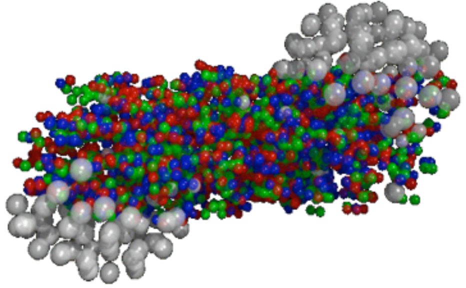
Form factors

$$F_{\pi^0}(M^2) = 1 + b_{\pi^0} M^2$$

$$F_\eta(M^2) = \left(1 - \frac{M^2}{\Lambda_\eta^2}\right)^{-1}$$

$$\left|F_\omega(M^2)\right|^2 = \frac{\Lambda_\omega^2(\Lambda_\omega^2 + \gamma_\omega^2)}{(\Lambda_\omega^2 - M^2)^2 + \Lambda_\omega^2 \gamma_\omega^2}$$

$$\left|F_{\eta'}(M^2)\right|^2 = \frac{\Lambda_{\eta'}^2(\Lambda_{\eta'}^2 + \gamma_{\eta'}^2)}{(\Lambda_{\eta'}^2 - M^2)^2 + \Lambda_{\eta'}^2 \gamma_{\eta'}^2}$$

