Electromagnetic Probes in Heavy-Ion Collisions III

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Outline

1. Transport theory and hydrodynamics
   - phase-space distribution
   - relativistic Boltzmann equation
   - the Boltzmann H theorem
   - hydrodynamics
2. transport simulations (UrQMD and GiBUU as examples)
   - GiBUU
   - UrQMD
   - Dalitz decays of hadron resonances
   - Baryon-resonance model at SIS energies
3. Dileptons in pp, pA, and AA collisions at SIS energies
4. Conclusions and Outlook
5. Quiz

Transport theory and hydrodynamics

Phase-space distribution

- classical many-body system of relativistic particles
- all particles are on their mass shell: $E = E_p := \sqrt{\vec{p}^2 + m^2}$
- Boltzmann equation [dvv80, CK02, Hee13]:
  dynamical equation for phase-space distribution function $f(t, \vec{x}, \vec{p})$
- relativistic covariance of phase-space distribution
  - $f(t, \vec{x}, \vec{p})$ defined as Lorentz scalar quantity
  - particle number N: $\mathrm{d}N = \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{p}\, f(t, \vec{x}, \vec{p})$
  - particle-number four-vector current $(N^\mu) = (n, \vec{N})$

$$N^\mu = \int_{\mathbb{R}^3} \mathrm{d}^3\vec{p} \frac{p^\mu}{E_p} f(t, \vec{x}, \vec{p})$$

  - flow-velocity of fluid cell ("Eckart frame")

$$\vec{v}_{\mathrm{Eck}}(x) = \frac{\vec{N}(x)}{N^0(x)}, \quad u^\mu_{\mathrm{Eck}} = \frac{N^\mu}{\sqrt{N_\mu N^\mu}} = \frac{N^\mu}{n_0}$$

  - n_0: particle density in local fluid (Eckart) restframe

Relativistic Boltzmann equation

- particles moving along trajectories $(\vec{x}(t),\vec{p}(t))$
- for infinitesimal time step $\mathrm{d}t$

$$\mathrm{d}N(t+\mathrm{d}t) = f(t+\mathrm{d}t,\vec{x}+\mathrm{d}t\vec{v},\vec{p}+\mathrm{d}t\vec{F})\mathrm{d}^6\xi\,(t+\mathrm{d}t), \quad \mathrm{d}^6\xi = \mathrm{d}^3\vec{x}\mathrm{d}^3\vec{p}$$

- Jacobian for phase-space volume

$$\mathrm{d}^6\xi\,(t+\mathrm{d}t) = \mathrm{d}^6\xi\,(t)\det\left(\frac{\partial(\vec{x}+\mathrm{d}t\vec{v},\vec{p}+\mathrm{d}t\vec{F})}{\partial(\vec{x},\vec{p})}\right) = \mathrm{d}^6\xi\,(t)\left(1+\mathrm{d}t\vec{\nabla}_p\cdot\vec{F}\right)+\mathscr{O}(\mathrm{d}t^2)$$

- total change of $\mathrm{d}N$

$$\mathrm{d}N(t+\mathrm{d}t)-\mathrm{d}N(t) = \mathrm{d}^6\xi\,(t)\mathrm{d}t\left[\frac{\partial f}{\partial t}+\frac{\vec{p}}{E}\cdot\frac{\partial\vec{f}}{\partial\vec{x}}+\frac{\partial(\vec{F}f)}{\partial\vec{p}}\right]$$

Relativistic Boltzmann equation

- covariance: $\mathrm{d}\tau = \mathrm{d}t\sqrt{1-\vec{v}^2}$ proper time, $\vec{v} = \vec{p}/E_p$, $\sqrt{1-\vec{v}^2} = m/E_p$

$$\mathrm{d}t\left[\frac{\partial f}{\partial t} + \frac{\vec{p}}{E}\cdot\frac{\partial \vec{f}}{\partial \vec{x}}\right] = \mathrm{d}\tau\frac{p^\mu}{m}\frac{\partial f}{\partial x^\mu} \;\Rightarrow\; \text{covariant!}$$

- covariant equation of motion for point particle

$$\frac{\mathrm{d}p^\mu}{\mathrm{d}\tau} = K^\mu, \quad p_\mu p^\mu = m^2 = \text{const} \;\Rightarrow$$

$$K^0 = \frac{\vec{p}}{E_p}\cdot\vec{K} \;\Rightarrow\; \frac{\mathrm{d}\vec{p}}{\mathrm{d}t} = \vec{F} = \vec{K}\frac{m}{E_p}$$

$$\frac{E_p}{m}\vec{\nabla}_p(\vec{F}f) = \frac{\partial}{\partial p^\mu}(K^\mu f) \;\Rightarrow\; \text{covariant!}$$

$$\mathrm{d}N(t+\mathrm{d}t) - \mathrm{d}N(t) = \mathrm{d}t\left[\frac{\partial f}{\partial t} + \frac{\vec{p}}{E}\cdot\frac{\partial \vec{f}}{\partial \vec{x}} + \frac{\partial(\vec{F}f)}{\partial \vec{p}}\right] = \mathrm{d}\tau\left[\frac{p^\mu}{m}\frac{\partial f}{\partial x^\mu} + \frac{\partial(K^\mu f)}{\partial p^\mu}\right]$$

Relativistic Boltzmann equation

- change of particle number due to collisions
  - short-range interactions: collisions at one point (local) in space
  - invariant cross section

$$\mathrm{d}N_{\mathrm{coll}}(p_1',p_2' \leftarrow p_1,p_2) = \mathrm{d}^4x \frac{\mathrm{d}^3\vec{p}_1}{E_1}\frac{\mathrm{d}^3\vec{p}_2}{E_2}\frac{\mathrm{d}^3\vec{p}\,'_1}{E_1'}\frac{\mathrm{d}^3\vec{p}\,'_2}{E_2'} f_1 f_2 W(p_1',p_2' \leftarrow p_1,p_2),$$

$$\mathrm{d}\sigma = \frac{W(p_1',p_2' \leftarrow p_1,p_2)\mathrm{d}^4x\frac{\mathrm{d}^3\vec{p}_1}{E_1}\frac{\mathrm{d}^3\vec{p}_2}{m}\frac{\mathrm{d}^3\vec{p}\,'_1}{E_1'}\frac{\mathrm{d}^3\vec{p}\,'_2}{E_2'} f_1 f_2}{\mathrm{d}^4x\mathrm{d}^3\vec{p}_1 v_{\mathrm{rel}} f_1 \mathrm{d}^3\vec{p}_2 f_2},$$

$$\mathrm{d}\sigma = \frac{\mathrm{d}^3\vec{p}\,'_1}{E_1'}\frac{\mathrm{d}^3\vec{p}\,'_2}{E_2'}\frac{W(p_1',p_2' \leftarrow p_1,p_2)}{I}, \quad I = \sqrt{(p_1\cdot p_2)^2 - m^4}$$

  - important: v_{rel} is velocity of particle 1 in rest frame of particle 2
  - from relativistic covariance (or unitarity of S-matrix!) $\Rightarrow$ detailed-balance relation

$$W(p_1',p_2' \leftarrow p_1,p_2) = W(p_1,p_2 \leftarrow p_1',p_2')$$

- Boltzmann equation (manifestly covariant form)

$$p^\mu \frac{\partial f}{\partial x^\mu} + m\frac{\partial (K^\mu f)}{\partial p^\mu} = \frac{1}{2}\int_{\mathbb{R}^3}\frac{\mathrm{d}^3\vec{p}_2}{E_2}\int_{\mathbb{R}^3}\frac{\mathrm{d}^3\vec{p}\,'_1}{E_1'}\int_{\mathbb{R}^3}\frac{\mathrm{d}^3\vec{p}\,'_2}{E_2'} W(p_1',p_2' \leftarrow p,p_2)(f_1'f_2' - ff_2)$$

- collision integral: "gain minus loss"

Entropy

- input from quantum mechancis: particle in a cubic box (periodic boundary cond.)

$$\vec{p} = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3$$

- $\Delta^6\xi_j = L^3\Delta^3\vec{p}$ ("microscopically large, macroscopically small")
- contains G_j single-particle states (g: degeneracy due to spin, isospin, ...)

$$G_j = g\frac{\Delta^6\xi_j}{(2\pi)^3}$$

- statistical weight for N_j particles in $\Delta^6\xi_j$:
- factor $1/N_j!$: indistinguishability of particles

$$\Delta\Gamma_j = \frac{1}{N_j!}G_j^{N_j}$$

- entropy a la Boltzmann

$$S = \sum_j \ln\Delta\Gamma_j \simeq \sum_j [N_j \ln G_j - N_j(\ln N_j - 1)]$$

$$= -\int \mathrm{d}^3\vec{x}\,\mathrm{d}^3\vec{p}\, f(x,p)\{\ln[(2\pi)^3 f(x,p)/g] - 1\}$$

The Boltzmann H theorem

- H = greek Eta: Boltzmann's notation for entropy
- covariant description of entropy: entropy four-flow

$$S^\mu(x) = -\int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}}{E} p^\mu f(x,p)\{\ln[(2\pi)^3 f(x,p)/g] - 1\}$$

- Boltzmann equation + symmetries of $W(p_1' p_2' \leftarrow p_1 p_2)$

$$\frac{\partial S^\mu}{\partial x^\mu} := \zeta = +\frac{1}{4}\int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}}{E}\int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}_2}{E_2}\int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}\,'_1}{E'_1}\int_{\mathbb{R}^3} \frac{\mathrm{d}^3\vec{p}\,'_2}{E'_2} ff_2 \times \left[\frac{f'_1 f'_2}{ff_2} - \ln\left(\frac{f'_1 f'_2}{ff_2}\right) - 1\right] W(p'_1 p'_2 \leftarrow p, p_1) \geq 0$$

- (on average) entropy can never decrease with time!
- equilibrium $\Leftrightarrow$ S maximal!
- bracket must vanish $\Rightarrow$ Maxwell-Boltzmann distribution

$$f_{\mathrm{eq}}(x,p) = \frac{g}{(2\pi)^3}\exp\left[-\beta(x)\left(u(x)\cdot p - \mu(x)\right)\right], \quad p^0 = E = \sqrt{m^2 + \vec{p}^2}$$

- $\beta = 1/T$: inverse temperature, u: fluid four-velocity, μ: chemical potential
- temperature, chemical potential are Lorentz scalars!

Hydrodynamics

- in the limit of very small mean-free path: system in local thermal equilibrium
- switch to ideal hydrodynamics description
- forget about "particles" $\Rightarrow$ fluid description
- equations of motion for $\vec{v}(t,\vec{x})$: conservation laws

$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0$$

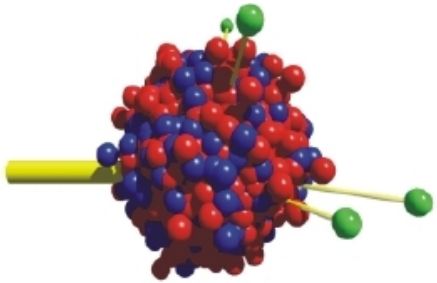
- N^μ: net-baryon number, $T^{\mu\nu}$: energy-momentum tensor
- ideal hydrodynamics

$$N^\mu = n u^\mu, \quad T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu - P \eta^{\mu\nu}$$
$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu\nu} = 0$$

- n: proper net-baryon density, ε: proper energy density, P: pressure
- 5 equations of motion, 6 unknowns: $\vec{v}$, n, ε, P
- need also equation of state $\varepsilon = \varepsilon(P)$
- hadron-resonance gas EoS (low energies)
  lQCD based cross-over phase transition (high energies)

Transport simulations (UrQMD and GiBUU)

The GiBUU Model



GiBUU

The Giessen Boltzmann-Uehling-Uhlenbeck Project

- Boltzmann-Uehling-Uhlenbeck (BUU) framework for hadronic transport
- reaction types: pA, πA, γA, eA, νA, AA
- open-source modular Fortran 95/2003 code
- version control via Subversion
- publicly available realeases: `https://gibuu.hepforge.org`
- Review on hadronic transport (GiBUU): [BGG+12]
- all calculations for dileptons: J. Weil

The Boltzmann-Uehling-Uhlenbeck Equation

- time evolution of phase-space distribution functions

$$[\partial_t + (\vec{\nabla}_p H_i)\cdot\vec{\nabla}_x - (\vec{\nabla}_x H_i)\cdot\vec{\nabla}_p] f_i(t,\vec{x},\vec{p}) = I_{\rm coll}[f_1,\ldots,f_i,\ldots,f_j]$$

- use Monte-Carlo simulation for test particles
- transition probability W in collision term used to define stochastic process ("random numbers" on the computer)
- Hamiltonian H_i
  - selfconsistent hadronic mean fields, Coulomb potential, "off-shell potential"
- collision term $I_{\rm coll}$
  - two- and three-body decays/collisions
  - multiple coupled-channel problem
  - resonances described with relativistic Breit-Wigner distribution

$$\mathscr{A}(x,p) = -\frac{1}{\pi}\frac{\mathrm{Im}\,\Pi}{(p^2 - M^2 - \mathrm{Re}\,\Pi)^2 + (\mathrm{Im}\,\Pi)^2}; \quad \mathrm{Im}\,\Pi = -\sqrt{p^2}\Gamma$$

  - off-shell propagation: test particles with off-shell potential

Ultra-relativistic Molecular Dynamics (UrQMD)

- transport model for hadrons
  - all hadrons (resonances) with masses up to 2.2 GeV included
  - cross sections adapted to experimental data
  - no explicit medium modifications of hadrons implemented
- quantum molecular dynamics
  - hadrons represented by quantum-mechanical Gaussian wave packets

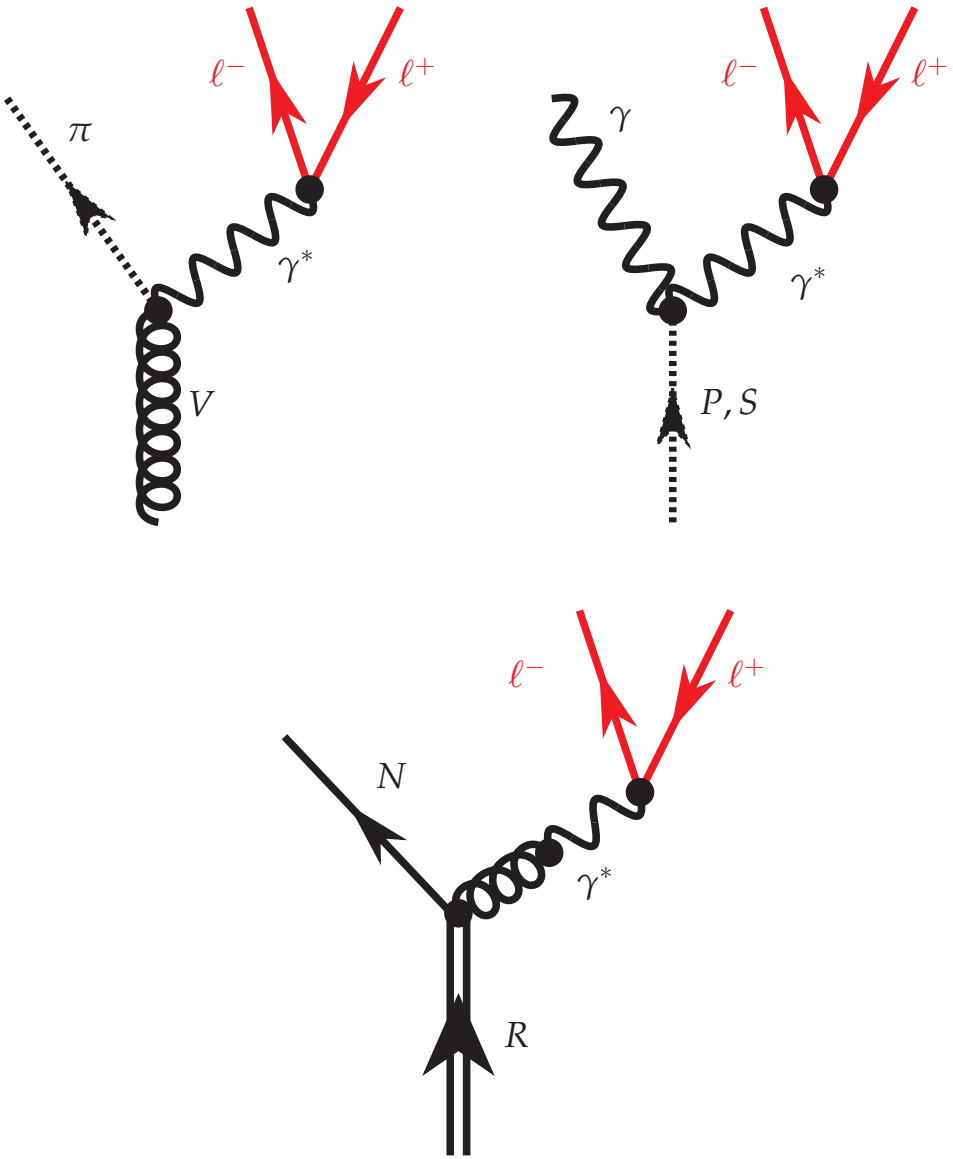
$$\psi_i(t,\vec{x}) = \left(\frac{2}{\pi L}\right)^{1/4} \exp\left\{-\frac{2}{L}[\vec{x}-q_i(t)]^2 + \mathrm{i}\vec{p}_i(t)\cdot\vec{x}\right\}$$

