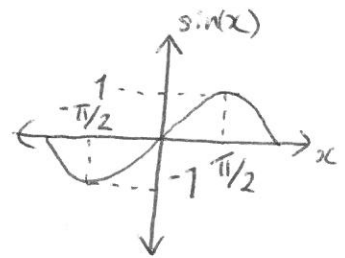
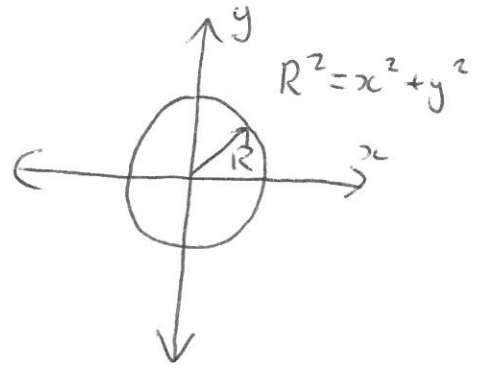


Woche 3

Aufgabe 1

$$\begin{aligned}
 A &= \int_{x=-R}^{x=R} \int_{y=-\sqrt{R^2-x^2}}^{y=\sqrt{R^2-x^2}} 1 \, dy \, dx \\
 &= \int_{-R}^R 2\sqrt{R^2-x^2} \, dx \\
 &= \left[x\sqrt{R^2-x^2} + R^2 \arcsin\left(\frac{x}{R}\right) \right]_{x=-R}^{x=R} \\
 &= (\cancel{R\sqrt{0}} + R^2 \arcsin(1)) - (\cancel{R\sqrt{0}} + R^2 \arcsin(-1)) \\
 &= R^2 (\arcsin(1) - \arcsin(-1)) \\
 &= \underline{\underline{\pi R^2}}
 \end{aligned}$$



Aufgabe 2

$$\begin{aligned}
 A &= \int_{r=0}^{r=R} \int_{\varphi=0}^{\varphi=2\pi} 1 \cdot r \, d\varphi \cdot dr \\
 &= 2\pi \left[\frac{r^2}{2} \right]_{r=0}^{r=R} \\
 &= \underline{\underline{\pi R^2}}
 \end{aligned}$$

Aufgabe 3

$$\begin{aligned}
 Q &= \iiint dq \\
 &= \iiint \rho \, dVol \\
 &= \int_{z=0}^{z=L} \int_{r=0}^{r=R} \int_{\varphi=0}^{\varphi=2\pi} \frac{A_z}{r} \, dV \\
 &= \int_{z=0}^{z=L} \int_{r=0}^{r=R} \int_{\varphi=0}^{\varphi=2\pi} A_z \, r \, d\varphi \, dr \, dz \\
 &= 2\pi R A \left[\frac{z^2}{2} \right]_{z=0}^{z=L} \\
 &= \underline{\underline{\pi L^2 R A}}
 \end{aligned}$$

Aufgabe 4

$$\begin{aligned}
 dq &= \rho \, dV \\
 dE &= \frac{dq}{4\pi\epsilon_0(r^2+x^2)} \quad (\text{Coulombsches Gesetz}) \\
 \frac{dE_y}{dE} &= \frac{r}{\sqrt{r^2+x^2}} \quad (\text{Geometrie}) \\
 \lambda &= \frac{Q}{L} \Rightarrow \lambda = \frac{dq}{dx} \Rightarrow dq = \lambda \, dx \\
 E_x &= 0 \quad (\text{Symmetrie})
 \end{aligned}$$

$$\begin{aligned}
 E_y &= \int dE_y \\
 &= \int \frac{r}{\sqrt{r^2+x^2}} \, dE \\
 &= \int \frac{r}{4\pi\epsilon_0(r^2+x^2)^{3/2}} \, dq \\
 &= \frac{\lambda}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{r}{(r^2+x^2)^{3/2}} \, dx \\
 &= \frac{\lambda}{2\pi\epsilon_0} \frac{1}{r}
 \end{aligned}$$