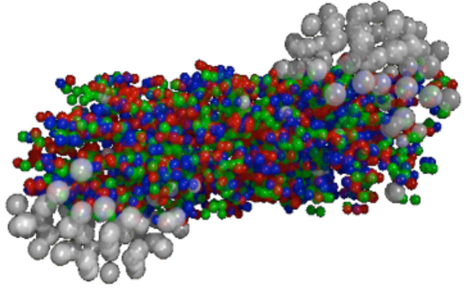


Charm decays

$$g + g \to c + \bar{c}$$

$$q + \bar{q} \to g^* \to c + \bar{c}$$

Huge mass, need lots of energy
⇒ initial collisions

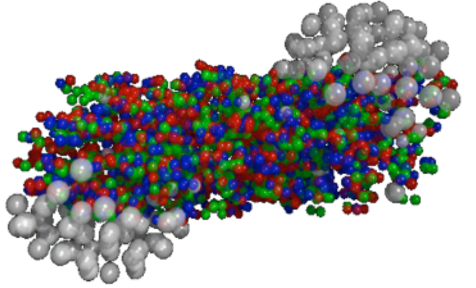


Bremsstrahlung

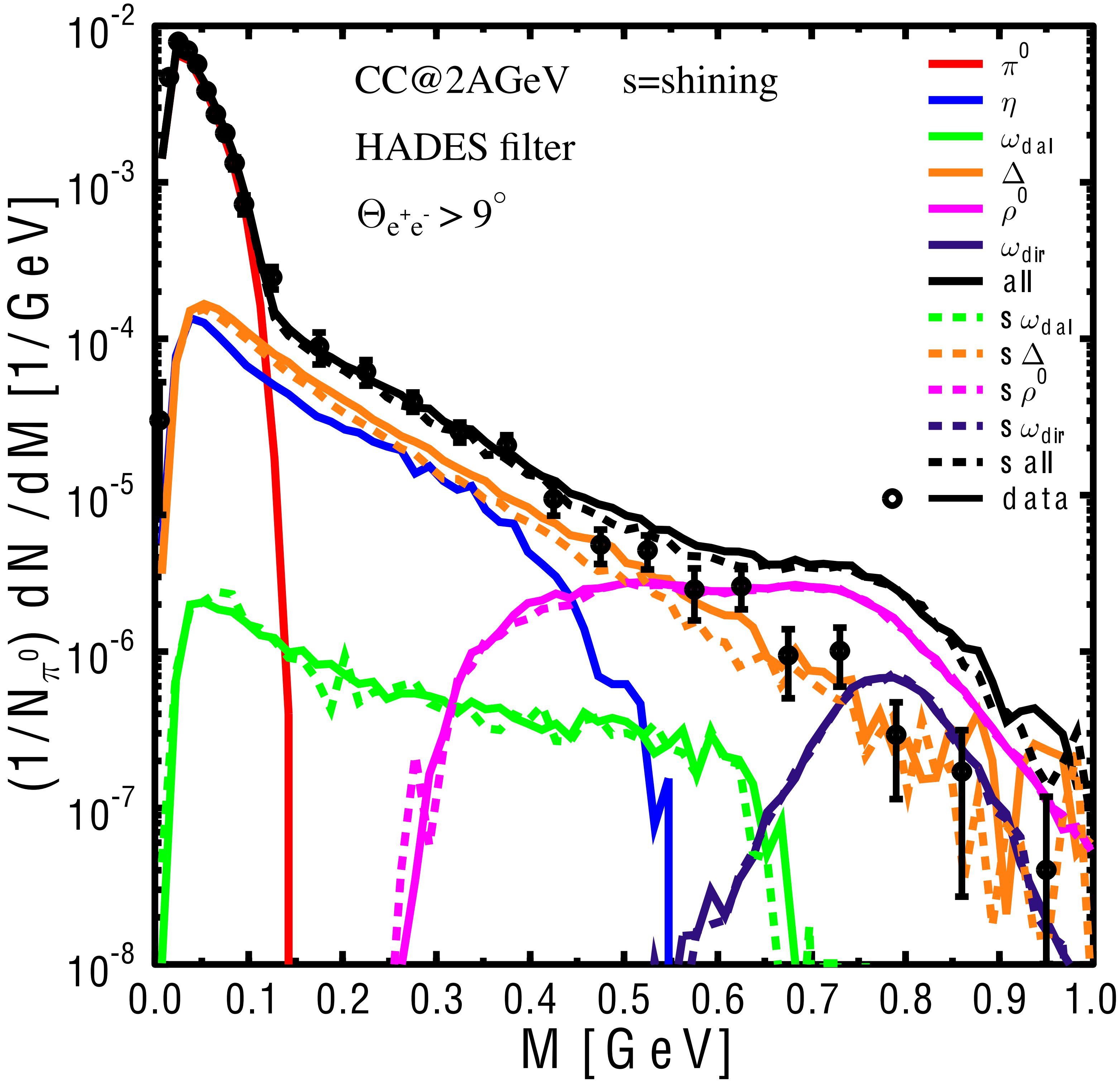
Long thought to be negligible.

Different treatment of proton and neutrons might enhance importance...

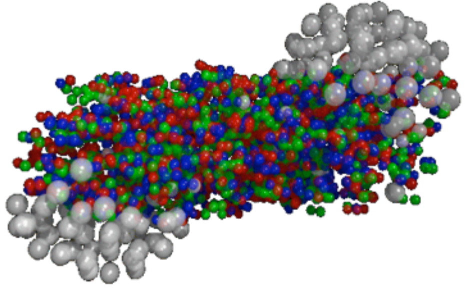
(see Tatyana?)