  - N-body state = product state (no Bose/Fermi symmetrization!)
  - classical equations of motion from Lagrangian

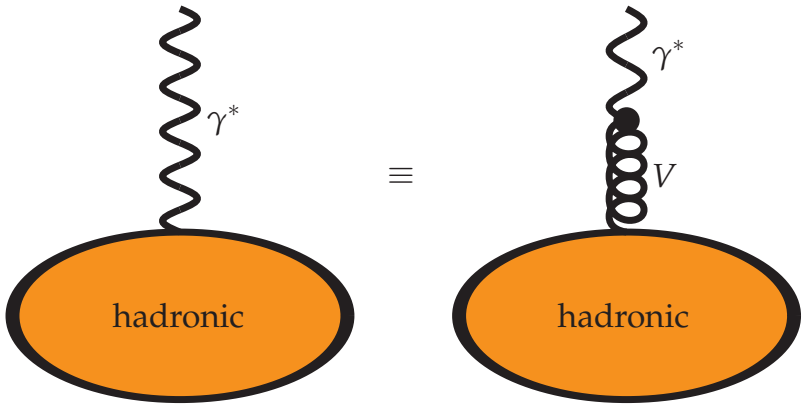
$$L = \sum_i \left[-\dot{q}_i\cdot\vec{p}_i + \langle T_i\rangle + \frac{1}{2}\sum_{ij}\left\langle V_{ij}^{(2)}\right\rangle - \frac{3}{2Lm}\right]$$

  - interaction potentials: similar resonance model as in GiBUU
- all calculations for dileptons: S. Endres

Dalitz decays



- Dalitz decay:
  1 particle $\rightarrow$ 3 particles
- V: $\omega \rightarrow \pi + \gamma^* \rightarrow \pi + \ell^+ + \ell^-$
- P, S: $\pi, \eta \rightarrow \gamma + \gamma^* \rightarrow \gamma + \ell^+ + \ell^-$
- R: Baryon resonances
  $\Delta, N^* \rightarrow N + V \rightarrow N + \gamma^* \rightarrow$
  $N + \ell^+ + \ell^-$
- vector-meson dominance

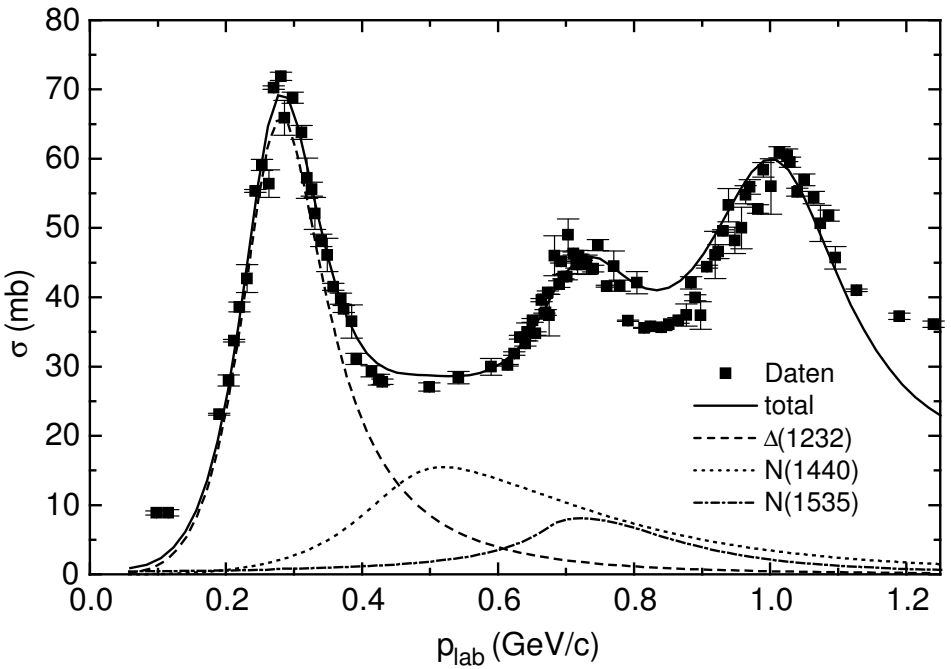


Resonance Model

- reactions dominated by resonance scattering: $ab \to R \to cd$
- Breit-Wigner cross-section formula

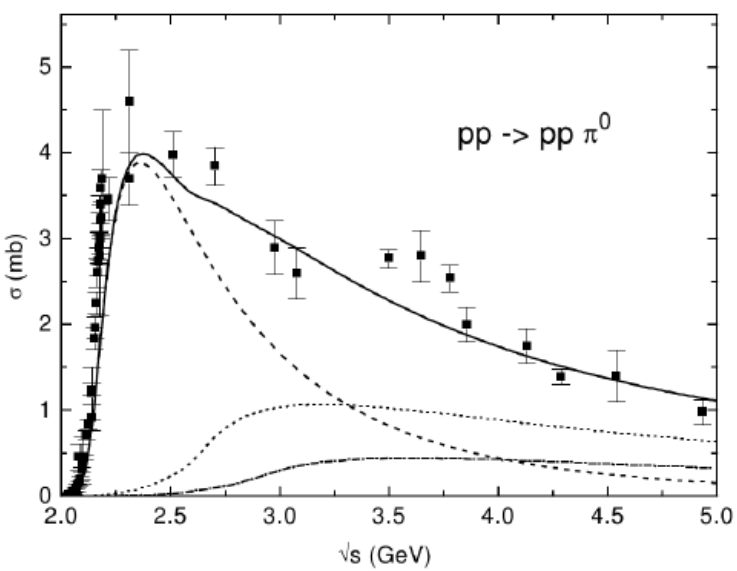
$$\sigma_{ab\to R\to cd} = \frac{2s_R+1}{(2s_a+1)(2s_b+1)} \frac{4\pi}{p_{\text{lab}}^2} \frac{s\Gamma_{ab\to R}\Gamma_{R\to cd}}{(s-m_R^2)^2 + s\Gamma_{\text{tot}}^2}$$

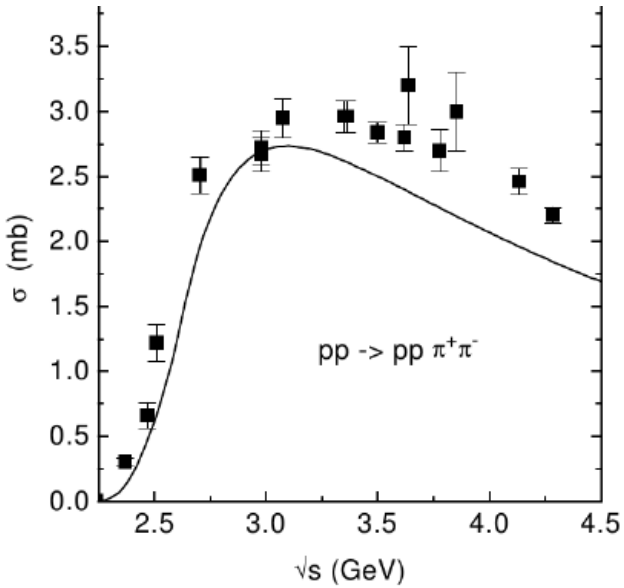
- applicable for low-energy nuclear reactions $E_{\text{kin}} \lesssim 1.1$ GeV
- example: $\sigma_{\pi^- p\to\pi^- p}$ [Teis (PhD thesis 1996), data: Baldini et al, Landolt-Börnstein **12** (1987)]

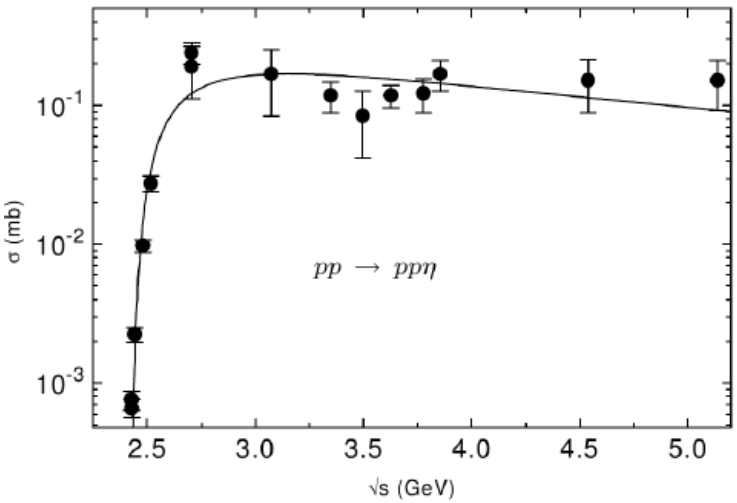


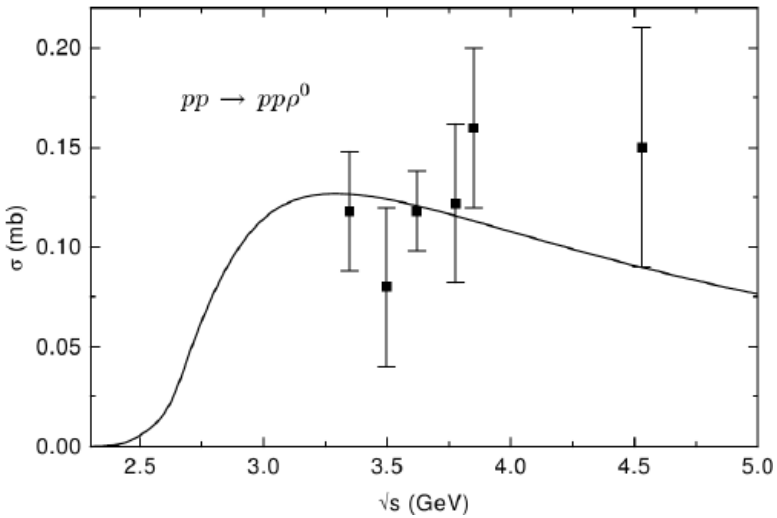
GiBUU: Resonance Model

- further cross sections









GiBUU: Extension to HADES energies

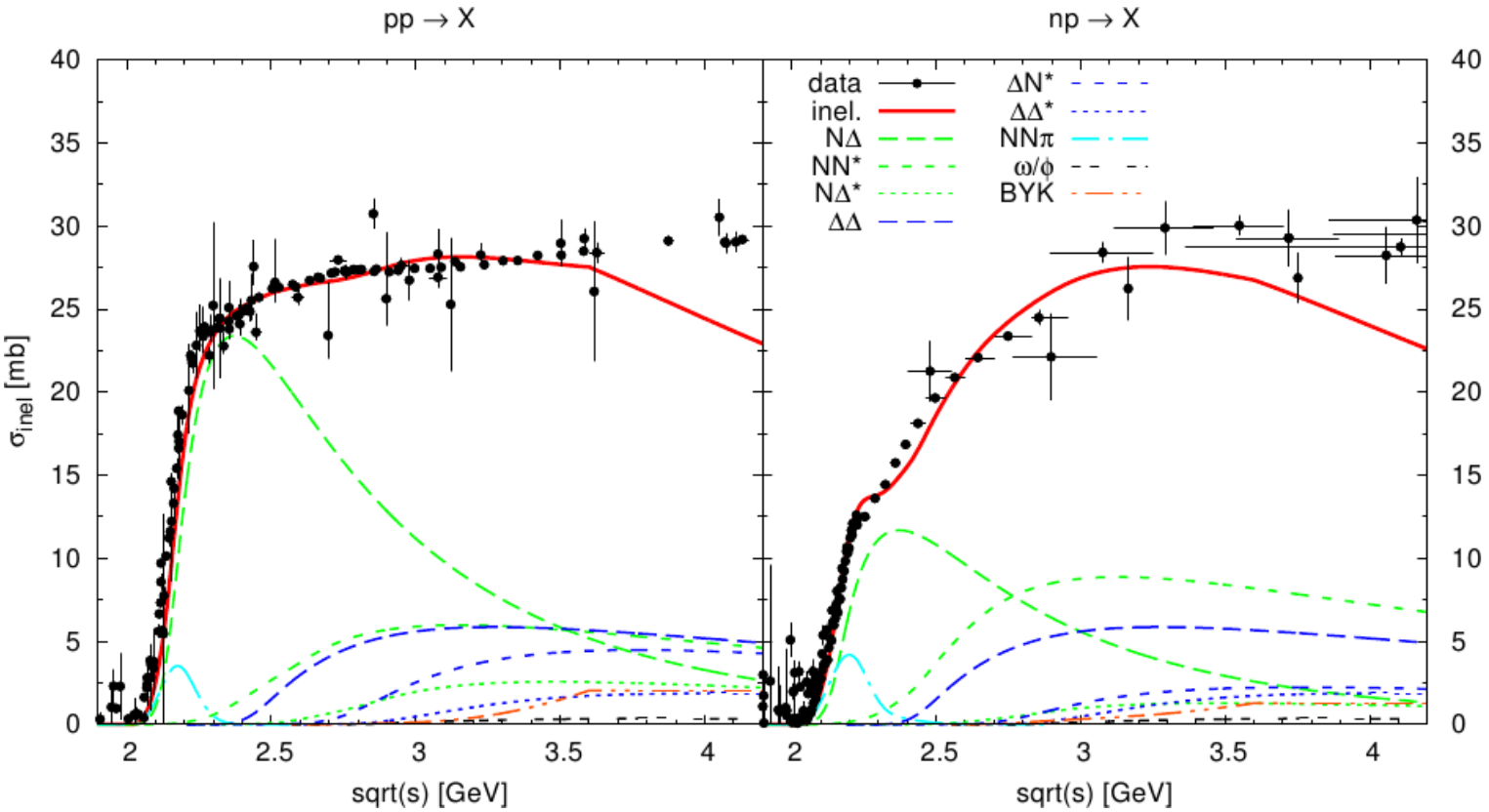
- [WHM12, WM13]
- keep same resonances (parameters from Manley analysis)

	rating	M_0 [MeV]	Γ_0 [MeV]	$\lvert\mathcal{M}^2\rvert/16\pi$ [mb GeV^2] NR	ΔR	branching ratio in % πN	ηN	$\pi\Delta$	ρN	σN	$\pi N^*(1440)$	$\sigma\Delta$
$P_{11}(1440)$	****	1462	391	70	—	69	—	22_P	—	9	—	—
$S_{11}(1535)$	***	1534	151	8	60	51	43	—	2_S+1_D	1	2	—
$S_{11}(1650)$	****	1659	173	4	12	89	3	2_D	3_D	2	1	—
$D_{13}(1520)$	****	1524	124	4	12	59	—	5_S+15_D	21_S	—	—	—
$D_{15}(1675)$	****	1676	159	17	—	47	—	53_D	—	—	—	—
$P_{13}(1720)$	*	1717	383	4	12	13	—	—	87_P	—	—	—
$F_{15}(1680)$	****	1684	139	4	12	70	—	10_P+1_F	5_P+2_F	12	—	—
$P_{33}(1232)$	****	1232	118	OBE	210	100	—	—	—	—	—	—
$S_{31}(1620)$	**	1672	154	7	21	9	—	62_D	25_S+4_D	—	—	—
$D_{33}(1700)$	*	1762	599	7	21	14	—	74_S+4_D	8_S	—	—	—
$P_{31}(1910)$	****	1882	239	14	—	23	—	—	—	—	67	10_P
$P_{33}(1600)$	***	1706	430	14	—	12	—	68_P	—	—	20	—
$F_{35}(1905)$	***	1881	327	7	21	12	—	1_P	87_P	—	—	—
$F_{37}(1950)$	****	1945	300	14	—	38	—	18_F	—	—	—	44_F

