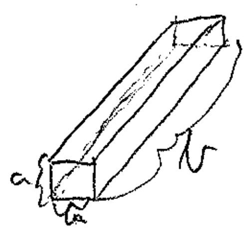
Problems Chapt. VII

(1)



$$R_L = \rho \frac{b}{a^2}$$

$$R_A = \rho \frac{a}{ab} = \frac{\rho}{b}$$

$$\frac{R_L}{R_a} = \frac{b^2}{a^2} = \left(\frac{30}{1}\right)^2 = 900$$

(2) $Q_{total} = 1A \cdot 100h = 1A \cdot 6 \cdot 10^3 \, min = 3.6 \cdot 10^5 C$

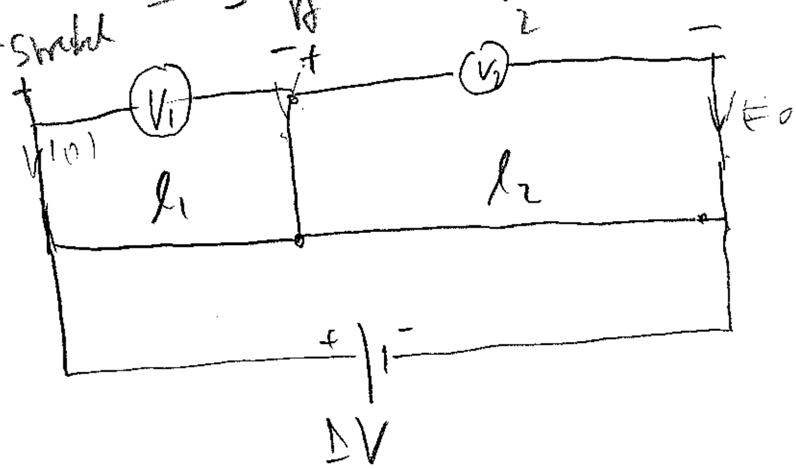
$$N = \frac{Q_{total}}{e} = \frac{3.6 \cdot 10^5}{1.6 \cdot 10^{-19}} = 2.25 \cdot 10^{24}$$

(3) Volume unchanged

$$V = A_0 l_0 = A l = 2 A l_0 \Rightarrow A = \frac{A_0}{2}$$

$$R_{stretched} = \rho \frac{l}{A} = \rho \frac{2 l_0}{\frac{A_0}{2}} = 4 \rho \frac{l_0}{A_0} = 4 \cdot 10 \Omega = 40 \Omega$$

(4) $V(0)$



(a) $\vec{j}$ = const along wires

$$j_1 = j_2 = j$$

(b) $j = \sigma_1 E_1 = \sigma_2 E_2 \Rightarrow E_1 = \rho_1 j$; $E_2 = \rho_2 j_2$

$$\Rightarrow V_1 = \rho_1 l_1 j \; ; \; V_2 = \rho_2 l_2 j$$

$$V = (\rho_1 l_1 + \rho_2 l_2) j = \frac{\rho_1 l_1 + \rho l_2}{A} i = (R_1 + R_2) i$$ ②

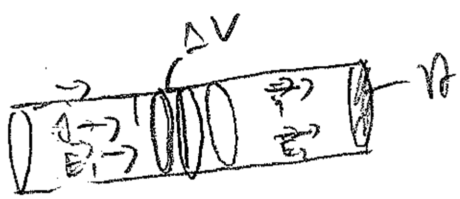
$$V_1 = R_1 i = \frac{R_1}{R_1+R_2} V$$

$$V_2 = R_2 i = \frac{R_2}{R_1+R_2} V$$

(serial resistors).