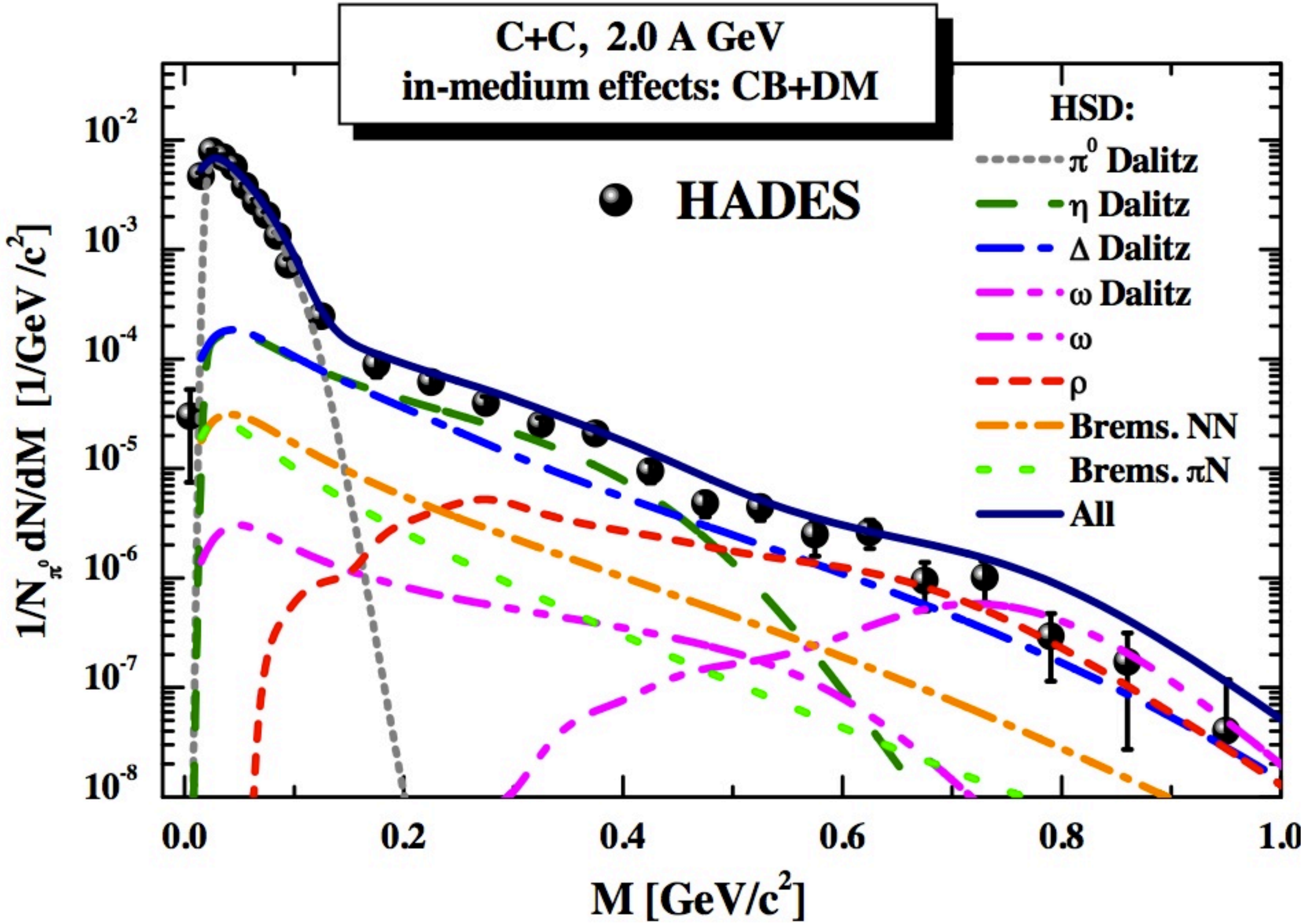


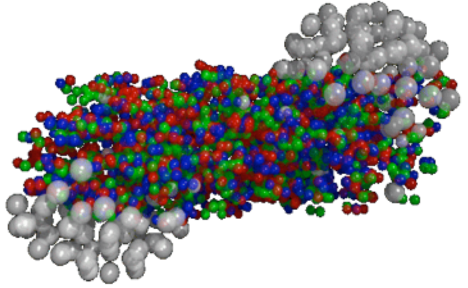
Cocktail ingredients



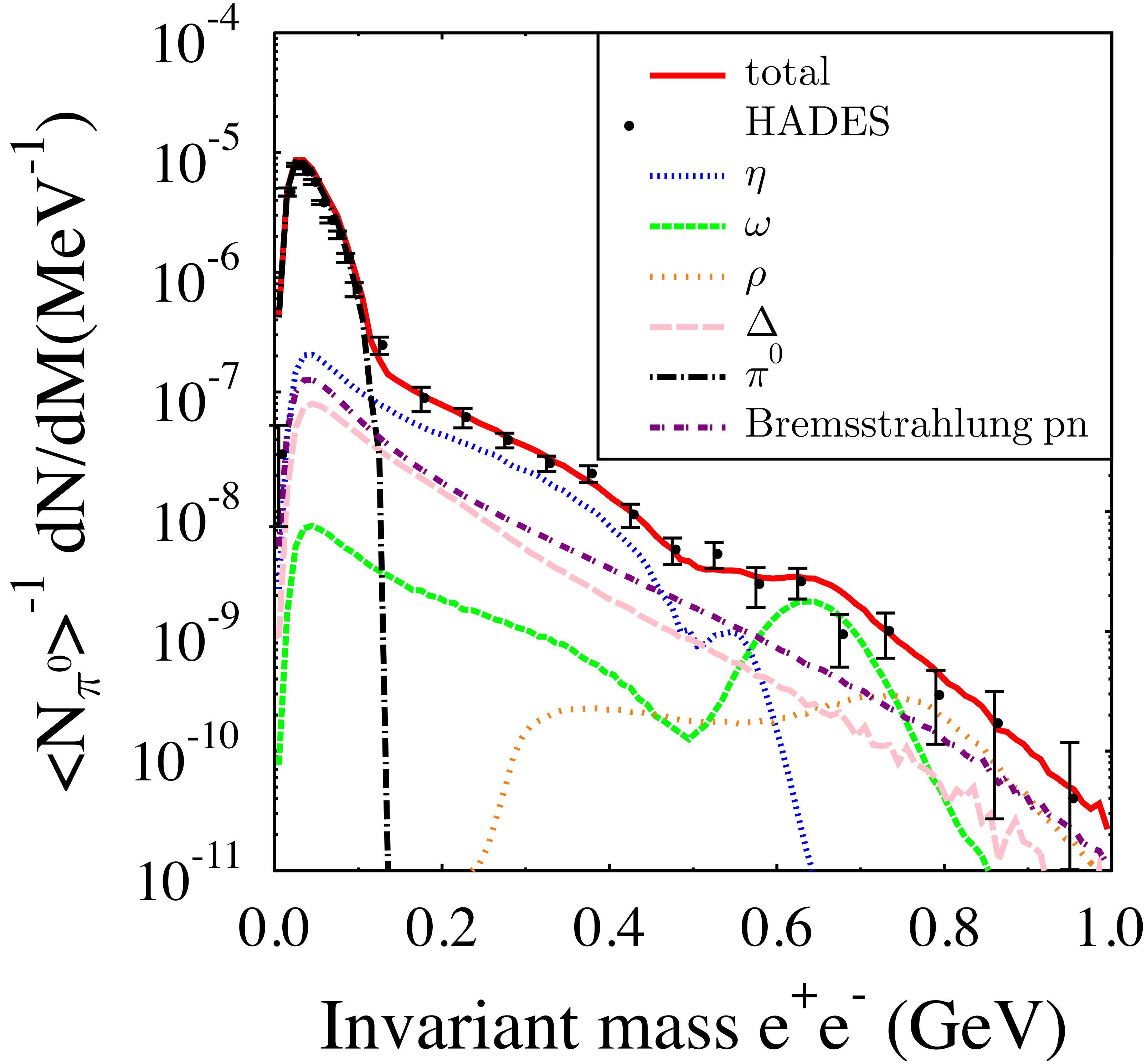
Cocktail ingredients



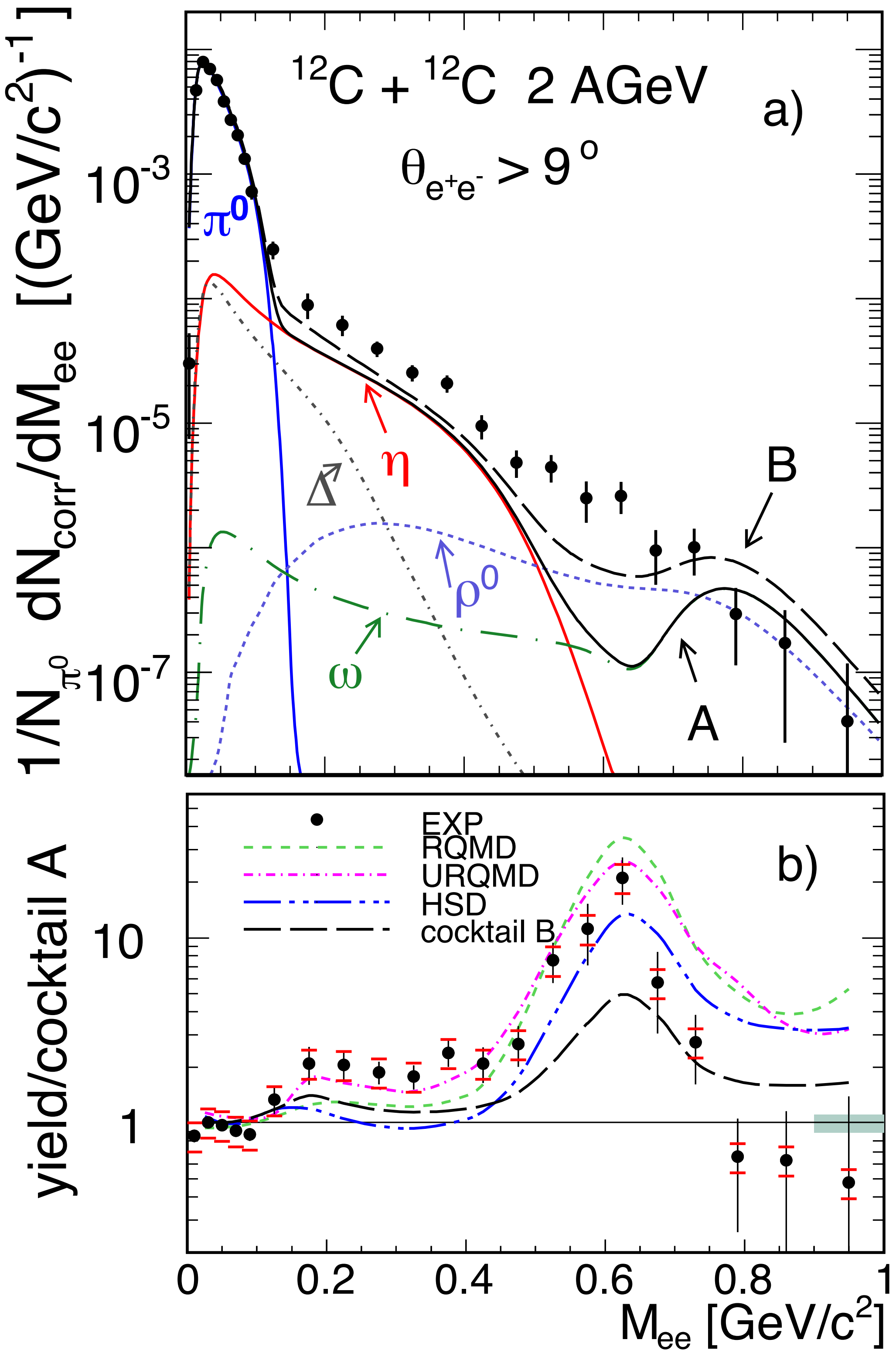




Cocktail ingredients