- production channels in Teis: $NN \to N\Delta$, $NN \to NN^*, N\Delta^*$, $NN \to \Delta\Delta$
- extension to $NN \to \Delta N^*, \Delta\Delta^*$, $NN \to NN\pi$, $NN \to NN\rho, NN\omega, NN\pi\omega, NN\phi$, $NN \to BYK$ ($B = N, \Delta$, $Y = \Lambda, \Sigma$)

GiBUU Extension to HADES energies

- good description of total pp, pn (inelastic) cross section

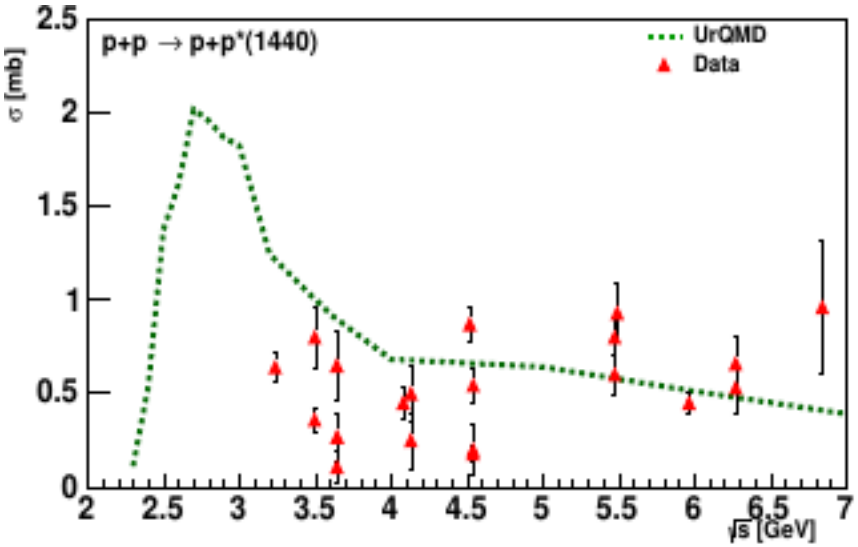


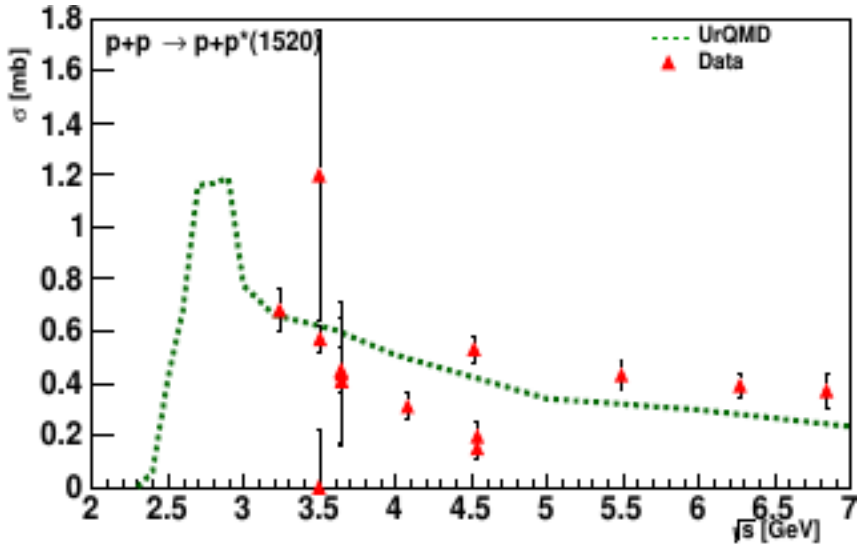
- dilepton sources
  - Dalitz decays: $\pi^0, \eta \to \gamma \ell^+\ell^-$; $\omega \to \pi^0 \ell^+\ell^-$, $\Delta \to N \ell^+\ell^-$
  - $\rho, \omega, \phi \to \ell^+\ell^-$: invariant mass $\ell^+\ell^-$ spectra $\Rightarrow$ spectral properties of vector mesons
  - for details, see [WHM12]

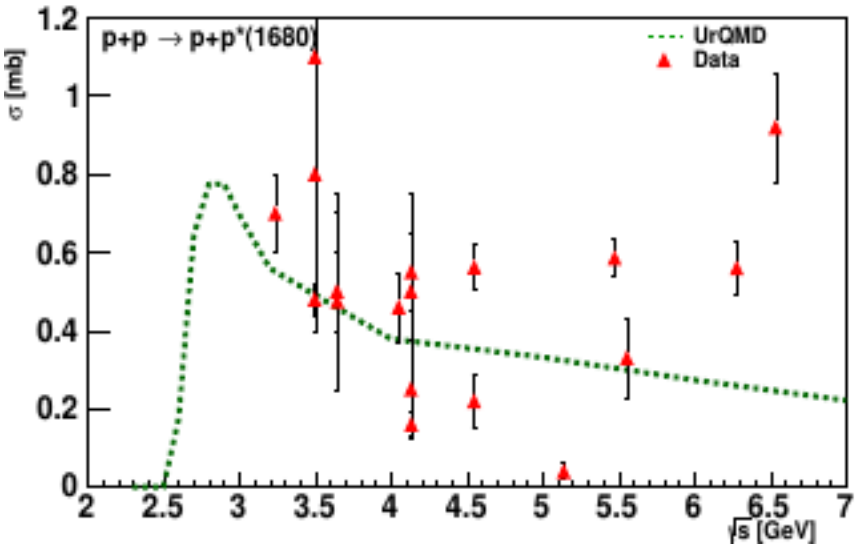
UrQMD: Baryon resonances

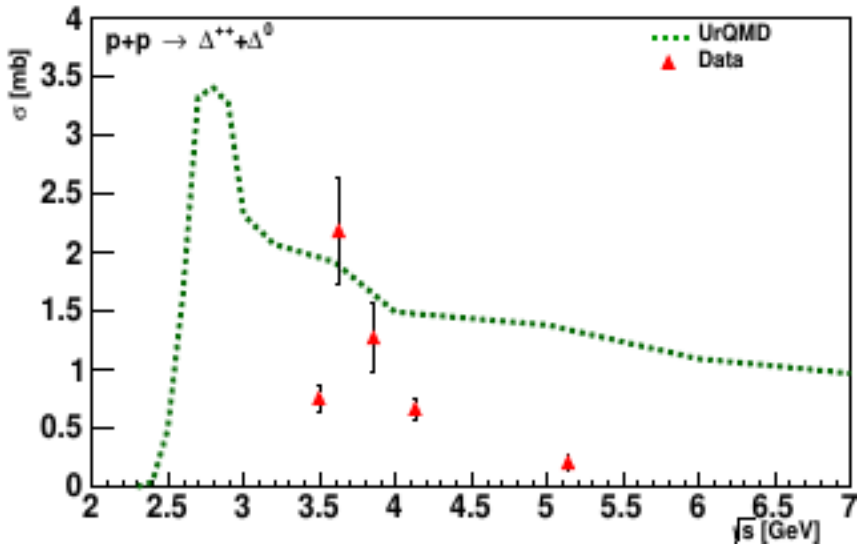
Resonance	Mass	Width	$N\pi$	$N\eta$	$N\omega$	$N\varrho$	$N\pi\pi$	$\Delta_{1232}\pi$	$N^*_{1440}\pi$	ΛK	ΣK	$f_0 N$	$a_0 N$
N^*_{1440}	1.440	350	0.65				0.10	0.25					
N^*_{1520}	1.515	120	0.60			0.15	0.05	0.20					
N^*_{1535}	1.550	140	0.60	0.30			0.05		0.05				
N^*_{1650}	1.645	160	0.60	0.06		0.06	0.04	0.10	0.05	0.07	0.02		
N^*_{1675}	1.675	140	0.40					0.55	0.05				
N^*_{1680}	1.680	140	0.60			0.10	0.10	0.15	0.05				
N^*_{1700}	1.730	150	0.05			0.20	0.30	0.40	0.05				
N^*_{1710}	1.710	500	0.16	0.15		0.05	0.21	0.20	0.10	0.10	0.03		
N^*_{1720}	1.720	550	0.10			0.73	0.05			0.10	0.02		
N^*_{1900}	1.850	350	0.30	0.14	0.39	0.15				0.02			
N^*_{1990}	1.950	500	0.12			0.43	0.19	0.14	0.05	0.03		0.04	
N^*_{2080}	2.000	550	0.42	0.04	0.15	0.12	0.05	0.10		0.12			
N^*_{2190}	2.150	470	0.29			0.24	0.10	0.15	0.05	0.12			
N^*_{2220}	2.220	550	0.29		0.05	0.22	0.17	0.20		0.12			
N^*_{2250}	2.250	470	0.18			0.25	0.20	0.20	0.05	0.12			
Δ_{1232}	1.232	115	1.00										
Δ^*_{1600}	1.700	350	0.10					0.65	0.25				
Δ^*_{1620}	1.675	160	0.15			0.05		0.65	0.15				
Δ^*_{1700}	1.750	350	0.20			0.25		0.55					
Δ^*_{1900}	1.840	260	0.25			0.25		0.25	0.25				
Δ^*_{1905}	1.880	350	0.18			0.80		0.02					
Δ^*_{1910}	1.900	250	0.30			0.10		0.35	0.25				
Δ^*_{1920}	1.920	200	0.27					0.40	0.30	0.03			
Δ^*_{1930}	1.970	350	0.15			0.22		0.20	0.28	0.15			
Δ^*_{1950}	1.990	350	0.38			0.08		0.20	0.18	0.12			0.04

UrQMD: Baryon resonances



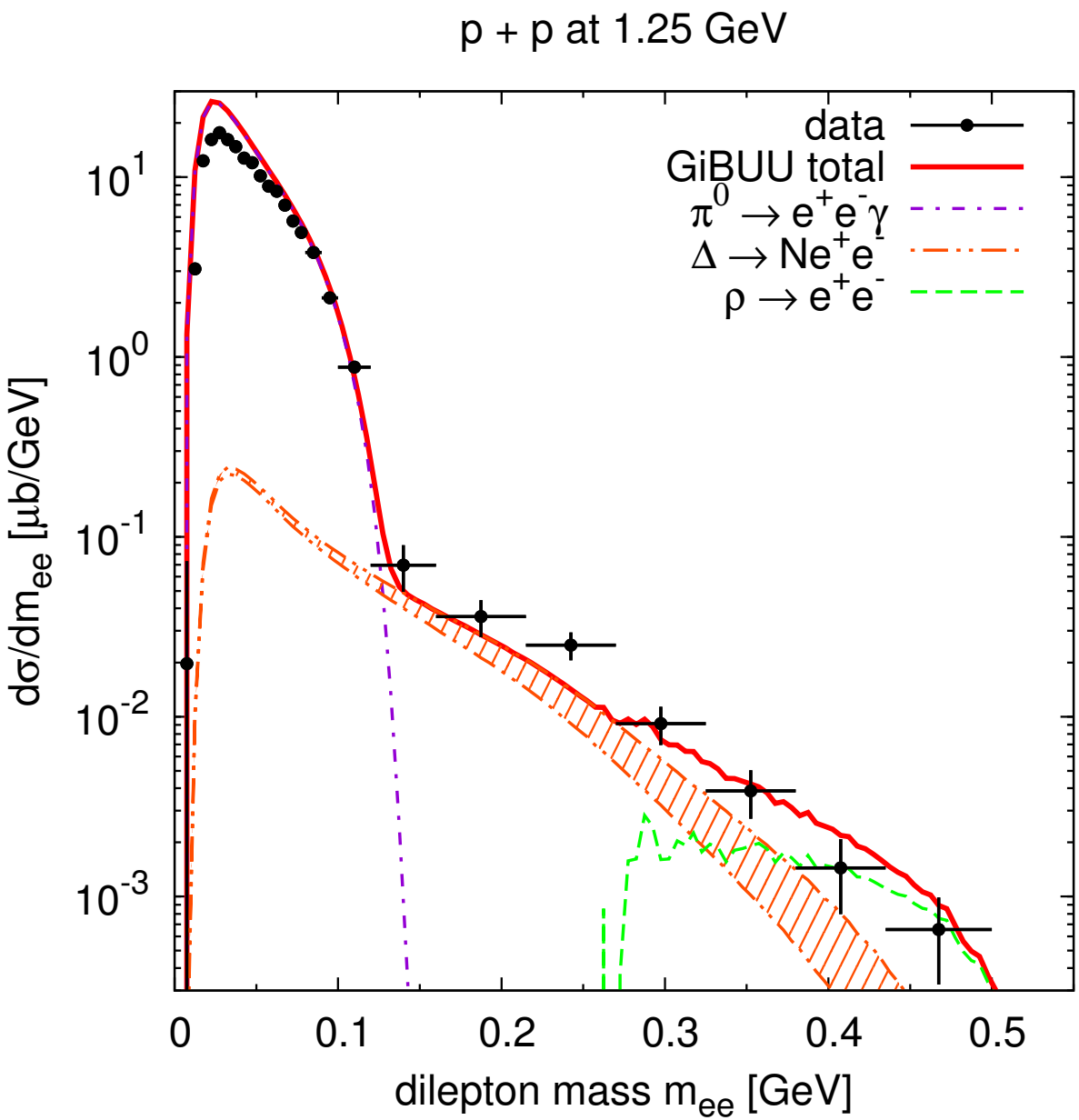




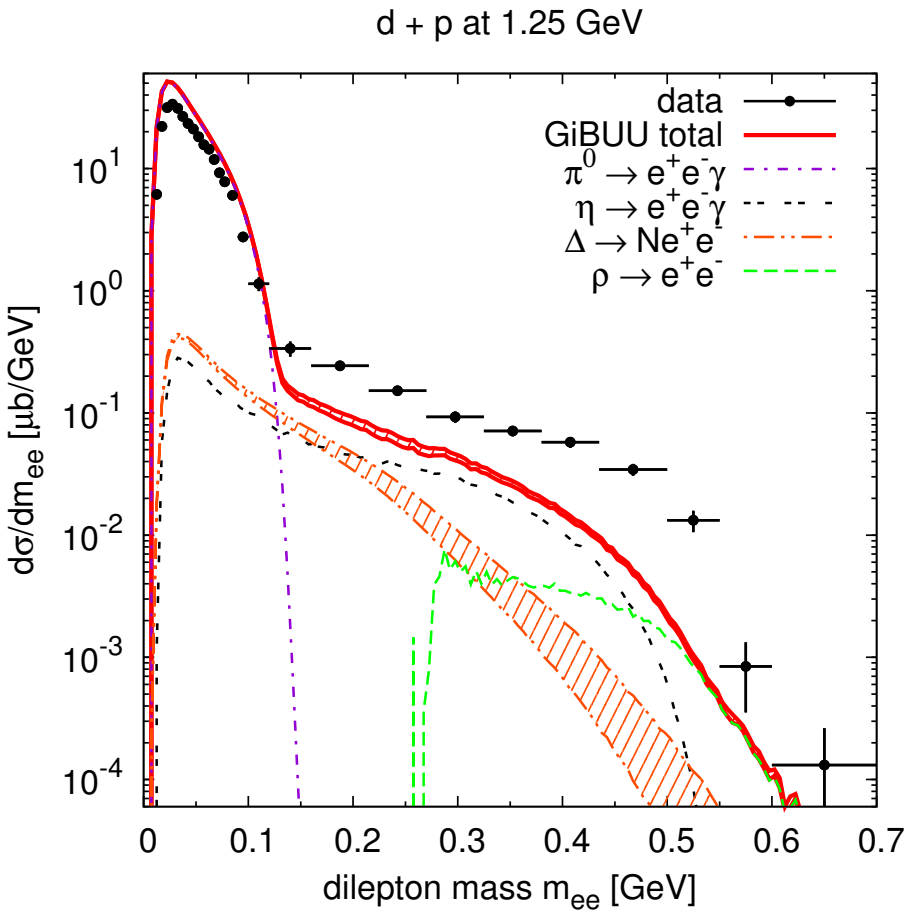


Dileptons in pp, pA, and AA collisions at SIS energies

GiBUU: p p at HADES ($E_{\mathrm{kin}} = 1.25$ GeV)

