Name: .... Section: ....

Physics 208 Quiz 6

March 26, 2008; due April 4, 2008

Problem 1 (50 points)

In the lecture (on March 24) we have seen that a magnetic dipole field can be described by

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{r}\vec{p}_m)\vec{r}}{r^5} - \frac{\vec{p}_m}{r^3}\right].$$

Here, $\mu_0 = \text{const}$ is a constant, we will learn about in detail in the next chapter; $\vec{p}_m$ is a constant vector called the magnetic dipole moment, and $\vec{r}$ is the position vector of an arbitrary point.

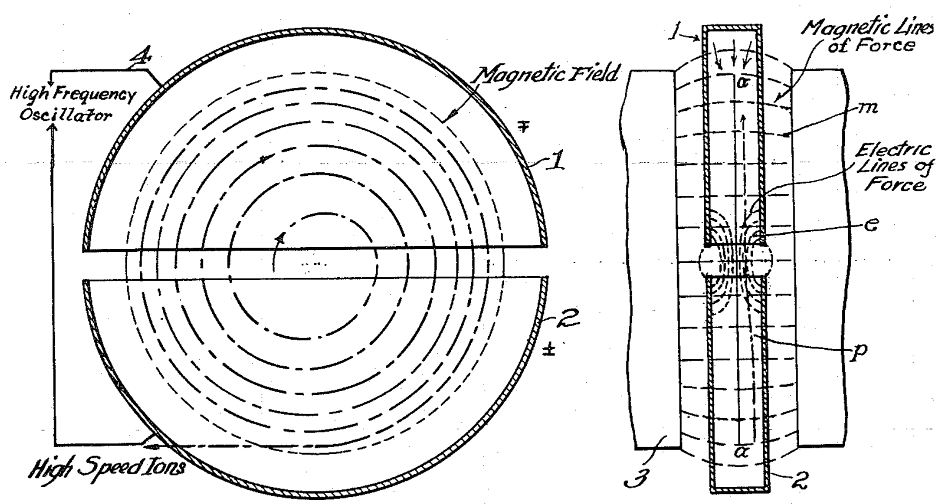
Let S be a sphere with radius R around the origin. Show that there is indeed no "magnetic charge" present as it should be, i.e.,

$$\int_S \mathrm{d}\vec{S}\vec{B} = 0.$$

Hint: Is is helpful to choose appropriate coordinates, namely spherical coordinates with the polar axis (in our standard notation that is the z axis) in the same direction as $\vec{p}_m$.

Problem 2 (50 points)

In the early 1930ies, Ernest Lawrence invented a new type of particle accelerator called Cyclotron. To understand the principle, we show a figure contained in Lawrence's patent application:



The apparatus consists of two D-shaped electrodes (called simply D's by accelerator physicists). They are connected to a radio-frequency (RF) voltage of frequency f. The voltage at the electrodes is thus changing with time by

$$V(t) = V_{\max} \cos(\omega t), \quad \omega = 2\pi f.$$

Now by some mechanism, at $t = 0$ a proton (charge $q = 1.6 \cdot 10^{-19}$ C, mass $m = 1.67 \cdot 10^{-27}$ kg) is produced in the gap between the poles. This is the beginning of the dashed line, denoted "high speed ions" in the picture. The voltage is such that the lower plate at this moment is the + terminal, and the upper plate the − terminal (i.e., the upper signs indicated in the figure). Thus the proton becomes accelerated upwards as shown in the left part of the figure.

The whole apparatus is evacuated (such that the protons suffer no friction with air), and a magnetic field, $\vec{B}$, perpendicular to the plane pointing outwards, is applied. As we know from the lecture, this forces the protons on a circle. After half a cycle, the protons enter again the gap between the poles, and the magnitude of the B field is chosen such that now the electric field is pointed precisely in the other direction (indicated by the lower signs in the figure) with a voltage difference $V_{\max}$. Thus, again the protons gain the maximal possible acceleration. This condition is called the "resonance condition".

With the knowledge from the lecture, you can easily calculate Lawrence's original setup (which, in 1939 earned Lawrence the Nobel Prize in physics!).

1. It is given that the magnetic field in Lawrence's apparatus in one case has been of magnitude $B = 0.693$ T $= 0.693$ Wb/m^2 and that Lawrence has chosen the frequency of the voltage such that the protons cycle around exactly once in one period of the RF voltage. What is this frequency, $f = \omega/(2\pi)$, of the voltage, Lawrence has used in this case? Explain briefly, why this is a good choice, meeting the "resonance condition", explained above.

2. The radius of the apparatus, within which the principle works, has been about $r_{\max} = 28$ cm. The maximum voltage of the RF generator used has been $V_{\max} = 4000$ V. Calculate the maximal energy of the protons, that Lawrence could reach with his apparatus. *Hint:* Note that the protons are accelerated twice per cycle and, in the ideal case assumed here, in each acceleration run through the full voltage difference, $V_{\max}$.

3. How many cycles have the protons made to full acceleration?

4. (*for extra credit*): Suppose, you want to build a cyclotron, but you cannot reach as high magnetic fields as Lawrence could. Which are smaller allowed values for B to meet the "resonance condition", i.e., that the protons are always accelerated by the maximum available voltage, $V_{\max}$, when they run through the gap of the poles? What is the disadvantage?

Hint: You find the Lawrence's original paper in the Physical Review:
E. O. Lawrence, M. S. Livingston, Phys. Rev. **40** (1932) 19.

It is available online (if you use an internet connection within the university) at the following URL:

http://link.aps.org/abstract/PR/v40/p19

If you read the paper, note that Lawrence uses different units, called Gaussian CGS units, than we do!