(c) It's a typical application for Gauss's law, because we know $E_1 = \rho_1 j$ and $E_2 = \rho_2 j$

Gaussian volume a infinitesimally thin around connection



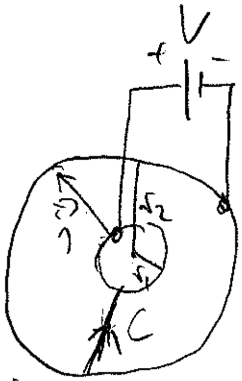
$$\int_{\partial \Delta V} \vec{E}\, d\vec{S} = (-E_1 + E_2) A = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \sigma_Q = \frac{Q}{A} = \varepsilon_0 (E_2 - E_1)$$

$$\sigma_Q = \varepsilon_0 (\rho_2 - \rho_1) j$$

$$\sigma_Q = \varepsilon_0 (\rho_2 - \rho_1) \frac{V}{\rho_1 l_1 + \rho_2 l_2}$$

(5)



Spherical coordinates

The current through each sphere inside the conducting shell is a constant

$$\Rightarrow \vec{j}(\vec{r}) = \frac{i}{4\pi r^2}\hat{e}_r$$

$$\vec{E} = \rho\,\vec{j} \Rightarrow V = -\int_C d\vec{r}\cdot\vec{E}(\vec{r})$$

$$= -\frac{i\rho}{4\pi}\int_0^{r_2-r_1} d\lambda\,\frac{1}{(\lambda-r_2)^2} = \frac{i\rho}{4\pi}\,\frac{1}{\lambda-r_2}\Big|_0^{r_2-r_1}$$

$$\boxed{C: \vec{r} = (r_2-\lambda)\hat{e}_r \quad \lambda \in (0, r_2-r_1)}$$

$$V = \frac{i\rho}{4\pi}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) := R\,i$$

$$R = \frac{\rho}{4\pi}\left(\frac{1}{r_1}-\frac{1}{r_2}\right) = \frac{\rho}{4\pi}\cdot\frac{r_2-r_1}{r_1 r_2}$$

Exercises – Chapt. VII

(1) $R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$; $r = \frac{d}{2} = 2.5\cdot 10^{-3}$ m

$$L = \frac{\pi r^2 R}{\rho} =$$

(2) $V = Ri = \frac{\rho L}{A} i = \rho L j$

$$\Rightarrow j = \frac{V}{\rho L}$$

(3) $i = \frac{V}{R} = \text{const.}$

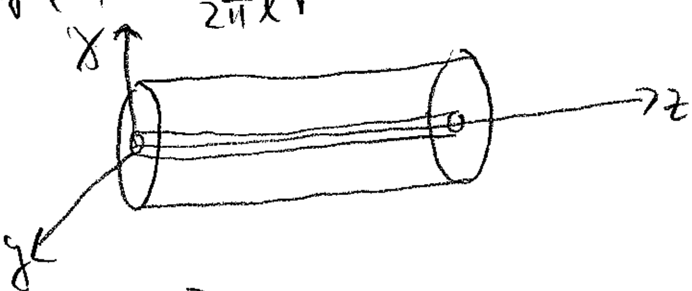
$$Q = \int_0^{\Delta t} dt\, i = i \Delta t = \frac{V}{R}\Delta t$$

$$N_e = \frac{Q}{e} = \frac{V\Delta t}{R} = \frac{1.5\cdot 60}{3\cdot 1.6\cdot 10^{-19}} =$$

(4) $R = R_1 + R_2 + R_3$

$$R_j = \rho \frac{l_j}{A_j} \Rightarrow R = \rho\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

(5) $\vec{j}(\vec{r}) = \frac{\vec{i}_{\tilde{r}}\, i}{2\pi l \tilde{r}}$ with $\tilde{r} = \sqrt{x^2+y^2}$



$$\vec{E} = \rho \vec{j}$$

$V = -\int d\vec{r}\cdot\vec{E}$ ($d\vec{r}$ radially in against direction of $\vec{j}$)

$$= \frac{\rho i}{2\pi l}\int_a^b \frac{d\tilde{r}}{\tilde{r}} = \frac{\rho i}{2\pi l}\ln\left(\frac{b}{a}\right) \Rightarrow R = \frac{V}{i} = \frac{\rho}{2\pi l}\ln\left(\frac{b}{a}\right)$$