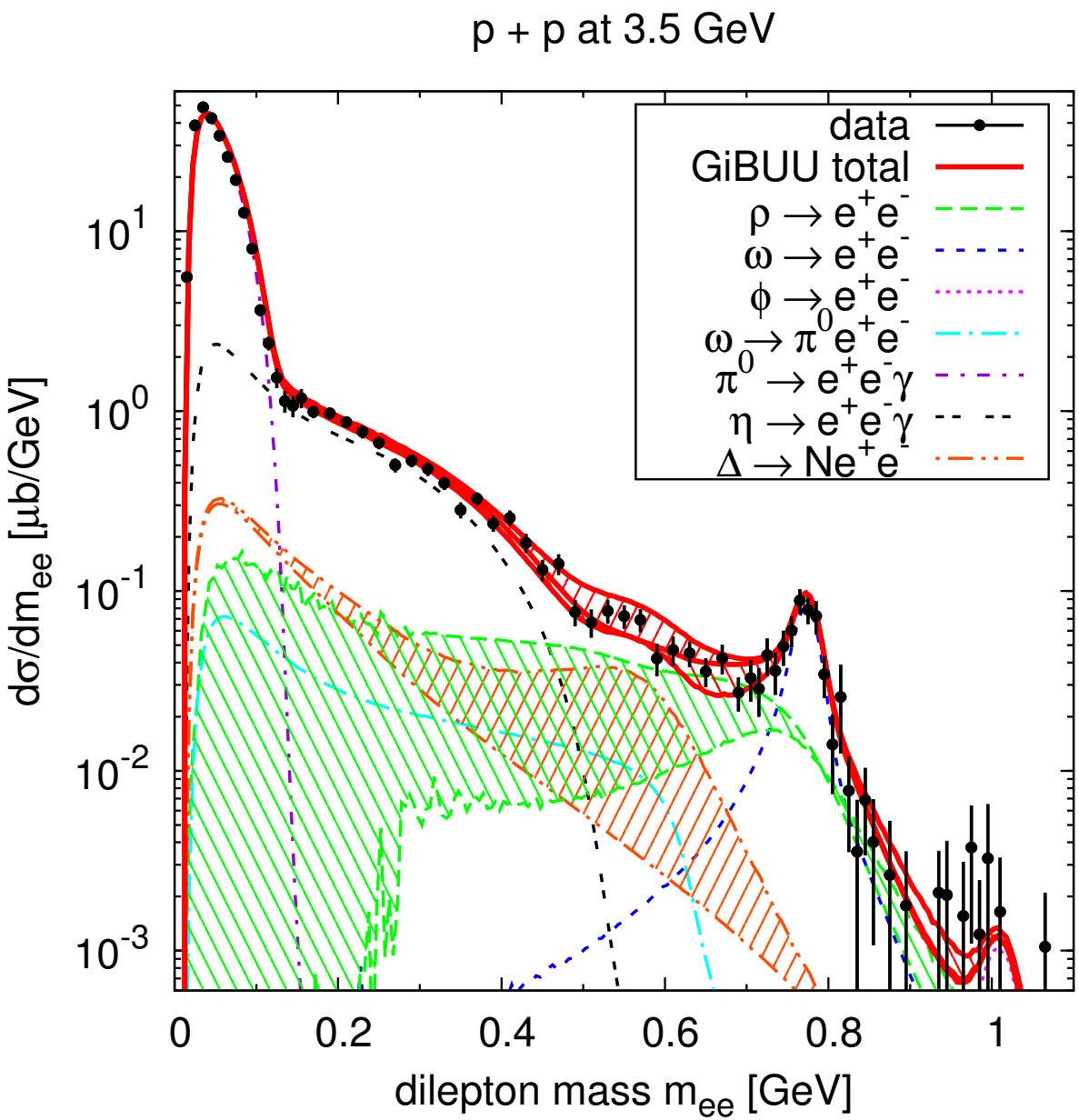


d p at HADES ($E_{\mathrm{kin}} = 1.25$ GeV)

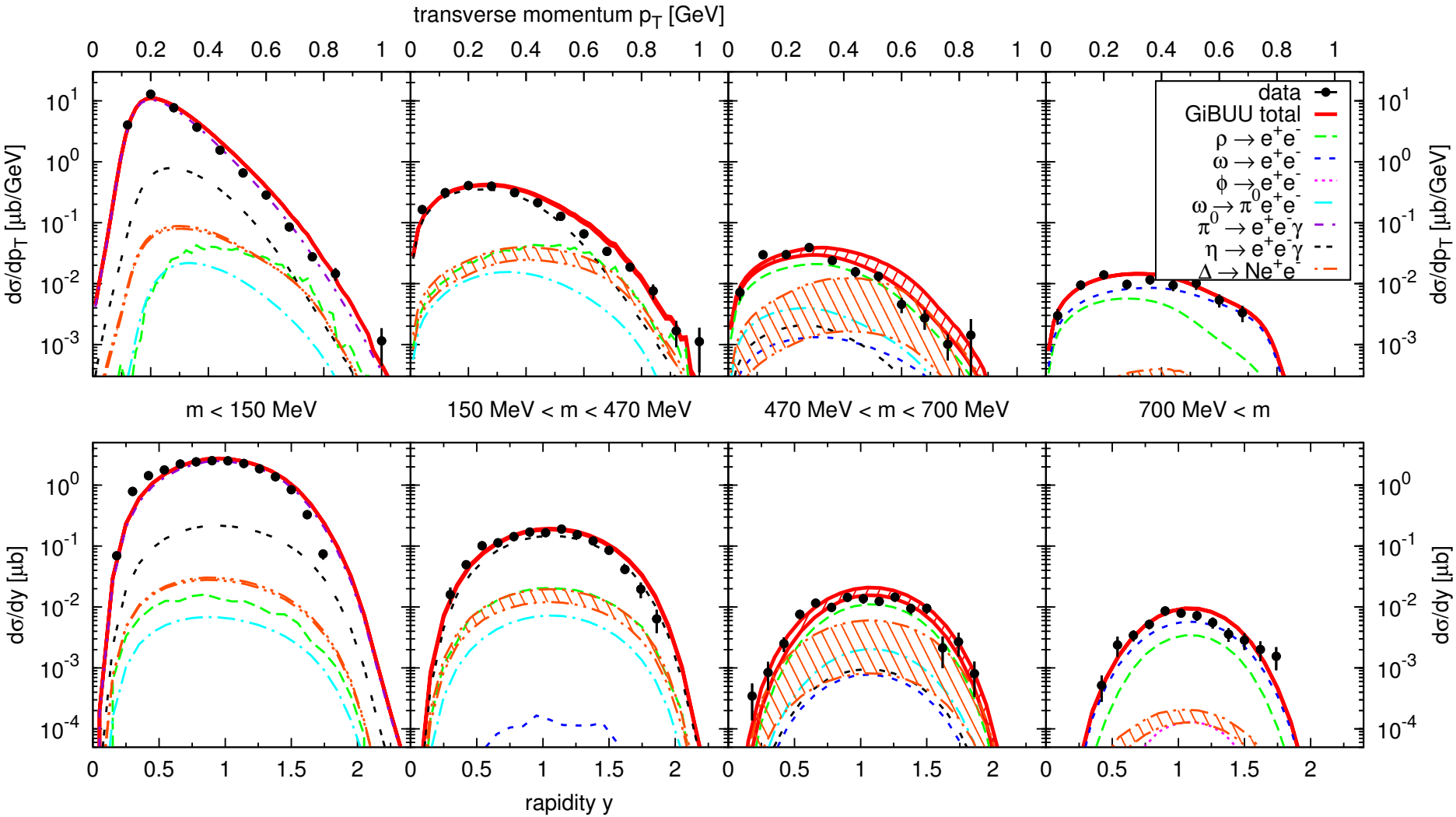


- triggered on forward protons $\rightarrow$ quasifree np scattering
- model uncertainties:
  - ρ production through $D_{13}(1525)$ (isospin symmetric?)
  - $S_{11}(1535)$ [enhanced in np; (from η production)]
  - d-wave function treatable as quasiclassical "distribution"?
  - bremsstrahlung contributions

GiBUU: p p at HADES ($E_{\mathrm{kin}} = 3.5$ GeV)

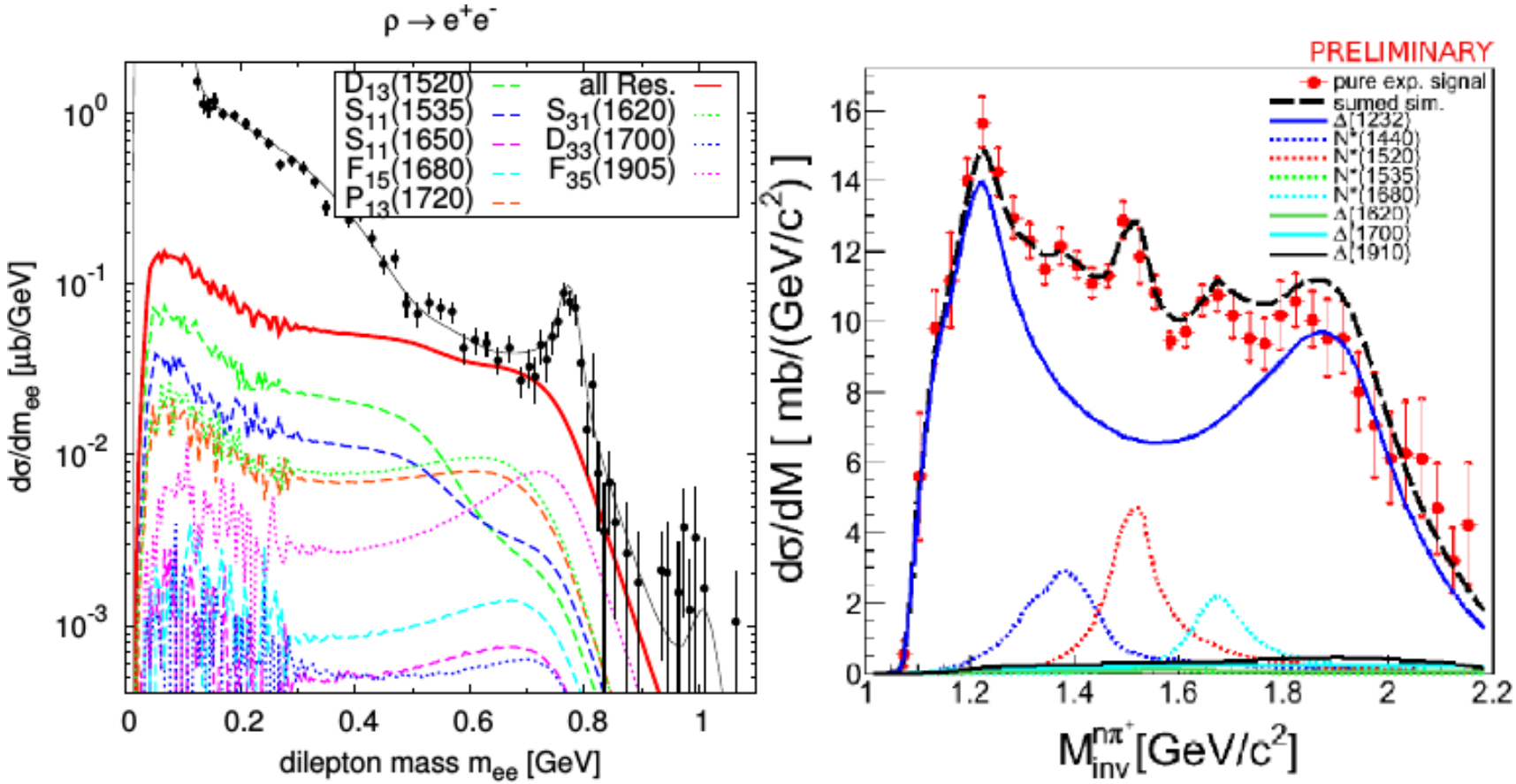


GiBUU: p p at HADES ($E_{\mathrm{kin}} = 3.5$ GeV)