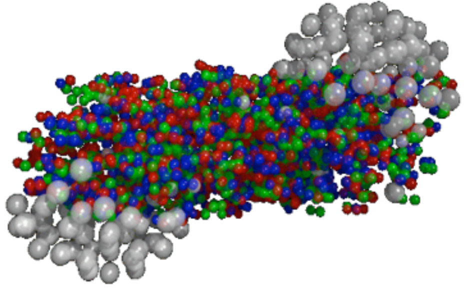
Cocktail ingredients



Different results?

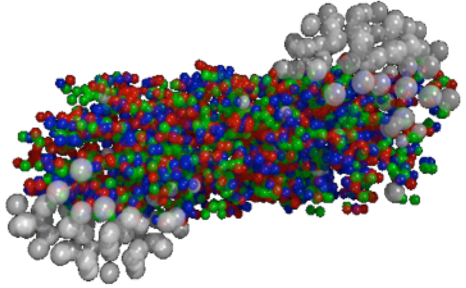
Medium evolution

Dilepton calculation



Take home messages

- Plenty of dilepton sources
- Disentangling them is hard (more by Hendrik tomorrow)
- Often model dependent → experimental input needed



Take home messages

- Plenty of dilepton sources
- Disentangling them is hard (more by Hendrik tomorrow)
- Often model dependent → experimental input needed

Thanks!