GiBUU: "ρ meson" in pp

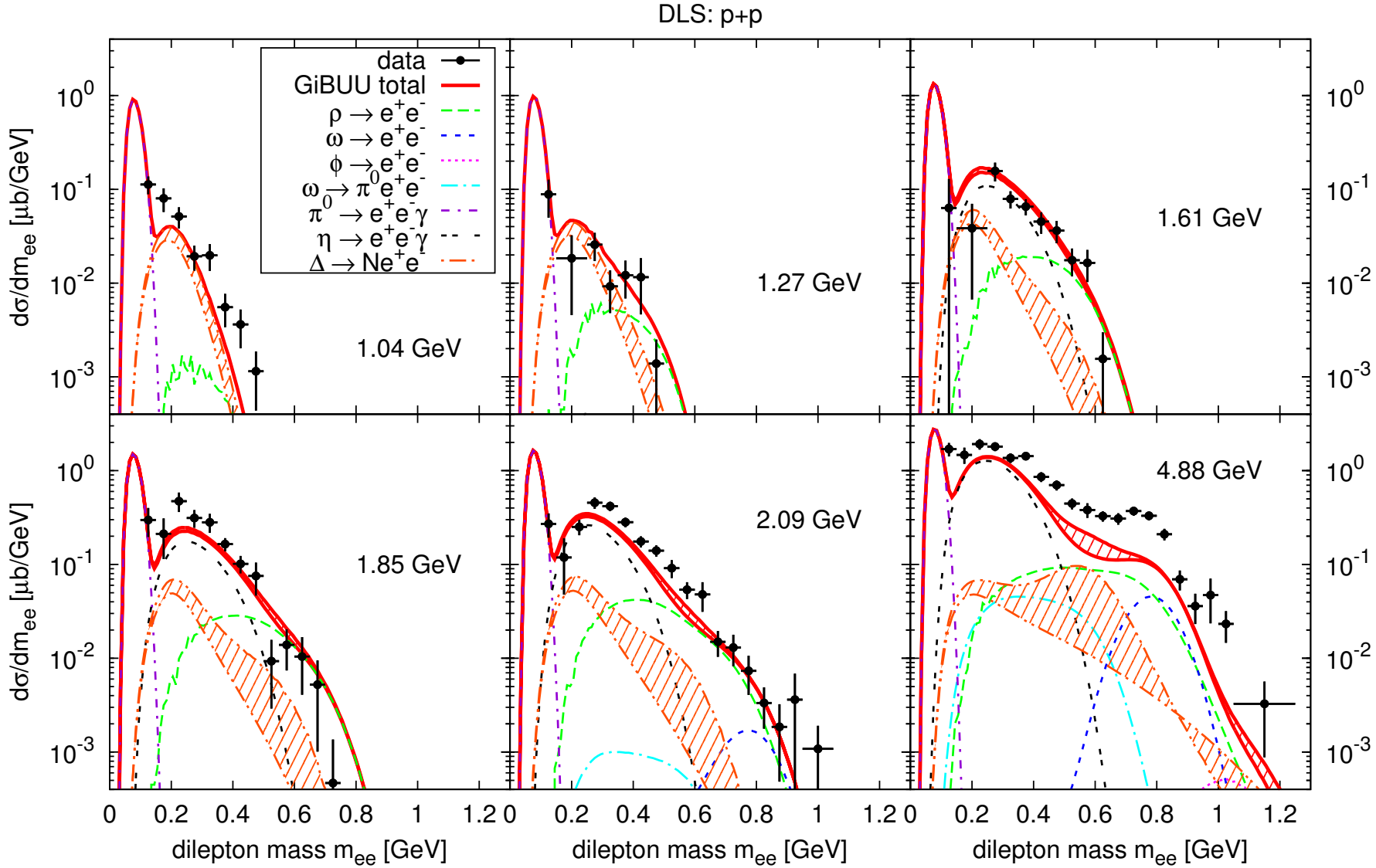
- production through hadron resonances
  $NN \to NR \to NN\rho$, $NN \to N\Delta \to NN\pi\rho$



- "ρ"-line shape "modified" already in elementary hadronic reactions
- due to production mechanism via resonances

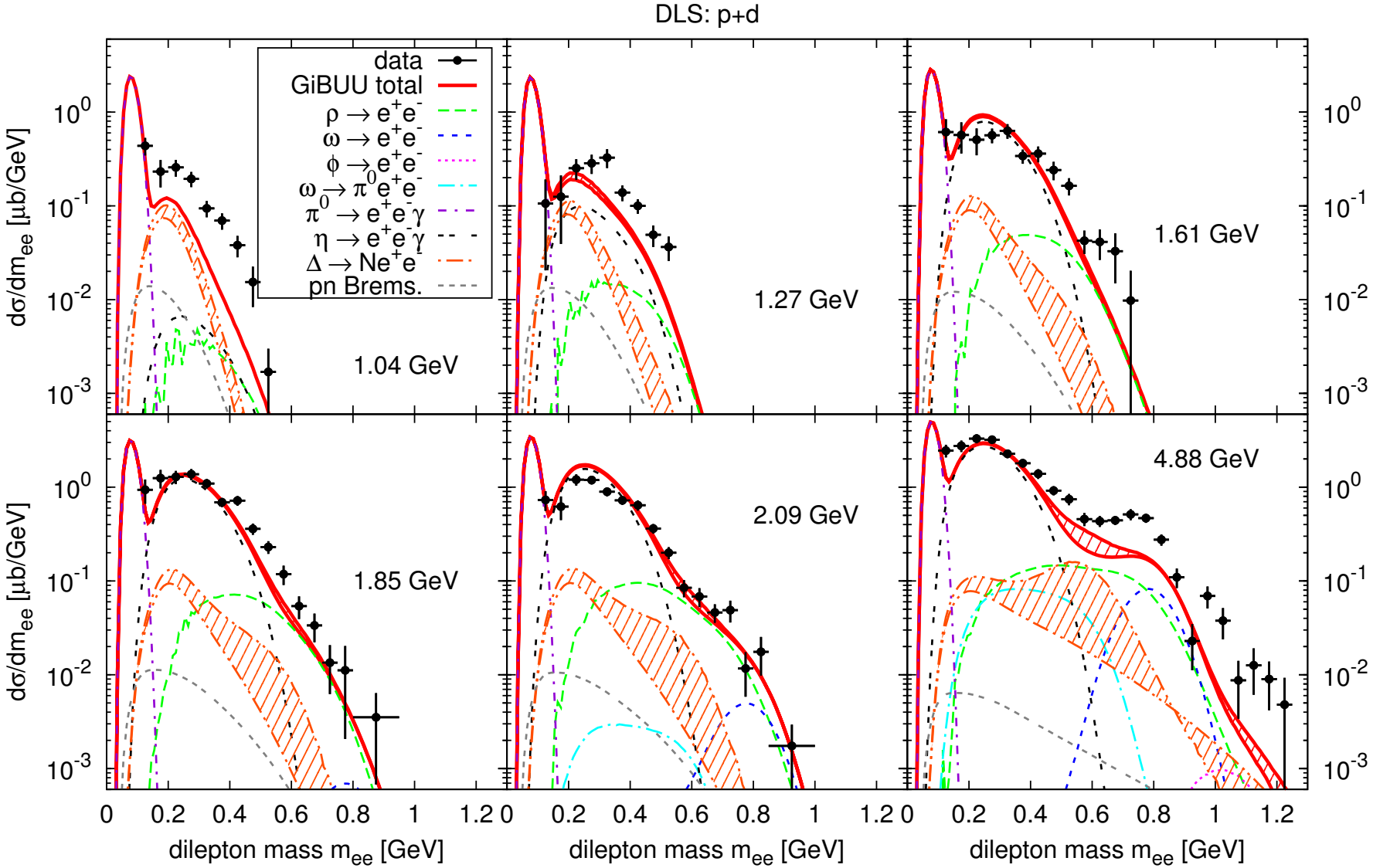
GiBUU: Comparison to old DLS data (pp)

- HADES data consistent with DLS data
- checked by comparing HADES data within DLS acceptance



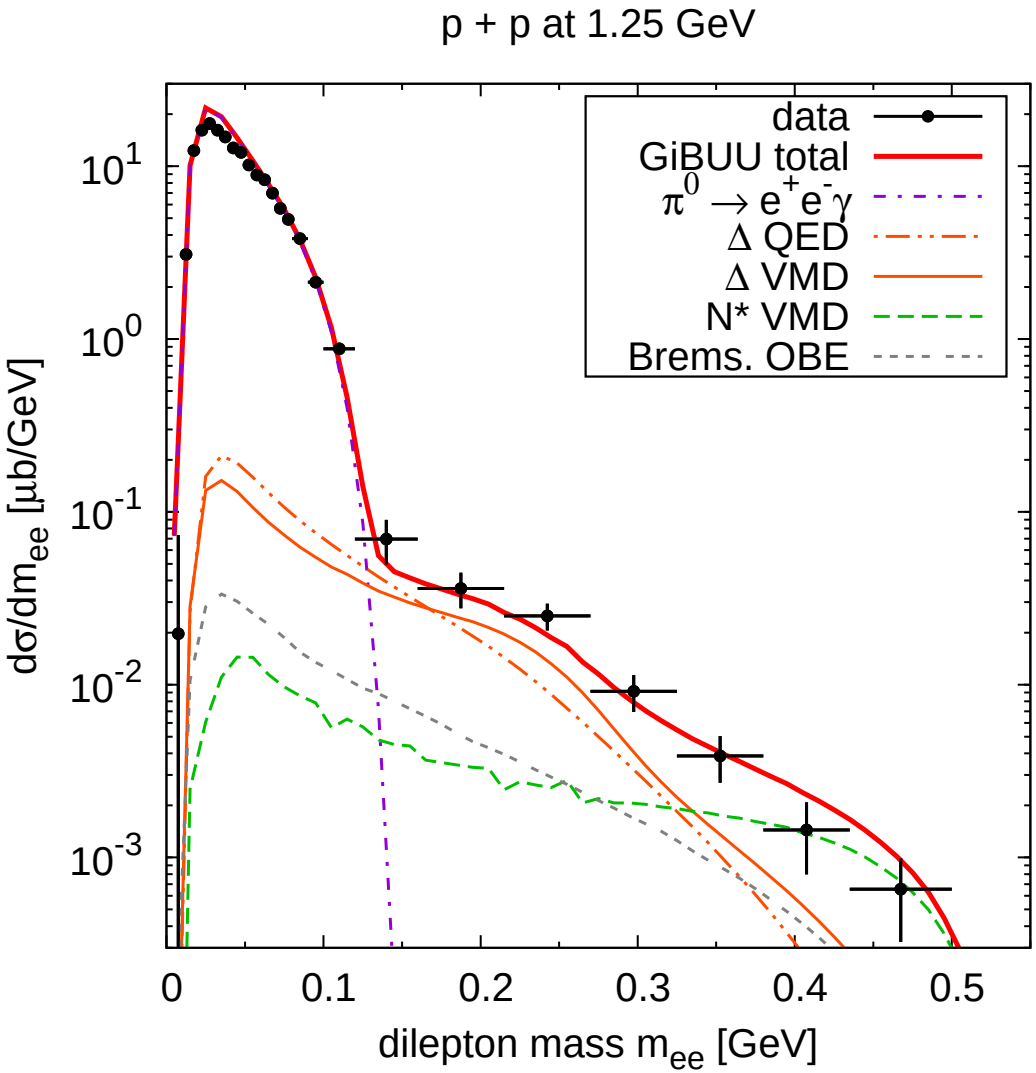
GiBUU: Comparison to old DLS data (pd)

- HADES data consistent with DLS data
- checked by comparing HADES data within DLS acceptance



GiBUU: Newest development: Δ meson in VMD model

- so far: Δ-Dalitz decay treated separately from other resonances
- now: treating Δ as all other resonances via VMD model



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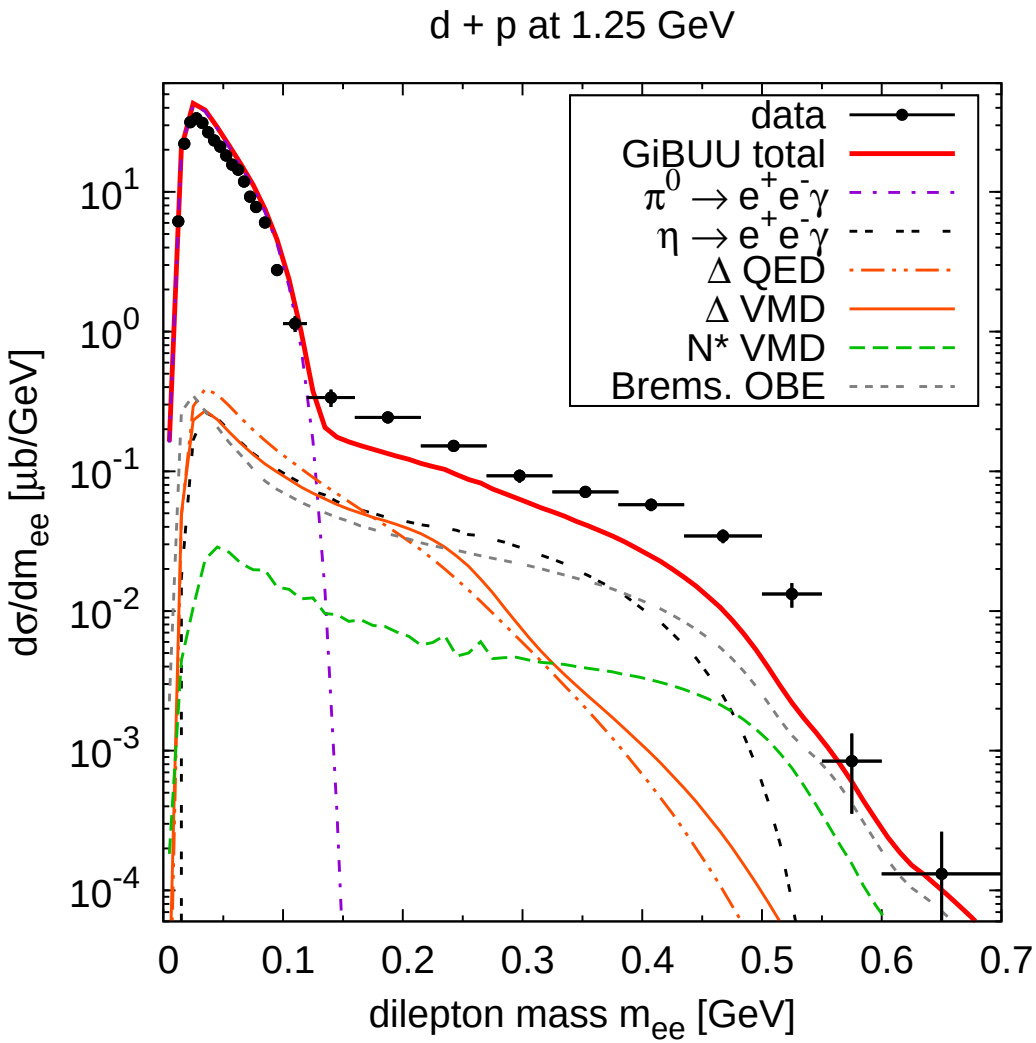

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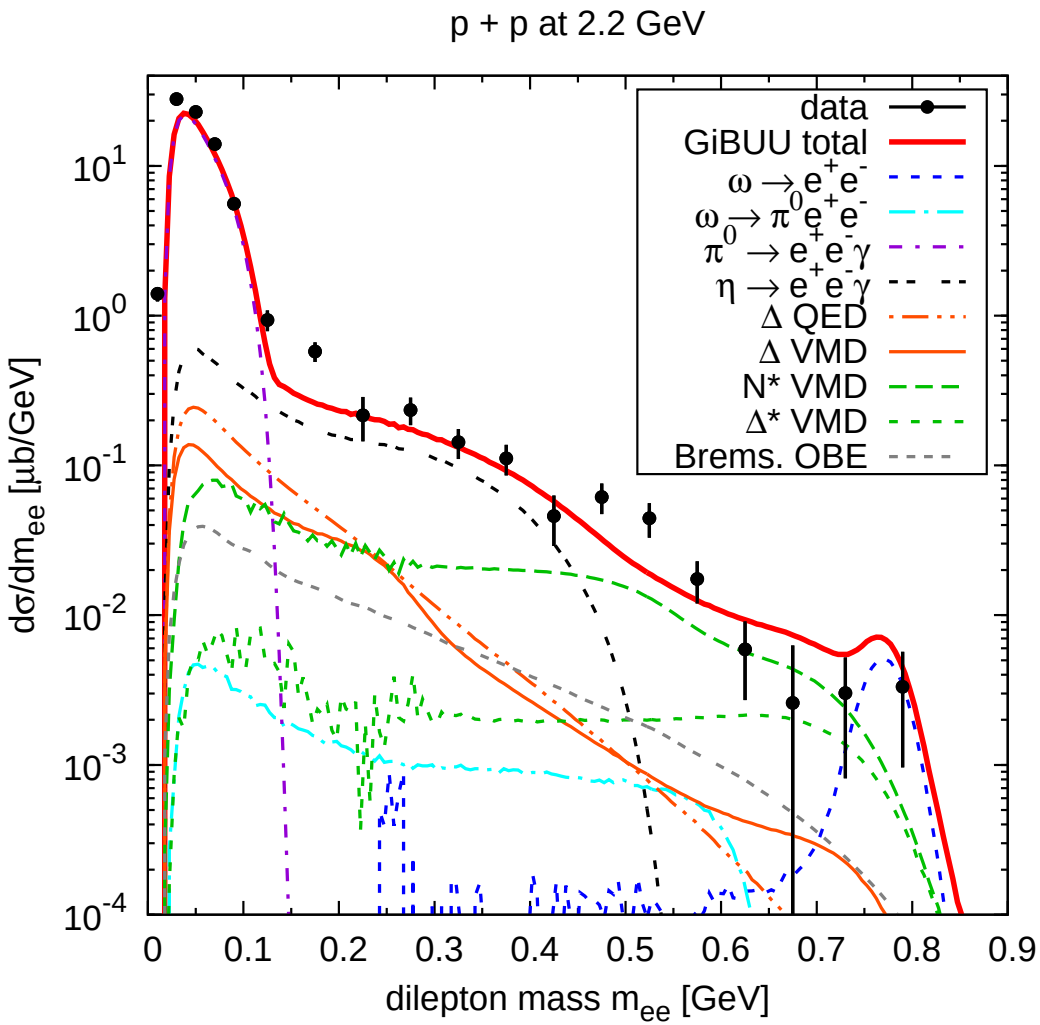

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- so far: Δ-Dalitz decay treated separately from other resonances
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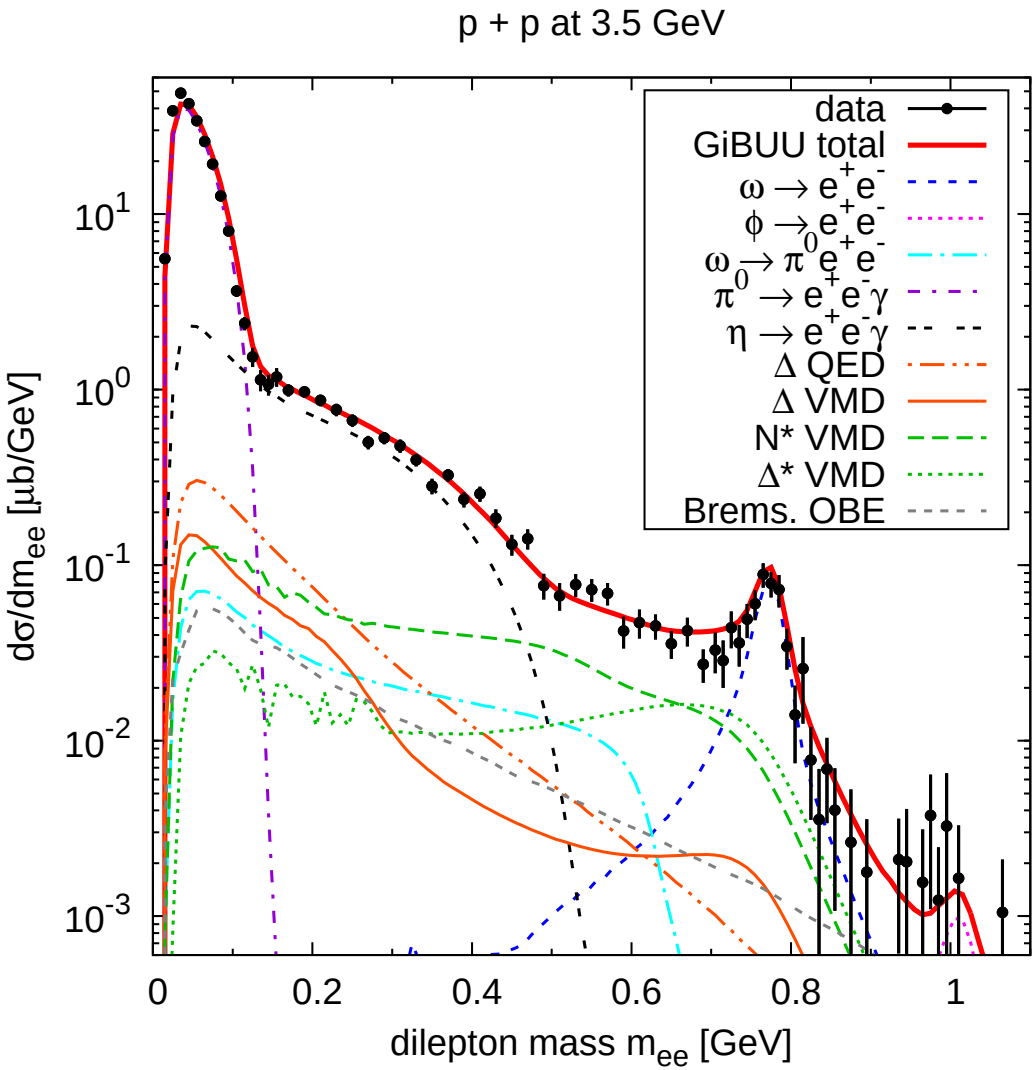
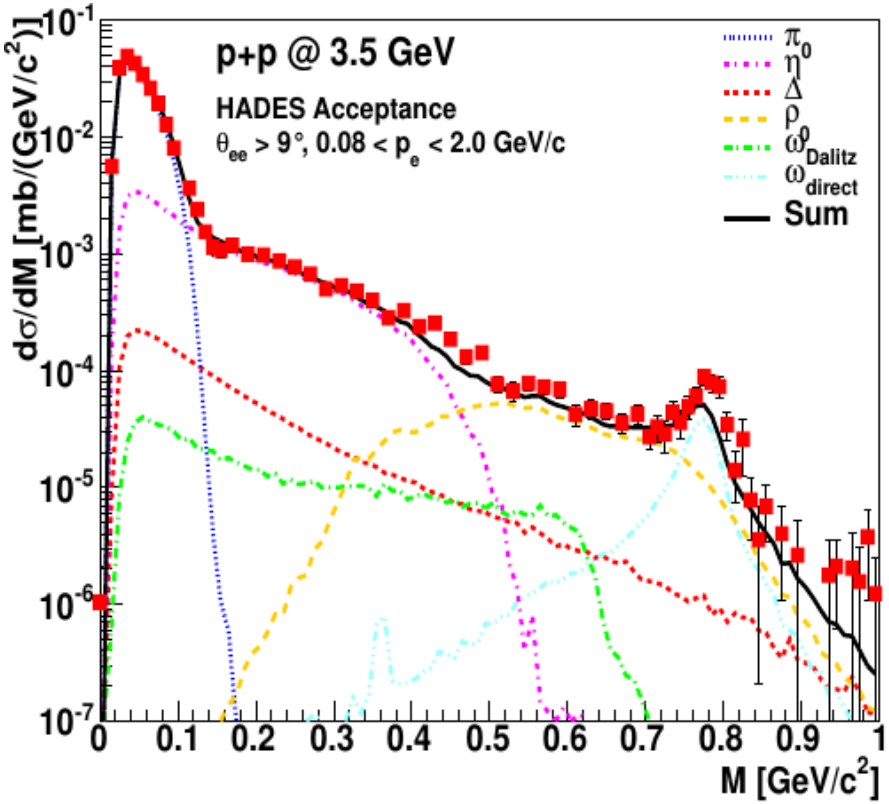
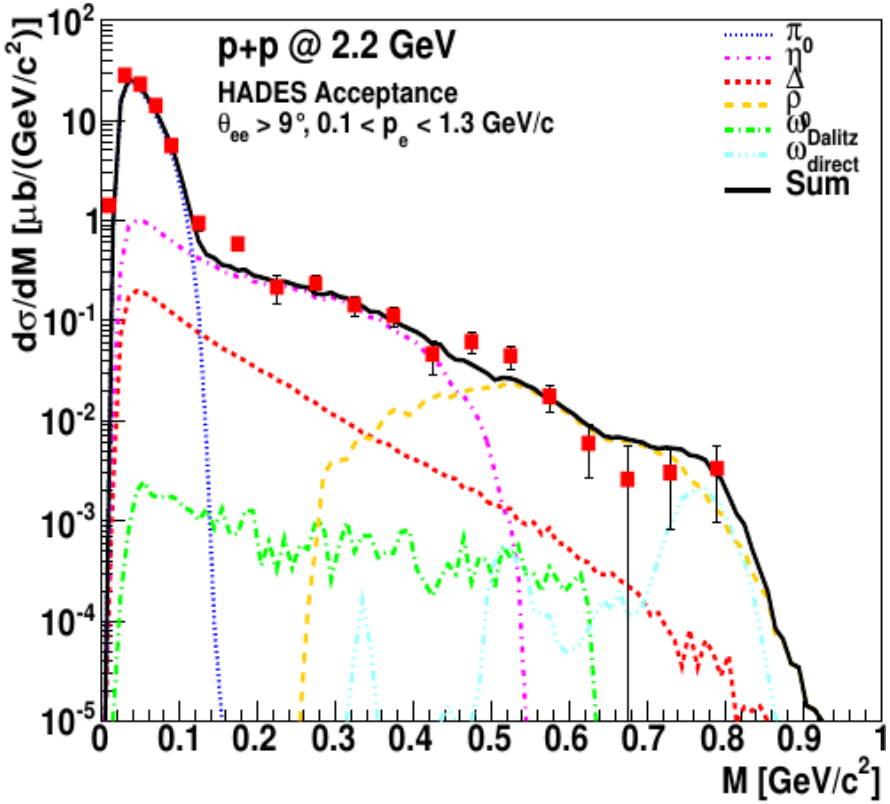


UrQMD: p p at HADES ($E_{\mathrm{kin}} = 2.2$ GeV and 3.5 GeV





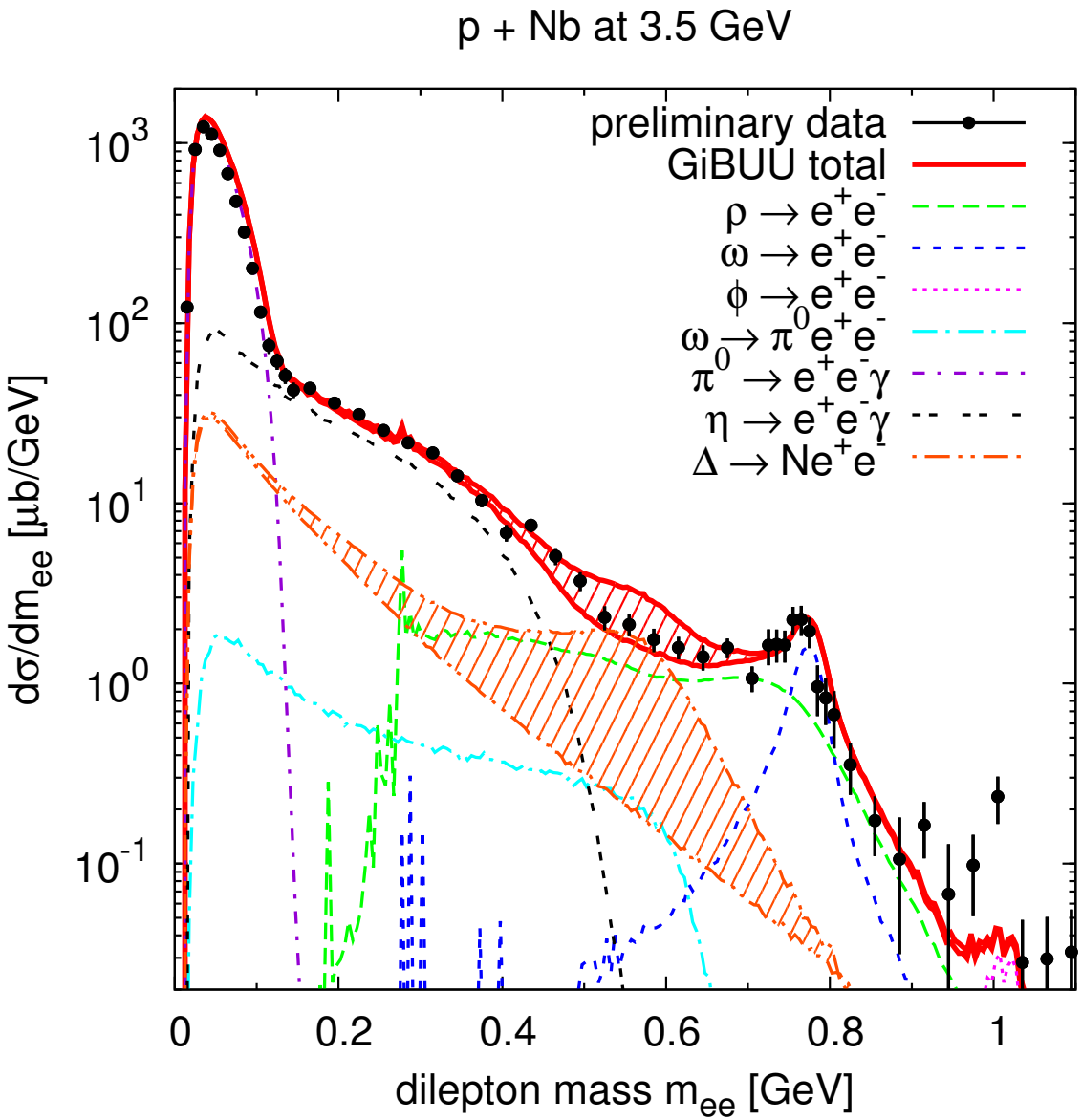
GiBUU: p Nb at HADES (3.5 GeV)

GiBUU:

- medium effects built in transport model
  - binding effects, Fermi smearing, Pauli blocking
  - final-state interactions
  - production from secondary collisions
- sensitivity to additional in-medium modifications of vector mesons?

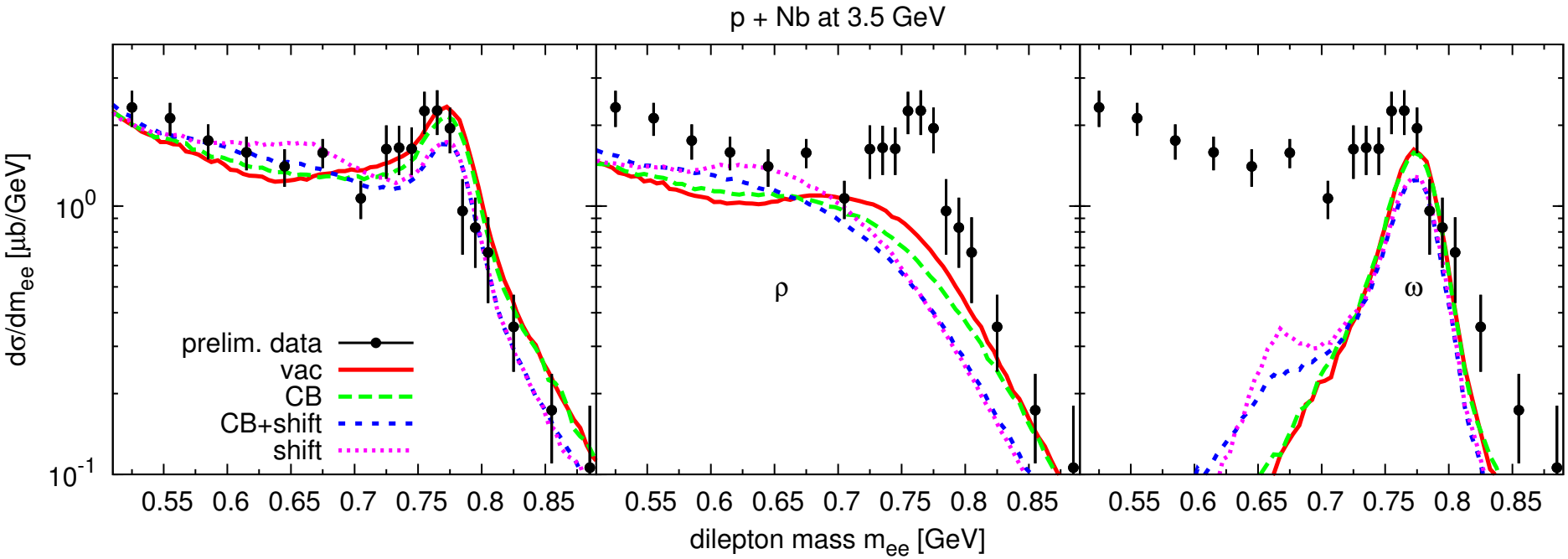
GiBUU: p Nb at HADES (3.5 GeV)

- with vacuum spectral functions:



GiBUU: p Nb at HADES (3.5 GeV)

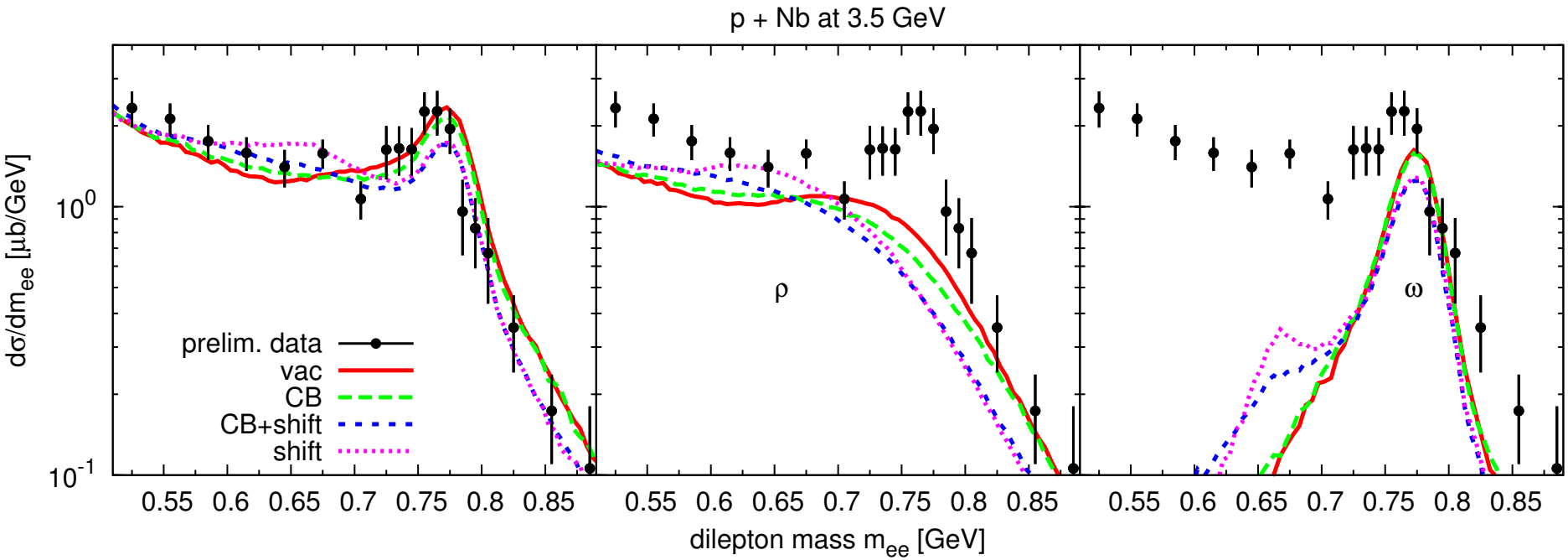
- with medium modified spectral functions:



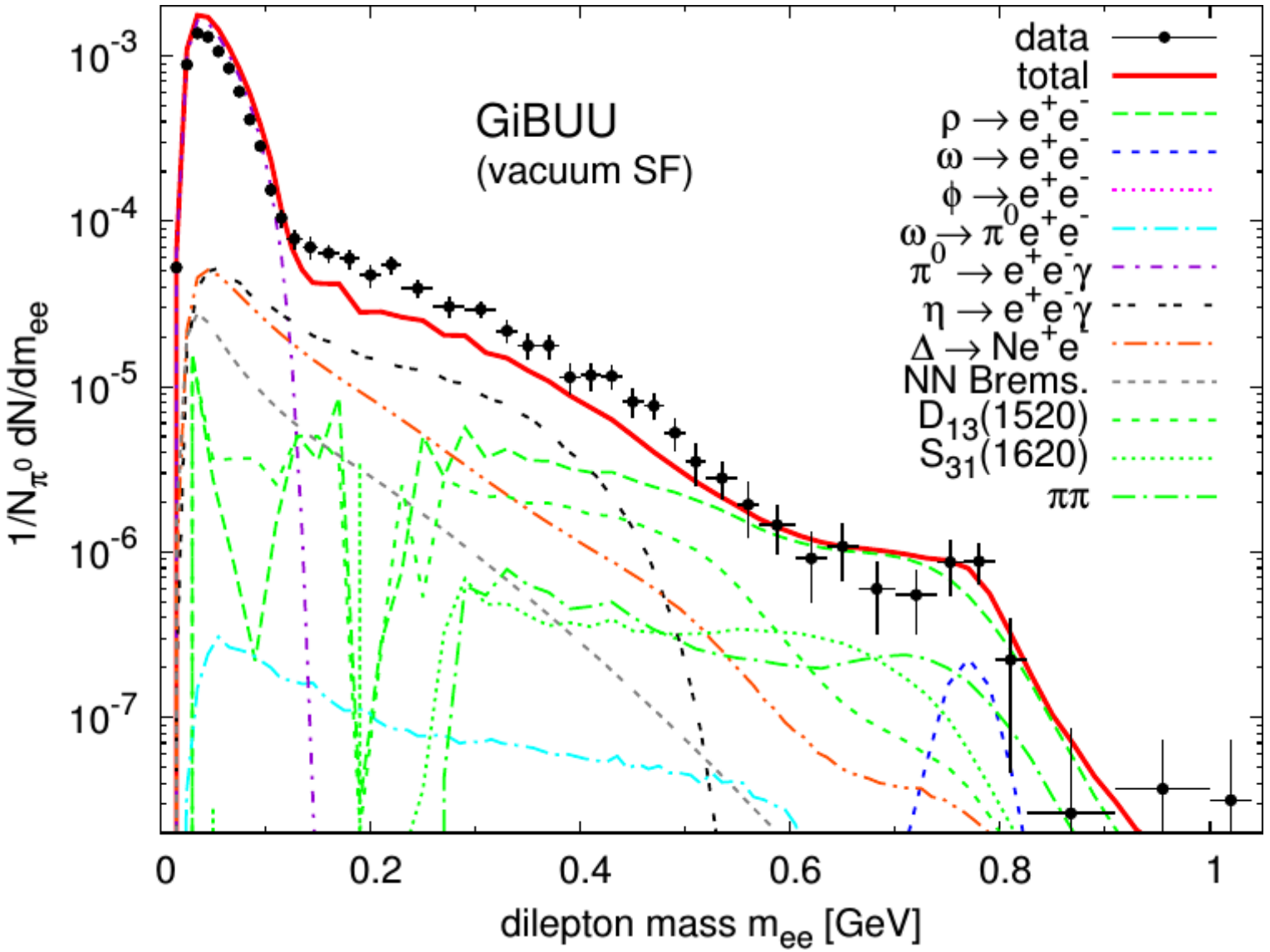
- no definite hint for medium modifications in p Nb

GiBUU: p Nb at HADES (3.5 GeV)

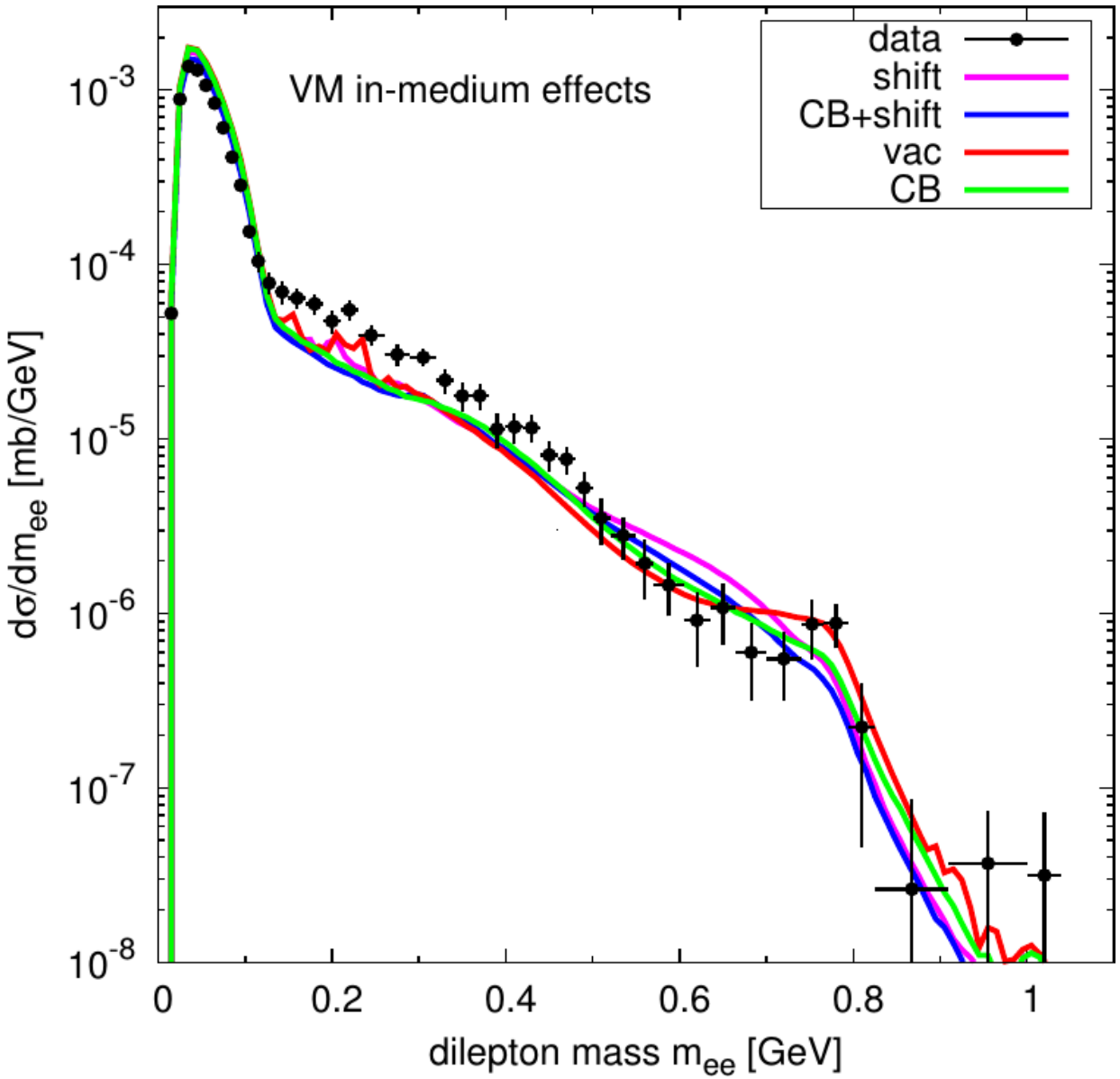
- medium effects built in transport model
  - binding effects, Fermi smearing, Pauli blocking
  - final-state interactions
  - production from secondary collisions
- sensitivity on medium effects of vector-meson spectral functions?



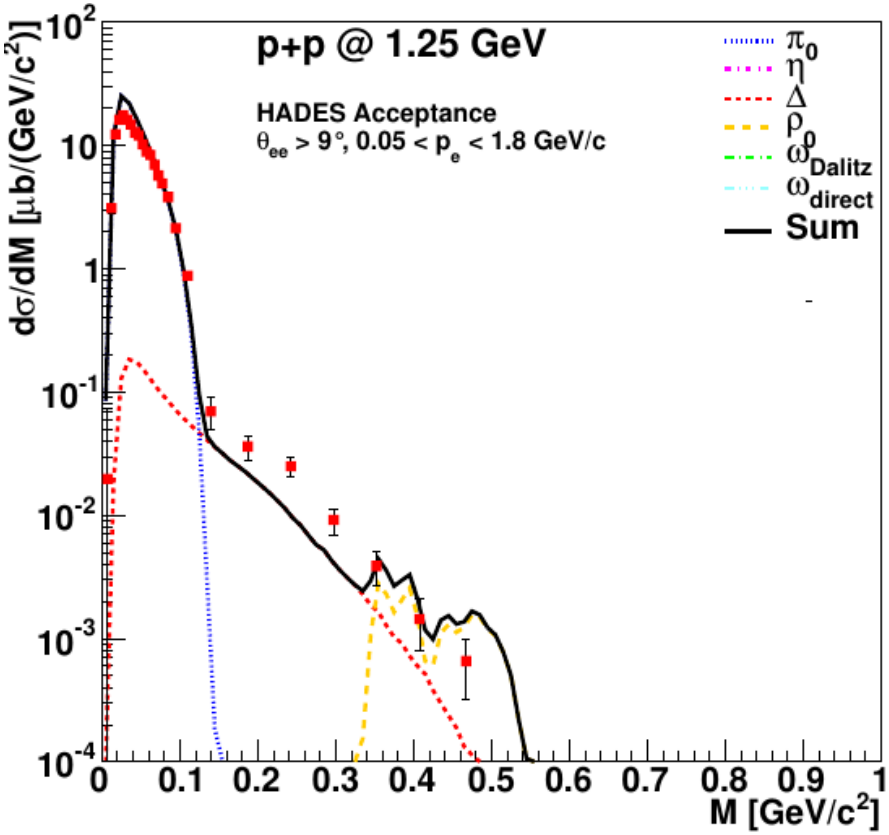
GiBUU: Ar KCl at HADES

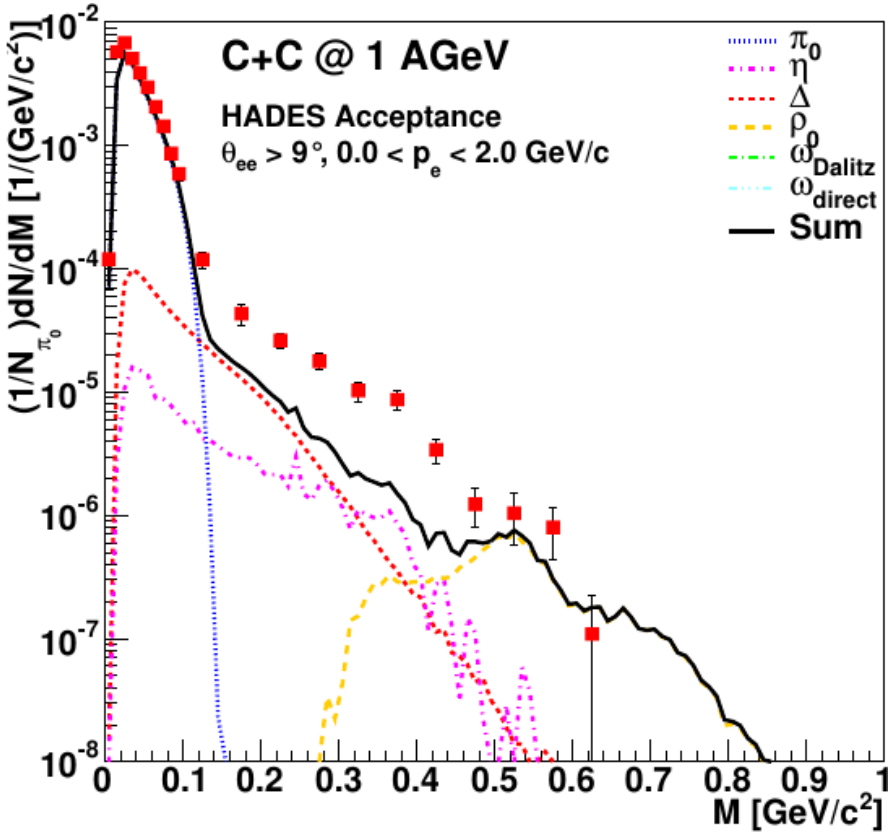


GiBUU: Ar KCl at HADES

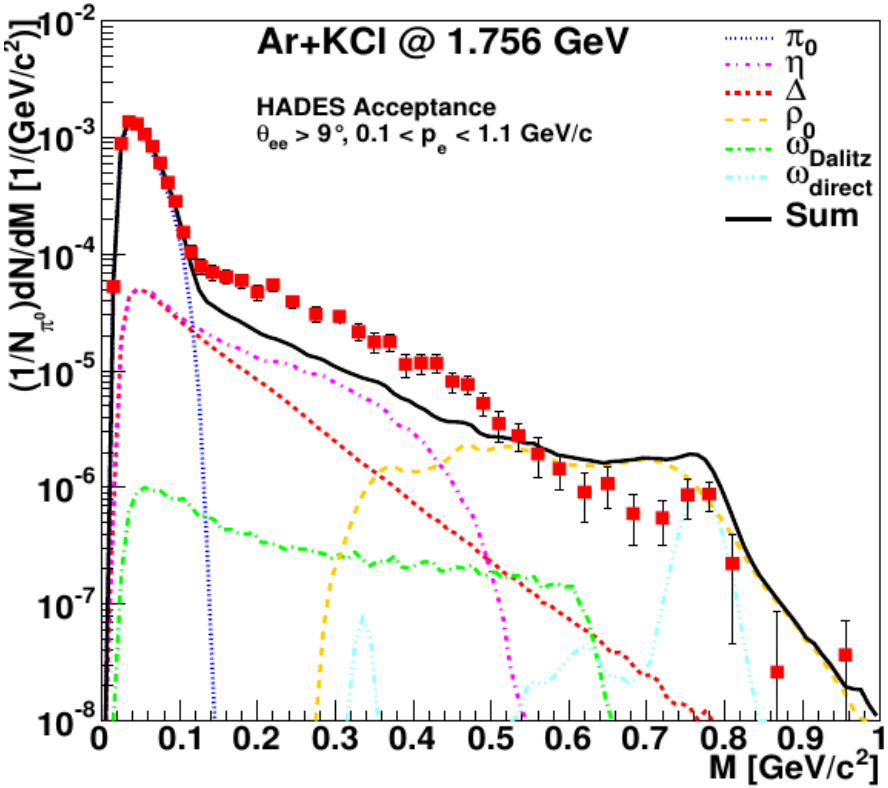


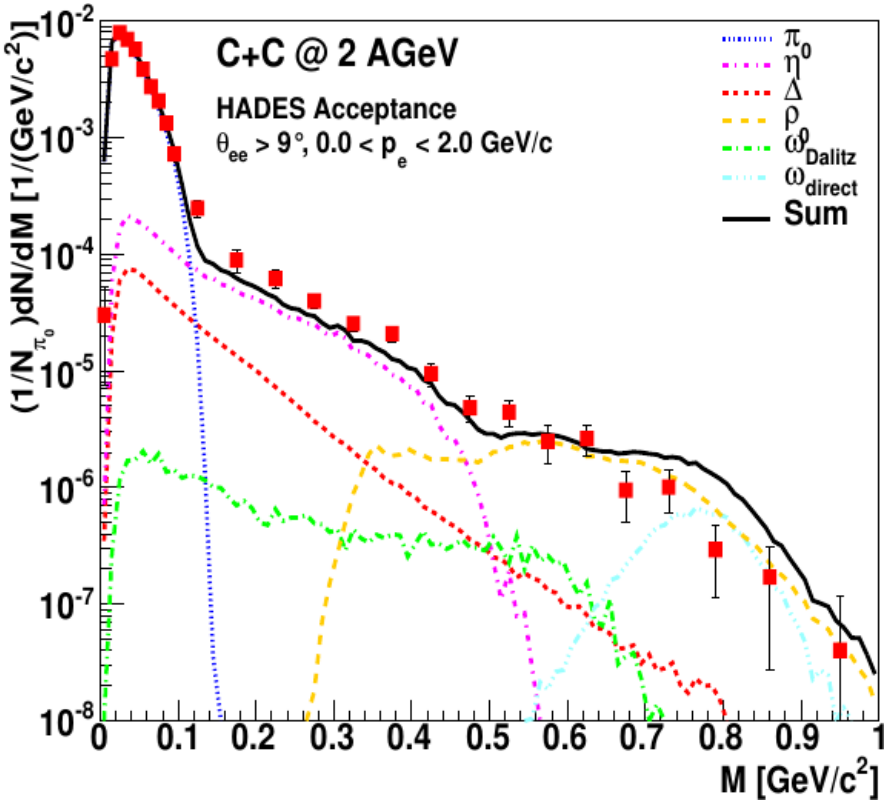
UrQMD: pp and CC at HADES (lowest energies)





UrQMD: Ar KCl and CC at HADES

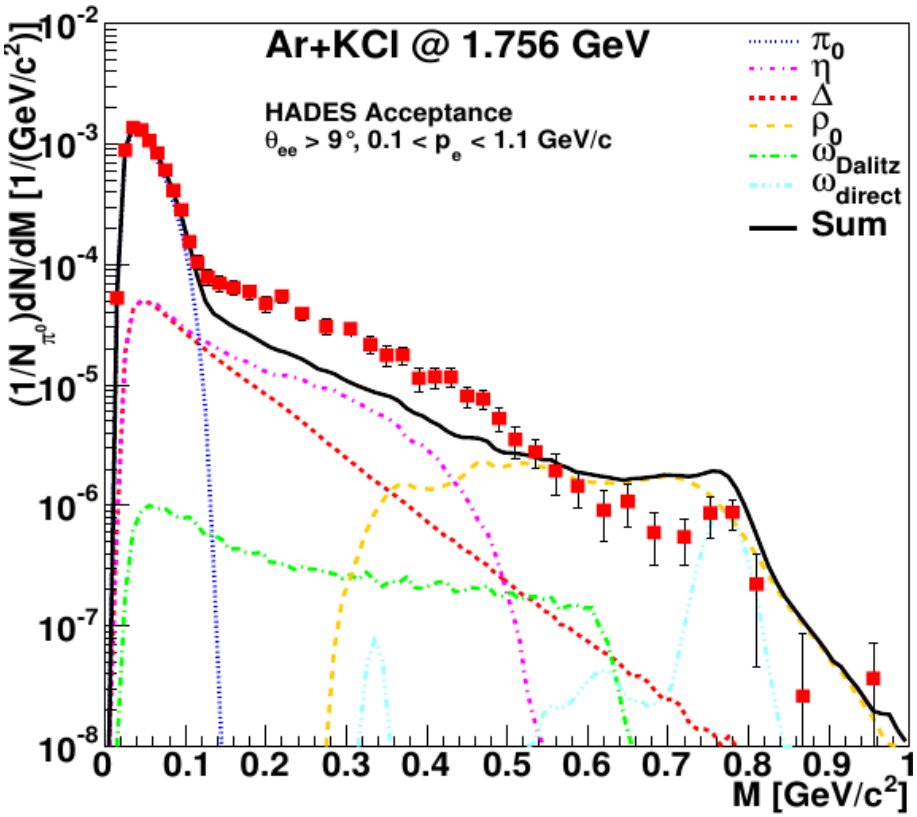


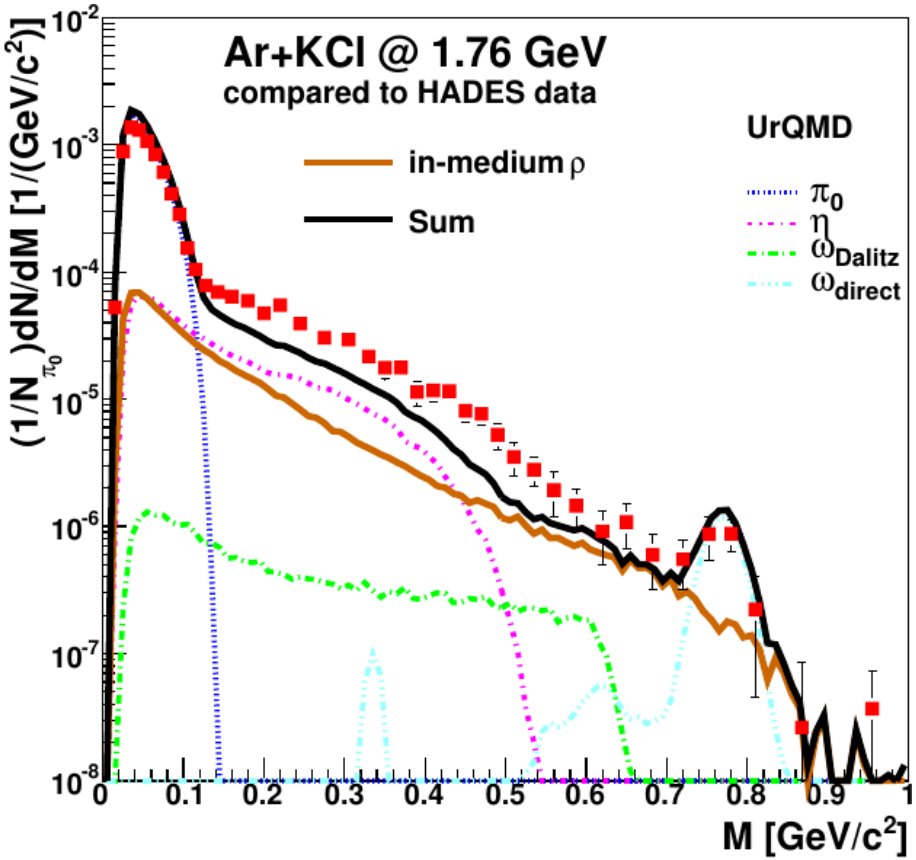


UrQMD: Coarse graining

- problem in transport models:
  how to implement medium modifications of hadrons?
- detailed calculations from equilibrium many-body QFT
- Coarse-grained transport
  - define grid of fluid cells in space-time
  - ensemble of UrQMD runs
  - determine $T(t,\vec{x})$, $\mu_B(t,\vec{x})$ from averaged net-baryon current (Eckart frame) using equation of state
  - use dilepton rates from many-body QFT
  - problem: consistency between particle content in UrQMD, QFT model, and EoS
- Eletsky-Joffe model for dileptons [EBEK01]
  - uses empirical scattering amplitudes (resonance dominated) from Manley et al
  - calculate modifications of ρ self-energies in leading-order low-density ($T\rho$) approximation
- Rapp-Wambach model [RW99, GR99, RW00]
  - detailed HMBT model for ρ meson self-energies
  - implements a lot of meson and baryon resonances
  - adjusted to elementary data ($\pi N \to \rho N$, γ absorption at nucleons and nuclei)
  - goes beyond leading-order "$T\rho$ approximation"
  - more details in Lecture IV

UrQMD: Ar KCl at HADES





- significant improvement with use of medium modified ρ
- comparison between GiBUU and UrQMD: need better constraints for hadronic models for conclusive interpretation
- a lot to do for both experimentalists and theorists!

Conclusions and Outlook

- dilepton spectra $\Leftrightarrow$ in-medium em. current correlator
- effective hadronic models for dilepton sources
  - Dalitz decays of baryon resonances $\Leftrightarrow$ vector-meson (ρ, ω, ϕ) spectral functions
  - medium modification of vector mesons
- transport models and hydrodynamics for heavy-ion collisions
- Dileptons at SIS energies
  - GiBUU and UrQMD for pp, pn, pA, AA with resonance model
  - pn still a problem?

Bibliography I

[BGG+12] O. Buss, et al., Transport-theoretical Description of Nuclear Reactions, Phys. Rept. **512** (2012) 1. http://dx.doi.org/10.1016/j.physrep.2011.12.001

[CK02] C. Cercignani, G. M. Kremer, The relativistic Boltzmann Equation: Theory and Applications, Springer, Basel (2002). http://dx.doi.org/10.1007/978-3-0348-8165-4

[dvv80] S. R. de Groot, W. A. van Leeuwen, C. G. van Weert, Relativistic kinetic theory: principles and applications, North-Holland (1980).

[EBEK01] V. Eletsky, M. Belkacem, P. Ellis, J. I. Kapusta, Properties of rho and omega mesons at finite temperature and density as inferred from experiment, Phys. Rev. C **64** (2001) 035202.

[GR99] C. Gale, R. Rapp, Rho Properties in a hot Gas: Dynamics of Meson-Resonances, Phys. Rev. C **60** (1999) 024903. http://publish.aps.org/abstract/PRC/v60/e024903

Bibliography II

[Hee13] H. van Hees, Introduction to relativistic transport theory (2013). http://fias.uni-frankfurt.de/~hees/publ/roorkee.pdf

[RW99] R. Rapp, J. Wambach, Low mass dileptons at the CERN-SPS: Evidence for chiral restoration?, Eur. Phys. J. A **6** (1999) 415. http://dx.doi.org/10.1007/s100500050364

[RW00] R. Rapp, J. Wambach, Chiral symmetry restoration and dileptons in relativistic heavy-ion collisions, Adv. Nucl. Phys. **25** (2000) 1. http://arxiv.org/abs/hep-ph/9909229

[WHM12] J. Weil, H. van Hees, U. Mosel, Dilepton production in proton-induced reactions at SIS energies with the GiBUU transport model, Eur. Phys. J. A **48** (2012) 111. http://dx.doi.org/10.1140/epja/i2012-12111-9,10.1140/epja/i2012-12150-2

Bibliography III

[WM13] J. Weil, U. Mosel, Dilepton production at SIS energies with the GiBUU transport model, J. Phys. Conf. Ser. **426** (2013) 012035. http://dx.doi.org/10.1088/1742-6596/426/1/012035

Quiz

Quiz

1. show that $\mathrm{d}^3\vec{p}/E_p$ is a Lorentz invariant! Hint: discuss $\mathrm{d}^4 p\,\delta(p\cdot p - m^2)$
2. ad p. 7: show that $v_{\mathrm{rel}} E_1 m = I = \sqrt{(p_1\cdot p_2)^2 - m^4}$; hint: W is defined as Lorentz-invariant quantity defined in the frame, where the incoming particle 1 hits the particle 2 at rest with velocity $\vec{p}_1/E_1 =: v_{\mathrm{rel}}$
3. how can we "translate" the result of a hydro calculation giving $n(x)$, $\varepsilon(x)$, $P(x)$, $u(x)$ into a particle distribution, given an assumption of the particle content of the fluid?
4. what's the difference between the simulation algorithms used in GiBUU (test-particle Monte Carlo simulation) and in UrQMD (quantum molecular dynamics simulation)?
5. which is the most important empirical input we need for transport models in low-energy heavy-ion collisions?
6. why are the ρ-meson properties in the particle-data booklet defined solely through reactions like $e^+ + e^- \to \pi + \pi$ and not with $p + p \to$ hadrons?
7. which kind of medium effects are already included in transport simulations of AA collisions when only vacuum spectral functions of particles/resonances are used?
8. what's the fundamental difficulty in making use of (quantitative) many-body-QFT calculations of medium-modified spectral functions?
9. how can one solve this approximately and what are the caveats?